1. Sketch the level curves of the function $z = x + y$ for the values $c = -1, 0, 2, 4$.

Solution:

To determine the level curves at the given values we have

$$-1 = x + y \implies y = -1 - x$$
$$0 = x + y \implies y = -x$$
$$2 = x + y \implies y = 2 - x$$
$$4 = x + y \implies y = 4 - x$$

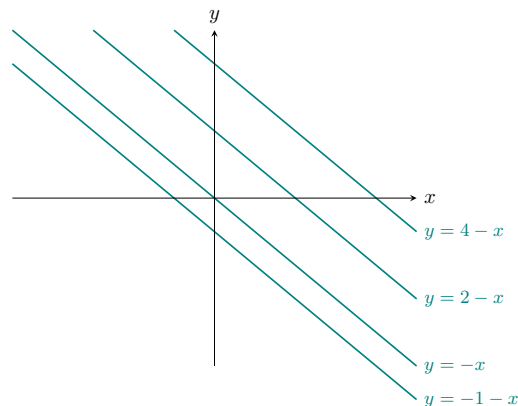We can see that each of these level curves are a line.

Figure 1: Level curves of $z = x + y$

2. Given $f(x, y, z) = \dfrac{2xz}{x + y}$, find $\dfrac{\partial f}{\partial x}$, $\dfrac{\partial f}{\partial y}$, and $\dfrac{\partial f}{\partial z}$.

Solution:

$$\frac{\partial f}{\partial x} = \frac{2yz}{(x + y)^2}$$

$$\frac{\partial f}{\partial y} = -\frac{2xz}{(x + y)^2}$$

$$\frac{\partial f}{\partial z} = \frac{2x}{x + y}$$

3. Find the four second partial derivatives of $f(x, y) = 2xe^y - 3ye^{-x}$.

Solution:

$$f_x = \frac{\partial f}{\partial x} = 3e^{-x}y + 2e^y \qquad\qquad f_y = \frac{\partial f}{\partial y} = 2xe^y - 3e^{-x}$$

$$f_{xx} = \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}f\right) = -3e^{-x}y \qquad\qquad f_{yy} = \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}f\right) = 2xe^y$$

$$f_{xy} = \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}f\right) = 3e^{-x} + 2e^y \qquad\qquad f_{yx} = \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}f\right) = 3e^{-x} + 2e^y$$

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4. Show that the mixed partial derivaitves f_{xyy}, f_{yxy} and f_{yyx} are equal for the function

$$f(x, y, z) = x^2 - 3xy + 4yz + z^3$$

Solution:

$$f_x = 2x - 3y$$
$$f_{xy} = -3$$
$$f_{xyy} = 0$$

$$f_y = -3x + 4z$$
$$f_{yy} = 0$$
$$f_{yyx} = 0$$
$$f_{yxy} = 0$$

and so we see that $f_{xyy} = f_{yxy} = f_{yyx} = 0$.