1. Circle True (T) or False (F). If a statement is true explain why; if false explain why or give a counterexample.

   (a) **T** $\boxed{\textbf{F}}$ The double integral

$$\int_0^2 \int_0^2 [f(x) + g(y)]\, dx\, dy = \int_0^2 f(x)\, dx \int_0^2 g(y)\, dy$$

> **Solution:** FALSE: If the double integral represents a volume then f and g have units of length. But then the right side has units of length^4. Look at the properties of integrals.

   (b) $\boxed{\textbf{T}}$ **F** The double integral

$$\int_0^4 \int_0^4 f(x)f(y)\, dx\, dy = \left(\int_0^4 f(x)\, dx\right)^2$$

> **Solution:** TRUE: because
>
> $$\int_0^4 \int_0^4 f(x)f(y)\, dx\, dy = \int_0^4 f(y)\, dy \int_0^4 f(x)\, dx$$

   (c) **T** $\boxed{\textbf{F}}$ If the double integral $\iint_R f(x,y)\, dA = 0$ then $f(x,y) = 0$ at every point in R.

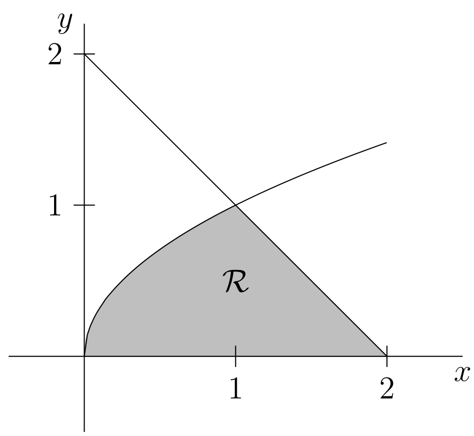
> **Solution:** FALSE: Consider the function $f(x,y) = \sin(x)$ and the region $R = \{(x,y) : -\pi \le x \le \pi, 0 \le y \le 1\}$.

2. Evaluate the integral

$$\iint_R 12y \, \mathrm{d}A$$

where R is the region bounded by $y = 2 - x$, $y = \sqrt{x}$ and $y = 0$.

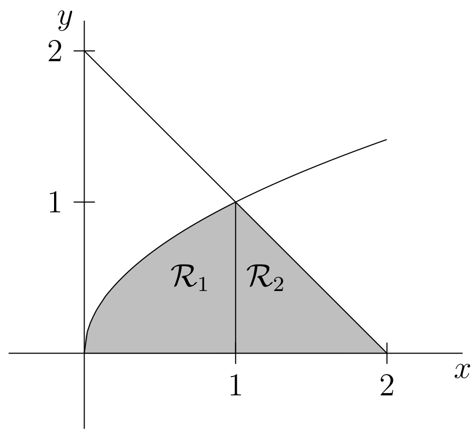
Solution: Sketching the region R we find:



Looking at a graph of the region we are integrating over we see that if we integrate with respect to x first we can avoid the need to split the integration into to parts. Solving $y = 2 - x$, $y = \sqrt{x}$ for x we get $x = 2 - y$ and $x = y^2$. Along the y axis the integration will be over the interval $[0, 1]$. Thus,

$$\begin{aligned}
\iint_R 12y \, \mathrm{d}A &= \int_0^1 \int_{y^2}^{2-y} 12y \; dxdy \\
&= \int_0^1 (12yx)\Big|_{x=y^2}^{x=2-y} dy \\
&= \int_0^1 12\left(2y - y^2 - y^3\right) dy \\
&= 12(y^2 - \frac{y^3}{3} - \frac{y^4}{4})\Big|_{y=0}^{y=1} \\
&= 12(\frac{5}{12}) \\
&= 5
\end{aligned}$$

