1. The graph of h in the figure has vertical asymptotes at $x = -1$ and $x = 4$. Analyze the following limits.

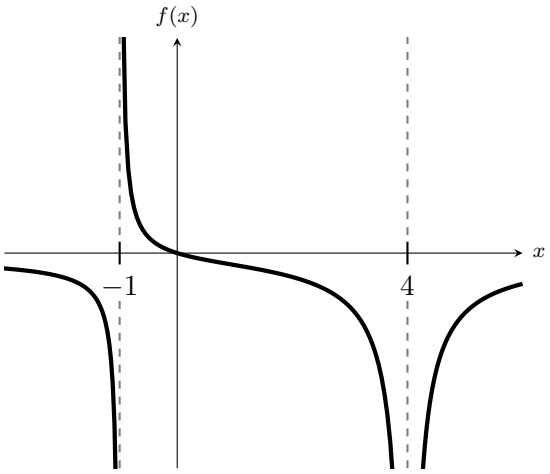

Figure 1: Graph of $y = h(x)$

(a) $\lim_{x \to -1^-} h(x)$

(b) $\lim_{x \to -1^+} h(x)$

(c) $\lim_{x \to -1} h(x)$

(d) $\lim_{x \to 4^-} h(x)$

(e) $\lim_{x \to 4^+} h(x)$

(f) $\lim_{x \to 4} h(x)$

2. Determine the following limits or explain why they do not exist.

   (a) $\lim_{x \to 3^+} \dfrac{2}{(x-3)^3}$

   (b) $\lim_{x \to 3^-} \dfrac{2}{(x-3)^3}$

   (c) $\lim_{x \to 3} \dfrac{2}{(x-3)^3}$

3. Consider the function

$$f(x) = \frac{x+7}{x^4 - 49x^2}$$

and evaluate the given limits. In each case, for $\lim_{x \to a} f(x)$ determine if the point $x = a$ is a vertical asymptote or a hole. Be sure to justify your answer.

(a) $\lim_{x \to 7^-} f(x)$

(b) $\lim_{x \to 7^+} f(x)$

(c) $\lim_{x \to -7} f(x)$

(d) $\lim_{x \to 0} f(x)$

4. Determine the vertical asymptotes and/or "holes" of

$$f(x) = \frac{x^3 - 10x^2 + 16x}{x^2 - 8x}$$

using your knowledge of *limits*. In other words, for each value $x = a$ for which $f(x)$ is undefined, you must evaluate $\lim_{x \to a^-} f(x)$, $\lim_{x \to a^+} f(x)$, and $\lim_{x \to a} f(x)$ and then classify $x = a$ as either a vertical asymptote or a hole.

Warning: You must justify your answer by evaluating ALL appropriate limits. Results using algebra knowledge/explanation alone will receive no credit.

5. Consider $f(x) = \dfrac{3x^2 - 7}{x^2 + 5x}$ and find the following.

   (a) $\lim_{x \to -\infty} f(x)$

   (b) $\lim_{x \to \infty} f(x)$

   (c) Use your results in (a) and (b) to determine the horizontal asymptotes of $f(x)$ (if any). Justify your answer.

6. Consider $f(x) = \dfrac{x^2 - 4x + 3}{x - 1}$ and find the following.

   (a) $\lim_{x \to -\infty} f(x)$

   (b) $\lim_{x \to \infty} f(x)$

   (c) Use your results in (a) and (b) to determine the horizontal asymptotes of $f(x)$ (if any). Justify your answer.

For questions **7** to **10** Evaluate the following limits using appropriate algebraic/analytical methods. Results using graphs or tables of values will receive NO credit.

7. $\lim_{x \to -\infty} (3x^7 + x^2)$

8. $\lim_{x \to -\infty} x^{-11}$

9. $\lim_{x \to -\infty} \dfrac{\sqrt{9x^6 - x}}{x^3 + 1}$

10. $\lim_{x\to\infty} \dfrac{\sqrt{16x^4+64x^2}+x^2}{2x^2-4}$