1. For the following sequences find a formula for the general term a_n

(a) $\{\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}, \ldots\}$

Solution:

$a_1 = \frac{1}{2} = \frac{1}{2\cdot 1}$

$a_2 = \frac{1}{4} = \frac{1}{2\cdot 2}$

$a_3 = \frac{1}{6} = \frac{1}{2\cdot 3}$

$a_4 = \frac{1}{8} = \frac{1}{2\cdot 4}$

$a_5 = \frac{1}{10} = \frac{1}{2\cdot 5}$

This pattern implies that

$$a_n = \frac{1}{2\cdot n}$$

(b) $\{4, -1, \frac{1}{4}, -\frac{1}{16}, \frac{1}{64}, \ldots\}$

Solution:

$a_1 = 4 = 4\left(-\frac{1}{4}\right)^0$

$a_2 = -1 = 4\left(\frac{-1}{4}\right) = 4\left(-\frac{1}{4}\right)^1$

$a_3 = \frac{1}{4} = 4\left(\frac{1}{16}\right) = 4\left(\frac{-1}{4}\right)^2$

$a_4 = -\frac{1}{16} = 4\left(-\frac{1}{64}\right) = 4\left(-\frac{1}{4}\right)^3$

$a_5 = \frac{1}{64} = 4\left(\frac{1}{256}\right) = 4\left(-\frac{1}{4}\right)^4$

This implies that $a_n = 4\left(-\frac{1}{4}\right)^n$ for $n = 0, 1, 2, \ldots$

or $a_n = 4\left(-\frac{1}{4}\right)^{n-1}$ for $n = 1, 2, 3, \ldots$

2. Determine if the following sequences converge or diverge.

(a) $\{a_n\} = \left\{ \dfrac{3+5n^2}{n+n^2} \right\}$

Solution: To determine convergence we eval $\lim\limits_{n\to\infty} a_n$

$$\Rightarrow \lim_{n\to\infty} \frac{3+5n^2}{n+n^2} = \lim_{n\to\infty} \frac{\cancel{n^2}\left(\frac{3}{n^2}+5\right)}{\cancel{n^2}\left(\frac{1}{n}+1\right)}$$

$$= \lim_{n\to\infty} \frac{\frac{3}{n^2}+5}{\frac{1}{n}+1}$$

$$= \frac{\lim\limits_{n\to\infty} \frac{3}{n^2} + \lim\limits_{n\to\infty} 5}{\lim\limits_{n\to\infty} \frac{1}{n} + \lim\limits_{n\to\infty} 1} = 5$$

Since $\lim\limits_{n\to\infty} a_n$ exists, the seq. converges

(b) $\{b_n\} = \left\{ \dfrac{3+5n^2}{1+n} \right\}$

Solution: To determine convergence we eval $\lim\limits_{n\to\infty} a_n$

$$\Rightarrow \lim_{n\to\infty} \frac{3+5n^2}{1+n} = \lim_{n\to\infty} \frac{10n}{1}$$

$$= 10 \lim_{n\to\infty} n$$

$$= 10(\infty) = \infty$$

Since $\lim\limits_{n\to\infty} a_n$ DNE, the seq. diverges

3. Determine if the following sequences converge or diverge.

   (a) $\left\{\left(\frac{1}{2}\right)^n\right\}$

   Solution:

   This seq. has form ar^n so its a geometric seq.

   Since $|r| = \left|\frac{1}{2}\right| < 1$ then

   $$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \left(\frac{1}{2}\right)^n = 0$$

   & the seq. converges

   (b) $\{2^{n+1}3^{-n}\}$

   Solution: Rewriting

   $$2^{n+1}\, 3^{-n} = 2\cdot 2^n \cdot 3^{-n} = 2\cdot \frac{2^n}{3^n} = 2\left(\frac{2}{3}\right)^n$$

   This seq. has form ar^n so its a geometric seq.

   Since $|r| = \left|\frac{2}{3}\right| < 1$ then

   $$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \left(\frac{2}{3}\right)^n = 0$$

   & the seq. converges

4. Determine if the following sequences converge or diverge.

(a) $\{a_n\} = \left\{ \dfrac{(-1)^n}{2\sqrt{n}} \right\}$

Solution:

To determine convergence we eval $\lim_{n\to\infty} a_n$

$$\lim_{n\to\infty} \frac{(-1)^n}{2\sqrt{n}} = \lim_{n\to\infty} \frac{(-1)^n}{2n^{1/2}} = 0$$

Since $\lim_{n\to\infty} a_n$ exists, the seq. converges

(b) $\{b_n\} = \left\{ \dfrac{4^n}{1+9^n} \right\}$

Solution:

To determine convergence we eval $\lim_{n\to\infty} b_n$

$$\lim_{n\to\infty} \frac{4^n}{1+9^n} = \lim_{n\to\infty} \frac{9^n\left(\frac{4^n}{9^n}\right)}{9^n\left(\frac{1}{9^n}+1\right)}$$

$$= \lim_{n\to\infty} \frac{\left(\frac{4}{9}\right)^n}{\left(\frac{1}{9}\right)^n+1}$$

$$= \frac{\lim_{n\to\infty}\left(\frac{4}{9}\right)^n}{\lim_{n\to\infty}\left(\frac{1}{9}\right)^n + \lim_{n\to\infty} 1}$$

$$= \frac{0}{0+1} = 0$$

Since $\lim_{n\to\infty}\left(\frac{4}{9}\right)^n = 0$ & $\lim_{n\to\infty}\left(\frac{1}{9}\right)^n = 0$

Geom. seq. w $|r|<1$

Since $\lim_{n\to\infty} b_n$ exists, the seq. converges