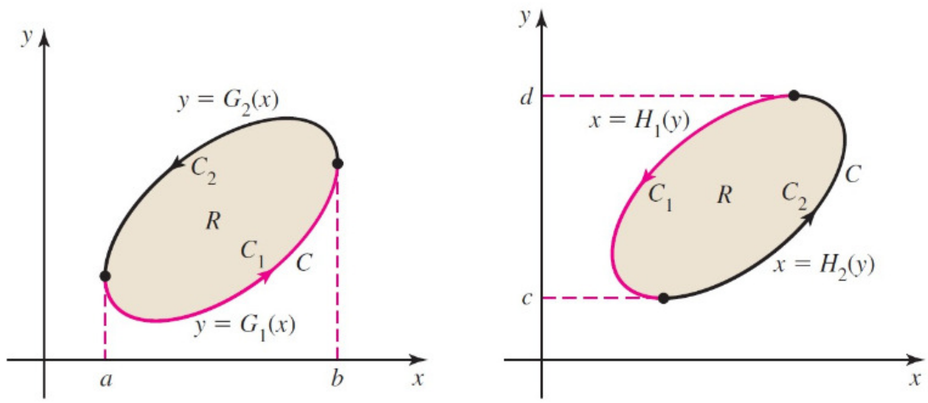
Name: ______________________________ Sort #: ________

Worksheet 18

Green's Theorem & Curl and Divergence

MATH 2210, Fall 2018

1. Consider the situation below where you have a simple, closed, and smooth curve C that is oriented counter clockwise and defines the boundary of a simply connected region R. Let $\boldsymbol{F} = \langle f, g \rangle$ be a vector field over R that has continuous partial derivatives.



(a) Use the figure on the left and the Fundamental Theorem of Calculus to show that

$$\iint_R \frac{\partial f}{\partial y}\, \mathrm{d}A = -\oint_C f\, dx.$$

(b) Use the figure on the right and the Fundamental Theorem of Calculus to show that

$$\iint_R \frac{\partial g}{\partial x}\ \mathrm{d}A = \oint_C g dy.$$

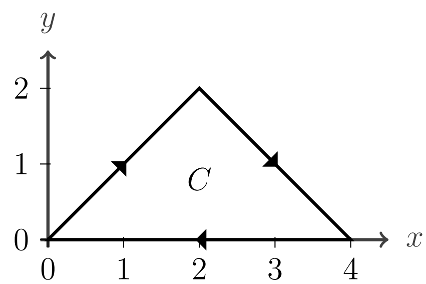
(c) Combine the results from part (a) and (b) to show that

$$\iint_R \left(\frac{\partial g}{\partial x} - \frac{\partial f}{\partial y}\right)\ \mathrm{d}A = \oint_C f\ \mathrm{d}x + g\ \mathrm{d}y.$$

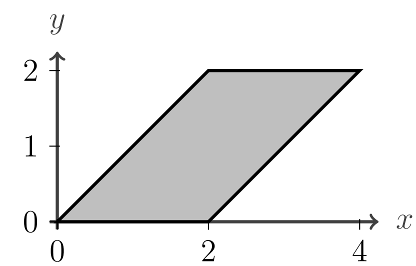
2. A mass moves in the xy-plane while under the influence of a force. The work done by the force is

$$\mathrm{W} = \int_C (x^2 - y^2)\,dx + (1 + 4xy)\,dy.$$

The negatively oriented curve C that the mass travels along is a triangle formed by the lines $y = 0$, $y - x = 0$, and $y + x = 4$. C is a simple closed curve, so use Green's theorem to compute W.



3. Evaluate the flux integral
$$\oint_C \mathbf{F} \cdot \mathbf{n} \mathrm{d}r,$$
where $\mathbf{F} = \langle y^2 - 2xy, x^2 + 2xy \rangle$ and C is the counterclockwise oriented boundary of the parallelogram shown here.



4. Decide if the following expressions are defined. If they are defined, state whether the result is a scalar of or a vector. Assume that $\mathbf{F}$ is a sufficiently differentiable vector field in $\mathbb{R}^2$ or $\mathbb{R}^3$ and φ is a differentiable scalar valued function.

   (a) $\nabla \cdot (\nabla\varphi)$

   (b) $\nabla \times (\nabla \cdot \mathbf{F})$

   (c) $\nabla \times (\nabla\varphi)$

5. (Bonus 10 Points) The vector function $\operatorname{curl}\mathbf{F}$ measures the rotation of the vector field $\mathbf{F}$ at a point (x, y, z). If $\operatorname{curl}\mathbf{F} = \mathbf{0}$ everywhere, then $\mathbf{F}$ is said to be irrotational. Show that conservative vector fields, $\mathbf{F} = \nabla f = \langle f_x, f_y, f_z\rangle$, are irrotational if $f(x, y, z)$ is a smooth function with continuous derivatives. (Write your solution on a separate page and attach. Be sure to clearly label the problem.)