

Name: \_\_\_\_\_ Sort #: \_\_\_\_\_

## Worksheet 19

### Surface Integrals & Stokes' Theorem

MATH 2210, Fall 2018

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1. Give a parametric description of the form  $\mathbf{r}(u, v) = \langle x(u, v), y(u, v), z(u, v) \rangle$  for the following surfaces. Specify the required rectangle in the  $uv$ -plane.

(a) The cap of the sphere  $x^2 + y^2 + z^2 = 16$  for  $\sqrt{8} \leq z \leq 4$ .

(b) The cylinder  $y^2 + z^2 = 36$ , for  $0 \leq x \leq 9$ .

2. Find the area of the surface  $S$  that lies in the plane  $z = 12 - 4x - 3y$  directly above the region  $R$  bounded by the ellipse  $\frac{x^2}{4} + y^2 = 1$ .

3. In the following problem the surface  $S$  is the part of the paraboloid  $z = 4 - x^2 - y^2$  that lies above the  $xy$ -plane, the boundary of  $S$  is the circle  $x^2 + y^2 = 4$  in the  $xy$ -plane, and  $\mathbf{F} = \langle y, -x, xy \rangle$ .

(a) Directly compute  $\iint_S (\nabla \times \mathbf{F}) \cdot d\mathbf{S}$ .

(b) Compute  $\iint_S (\nabla \times \mathbf{F}) \cdot d\mathbf{S}$  using Stoke's Theorem.

4. Use Stokes' Theorem to evaluate  $\oint_C \mathbf{F} \cdot d\mathbf{r}$  where  $C$  is the triangle with vertices  $(2, 0, 0)$ ,  $(0, 2, 0)$ , and  $(0, 0, 2)$  is oriented counterclockwise as viewed from above and  $\mathbf{F} = \langle y^2, z^2, x^2 \rangle$ .