Evaluate the following integrals

1. $\int x\sqrt{x+2}\,dx$

Solution: Let $u = x+2 \implies x = u-2$
$du = dx$

$$\implies \int x\sqrt{x+2}\,dx = \int (u-2)(u)^{1/2}\,du$$
$$= \int (u^{3/2} - 2u^{1/2})\,du$$
$$= \frac{u^{5/2}}{\frac{5}{2}} - \frac{2u^{3/2}}{\frac{3}{2}} + c$$
$$= \frac{2}{5}u^{5/2} - \frac{4}{3}u^{3/2} + c = \frac{2}{5}(x+2)^{5/2} - \frac{4}{3}(x+2)^{3/2} + c$$

2. $\int \sin(\theta)\sin(\cos(\theta))\,d\theta$

Solution: Let $u = \cos(\theta)$
$du = -\sin(\theta)\,d\theta \implies -du = \sin(\theta)\,d\theta$

$$\implies \int \sin(\theta)\sin(\cos\theta)\,d\theta = \int \sin(u)(-du)$$
$$= -\int \sin(u)\,du$$
$$= \cos(u) + c$$
$$= \cos(\cos(\theta)) + c$$

3. $\int \frac{t}{1+t^4}\,dt$

Solution: Let $u = t^2$
$du = 2t\,dt \implies \frac{1}{2}du = dt$

$$\implies \int \frac{t}{1+(t^2)^2}\,dt = \int \frac{1}{1+u^2}\left(\frac{1}{2}du\right)$$
$$= \frac{1}{2}\int \frac{1}{1+u^2}\,du$$
$$= \frac{1}{2}\arctan(u) + c$$
$$= \frac{1}{2}\arctan(t^2) + c$$

4. $\displaystyle\int_e^{e^4} \frac{1}{x\sqrt{\ln x}}\,dx$

Solution: Let $u = \ln(x)$
$du = \frac{1}{x}\,dx$

$$\Rightarrow \int \frac{1}{(\ln(x))^{1/2}}\left(\frac{1}{x}\,dx\right) = \int \frac{1}{u^{1/2}}\,du = \int u^{-1/2}\,du = \frac{u^{1/2}}{1/2} + C$$

$$= 2u^{1/2} + C = 2(\ln(x))^{1/2} + C$$

$$\Rightarrow \int_e^{e^4} \frac{1}{x\sqrt{\ln(x)}}\,dx = \left[2(\ln(x))^{1/2}\right]_e^{e^4} = 2\left[(\ln(e^4))^{1/2} - (\ln(e))^{1/2}\right]$$

$$= 2\left[(4)^{1/2} - 1\right]$$

$$= 2$$

5. If f is continuous and $\displaystyle\int_0^9 f(x)\,dx = 4$, find $\displaystyle\int_0^3 xf(x^2)\,dx$

Solution:

Let $u = x^2$
$du = 2x\,dx \Rightarrow \frac{1}{2}\,du = x\,dx$

Bounds:
$x = 3 \qquad u = (3)^2 = 9$

$x = 0 \qquad u = (0)^2 = 0$

$$\Rightarrow \int_0^3 x f(x)\,dx = \frac{1}{2}\int_0^9 f(u)\,du = \frac{1}{2}(4) = 2$$