Lesson 3 Practice Problems

Continuity & the Definition of the Derivative MATH 2200-98

Determining Continuity at a Point

1. Determine whether the following functions are continuous at the given value a. Use the continuity checklist to justify your answer.

   (a) $f(x) = \begin{cases} \dfrac{x^2-4x+3}{x-3}, & \text{if } x \neq 3 \\ 2, & \text{if } x = 3 \end{cases}; \quad a = 3$

   (b) $g(x) = \begin{cases} \dfrac{x^2+x}{x+1}, & \text{if } x \neq -1 \\ 2, & \text{if } x = -1 \end{cases}; \quad a = -1$

Limit of a Function Composition

2. $\lim\limits_{x\to 0} e^{-1/x^2}$

Applying the Intermediate Value Theorem

3. Use the Intermediate Value Theorem to show that the equation

$$x^3 - 5x^2 + 2x = -1$$

   has a solution on the interval $(-1, 5)$.

Using the Definition of the Derivative

4. Consider $f(x) = 3x^2 - x$. Find $f'(1)$ using the version of the definition of the derivative given in **Definition 3.7** of the lesson notes. Then use the value of $f'(1)$ to find the equation of the line tangent to the curve $y = 3x^2 - x^3$ at the point $(1, 2)$.

5. Using **Definition 3.6** of the lesson notes, find the derivatives of the given functions.

   (a) $f(t) = 5t - 9t^2$

   (b) $g(x) = \sqrt{9-x}$

Determining Differentiability

6. Determine whether the function is differentiable at the given value of x. (Hint: use Definition 3.7 of the *lesson notes*).

   (a) $f(x) = \begin{cases} \dfrac{1}{2}x + 1, & x < 2 \\ \sqrt{2x}, & x \geq 2 \end{cases}$

   (b) $g(x) = \begin{cases} x, & x \leq 1 \\ x^2, & x > 1 \end{cases}$