1. Find the average value of the function $f(t) = e^{\sin(t)}\cos(t)$ on $[0, \frac{\pi}{2}]$.

Solution:

Average value of f(x) on [a,b] is

$$\frac{1}{b-a}\int_a^b f(x)\,dx$$

$$\Rightarrow \frac{1}{\frac{\pi}{2}-0}\int_0^{\pi/2} e^{\sin(t)}\cos(t)\,dt$$

$$= \frac{2}{\pi}\int_0^{\pi/2} e^{\sin(t)}(\cos(t)\,dt)$$

Using u-sub:

Let $u = \sin(t)$

$du = \cos(t)\,dt$

Bounds:

$x = \frac{\pi}{2}$ $\quad u(\frac{\pi}{2}) = \sin(\frac{\pi}{2}) = 1$

$x = 0$ $\quad u(0) = \sin(0) = 0$

$$\frac{2}{\pi}\int_0^{\pi/2} e^{\sin(t)}(\cos(t)\,dt) = \frac{2}{\pi}\int_0^1 e^u\,du$$

$$= \frac{2}{\pi}\, e^u\Big|_0^1$$

$$= \frac{2}{\pi}\left[e^1 - e^0\right] = \frac{2e}{\pi}$$

For Problems 2 to 4: Write up the solutions to these problems using the GASCAP format (posted in the document on Canvas)

2. A spring has a natural length of 20 cm. If a 25-N force is required to keep it stretched to a length of 30 cm, how much work is required to stretch it from 20 cm to 25 cm? Round your final result to 3 decimal places.

Solution:

Given Spring 20 cm

Force $25\ N = 25\ kg \cdot m/s^2$

to stretch to 30 cm

i.e. $30cm - 20cm = 10cm = 0.1m$

Asked Work to stretch spring

$25cm - 20cm = 5cm = 0.05\ m$

Compute

Hooke's Law: $F(x) = kx$

Find Spring Constant k:

$25 \stackrel{?}{=} k(0.1m) \implies k = 250\ N/m$

So force to stretch spring is $F(x) = 250x$

Work needed to stretch spring 0.05m

$$W = \int_0^{0.05} F(x)\,dx$$

$$= \int_0^{0.05} 250x\,dx$$

$$= 250\frac{x^2}{2}\Big|_0^{0.05}$$

$$= 125x^2\Big|_0^{0.05} = 125(0.05)^2 - 0 = 0.3125\ J$$

Answer

It requires 0.3125 J to stretch the spring 0.05 m.

3. A steady wind blows a kite due west. The kite's height above ground from horizontal position $x = 0$ to $x = 80$ ft is given by

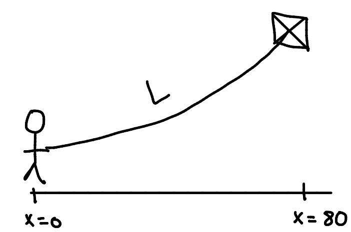
$$y = 150 - \frac{1}{40}(x - 50)^2$$

Set up the integral needed to find the distance traveled by the kite. Use technology to evaluate this integral. Round the final result to two decimal places.

Solution:

Given: Kite's height from $x=0$ to $x=80$

$$y = 150 - \frac{1}{40}(x-50)^2$$



Asked: distance kite travels

$\Rightarrow$ arc length!

Compute:

arc length: $L = \int_a^b \left[1 + (f'(x))^2\right]^{1/2} dx$

$f(x) = 150 - \frac{1}{40}(x-50)^2 \Rightarrow f'(x) = -\frac{1}{20}(x-50)$

So $L = \int_0^{80} \left[1 + \left(-\frac{1}{20}(x-50)\right)^2\right]^{1/2} dx$

Using mathematica: $L \approx 122.776$ ft

Answer

The distance traveled by the kite is 122.78 ft

4. The velocity v of blood that flows in a blood vessel with radius R and length l at a distance r from the central axis is

$$v(r) = \frac{P}{4\eta l}(R^2 - r^2)$$

where P is the pressure difference between the ends of the vessel and η is the viscosity of the blood. Find the average velocity (with respect to r) over the interval $0 < r < R$.

Solution:

Given $v(r) \to$ Velocity of blood

Asked Average velocity $\Longrightarrow$ Average value of $v(r)$

Compute Average value given by $\frac{1}{b-a}\int_a^b f(x)\,dx$

Average velocity:

$$V_{avg} = \frac{1}{R-0}\int_0^R \frac{P}{4\eta l}(R^2 - r^2)\,dr$$

$$= \frac{1}{R}\left(\frac{P}{4\eta l}\right)\int_0^R (R^2 - r^2)\,dr$$

$$= \frac{P}{4\eta l R}\left[R^2 r - \frac{r^3}{3}\right]_0^R$$

$$= \frac{P}{4\eta l R}\left[\left(R^2(R) - \frac{(R)^3}{3}\right) - 0\right]$$

$$= \frac{P}{4\eta l R}\left[R^3 - \frac{R^3}{3}\right]$$

$$= \frac{P}{4\eta l R}\left[\frac{2}{3}R^3\right]$$

$$= \frac{R^2}{6\eta l}$$

Answer: The average velocity of blood flow is $\frac{R^2}{6\eta l}$