Name: _____ Sort #: _____

Written Homework 4

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Problem 1: Provide (non-constant) examples of sequences satisfying the following conditions.

(a) A bounded sequence that diverges

(b) An alternating sequence that converges

(c) An increasing sequence that converges to 0

How you come up with your sequence is not important. A properly written solution will state your sequence, and then demonstrate that the sequence meets the required condition(s). Clearly label each part as (a), (b), (c). Do not question parts in columns!

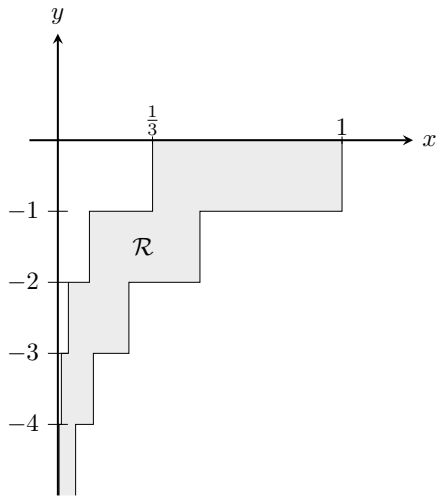
Problem 2: Provide sequences $\{a_n\}$ and $\{b_n\}$ with following properties

(a) $\lim_{n\to\infty} \cos(a_n)$ exists and

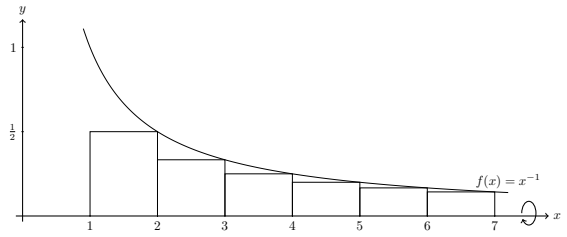
(b) $\lim_{n\to\infty} \cos(b_n)$ does not exist.

How you come up with your sequence is not important. A properly written solution will state your sequence, and then demonstrate that the sequence meets the required condition(s). Clearly label each part as (a), (b), (c). Do not question parts in columns!

Problem 3: Compute the area of the infinitely long region $\mathcal{R}$ indiated in the picture. The oustide width of each step decreases according to 2^{-k}, $k = 0, 1, 2, 3, \ldots$. The inside width of each step decreases according to 3^{-k}, $k = 1, 2, 3, \ldots$.



Problem 4: The following picture shows an infinite sequence of rectangles with heights given by $\left\{\frac{1}{n}\right\}_{n=2}^{\infty}$ and the curve $f(x) = x^{-1}$.



You are to compare the volume of the solid obtained by revolving the rectangles about the x-axis with that of solid obtained by revolving the region under the curve f about x-axis.

- The volume of the rectangles should be expressed as an infinite series (Hint: This solid will look like a bunch of stacked discs.)
- The volume of the solid obtained by revolving the curve $f(x) = x^{-1}$ about the x-axis from $[1, \infty)$ should be computed with an appropriate integral.

What relationship do you see concerning these two volumes? What can you conclude based on this relationship?