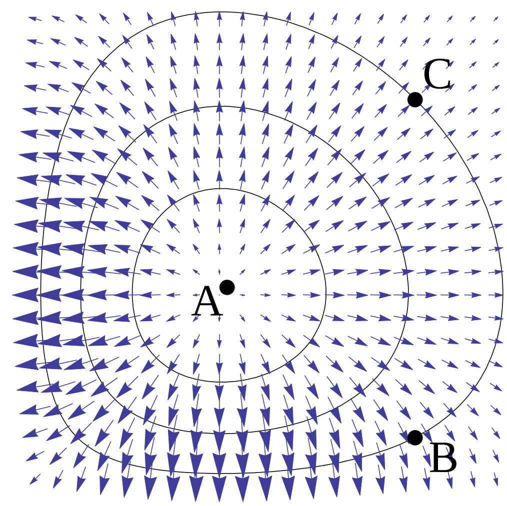
Name: ______________________ Sort #: ________

Worksheet 17
Line Integrals

MATH 2210, Fall 2018

1. This figure shows the vector function $\mathbf{F}(x, y) = \nabla f(x, y)$, three contour (or level) curves of the potential function $f(x, y)$, and three points (A, B, and C). Place the appropriate symbol, <, =, or >, in each box.



A. C_1 is the line segment from A to B: $\displaystyle\int_{C_1} \mathbf{F} \cdot d\mathbf{r}$ $\boxed{}$ 0.

B. C_2 is the line segment from A to C: $\displaystyle\int_{C_2} \mathbf{F} \cdot d\mathbf{r}$ $\boxed{}$ 0.

C. C_3 is any path from B to C: $\displaystyle\int_{C_3} \mathbf{F} \cdot d\mathbf{r}$ $\boxed{}$ 0.

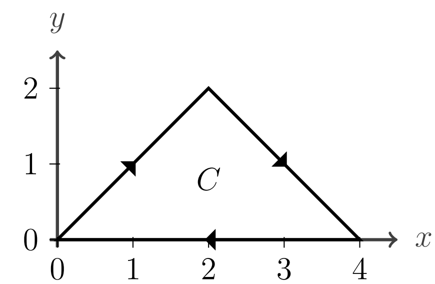
2. The closed curve C is the intersection of the hyperboloid $x^2 + y^2 - z^2 = 3$ with the plane $z = 1$. Compute the integral
$$\oint_C (x - z)\, ds.$$

3. In these problems C consists of the arc of the circle $x^2 + y^2 = 4$ from $(2, 0)$ counterclockwise to $(0, 2)$.

   (a) Give a parametrization for C. Specify the domain for the parametrization.

   (b) A mass moves along C while under the influence of the force field $\mathbf{F}(x, y) = \langle y, 1 \rangle$. Determine the amount of work done by $\mathbf{F}$ on the mass by computing an appropriate line integral.

4. A mass moves in the xy-plane while under the influence of a force. The work done by the force is
$$\mathrm{W} = \int_C (x^2 - y^2)\,dx + (1 + 4xy)\,dy.$$
The positively oriented curve C that the mass travels along is a triangle formed by the lines $y = 0$, $y - x = 0$, and $y + x = 4$. Compute the work done by this force by breaking this integral into three pieces.



5. A mass moves along a curve C, the portion of the parabola $y = x^2$ from $(-1, 1)$ to $(1, 1)$, while under the influence of the force field $\mathbf{F}(x, y) = \langle y, -x \rangle$. Determine the amount of work done by $\mathbf{F}$ on the mass by computing the line integral

$$\text{Work} = \int_C y\,dx - x\,dy.$$