Name: ________________________ Sort #: ________

Written Homework 6

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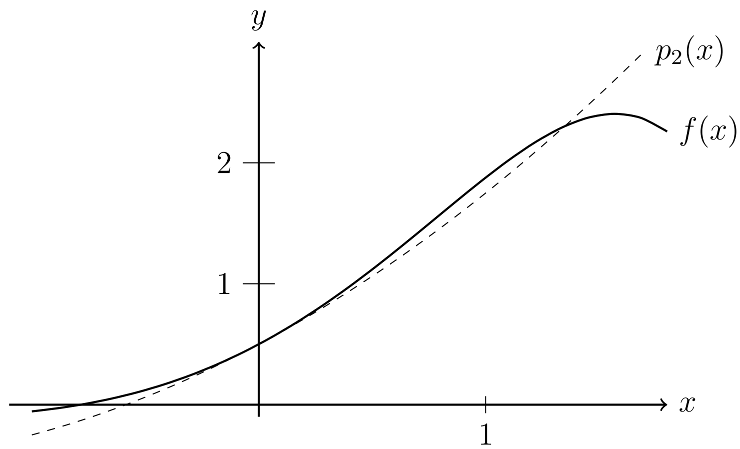
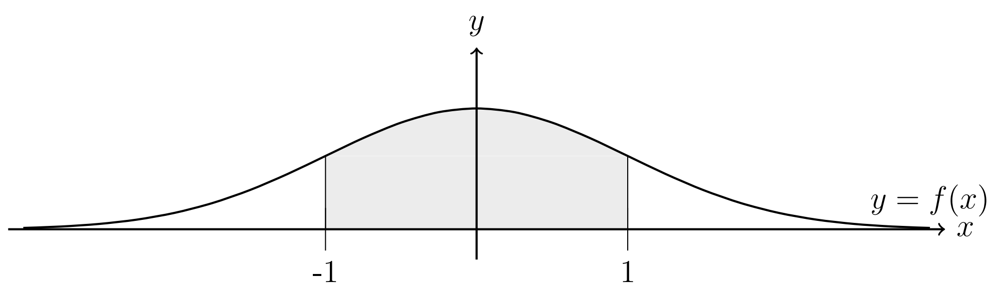
Problem 1: Consider the function $f(x) = \frac{e^x}{2}\big(\sin(x)+\cos(x)\big)$. Use the second degree Taylor polynomial for f centered at $a = 0$ to approximate the value of f at $x = 1$. Use the estimate for the remainder to determine how far your approximation could be from the exact value. For reference, the graphs of f (solid) and p_2 (dashed) are given below.



Problem 2: In probability, the normal distribution is used to calculate the probabilities that certain observations could occur, e.g., that a randomly chosen person will be of a certain height. In such a scenario, the observations have numerical values (e.g., height) and the probability that any particular observation has a value lying in a given range (e.g., the height is between 65 and 69 inches) is given by the area under a curve from 65 to 69. To make things more precise, one such curve is given by the function $f(x) = \frac{1}{\sqrt{2\pi}}e^{-x^2/2}$. This curve has a mean of 0 and a standard deviation of 1. Let's consider one example. We want to know the probability that the randomly obtained observation will lie within 1 standard deviation of the mean. The corresponding area is shaded in the picture. In symbols, this probability will be equal to

$$\int_{-1}^{1} \frac{1}{\sqrt{2\pi}}e^{-x^2/2}dx,$$

but we have no way of computing this using the Fundamental Theorem of Calculus. Use the second degree Taylor polynomial for f centered at $a = 0$ to approximate this probability. It may be useful to know that $\frac{1}{\sqrt{2\pi}} \approx 0.4$. For reference, the commonly used value is 0.6827.



Problem 3: Consider the number $\sqrt{e}$. We can approximate the decimal expansion of this number using the Taylor series for $e^{x/2}$ centered at 0. How many terms of this Taylor series does one need to be certain that $\sqrt{e} > 1.6$? You will need to justify your claim using remainder estimates.

Problem 4: Approximate the area between the curve $f(x) = \sin(x^2)$ and the x-axis from $x = 0$ to $x = 1$ with an error less than 10^{-3}. For reference, the graph of f is given below.