Lesson 6 Practice Problems

Rates of Change & Related Rates   MATH 2200-98

Rates of Change

1. A particle moves on a vertical line so that its coordinate at time t is $s(t) = t^3 - 12t + 3$, $t \geq 0$.

    (a) Find the velocity and acceleration functions.

    (b) When is the particle moving upward and downward?

    (c) Find the distance the particle travels in the time interval $0 \leq t \leq 3$.

    (d) Determine when the particle is speeding up and slowing down on the interval $0 \leq t \leq 3$. (It may help to graph it.)

2. The number of gallons of water in a tank t minutes after the tank has started to drain is $Q(t) = 200(30 - t^2)$

    (a) Find the function that governs the *rate of change* of the amount of water in the tank with respect to time.

    (b) What is the rate at which water is leaving the tank at the end of 10 minutes?
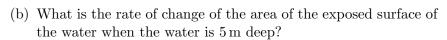
Related Rates

3. Sand falls from an overhead bin and accumulates in a conical pile with a radius that is always three times its height. Suppose the height of the pile increases at a rate of $2\,\text{cm/s}$ when the pile is $12\,\text{cm}$ high. At what rate is the sand leaving the bin at that instant.
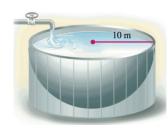
4. A spherical ballon is inflated and its volume increases at a rate of $15\,\text{in}^3/\text{min}$. How fast is the radius increasing when the radius is $10\,\text{in}$.

5. A hemispherical tank with a radius of $10\,\text{m}$ is filled from an inflow pipe at a rate of $3\,\text{m}^3/\text{min}$.

    (a) How fast is the water level rising when the water level is $5\,\text{m}$ from the bottom of the tank? (Hint: the volume of a cap of thickness h sliced from a sphere of radius r is $\frac{\pi}{3}h^2(3r - h)$)

    (b) What is the rate of change of the area of the exposed surface of the water when the water is $5\,\text{m}$ deep?

6. A 13 foot ladder is leaning against a vertical wall when Jack begins pulling the foot of the ladder away form the wall at a rate of $0.5\,\text{ft/sec}$. How fast is the top of the ladder sliding down the wall when the foot of the ladder is 5 feet from the wall?