1. Find the volume of the solid obtained when the shaded region is revolved about the y-axis in Figure 1 using the shell method.

Solution:

Due to symmetry, we consider the region for $x \geq 0$.

Finding the volume here & doubling it will give the volume of the entire region

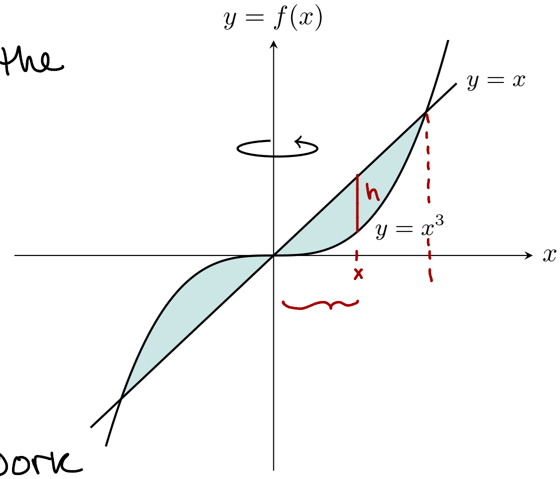


Figure 1

We know from previous work in worksheet #3 that the pts of intersection occur @ $x=0$ & $x=3$

Using shells:

$$\text{Volume} = \int_a^b 2\pi x\,(f(x) - g(x))\,dx$$

So our volume will be given by

$$V = 2\int_a^b 2\pi x\,(f(x) - g(x))\,dx$$

Where $f(x) = x$ and $g(x) = x^3$

So

$$V = 2\int_0^1 2\pi x\,(x - x^3)\,dx$$

$$= 4\pi\int_0^1 x(x - x^3)\,dx$$

$$= 4\pi\int_0^1 (x^2 - x^4)\,dx$$

$$= 4\pi\left[\frac{x^3}{3} - \frac{x^5}{5}\right]_0^1 = 4\pi\left[\left(\frac{(1)^3}{3} - \frac{(1)^5}{5}\right) - 0\right] = 4\pi\left[\frac{5}{15} - \frac{3}{15}\right]$$

$$= 4\pi\left[\frac{2}{15}\right] = \frac{8\pi}{15}$$

2. Find the volume of the solid obtained when the shaded region is revolved about the x-axis in Figure 2. Use the shell method.

Solution:

Using the shell method

$$V = \int_a^b 2\pi y \left(p(y) - q(y)\right) dy$$

where $p(y) = 4 - y$

& $q(y) = 0$

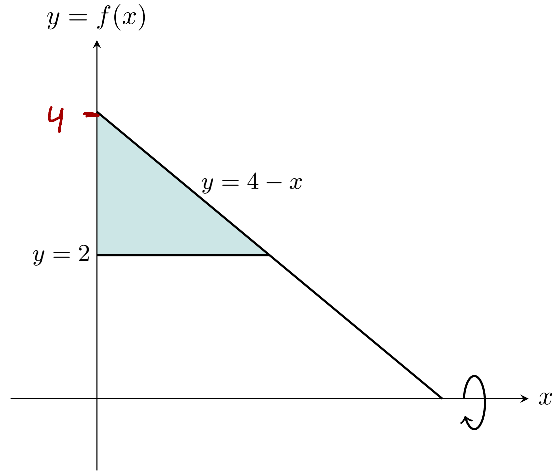


Figure 2

So volume is given by

$$V = \int_2^4 2\pi y (4 - y)\, dy$$

$$= 2\pi \int_2^4 (4y - y^2)\, dy$$

$$= 2\pi \left[2y^2 - \frac{y^3}{3} \right]_2^4$$

$$= \frac{32\pi}{3}$$

3. Find the volume of the solid obtained when the shaded region is revolved about the y-axis seen in Figure 3. Use the shell method.

Solution:

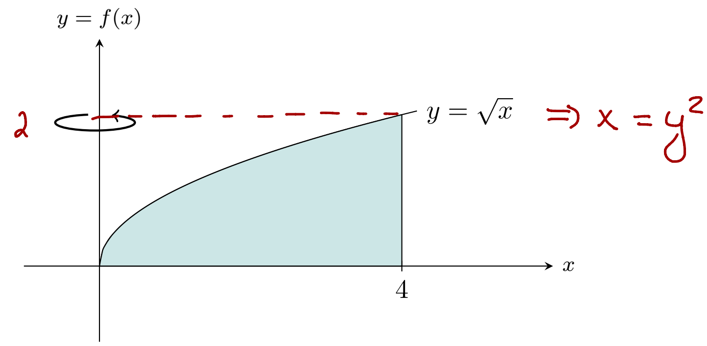


Figure 3

Using the shell method

$$V = \int_a^b 2\pi y\,(p(y) - g(y))\,dy$$

Where $p(y) = 4$

& $g(y) = y^2$

So volume is given by

$$V = \int_0^2 2\pi y\,(4 - y^2)\,dy$$

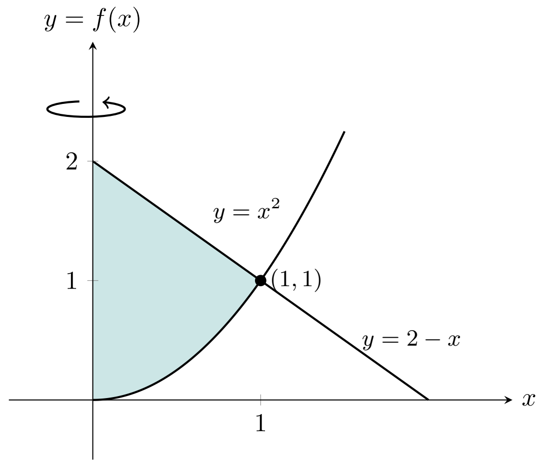
$$= 2\pi \int_0^2 (4y - y^3)\,dy$$

$$= 2\pi \left[2y^2 - \frac{y^4}{4} \right]_0^2$$

$$= 8\pi$$

4. Find the volume of the region bounded by $y = x^2$, $y = 2 - x$ and $x = 0$ in the first quadrant, revolved around the y axis. Use whichever method you like.

Solution:



If we use disks we'll need to split the region in 2. It will be easier to use shells here.

Find the pt of intersection

$$x^2 = 2 - x$$

$$\Rightarrow x^2 + x - 2 = 0$$

$$\Rightarrow (x-1)(x+2) = 0 \quad \Rightarrow \text{only need } x = 1$$

Volume using the shell method is

$$V = \int_0^1 2\pi x (f(x) - g(x))\, dx$$

$$= 2\pi \int_0^1 x[(2-x) - x^2)\, dx$$

$$= 2\pi \int_0^1 [2x - x^2 - x^3]\, dx$$

$$= 2\pi \left[x^2 - \frac{x^3}{3} - \frac{x^4}{4} \right]_0^1$$

$$= \frac{5\pi}{6}$$