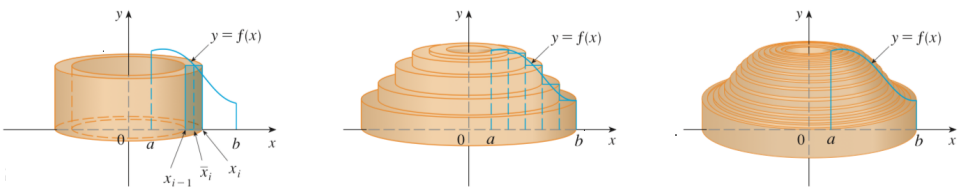
Lecture # 04 : Volume by Shells

Date: Wed. 9/19/18

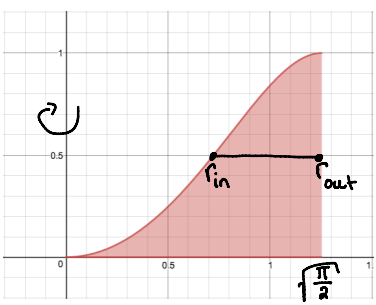
Previously, we found the volume of a solid of revolution using planes.

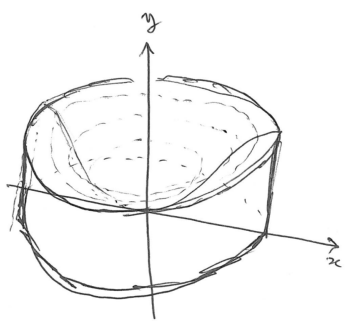
We can also find the volume using cylindrical shells



Question: Why do we need a different method?

Ex) Let R be the region bdd by $f(x) = \sin(x^2)$





We can try using washers

we would need $y = \sin(x^2)$ in form $x = \ldots$ to get r_{in}. That will be difficult to find

Instead we'll use slices that are cylinders.

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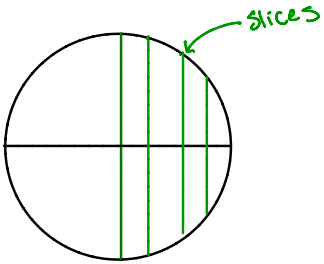
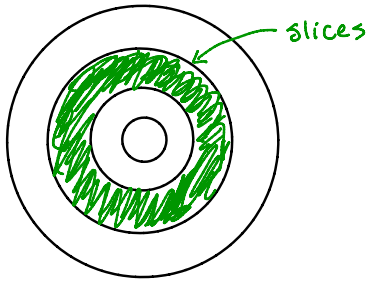
In 2D:

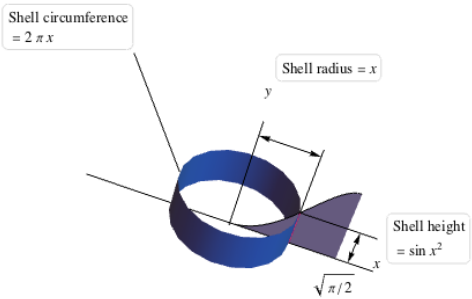


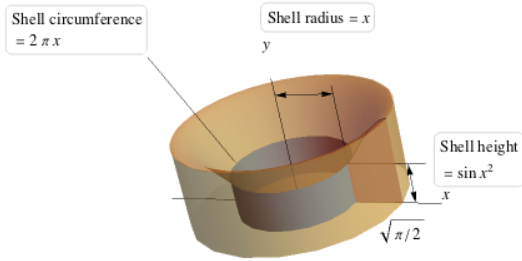
Straight line (planar) slices



Circular slices like rings of a tree.

Slicing are "sine bowl" using a cylinder gives





Which essentially splits the bowl into infinely many concentric, nested shells

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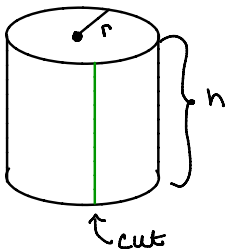
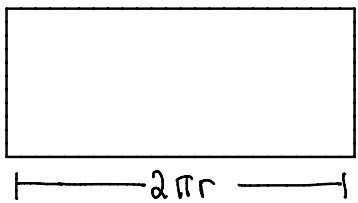
Question: Can we calculate the area of this shell?

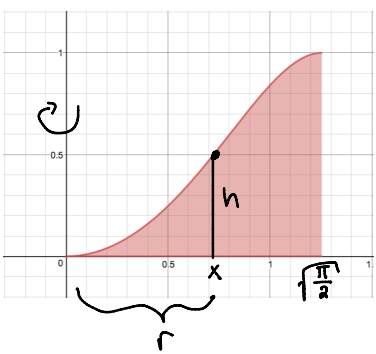


$\Longrightarrow$



area $= 2\pi r h$

So, we need to know 2 things for each shell the radius & the height



The vertical line segment generates the shell

$r = x, \quad h = \sin(x^2)$

$\Rightarrow$ area $= 2\pi r h$

$= 2\pi x \sin(x^2)$

Since x is keeping track of the shells & the range of x-values is 0 to $\sqrt{\frac{\pi}{2}}$:

$$\text{Volume} = \int_0^{\sqrt{\frac{\pi}{2}}} \underbrace{2\pi x \sin(x^2)}_{\text{area of shell}}\, dx$$

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$$\text{Volume} = \int_0^{\sqrt{\frac{\pi}{2}}} 2\pi x \sin(x^2)\, dx$$

$$= 2\pi \int_0^{\sqrt{\frac{\pi}{2}}} x \sin(x^2)\, dx$$

$$u = x^2$$

$$du = 2x\,dx \implies \frac{1}{2}du = x\,dx$$

$$x = \sqrt{\frac{\pi}{2}} \implies u = \left(\sqrt{\frac{\pi}{2}}\right)^2 = \frac{\pi}{2}$$

$$x = 0 \implies u = (0)^2 = 0$$

$$\implies V = 2\pi \int_0^{\pi/2} \sin(u)\left(\frac{1}{2}du\right)$$

$$= \pi \int_0^{\pi/2} \sin(u)\,du$$

$$= -\pi \cos(u)\Big|_0^{\pi/2}$$

$$= -\pi\left[\cos\left(\frac{\pi}{2}\right) - \cos(0)\right] = \pi$$

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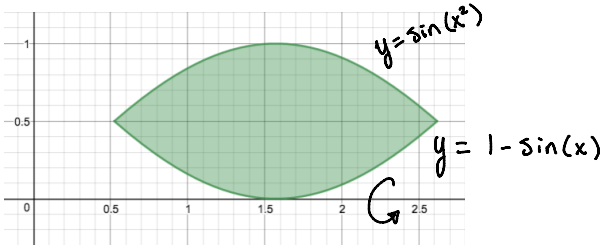
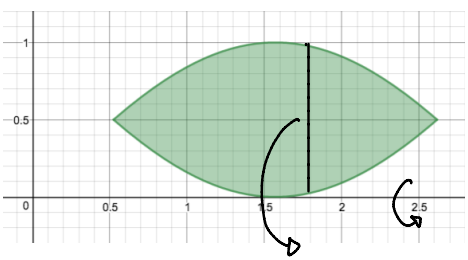
Ex) Find the volume of the solid formed when the region bdd by $y = \sin(x)$ & $y = 1 - \sin(x)$ btwn $x = \frac{\pi}{6}$ & $x = \frac{5\pi}{6}$ is revolved about the x-axis



Which type of slice should we use?

Vertical washers?



Horizontal shells