Indefinite Integration

1. Integrate the following

   (a) $\displaystyle\int \left(\frac{3}{x^4} + 2 - \frac{3}{x^2}\right) dx$

   Solution:

   Note that $\frac{3}{x^4} + 2 - \frac{3}{x^2} = 3x^{-4} + 2 - 3x^{-2}$

   $$\Rightarrow \int \left(\frac{3}{x^4} + 2 - \frac{3}{x^2}\right) dx = \int (3x^{-4} + 2 - 3x^{-2})\, dx$$
   $$= 3\int x^{-4}\,dx + 2\int 1\,dx - 3\int x^{-2}\,dx$$
   $$= 3\frac{x^{-4+1}}{-4+1} + 2x - \frac{3x^{-2+1}}{-2+1} + C$$
   $$= \frac{3}{-3}x^{-4} + 2x - \frac{3}{-1}x^{-1} + C$$
   $$= -x^{-4} + 2x - 3x^{-1} + C$$

   (b) $\displaystyle\int \frac{12t^8 - t}{t^3}\, dt$

   Solution:

   $$\int \frac{12t^8 - t}{t^3}\,dt = \int \left(\frac{12t^8}{t^3} + \frac{1}{t^3}\right) dt$$
   $$= \int \left[12t^5 - \frac{1}{t^3}\right] dt$$
   $$= 12\int t^5\,dt - \int t^{-3}\,dt$$
   $$= 12\frac{t^{5+1}}{5+1} - \frac{t^{-3+1}}{-3+1} + C = 2t^6 + \frac{t^{-2}}{2} + C$$

   (c) $\displaystyle\int 5m(12m^3 - 10m)\, dm$

   Solution:

   $$\int 5m(12m^3 - 10m)\,dm = \int (60m^4 - 50m^2)\,dm$$
   $$= 60\int m^4\,dm - 50\int m^2\,dm$$
   $$= 60\frac{m^{4+1}}{4+1} - \frac{50m^{2+1}}{2+1} + C$$
   $$= 12m^5 - \frac{50}{3}m^3 + C$$

Evaluating Riemann Sums

2. Speedometer readings for a motorcycle over a 4 second time period are given in the table below

$t(s)$	1	1.5	2	2.5	3	3.5	4	4.5	5
v (m/s)	50	50	60	60	55	65	50	60	70

* left endpts
* Right endpts
* midpoints

Find the indicated Riemann sum approximations to the displacement on $[1,5]$ with $n = 4$ subintervals.

(a) left Riemann sum. Note: $\Delta x = \frac{b-a}{n} = \frac{5-1}{4} = 1$

Solution:

Left endpts: 1, 2, 3, 4

$L_4 = \Delta x\,(v(1) + v(2) + v(3) + v(4))$

$= 1\,(50 + 60 + 55 + 50)$

$= 215$

(b) right Riemann sum.

Solution:

Right endpts: 2, 3, 4, 5

$R_4 = \Delta x\,(v(2) + v(3) + v(4) + v(5))$

$= 1\,(60 + 55 + 50 + 70)$

$= 235$

(c) midpoint Riemann sum.

Solution:

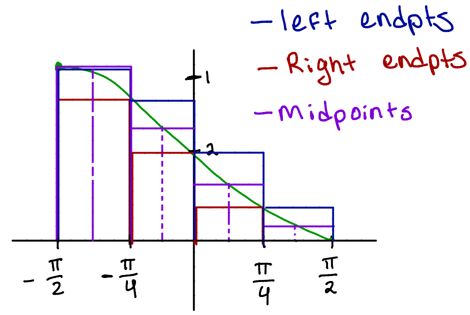
midpoints: 1.5, 2.5, 3.5, 4.5

$M_4 = \Delta x\,(v(1.5) + v(2.5) + v(3.5) + v(4.5))$

$= 1\,(50 + 60 + 65 + 60)$

$= 235$

3. using a Riemann sum with $n = 4$ approximate the area of the region bounded by the graph of $f(x) = 2 - 2\sin(x)$ and the x-axis on the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ with



For $n=4$, $a = -\frac{\pi}{2}$, $b = \frac{\pi}{2}$

$$\Delta x = \frac{b-a}{n} = \frac{\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)}{4} = \frac{\pi}{4}$$

$\Rightarrow$ endpts are

$-\frac{\pi}{2}, -\frac{\pi}{4}, 0, \frac{\pi}{4}, \frac{\pi}{2}$

(a) left endpoints.

