Name: ______________________ Sort #: ________

Worksheet 20

Divergence Theorem

MATH 2210, Fall 2018

1. If $\mathbf{F}$ is an electrostatic field then the scalar function $\operatorname{div}\mathbf{F}$ is proportional to the charge density at a point (x, y, z). Consider the field

$$\mathbf{F}(x, y, z) = \frac{\kappa\langle x, y, 0\rangle}{x^2 + y^2}$$

where κ is a constant. Show that $\operatorname{div}\mathbf{F} = 0$ if $x^2 + y^2 > 0$. What can you conclude about the location of the electrical charges responsible for this field?

2. For the vector function

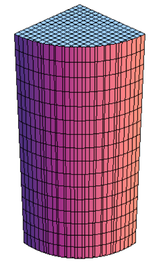
$$\mathbf{F}(x, y, z) = \langle x^3 - x^2, y^3 + xy, z^3 + xz \rangle.$$

use the Divergence Theorem to evaluate the flux integral $\iint_S \mathbf{F} \cdot d\mathbf{S}$ where S is the sphere $x^2 + y^2 + z^2 = 1$.

3. Consider the vector function

$$\mathbf{F} = \langle x^2y^2, \sin(xz), -xy^2z \rangle$$

and let E be the solid region in the first octant bounded by the cylinder $x^2 + y^2 = 1$, the plane $z = 3$, and the coordinate planes.



Compute the outward flux of $\mathbf{F}$ across the S, the boundary of the solid region E.

4. E is the solid region in the first octant that lies inside the the sphere $x^2+y^2+z^2=4$ and above the cone $z=\sqrt{x^2+y^2}$. The closed surface S is the boundary of E. *Use the divergence theorem* to calculate the outward oriented flux

$$\iint_S \langle y^2+3x, 2x^2-2y, 3z^2-z\rangle \cdot d\mathbf{S}.$$

The graph shows a cross section of E.