### Determining Limits from a Graph

1. Below is the graph of $f(x)$. For each of the given points determine the value of $f(a)$ and $\lim_{x \to a} f(x)$. If any of the quantities do not exist clearly explain why.

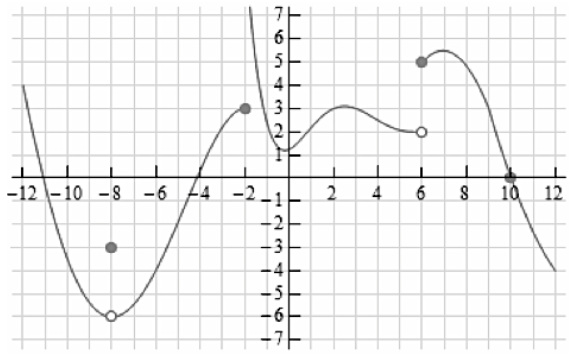

Figure 1: Graph of $f(x)$

(a) $a = -8$

**Solution:** From the graph we can see that, $f(-8) = -3$.
As we approach $x = -8$ from the left we have $\lim_{x \to -8^-} f(x) = -6$
As we approach $x = -8$ from the right we have $\lim_{x \to -8^+} f(x) = -6$. In other words, we have

$$\lim_{x \to -8^-} f(x) = \lim_{x \to -8^+} f(x)$$

and so $\lim_{x \to -8} f(x) = -6$.

(b) $a = -2$

**Solution:** We see that, $f(-2) = 3$ because the closed dot is at the value of $y = 3$. We can also see that as we approach $x = -2$ from both sides the graph is approaching the different values (3 from the left and doesnt approach any value from the right). Because of this we get

$$\lim_{x \to -2} f(x) \text{ Does not exist}$$

(c) $a = 6$

**Solution:** From the graph we can see that, $f(6) = 5$ because the closed dot is at the value of $y = 5$. We can also see that as we approach $x = 6$ from both sides the graph is approaching the different values (2 from the left and 5 from the right). Because of this we get

$$\lim_{x \to 6} f(x) \text{ Does not exist}$$

(d) $a = 10$

**Solution:** From the graph we can see that, $f(10) = 0$ because the closed dot is at the value of $y = 0$. We can also see that as we approach $x = 10$ from both sides the graph is approaching the same value 0 so we get

$$\lim_{x \to 10} f(x) = 0$$

## Evaluating Limits with Limit Laws & Properties

2. $\displaystyle\lim_{t\to-2}\frac{t^4-2}{2t^2-3t+2}$

**Solution:**

$$\lim_{t\to-2}\frac{t^4-2}{2t^2-3t+2} = \frac{\lim\limits_{t\to-2}(t^4-2)}{\lim\limits_{t\to-2}(2t^2-3t+2)}$$

Factor & Cancel

$$= \frac{\lim\limits_{t\to-2}(t^4) - \lim\limits_{t\to-2} 2}{\lim\limits_{t\to-2}(2t^2) - \lim\limits_{t\to-2}(3t) + \lim\limits_{t\to-2} 2}$$

$$= \frac{\left(\lim\limits_{t\to-2} t\right)^4 - \lim\limits_{t\to-2} 2}{2\left(\lim\limits_{t\to-2} t\right)^4 - 3\left(\lim\limits_{t\to-2} t\right) + \lim\limits_{t\to-2} 2}$$

$$= \frac{(2)^4 - 2}{2(2)^4 - 3(2) + 2}$$

Direct Substitution

$$= \frac{7}{8}$$

3. $\lim_{h\to 0} \dfrac{\sqrt{9+h}-3}{h}$

**Solution:**

$$\lim_{h\to 0} \frac{\sqrt{9+h}-3}{h} = \lim_{h\to 0} \frac{\sqrt{9+h}-3}{h}\left(\frac{\sqrt{9+h}+3}{\sqrt{9+h}+3}\right)$$

