

Worksheet 19

Surface Integrals & Stokes' Theorem

MATH 2210, Fall 2018

1. Give a parametric description of the form $\mathbf{r}(u, v) = \langle x(u, v), y(u, v), z(u, v) \rangle$ for the following surfaces. Specify the required rectangle in the uv -plane.

(a) The cap of the sphere $x^2 + y^2 + z^2 = 16$ for $\sqrt{8} \leq z \leq 4$.

(b) The cylinder $y^2 + z^2 = 36$, for $0 \leq x \leq 9$.

2. Find the area of the surface S that lies in the plane $z = 12 - 4x - 3y$ directly above the region R bounded by the ellipse $\frac{x^2}{4} + y^2 = 1$.

3. In the following problem the surface S is the part of the paraboloid $z = 4 - x^2 - y^2$ that lies above the xy -plane, the boundary of S is the circle $x^2 + y^2 = 4$ in the xy -plane, and $\mathbf{F} = \langle y, -x, xy \rangle$.

(a) Directly compute $\iint_S (\nabla \times \mathbf{F}) \cdot d\mathbf{S}$.

- (b) Compute $\iint_S (\nabla \times \mathbf{F}) \cdot d\mathbf{S}$ using Stoke's Theorem.

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4. Use Stokes' Theorem to evaluate $\oint_C \mathbf{F} \cdot d\mathbf{r}$ where C is the triangle with vertices $(2, 0, 0)$, $(0, 2, 0)$, and $(0, 0, 2)$ is oriented counterclockwise as viewed from above and $\mathbf{F} = \langle y^2, z^2, x^2 \rangle$.