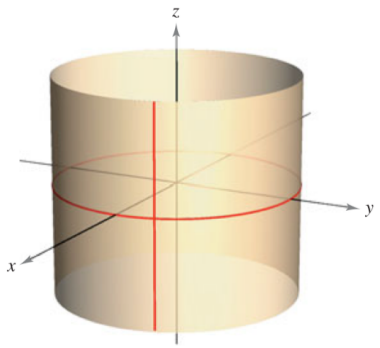
Lecture # 04 : Cylinders & Quadric surfaces

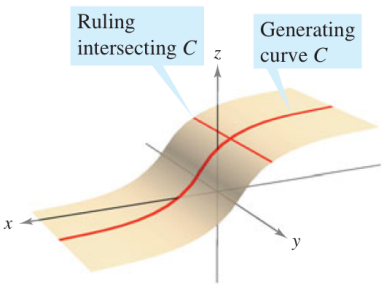
Date: Thu. 9/20/18

Cylinders



Right circular cylinder:
$x^2 + y^2 = a^2$

Rulings are parallel to z-axis.
Figure 11.56



Cylinder: Rulings intersect C and are parallel to the given line.
Figure 11.57

Def Let C be a curve in a plane & let L be a line not in a parallel plane. The set of all lines parallel to L & intersecting C is called a cylinder.

C is called the generating curve (or directrix) of the cylinder,

& the parallel lines are called rulings.

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Def

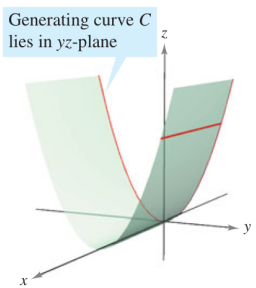
EQUATIONS OF CYLINDERS

The equation of a cylinder whose rulings are parallel to one of the coordinate axes contains only the variables corresponding to the other two axes.

Ex] Sketching a cylinder.

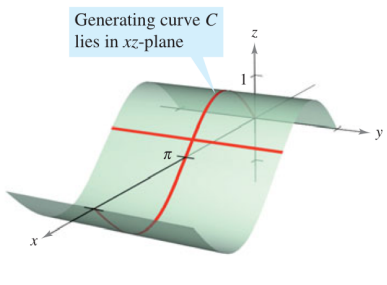
$z = y^2$

$z = \sin(x), \quad x \in [0, 2\pi]$



Cylinder: $z = y^2$

(a) Rulings are parallel to x-axis.



Cylinder: $z = \sin x$

(b) Rulings are parallel to y-axis.

Quadric Surfaces

QUADRIC SURFACE

The equation of a **quadric surface** in space is a second-degree equation in three variables. The **general form** of the equation is

$$Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Gx + Hy + Iz + J = 0.$$

There are six basic types of quadric surfaces: **ellipsoid, hyperboloid of one sheet, hyperboloid of two sheets, elliptic cone, elliptic paraboloid, and hyperbolic paraboloid.**

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Sketching Quadric Surfaces

Ex) Classify & sketch surface given by

$$4x^2 - 3y^2 + 12z^2 + 12 = 0$$

Write eqn is std. form

$$4x^2 - 3y^2 + 12z^2 + 12 = 0$$

$$\frac{x^2}{-3} + \frac{y^2}{4} - z^2 - 1 = 0$$

$$\Rightarrow \frac{y^2}{4} - \frac{x^2}{3} - \frac{z^2}{1} = 1 \qquad \text{(std. form)}$$

From table: hyperboloid of 2 sheets w/ general form

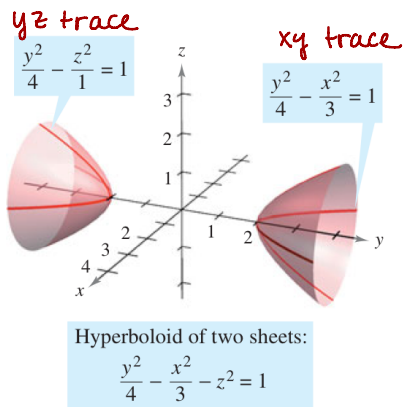
$$\frac{z^2}{c^2} - \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Find traces:

xy trace ($z = 0$): $\frac{y^2}{4} - \frac{x^2}{3} = 1$

xz trace ($y = 0$): $\frac{x^2}{3} + \frac{z^2}{1} = -1$

yz trace ($x = 0$): $\frac{y^2}{4} - \frac{z^2}{1} = 1$



Hyperboloid of two sheets:

$$\frac{y^2}{4} - \frac{x^2}{3} - z^2 = 1$$