1. Compute $f'(1)$, where $f(t) = \mathbf{u}(t) \cdot \boldsymbol{v}(t)$, $\mathbf{u}(1) = \langle 1, 2, -1 \rangle$, $\mathbf{u}'(1) = \langle 1, 2, -3 \rangle$, and $\mathbf{v}(t) = \langle t, t^2, t^3 \rangle$.

**Solution:**

Using the product rule

$$f'(t) = u'(t) \cdot v(t) + u(t) \cdot v'(t)$$

Since $v(t) = \langle t, t^2, t^3 \rangle$

$$= \langle 1, 2t, 3t^2 \rangle$$

So

$$f'(1) = u'(1) \cdot v(1) + u(1) \cdot v'(1)$$

$$= \langle 1, 2, -3 \rangle \cdot \langle 1, 1, 1 \rangle + \langle 1, 2, -1 \rangle \cdot \langle 1, 2, 3 \rangle$$

$$= (1)(1) + (2)(1) + (-3)(1) + (1)(1) + (2)(2) + (-1)(3)$$

$$= 2$$

2. Given $\mathbf{r}(t) = 4\cos(t)\mathbf{i} + 4\sin(t)\mathbf{j}$ find the following

(a) $\mathbf{r}'(t)$

**Solution:**

$$r'(t) = 4\frac{d}{dt}[\cos(t)]\hat{i} + 4\frac{d}{dt}[\sin(t)]\hat{j}$$

$$= -4\sin(t)\hat{i} + 4\cos(t)\hat{j}$$

$$= \langle -4\sin(t), 4\cos(t)\rangle$$

(b) $\mathbf{r}''(t)$

**Solution:**

$$r''(t) = -4\frac{d}{dt}[\sin(t)]\hat{i} + 4\frac{d}{dt}[\cos(t)]\hat{j}$$

$$= -4\cos(t)\hat{i} - 4\sin(t)\hat{j}$$

$$= \langle -4\cos(t), -4\sin(t)\hat{j}\rangle$$

(c) $\mathbf{r}'(t) \cdot \mathbf{r}''(t)$

**Solution:**

$$r'(t)\cdot r''(t) = \langle -4\sin(t), 4\cos(t)\rangle \cdot \langle -4\cos(t), -4\sin(t)\rangle$$

$$= (-4\sin(t))(-4\cos(t)) + (4\cos(t))(-4\sin(t))$$

$$= 16\sin(t)\cos(t) - 16\sin(t)\cos(t)$$

$$= 0$$

3. Evaluate $\int_0^1 (8t\mathbf{i} + t\mathbf{j} - \mathbf{k})\,dt$

   **Solution:**

$$\int_0^1 (8t\,\hat{i} + t\,\hat{j} - k)\,dt$$

$$= \left(\int_0^1 8t\,dt\right)\hat{i} + \left(\int_0^1 t\,dt\right)\hat{j} - \left(\int_0^1 1\,dt\right)\hat{k}$$

$$= 8\left[\frac{t^2}{2}\right]_0^1 \hat{i} + \left[\frac{t^2}{2}\right]_0^1 \hat{j} - \left[t\right]_0^1 \hat{k}$$

$$= 8\left(\frac{1}{2}\right)\hat{i} + \frac{1}{2}\hat{j} - \hat{k}$$

$$= 4\hat{i} + \frac{1}{2}\hat{j} - \hat{k}$$

4. The position function vector $\mathbf{r}(t) = t^2\mathbf{i} + \mathbf{t}\mathbf{j} + 2t^{3/2}\mathbf{k}$ describes the path of an object moving in space. Find the velocity, speed, and acceleration of the object.

   **Solution:**

Velocity: $v(t) = r'(t)$

$$= 2t\,\hat{i} + \hat{j} + 2\left(\frac{3}{2}\right)t^{1/2}\,\hat{k}$$

$$= 2t\,\hat{i} + \hat{j} + 3t^{1/2}\,\hat{k}$$

Speed: $\|v(t)\| = \left((2t)^2 + (1)^2 + \left(3t^{1/2}\right)^2\right)^{1/2}$

$$= \left(4t^2 + 9t + 1\right)^{1/2}$$

acceleration: $a(t) = v'(t)$

$$= 2\hat{i} + \frac{3}{2}t^{-1/2}\,\hat{k}$$