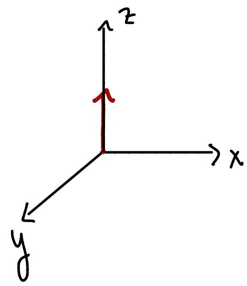
1. Find the set of parametric equations of the line that passes through the point $(2, 3, 4)$ and is parallel to the xz-plane and the yz-plane.

Solution:



To find eqn of a line we need a pt & vector $\vec{v}$ in same direction (i.e. parallel) to line.

vector parallel to both xz-plane & yz-plane occurs @ line where both of these planes intersect i.e. vector $\vec{v} = \langle 0, 0, 1 \rangle$

Parametric Eqns:

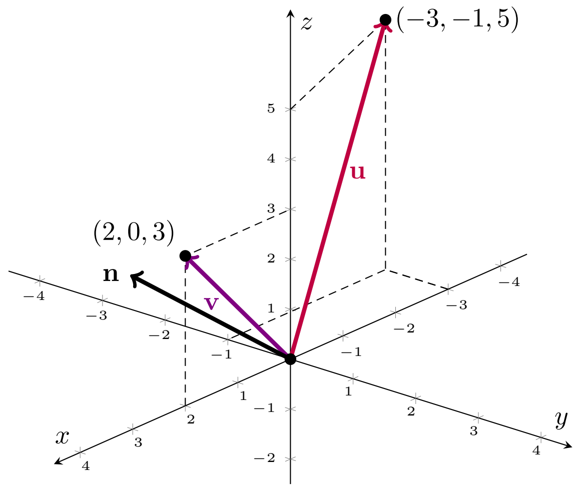
$$x = x_0 + at = 2 + 0t = 2$$

$$y = y_0 + bt = 3 + 0t = 3$$

$$z = z_0 + ct = 4 + t = 4 + t$$

2. Find the equation of the plane that passes through $(0, 0, 0)$, $(2, 0, 3)$, and $(-3, -1, 5)$.

Solution:



Find $\vec{u}$ & $\vec{v}$

$\vec{u} = \langle 2, 0, 3 \rangle$

$\vec{v} = \langle -3, 1, 5 \rangle$

To construct plane that includes both $\vec{u}$ & $\vec{v}$ we need a vector normal to both $\vec{u}$ & $\vec{v}$

$\Rightarrow$ Calculate $\vec{u} \times \vec{v}$

$$\vec{u} \times \vec{v} = \begin{vmatrix} i & j & k \\ 2 & 0 & 3 \\ -3 & 1 & 5 \end{vmatrix} = \langle -3, -19, 2 \rangle$$

So $\vec{n} = \langle -3, -19, 2 \rangle$ is a vector normal to $\vec{u}$ & $\vec{v}$

Standard form of a plane w/ $\vec{n} = \langle a, b, c \rangle$ & containing point (x_0, y_0, z_0)

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

$\Rightarrow$ plane w/ $\vec{n} = \langle -3, -19, 2 \rangle$ & containing pt $(0, 0, 0)$ is

$$-3x - 19y + 2z = 0$$

3. Find a set of parametric equations for the line of intersection of the planes defined by

$$3x + 2y - z = 7$$
$$x - 4y + 2z = 0$$

Solution:

We need to find the set of pts that satisfy both plane equations.

Solving the system:

$$2(3x + 2y - z = 7)$$
$$x - 4y + 2z = 0$$

$$\Rightarrow \quad 6x + 4y - 2z = 14$$
$$x - 4y + 2z = 0$$

$$7x = 14 \quad \Rightarrow \quad x = 2$$

Plug into ② & solve for y

$$\Rightarrow 2 - 4y + 2z = 0$$
$$4y = 2 - 2z$$
$$\Rightarrow y = \frac{1}{2} - \frac{1}{2}z$$

Let $z = t$

Then parametric eqns for line of intersection are

$$x = 2$$
$$y = \frac{1}{2} - \frac{1}{2}t$$
$$z = t$$

4. Find the point of intersection of the lines given below as well as the cosine of the angle of intersection

$$x = 4t + 2, \quad y = 3, \quad z = -t + 1$$
$$x = 2s + 2, \quad y = 2s + 3, \quad z = s + 1$$

Solution:

The point of intersection of the two lines will occur when the parametric eqns for each line are equal.

$$x: \quad 4t + 2 = 2s + 2$$
$$\Rightarrow \quad 4t = 2s$$
$$\Rightarrow \quad s = 2t$$

$$y: \quad 3 = 2s + 3$$
$$\Rightarrow \quad 0 = 2s$$
$$= \quad s = 0$$

$$z: \quad -t + 1 = s + 1$$
$$-t = s$$
$$t = -s$$

So lines are both equal for $t = 0$ & $s = 0$.

$$x = 4(0) + 2, \quad y = 3 \quad z = -(0) + 1$$
$$x = 2(0) + 2 \quad y = 2(0) + 3 \quad z = (0) + 1$$

This means the pt. of intersection is

$$\Rightarrow (2, 3, 1)$$