For problems 1 through 5 evaluate the given indefinite integrals.

1. $\displaystyle\int \left(\frac{6}{\sqrt{x}} + 6\sqrt{x}\right)\, dx$

Solution:
$$= \int \left(6x^{-1/2} + 6x^{1/2}\right) dx$$
$$= 6\int x^{-1/2}\, dx + 6\int x^{1/2}\, dx$$
$$= \frac{6x^{-1/2+1}}{-\frac{1}{2}+1} + \frac{6x^{1/2+1}}{\frac{1}{2}+1} + C$$
$$= \frac{6x^{1/2}}{\frac{1}{2}} + \frac{6x^{3/2}}{\frac{3}{2}} + C$$
$$= 12x^{1/2} + 4x^{3/2} + C$$

2. $\displaystyle\int \left(\frac{3}{s^2} - 4s^8\right)\, ds$

Solution:
$$= \int \left(3s^{-2} - 4s^8\right) ds$$
$$= 3\int s^{-2}\, ds - 4\int s^8\, ds$$
$$= \frac{3s^{-2+1}}{-2+1} - \frac{4s^{8+1}}{8+1} + C$$
$$= -3s^{-1} - \frac{4}{9}s^9 + C$$

3. $\displaystyle\int (9x+4)^2\, dx$

Solution:
$$= \int \left(81x^2 + 72x + 16\right) dx$$
$$= 81\int x^2\, dx + 72\int x\, dx + 16\int dx$$
$$= 81\frac{x^{2+1}}{2+1} + \frac{72x^{1+1}}{1+1} + 16x + C$$
$$= 27x^3 + 36x^2 + 16x + C$$

4. $\displaystyle\int \frac{3x^3 + 6x^2}{x}\, dx$

Solution: $= \displaystyle\int \left(\frac{3x^3}{x} + \frac{6x^2}{x} \right) dx$

$= 3\displaystyle\int x^2 dx + 6\int x\, dx$

$= \dfrac{3x^3}{2+1} + \dfrac{6x^{1+1}}{1+2} + C$

$= x^3 + 3x^2 + C$

5. $\displaystyle\int (\sec^2 t - 6)\, dt$

Solution: $= \displaystyle\int \sec^2(t)\, dt - 6\int dt$

$= \tan(t) - 6t + C$

6. Solve the initial value problem $f'(x) = x^2 - 2x$ with $f(1) = \dfrac{1}{3}$.

Solution:

$$f(x) = \int f'(x)dx = \int (x^2 - 2x)\,dx$$
$$= \int x^2 dx - 2\int x\,dx$$
$$= \frac{x^3}{3} - x^2 + C$$

Since $f(1) = \frac{1}{3}$ then

$$\frac{(1)^3}{3} - (1)^2 + C = \frac{1}{3}$$

Solving for C:

$$-\frac{2}{3} + C = \frac{1}{3} \Rightarrow C = 1$$

Solution to IVP is

$$f(x) = \frac{x^3}{3} - x^2 + 1$$

7. Given the acceleration function of an object moving along a line, find the position function with the given initial velocity and initial position. $a(t) = 4, \quad v(0) = -3, \quad s(0) = 2$

Solution: Since $a(t) = v'(t) = s''(t)$

Velocity is given by

$$v(t) = \int v'(t)dt = \int a(t) = \int 4\,dt = 4t + C$$

Given $v(0) = -3$ then

$$4(0) + C = -3 \Rightarrow C = -3$$

So velocity fcn is $v(t) = 4t - 3$

Position fcn given by

$$s(t) = \int s'(t)dt = \int v(t)\,dt = \int (4t-3)\,dt = 2t^2 - 3t + D$$

Given $s(0) = 2$

$$\Rightarrow 2(0)^2 - 3(0) + D = 2 \Rightarrow D = 2$$

So the position fcn is $s(t) = 2t^2 - 3t + 1$

For problems 8 through 15 use the Fundamental Theorem of Calculus to evaluate the given definite integrals.

