## Worksheet 12 Solutions
**Intro to Differential Equations** MATH 2205, Fall 2018

1. Plot the direction field and solution curves for several values of $y_0$.

   (a) $y' = -1 - 2y \quad y(0) = y_0$

   **Solution:**

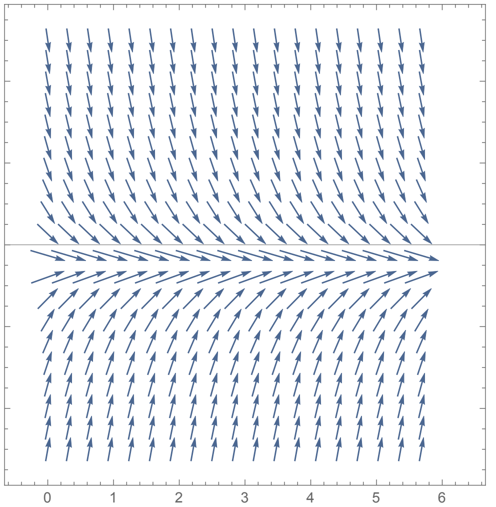


**Figure 1:** Direction Field for 2(b)

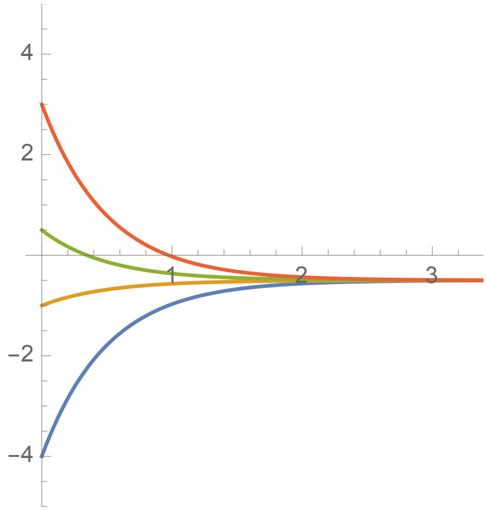


**Figure 2:** Some Solution Curves for 2(b)

   (b) $y' = 2y - 10; \quad y(0) = y_0$

   **Solution:**

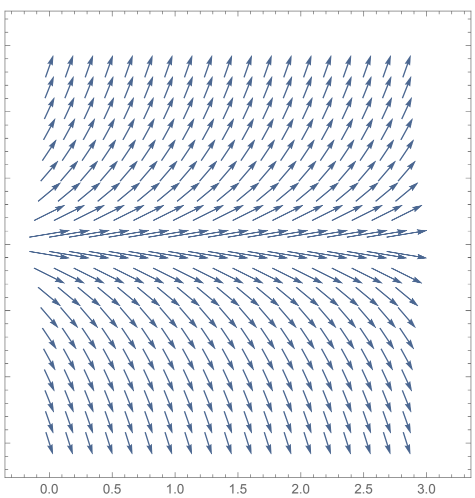


**Figure 3:** Direction Field for 2(b)

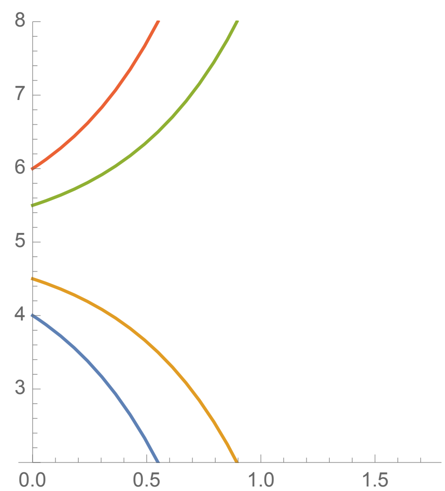


**Figure 4:** Some Solution Curves for 2(b)

2. Verify that equation

$$t^2y'' + 5ty' + 4y = 0, \quad t > 0$$

has solutions

$$y_1(t) = t^{-2}; \qquad y_2(t) = t^{-2}\ln(t)$$

**Solution:** Recall that to verify if a given equation is a valid solution to our ODE we simply calculate the needed derivatives, plug them into our ODE and verify that it results in a truthful statement.

My respective derivatives are

$$\begin{aligned} y_1(t) &= t^{-2} & y_2(t) &= t^{-2}\ln(t) \\ y_1'(t) &= -2t^{-3} & y_2'(t) &= -2t^{-3}\ln(t) + t^{-3} \\ y_1''(t) &= 6t^{-4} & y_2''(t) &= 6t^{-4}\ln(t) - 5t^{-4} \end{aligned}$$

Now we simply plug and chug to find that $y_1(t)$ and $y_2(t)$ are both solutions to the ODE.

3. For each of the given differential equations, determine its order and whether it is linear or nonlinear.

   (a) $t^2\dfrac{d^2y}{dt^2} + t\dfrac{dy}{dt} + 2y = \sin(t)$

   **Solution:** This is 2nd order and nonlinear. I always like to "translate" the $\dfrac{dy}{dt}$ notation to simple prime notation so feel free to do so as well. It can make patterns easier to see in my opinion.

   $$t^2y'' + ty' + 2y = \sin(t)$$

   The highest derivative is the 2nd derivative. There are no dependent variables multiplied by one another (i.e. no $y$'s multiplied by another $y$) so this equation is linear.

   (b) $\dfrac{dy}{dt} + ty^2 = 0$

   **Solution:** 1st order, nonlinear. Again "translating" the notation we have

   $$y' + ty^2 = 0$$

   We see that the highest derivative is the first derivative. In this case we have a $y^2$ term. In other words, we have $y \cdot y$. So this equation is nonlinear.

   (c) $\dfrac{d^3y}{dt^3} + t\dfrac{d^2y}{dt^2} + (\cos^2(t))y = t^3$

   **Solution:** 3rd order, linear. Again "translating" the notation we have

   $$y''' + ty'' + (\cos^2(t))y = t^3$$

   Our highest derivative is the 3rd derivative. Even though we have the $\cos(t)$ term it only involves our independent variable $t$. Thus, this is a linear equation.

   (d) $\dfrac{d^3y}{dt^3} + t\dfrac{d^2y}{dt^2} + \sin(t + y) = t^3$

   **Solution:** 3rd order, nonlinear. Again "translating" the notation we have

   $$y''' + ty'' + \sin(t + y)y = t^3$$

   Again we have the 3rd derivative. The term $\sin(t+y)$ involves our independent and our dependent variable. In other words we cannot write our differential equation explicitly in terms of the form $p(t) \cdot y^(n)$. Since we have a $y$ "trapped" inside the sin function this equation is nonlinear.