Identify and sketch the given quadric surfaces. State the surface type, it's general form, as well as it's xy, yz, and xz traces.

1. $x^2 + \dfrac{y^2}{4} + z^2 = 1$

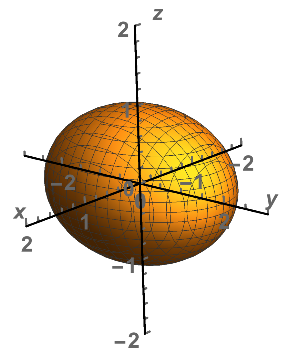
Solution: We rewrite the equation as $\dfrac{x^2}{1^2} + \dfrac{y^2}{2^2} + \dfrac{z^2}{1^2} = 1$

which has the general form of an ellipsoid:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

We have the following traces

$$\begin{aligned} xy\text{-trace}: &\quad x^2 + \frac{y^2}{2^2} = 1 \\ yz\text{-trace}: &\quad \frac{y^2}{2^2} + z^2 = 1 \\ xz\text{-trace}: &\quad x^2 + z^2 = 1 \end{aligned}$$



2. $16x^2 - y^2 + 16z^2 = 4$

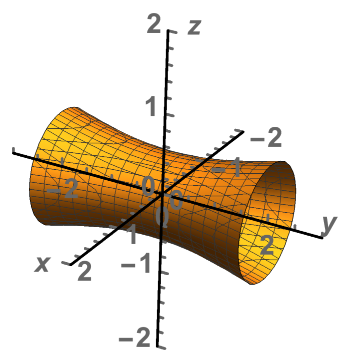
Solution: We rewrite the equation as $\dfrac{x^2}{(\frac{1}{2})^2} - \dfrac{y^2}{2^2} + \dfrac{z^2}{(\frac{1}{2})^2} = 1$

which has the general form of a hyperboloid of one sheet:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

where the axis of the surface is the y-axis (the term with the negative coefficient). We have the following traces

$$\begin{aligned} xy\text{-trace}: &\quad \frac{x^2}{(\frac{1}{2})^2} + \frac{y^2}{2^2} = 1 \\ yz\text{-trace}: &\quad \frac{y^2}{2^2} + \frac{z^2}{(\frac{1}{2})^2} = 1 \\ xz\text{-trace}: &\quad \frac{x^2}{(\frac{1}{2})^2} + \frac{z^2}{(\frac{1}{2})^2} = 1 \end{aligned}$$



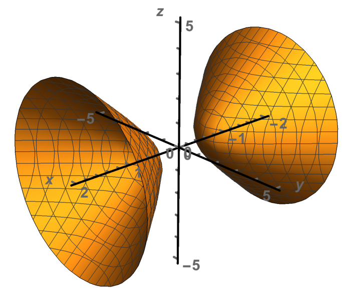
3. $4x^2 - y^2 - z^2 = 1$

Solution: We rewrite the equation as $\dfrac{x^2}{(\frac{1}{2})^2} - \dfrac{y^2}{1^2} - \dfrac{z^2}{1^2} = 1$

which has the general form of hyperboloid of two sheets:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

where the axis of the surface is the x-axis (the term with the positive coefficient). There is no trace perpendicular to this axis. We have the following traces



$$\begin{aligned} xy\text{-trace}: &\quad \frac{x^2}{(\frac{1}{2})^2} - \frac{y^2}{1^2} = 1 \\ yz\text{-trace}: &\quad \text{None} \\ xz\text{-trace}: &\quad \frac{x^2}{(\frac{1}{2})^2} - \frac{z^2}{1^2} = 1 \end{aligned}$$

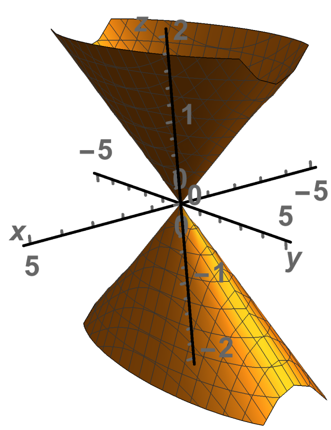
4. $z^2 = x^2 + \dfrac{y^2}{9}$

Solution: We rewrite the equation as $\dfrac{x^2}{1^2} + \dfrac{y^2}{3^2} - \dfrac{z^2}{1^2} = 0$

which has the general form of an elliptic cone:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 0$$

where the axis of the cone is the z-axis (the term with the negative coefficient). We have the following traces



$$\begin{aligned} xy\text{-trace}: &\quad \frac{x^2}{1^2} + \frac{y^2}{3^2} = 0 \\ yz\text{-trace}: &\quad \frac{y^2}{3^2} - \frac{z^2}{1^2} = 0 \\ xz\text{-trace}: &\quad \frac{x^2}{1^2} - \frac{z^2}{1^2} = 0 \end{aligned}$$