8. $\displaystyle\int_{-2}^{3} (x^2 - x - 6)\, dx$

Solution: $= \left[\frac{x^3}{3} - \frac{x^2}{2} - 6x \right]_{-2}^{3}$

$= \left[\frac{(3)^3}{3} - \frac{(3)^2}{2} - 6(3) \right] - \left[\frac{(-2)^3}{3} - \frac{(-2)^2}{2} - 6(-2) \right]$

$= -\frac{125}{6}$

9. $\displaystyle\int_{0}^{2} (3x^2 + 2x)\, dx$

Solution: $= \left[3\frac{x^3}{3} + \frac{2x^2}{2} \right]_{0}^{2}$

$= \left[x^3 + x^2 \right]_0^2$

$= \left[(2)^3 + (2)^2 \right] - [0]$

$= 12$

10. $\displaystyle\int_{0}^{\pi/4} 2\cos x\, dx$

Solution: $= 2\sin(x) \Big|_0^{\pi/4}$

$= 2\left[\sin\left(\frac{\pi}{4}\right) - \sin(0) \right]$

$= 2\left(\frac{\sqrt{2}}{2} \right)$

$= \sqrt{2}$

11. $\displaystyle\int_{0}^{\ln 8} e^x\, dx$

Solution: $= e^x \Big|_0^{\ln(8)}$

$= e^{\ln(8)} - e^0$

$= 8 - 1$

$= 7$

12. $\displaystyle\int_0^{\pi} (1 - \sin x)\, dx$

Solution: $= \Big[x + \cos(x) \Big]_0^{\pi}$

$= \pi + \cos(\pi) - (0 + \cos(0))$

$= \pi - 1 - 1$

$= \pi - 2$

13. $\displaystyle\int_0^{4} x(x-2)(x-4)\, dx$

Solution: $= \displaystyle\int_0^4 (x^4 - 6x^2 + 8x)\, dx$

$= \Big[\frac{x^4}{4} - \frac{6x^3}{3} + \frac{8x^2}{2} \Big]_0^4$

$= \frac{(4)^4}{4} - 2(4)^3 + 4(4)^2$

$= 0$

14. $\displaystyle\int_4^{9} \frac{x - \sqrt{x}}{x^3}\, dx$

Solution: $= \displaystyle\int_4^9 (x^{-2} - x^{-3/2})\, dx$

$= \Big[\frac{x^{-1}}{-1} - \frac{x^{-3/2}}{-\frac{3}{2}} \Big]_4^9$

$= \frac{2 - 3\sqrt{x}}{3x^{3/2}} \Big|_4^9 = \frac{2 - 3\sqrt{9}}{3\sqrt{9^3}} - \frac{2 - 3\sqrt{4}}{3(4)^{3/2}} = \frac{13}{162}$

15. $\displaystyle\int_0^{1/2} \frac{1}{\sqrt{1 - x^2}}\, dx$

Solution: $= \Big[\sin^{-1}(x) \Big]_0^{1/2}$

$= \sin^{-1}\left(\frac{1}{2}\right) - \sin^{-1}(0)$

$= \frac{\pi}{6} - 0$

16. Given $\int_2^6 f(x)\,dx = 10$ and $\int_2^6 g(x)\,dx = 2$, apply properties of integrals to evaluate

(a) $\int_2^6 \big(3g(x) - f(x)\big)\,dx$.

Solution:

$$= 3\int_2^6 g(x) - \int_2^6 f(x)\,dx$$

$$= 3(2) - 10$$

$$= 6$$

(b) $\int_2^3 \big(f(x) - g(x)\big)\,dx - \int_6^3 \big(f(x) - g(x)\big)\,dx$.

Solution:

$$= \int_2^3 (f(x) - g(x)) + \int_3^6 (f(x) - g(x))\,dx$$

$$= \int_2^6 (f(x) - g(x))\,dx$$

$$= \int_2^6 f(x)\,dx - \int_2^6 g(x)\,dx$$

$$= 10 - 2$$

$$= 8$$