

Written Homework 6

MATH 2200-98 ILSB Calculus 1

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Rates of change

1. The number of yeast cells in a laboratory culture increases rapidly initially but levels off eventually. The population is modeled by the function

$$f(t) = \frac{b}{1 + ae^{-0.7t}}$$

where t is measured in hours and a and b are *constants*.

- (a) Find the function that governs the *rate of change* of the population with respect to time.

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1. (Continued)

- (b) At time $t = 0$ the population is 20 cells and is increasing at a rate of 12 cells/hour. Using this information, find the values of a and b .

1. (Continued)

- (c) According to this model, what happens to the yeast population in the long run? (Hint: consider what happens as $t \rightarrow \infty$). Note that this needs to be a precise result, not just stating that it tapers off eventually as stated above!

2. The height (in meters) of a projectile shot vertically upward from a point 2 m above ground level with an initial velocity of 24.5 m/s is

$$h(t) = 2 + 24.5t - 4.9t^2$$

after t seconds. Round all results to 2 decimal places.

- (a) State the velocity function $v(t)$.

2. (Continued)

(b) When does the projectile reach its maximum height?

(c) What is the maximum height?

(d) When does the projectile hit the ground?

(e) With what velocity does it hit the ground?

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3. A stone is dropped into a lake creating a circular ripple that travels outward at a speed of 60 cm/sec . Find the rate at which the *area* within the circle is increasing after $t = 3 \text{ sec}$. You should leave π in your result (i.e. NO Decimals!).

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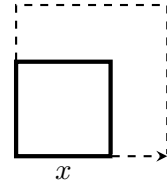
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4. On the moon, a feather will fall to the ground at the same rate as a heavy stone. Suppose a feather is dropped from a height of 40 m above the surface of the moon. Then its height h (in meters) above the ground after t seconds is given by $h(t) = 40 - 0.8t^2$. Determine the velocity and acceleration of the feather the moment it strikes the surface of the moon. Round your result to 3 decimal places.

Related Rates

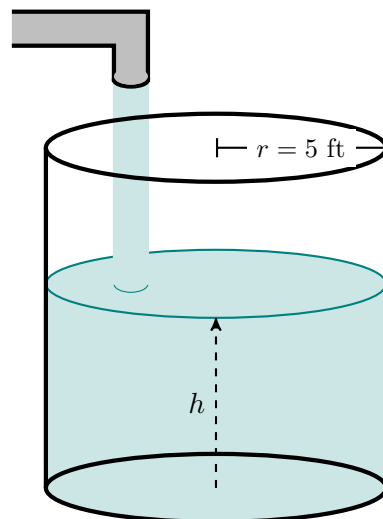
For Problems 5 through 10: You must use the layout demonstrated in GASCAP Problem Solving Str for the following problems. Failure to use this format will result in your problem not being graded. NO EXCEPTIONS! If a figure is given, it is expected that you will use the indicated variables in your solution.

5. Each side of a square is increasing at a rate of 6 cm/s . At what rate is the area of the square increasing when the area of the square is 16 cm^2 ?



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6. A cylindrical tank with radius 5 m is being filled with water at a rate of $3 \text{ m}^3/\text{min}$. How fast is the height of the water increasing? Your result should be given in exact form, meaning you should have π in your final answer (no decimals!).

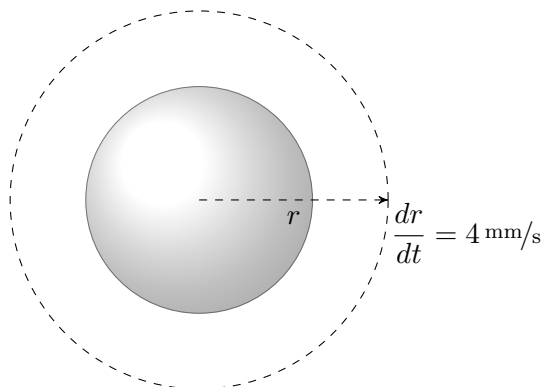


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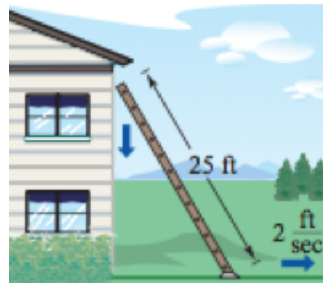
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7. The radius of a sphere is increasing at a rate of 4 mm/s . How fast is the volume increasing when the diameter is 80 mm ? Leave π in your final result.

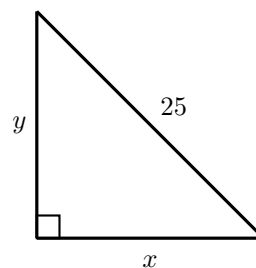


For Problems 8, 9 and 10:

The figure to the right shows a ladder 25 feet long leaning against the wall of a house. The base of the ladder is pulled away from the wall at a rate of 2 feet per second.



8. How fast is the top of the ladder moving down the wall when its base is 7 feet from the wall?
Please use the variables given in the figure below. Leave your final result in exact form.

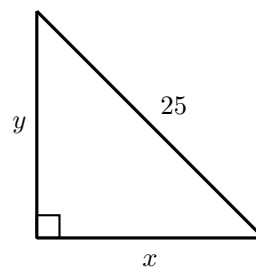


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9. Consider the triangle formed by the side of the house, the ladder, and the ground. Find the rate at which the area of the triangle is changing when the base of the ladder is 7 feet from the wall. Please use the variables given in the figure below. Leave your final result in exact form.



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10. Find the rate at which the angle between the ladder and the wall of the house is changing when the base of the ladder is 7 feet from the wall. Please use the variables given in the figure below. Leave your final result in exact form.

