Name: ______________________________ Sort #: ________

Written Homework 5

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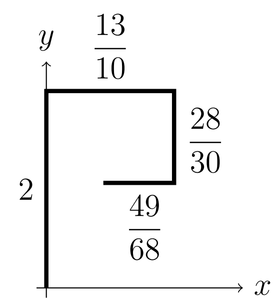
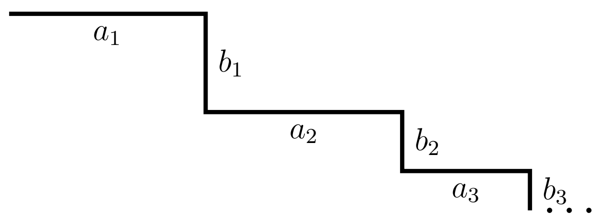
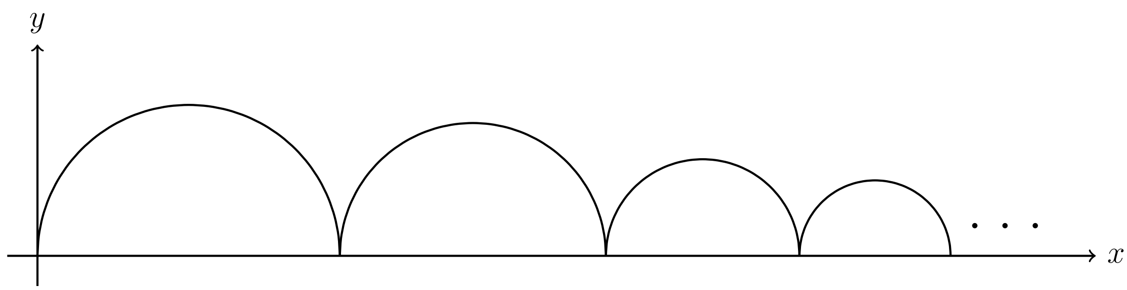
Problem 1: Your task is to compute the length (if it exists) of the following path. The path consists of an infinite number of legs, each being a straight line segment of a certain distance. Here are instructions for the first few legs. Begin at $(0, 0)$ and head in the positive y-direction for a distance of 2. Now make a 90 degree right turn and head in the positive x-direction for a distance of 13/10. Make another 90 degree right turn and head in the negative y-direction for a distance of 28/30. This process will continue forever with the subsequent distances traveled given by $\dfrac{3k^2+1}{k^3+k}$ for $k = 1, 2, 3, \dots$. After travelling the given distance on each leg, a 90 right turn is made and the next distance is travelled. The first 4 legs of the journey (labeled by their lengths) are illustrated below.



Problem 2: Consider the following path as illustrated in the diagram. The path consists of a horizontal piece, followed by a vertical piece, followed by a horizontal piece, and so on. The lengths of the successive horizontal pieces are given by $a_k = \dfrac{2^k}{k!}$, $k = 1, 2, 3, \ldots$. The lengths of the succesive vertical pieces are given by $b_k = \dfrac{\sqrt{k} + 2}{k^{5/2} + k + 1}$, $k = 1, 2, 3, \ldots$. Express the length of this path as an infinite series and determine whether it has an infinite or finite length.



Problem 3: Consider an infinite sequence of semicircles with decreasing radius lined up along the x-axis as illustrated in the diagram. The area of each semicircle follows the pattern $\frac{\pi}{2}k^4(k^3+1)^{-2}$ for $k = 1, 2, 3, \ldots$, so that the first semicircle has area $\frac{\pi}{8}$, the second has area $\frac{8\pi}{81}$, and so on. Also consider the path $\mathcal{L}$ that begins at (0,0) and traces the top of each semicricle in succession. Write down two different infinite series, one expressing the combined area of this sequence of semicircles, and another expressing the length of the path $\mathcal{L}$. Then determine whether each of these quantities is finite or infinite.



Problem 4: Consider the region $\mathcal{R}$ defined by an infinite sequence of isoceles triangles (each peak is a right angle) as illustrated in the diagram. The heights of the triangles decrease according to the pattern ke^{-k} for $k = 1, 2, 3, \ldots$. Express the area of $\mathcal{R}$ as an infinite series and determine if it is finite or infinite. Express the *net* area of $\mathcal{R}$ as an infinite series and determine if it is finite or infinite.