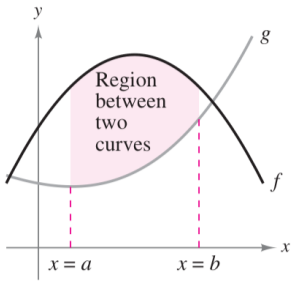
# Lecture # 03: Area btwn Curves

Date: 9/17/2018



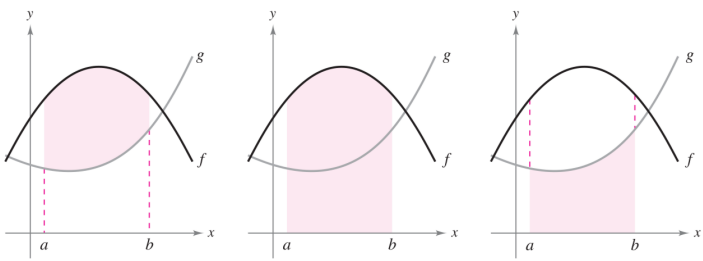

What is the area btwn f & g?

In this case,

$$\text{area btwn } f \text{ \& } g = \text{area under } f - \text{area under } g$$

$$= \int_a^b f(x)\,dx - \int_a^b g(x)\,dx$$

$$= \int_a^b (f(x) - g(x))\,dx$$




| Area of region between $f$ and $g$ | = | Area of region under $f$ | − | Area of region under $g$ |
|---|---|---|---|---|
| $\int_a^b [f(x) - g(x)]\,dx$ | = | $\int_a^b f(x)\,dx$ | − | $\int_a^b g(x)\,dx$ |

Note: In this case we must have that

$$f(x) \geq g(x)$$

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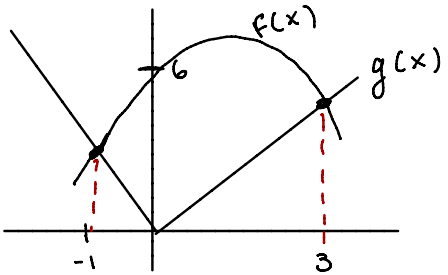
**Ex 1** Find the area btwn $f(x) = -x^2 + 3x + 6$ and $g(x) = |2x|$




We first need to find where f & g intersect. i.e.

$$f(x) = g(x)$$

$$\Rightarrow -x^2 + 3x + 6 = |2x|$$

**for $x > 0$**

$$-x^2 + 3x + 6 = 2x$$
$$x^2 - x - 6 = 0$$
$$(x-3)(x+2) = 0$$
$$\Rightarrow x = 3 \text{ or } x = -2 \times$$

**for $x < 0$**

$$-x^2 + 3x + 6 = -2x$$
$$x^2 - 5x - 6 = 0$$
$$(x-6)(x+1) = 0$$
$$x = 6 \text{ or } x = -1$$
$$\times$$

So region btwn two curves starts @ $x = -1$ & ends @ $x = 3$ then

$$\text{area btwn f \& g} = \int_a^b (f(x) - g(x))\,dx$$

$$= \int_{-1}^{3} (-x^2 + 3x + 6) - (2|x|)\,dx$$

↑ means we must split integral on interval

$$= \int_{-1}^{0} (-x^2 + 3x + 6) - (-2x)\,dx - \int_{0}^{3} (-x^2 + 3x + 6) - (2x)\,dx$$

$$= \int_{1}^{0} (-x^2 + 5x + 6)\,dx + \int_{0}^{3} (-x^2 + x + 6)\,dx = \frac{50}{3}$$

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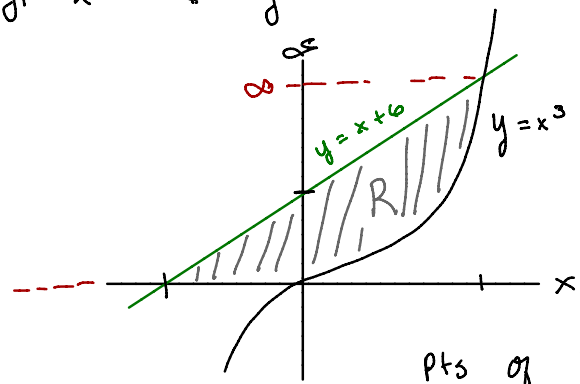
Sometimes it may be easier w.r.t. $y$ ← "with respect to" rather than $x$

**Ex. 2** Find the region R bdd by the graphs of $y_1 = x^3$ & $y = x+6$ & the x-axis




$y = x^3 \Rightarrow x = y^{1/3}$

$y = x+6 \Rightarrow x = y-6$

pts of intersection:

$y^{1/3} = y-6$

$y = (y-6)^3$

$0 = (y-6)^3 - y$

$\Rightarrow y = 0$ & $y = 8$

area btwn F & g $= \int (f(y) - g(y))\,dy$

$= \int_0^8 ((y^{1/3}) - (y-6))\,dy$

$= 28$