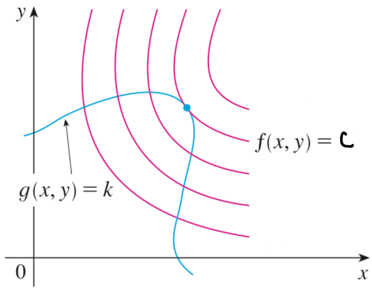
Lecture # 12: Lagrange Multipliers

Date: Thu. 10/25/18

Often it is necessary to find extrema of a fcn subject to some constraint.

We do this for multivariable fcns via the Lagrange multiplier method



Idea: want to find largest value of c s.t the level curve $f(x,y) = c$ intersects $g(x,y) = k$ (the constraint eqn)

This happens when the two curves touch each other.

i.e. they have a common tangent line.

i.e. normal lines @ (x_0, y_0) are the same

i.e. Gradient vectors are parallel @ (x_0, y_0)

i.e. Gradient vectors are scalar multiples of each other

i.e. $\nabla f(x_0, y_0) = \lambda \nabla g(x_0, y_0)$ for some scalar λ

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Thm (Lagrange)

Let f & g both have continuous first partial deriv.s s.t. f has an extreme value at a pt (x_0, y_0) on the smooth constraint curve $g(x,y) = c$.

If $\nabla g(x_0, y_0) \neq \vec{0}$ then there is a real number λ s.t.

$$\nabla f(x_0, y_0) = \lambda \nabla g(x_0, y_0)$$

where λ is known as a Lagrange multiplier

Method of Lagrange Multipliers

METHOD OF LAGRANGE MULTIPLIERS

Let f and g satisfy the hypothesis of Lagrange's Theorem, and let f have a minimum or maximum subject to the constraint $g(x, y) = c$. To find the minimum or maximum of f, use the following steps.

1. Simultaneously solve the equations $\nabla f(x, y) = \lambda \nabla g(x, y)$ and $g(x, y) = c$ by solving the following system of equations.

$$f_x(x, y) = \lambda g_x(x, y)$$
$$f_y(x, y) = \lambda g_y(x, y)$$
$$g(x, y) = c$$

2. Evaluate f at each solution point obtained in the first step. The largest value yields the maximum of f subject to the constraint $g(x, y) = c$, and the smallest value yields the minimum of f subject to the constraint $g(x, y) = c$.