Written Homework 3

1. Determine whether $g(x) = \dfrac{1}{x-3}$ is continuous at $a = 3$.

   You must show that the 3 parts of the continuity definition are confirmed, or state for which parts it fails.

2. Determine whether

$$f(x) = \begin{cases} \dfrac{x^2 - 4x + 3}{x - 3}, & \text{if } x \neq 3 \\ 2, & \text{if } x = 3 \end{cases}$$

is continuous at $a = 3$. You must show that the 3 parts of the continuity definition are confirmed, or state for which parts it fails.

3. Evaluate $\lim_{x \to 0} \ln\left(\frac{3\sin x}{x}\right)$. Show each limit law applied (answers that do not do this will receive NO credit!)

4. Use the Intermediate Value Theorem to show that the equation

$$\sqrt{x^4 + 25x^3 + 10} = 5$$

has a solution on the interval $(0, 1)$. (Hint: See the worksheet solutions for an example as well as Example 3.4 in the *lesson notes*).

5. Use Definition 3.7 of the *lesson notes* to find the equation of the line tangent to the graph of $f(x) = -3x^2 - 5x + 1$ at $P(1, -7)$.

6. Use Definition 3.6 of the *lesson notes* to find the equation of the line tangent to the graph of $f(x) = 8 - 2x^2$ at $P(0, 8)$.

For problems 7 and 8 you must use whichever version of the the definition of the derivative is appropriate. (For those who have taken calculus before: results using the rules of differentiation will recieve no credit!)

7. (a) Find the derivative of $f(x) = 3x^2$.

(b) Find the equation of the line tangent to $f(x)$ at $x = 0$.

8. (a) Find the derivative of $f(x) = \sqrt{x+2}$.

(b) Find the equation of the line tangent to the graph of $f(x)$ at the point $(a, f(a))$ for $a = 7$.

For problems 9 and 10: Determine whether the function is differentiable at the given value of x. If it is differentiable, find $f'(x)$ at the given value of x. (Hint: your solution should involve analysis of the left and right derivatives! See the worksheet solutions for an example).

9. Is $f(x) = |x - 1|$ differentiable at $x = 1$?

10. Is $g(x) = \begin{cases} x^2 + 1, & x \leq 2 \\ 4x - 3, & x > 2 \end{cases}$ differentiable at $x = 2$?