1. Find the area of the surface given by $f(x,y) = \ln|\sec(x)|$ over the region

$$R = \{(x,y) : 0 \le x \le \frac{\pi}{4},\ 0 \le y \le \tan x\}$$

Solution: Since $f(x,y) = \ln|\sec x|$ then

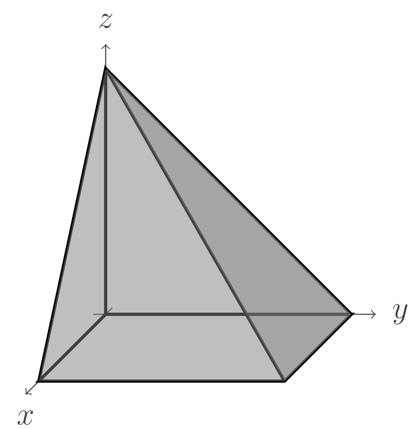
$$\begin{aligned} f_x &= \tan x \\ f_y &= 0 \\ \sqrt{1+(f_x)^2+(f_y)^2} &= \sqrt{1+(\tan x)^2+(0)^2} = \sqrt{1+\tan^2 x} \end{aligned}$$

and so the surface area is

$$\begin{aligned} S &= \iint_R \sqrt{1+(f_x)^2+(f_y)^2}\,dA \\ &= \int_0^{\frac{\pi}{4}} \int_0^{\tan x} \sec x\,dy\,dx \\ &= \int_0^{\frac{\pi}{4}} \sec x \tan x\,dx \\ &= \sec x\Big|_0^{\frac{\pi}{4}} \\ &= \sec\left(\frac{\pi}{4}\right) - \sec(0) \\ &= \sqrt{2} - 1 \end{aligned}$$

In the problems 2 to 4, E is the pyramid in the first octant bounded by the planes $2x+z=4$ and $y+z=4$.

2. Compute the volume of E using triple integral(s) in the order $dx\,dy\,dz$.



Solution:

$$\text{Volume}(E) = \int_0^4 \int_0^{4-z} \int_0^{2-\frac{z}{2}} dx\,dy\,dz$$

3. Compute the volume of E using triple integral(s) in the order $dz\,dy\,dx$.

Solution:

$$\text{Volume}(E) = \int_0^2 \int_0^{2x} \int_0^{4-2x} dz\,dy\,dx + \int_0^2 \int_{2x}^{4} \int_0^{4-y} dz\,dy\,dx$$

4. Compute the volume of E using triple integral(s) in the order $dy\,dz\,dx$.

Solution:

$$\begin{aligned}
\text{Volume}(E) &= \int_0^2 \int_0^{4-2x} \int_0^{4-z} dy\,dz\,dx \\
&= \int_0^2 \int_0^{4-2x} (4-z)\,dz\,dx \\
&= \int_0^2 (4z - \frac{1}{2}z^2)\Big|_0^{4-2x}\,dx \\
&= \int_0^2 (8-2x^2)\,dx \\
&= (8x - \frac{2}{3}x^3)\Big|_0^2 = 16 - \frac{16}{3} = 16\,(1-\frac{1}{3}) = \frac{32}{3}.
\end{aligned}$$