multiply by the conjugate

$$= \lim_{h\to 0} \frac{(\sqrt{9+h})^2-(3)^2}{h(\sqrt{9+h}+3)}$$

Apply difference of squares property

$$= \lim_{h\to 0} \frac{\cancel{9}+h-\cancel{9}}{h(\sqrt{9+h}+3)}$$

$$= \lim_{h\to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{9+h}+3)}$$

Cancel common terms

$$= \lim_{h\to 0} \frac{1}{\sqrt{9+h}+3}$$

$$= \frac{1}{\sqrt{9+(0)}+3}$$

Direct Substitution

$$= \frac{1}{6}$$

4. $\lim\limits_{x\to -1} \dfrac{2x^2+3x+1}{x^2-2x-3}$

**Solution:**

$$\lim_{x\to -1} \frac{2x^2+3x+1}{x^2-2x-3} = \lim_{x\to -1} \frac{(2x+1)\cancel{(x+1)}}{(x-3)\cancel{(x+1)}}$$ Factor & Cancel

$$= \lim_{x\to -1} \frac{2x+1}{x-3}$$

$$= \frac{2(-1)+1}{(-1)-3}$$ Direct Substitution

$$= \frac{-1}{-4}$$

$$= \frac{1}{4}$$

**Evaluating Limits Involving Absolute Value**

5. Let $g(x) = \dfrac{x^2 + x - 6}{|x-2|}$

   (a) Find $\lim\limits_{x \to 2^+} g(x)$ and $\lim\limits_{x \to 2^-} g(x)$

   **Solution:**

   Note that $|x-2| = \begin{cases} x-2, & x \geq 2 \\ -(x-2), & x \leq 2 \end{cases}$

$$\lim_{x \to 2^+} g(x) = \lim_{x \to 2^+} \frac{x^2 - x + 6}{x-2} = \lim_{x \to 2^+} \frac{\cancel{(x-2)}(x+3)}{\cancel{(x-2)}}$$

$$= \lim_{x \to 2^+} x+3$$

$$= 2+3 = 5$$

$$\lim_{x \to 2^-} g(x) = \lim_{x \to 2^-} \frac{x^2 - x + 6}{-(x-2)} = \lim_{x \to 2^-} \frac{\cancel{(x-2)}(x+3)}{-\cancel{(x-2)}}$$

$$= \lim_{x \to 2^-} \frac{x+3}{-1}$$

$$= \frac{(2+3)}{-1}$$

$$= -5$$

   (b) Does $\lim\limits_{x \to 2} g(x)$ exist? Explain why or why not.

   **Solution:** Since

$$\lim_{x \to 2^-} g(x) \neq \lim_{x \to 2^+} g(x)$$

   then

   $\lim\limits_{x \to 2} g(x)$ Does not exist.

**Applying the Squeeze Theorem**

6. Consider the graphs of $f(x) = x^2$, $h(x) = -x^2$, $g(x) = x^2 \sin\left(\frac{1}{x}\right)$ given below in Figure 2.

   Use this information to evaluate

   $$\lim_{x \to 0} x^2 \sin\left(\frac{1}{x}\right)$$

   by applying the Squeeze Theorem.

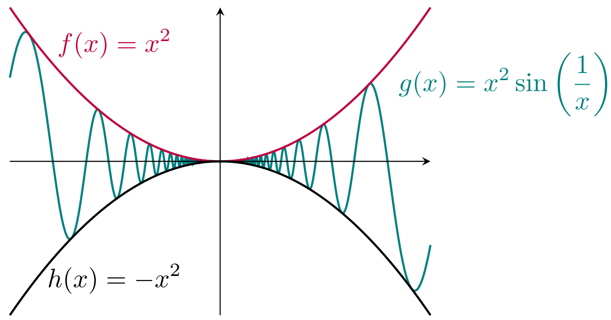


Figure 2: Graphs of $f(x)$, $g(x)$, $h(x)$

**Solution:**

From the graph we have that

$$-x^2 \leq x^2 \sin\left(\frac{1}{x}\right) \leq x^2$$

then

$$-\lim_{x \to 0} x^2 \leq \lim_{x \to 0} x^2 \sin\left(\frac{1}{x}\right) \leq \lim_{x \to 0} x^2$$

Since

$$-\lim_{x \to 0} x^2 = 0 \qquad \text{and} \qquad \lim_{x \to 0} x^2 = 0$$

then

$$0 \leq \lim_{x \to 0} x^2 \sin\left(\frac{1}{x}\right) \leq 0$$

Therefore, by the squeeze Theorem

$$\lim_{x \to 0} x^2 \sin\left(\frac{1}{x}\right) = 0$$