Solution:

Left endpts: $x_i = -\frac{\pi}{2}, -\frac{\pi}{4}, 0, \frac{\pi}{4}$

$$L_4 = \sum_{i=1}^{4} f(x_i)\Delta x = \Delta x\left[f\left(-\frac{\pi}{2}\right) + f\left(-\frac{\pi}{4}\right) + f(0) + f\left(\frac{\pi}{2}\right)\right]$$

$$= \frac{\pi}{4}\left[4 + (2+\sqrt{2}) + 2 + (2-\sqrt{2})\right]$$

$$= \frac{5\pi}{4}$$

(b) right endpoints.

Solution:

right endpts: $x_i = -\frac{\pi}{4}, 0, \frac{\pi}{4}, \frac{\pi}{2}$

$$R_4 = \sum_{i=1}^{4} f(x_i)\Delta x = \Delta x\left[f\left(-\frac{\pi}{2}\right) + f\left(-\frac{\pi}{4}\right) + f(0) + f\left(\frac{\pi}{2}\right)\right]$$

$$= \frac{\pi}{4}\left[(2+\sqrt{2}) + 2 + (2-\sqrt{2}) + 0\right]$$

$$= \frac{3\pi}{4}$$

(c) midpoints.

Solution:

Midpoints: $x_i^* = -\frac{3\pi}{8}, -\frac{\pi}{8}, \frac{\pi}{8}, \frac{3\pi}{8}$

$$M_4 = \Delta x\left[f\left(-\frac{3\pi}{8}\right) + f\left(-\frac{\pi}{8}\right) + f\left(\frac{\pi}{8}\right) + f\left(\frac{3\pi}{8}\right)\right]$$

$$= \frac{\pi}{4}[8]$$

$$= 2\pi$$

Initial Value Problems

4. Given the acceleration function of an object moving along a line, find the position function with the given initial velocity and initial position.

$$a(t) = 2t + 1, \quad v(0) = -2, \quad s(0) = 3$$

Solution:

We know that $a(t) = v'(t) = s''(t)$

First we find $v(t)$:

$\Rightarrow \quad v(t) = \int a(t)\,dt = \int (2t+1)\,dt = t^2 + t + C$

Since $v(0) = -2$:

$-2 = 0^2 + 0 + C \Rightarrow C = -2$

So $v(t) = t^2 + t - 2$

Now we find the position fcn $s(t)$:

$\Rightarrow \quad s(t) = \int v(t)\,dt = \int (t^2 + t - 2)\,dt$

$= \frac{t^3}{3} + \frac{t^2}{2} - 2t + D$

Since $s(0) = 3$

$\Rightarrow \quad 3 = \frac{(0)^3}{3} + \frac{(0)^2}{2} - 2(0) + D$

$\Rightarrow \quad D = 3$

So our position function is

$s(t) = \frac{t^3}{3} + \frac{t^2}{2} - 2t + 3$

5. Solve the initial value problem

$$g'(x) = \frac{x^4 - 1}{x^3}, \quad g(1) = 6$$

Solution:

To solve the IVP we find $g(x)$ by using FTOC i.e.

$$g(x) = \int g'(x)\,dx$$

$$\Rightarrow \quad g(x) = \int \frac{x^4 - 1}{x^3}\,dx$$

$$= \int [x - x^{-3}]\,dx$$

$$= \frac{x^2}{2} - \frac{x^{-3+1}}{-3+1} + C$$

$$= \frac{x^2}{2} + \frac{x^{-2}}{2} + C$$

The initial condition is $g(1) = 6$

$$\Rightarrow \frac{(1)^2}{2} + \frac{(1)^{-2}}{2} + C = 6$$

$$\Rightarrow \frac{1}{2} + \frac{1}{2} + C = 6$$

$$\Rightarrow 1 + C = 6$$

$$\Rightarrow C = 5$$

So the solution to the IVP is

$$g(x) = \frac{x^2}{2} + \frac{x^{-2}}{2} + 5$$