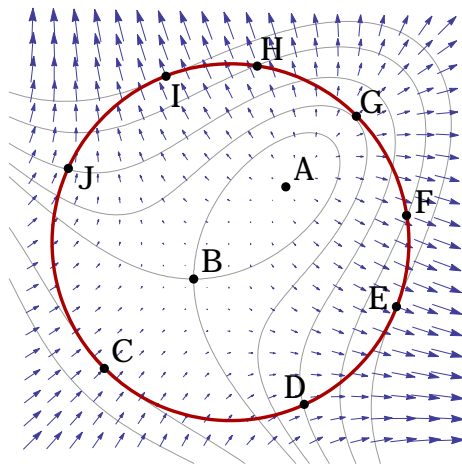


**Worksheet 12**  
**Lagrange Multipliers**

MATH 2210, Fall 2018

1. This figure shows several level curves,  $f(x, y) = k$ , and the gradient vector field  $\nabla f$ . The circle is the graph of a constraint equation  $g(x, y) = 0$ . Locations of ten points, A, B, C, D, E, F, G, H, I and J are marked.



- (a) List the points on the circle where  $\nabla f$  is perpendicular to the graph of the constraint equation  $g(x, y) = 0$ .
- (b) List the points where  $f(x, y)$  has an absolute minimum, subject to the constraint  $g(x, y) = 0$ . If there are no such points, write DNE.
- (c) List the points where  $f(x, y)$  has an absolute maximum, subject to the constraint  $g(x, y) = 0$ . If there are no such points, write DNE.

2. Find the extreme value of the function  $f(x, y, z) = xyz$  subject to the constraint  $3x + 3y + 2z = 12$ . Is the extreme value a maximum or a minimum?

3. Find the extreme value of  $f(x, y, z) = x^2 + y^2 + z^2$  subject to the constraint  $x + 2y - z = 12$ .  
Is the extreme value a maximum or a minimum?

4. Find the extreme value of  $f(x, y) = x^2 + y^2$  on the hyperbola  $4x^2 - y^2 = 4$ . Is the extreme value a maximum or minimum?