To spice things up a notch we could integrate with respect the y first. In this case we have to split the region R into 2 regions. Let $R_1 = [0,1] \times [0, \sqrt{x}]$ and $R_2 = [1,2] \times [0, 2-x]$



$$
\begin{aligned}
\iint_R 12y \, \mathrm{d}A &= \iint_{R_1} 12y \, \mathrm{d}A + \iint_{R_2} 12y \, \mathrm{d}A \\
&= \int_0^1 \int_0^{\sqrt{x}} 12y \; dydx + \int_1^2 \int_0^{2-x} 12y \; dydx \\
&= \int_0^1 6y^2 \Big|_{y=0}^{y=\sqrt{x}} \; dx + \int_1^2 6y^2 \Big|_{y=0}^{y=2-x} \; dx \\
&= 6 \int_0^1 x \; dx + 6 \int_1^2 (2-x)^2 \; dx \\
&= 6 \frac{x^2}{2} \Big|_{x=0}^{x=1} + 6 \left(\frac{x^3}{3} - 2x^2 + 4x \right) \Big|_{x=2}^{x=3} \\
&= 6 \left(\frac{1}{2} \right) + 6 \left(\frac{1}{3} \right) \\
&= 3 + 2 \\
&= 5
\end{aligned}
$$

3. Let $R = \{(x, y) : 0 \le x \le \pi, 0 \le y \le a\}$. For what values of a, with $0 \le a \le \pi$ will

$$\iint_R \sin(x + y)\ \mathrm{d}A = 1?$$

Solution: With the given information we can evaluate the left hand side of the equation

$$\begin{aligned}
\iint_R \sin(x+y)\ \mathrm{d}A &= \int_0^a \int_0^\pi \sin(x+y)\ dxdy \\
&= \int_0^a (-\cos(x+y))\big|_{x=0}^{x=\pi}\ dy \\
&= \int_0^a (\cos(y) - \cos(\pi + y))\, dy \\
&= (\cos(y) - \cos(\pi + y))\ \big|_0^a \\
&= \sin(a) - \sin(\pi + a) \\
&= 2\sin(a)
\end{aligned}$$

Substituting this result in (3) and solving for a we find that

$$2\sin(a) = 1 \quad \Rightarrow \quad \sin(a) = \frac{1}{2} \quad \Rightarrow \quad a = \frac{\pi}{6} \text{ or } \frac{5\pi}{6}.$$

4. Match each iterated integral with its region of integration. Place the appropriate letter (A, B, C, or D) in the box next to the appropriate graph.

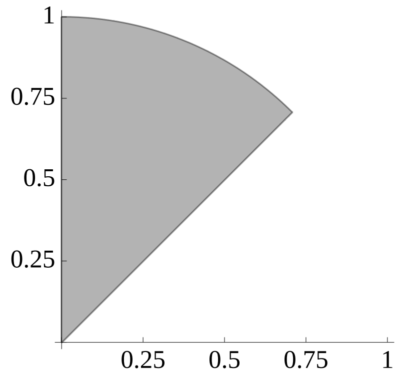
A. $\displaystyle\int_0^{\sqrt{2}/2}\int_y^{\sqrt{1-y^2}} x^2y\,dx\,dy$

C. $\displaystyle\int_0^{1}\int_{y/2}^{(2-y)/2} \sqrt{1-x^2}\,dx\,dy$

B. $\displaystyle\int_0^{1}\int_{x/2}^{(1+x)/2} (x^2+y^2)\,dy\,dx$

D. $\displaystyle\int_0^{\sqrt{2}/2}\int_x^{\sqrt{1-x^2}} x\,dy\,dx$

$\boxed{\text{D}}$



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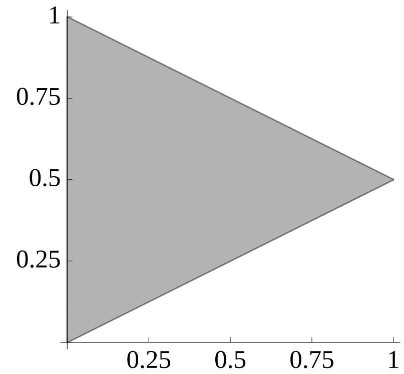
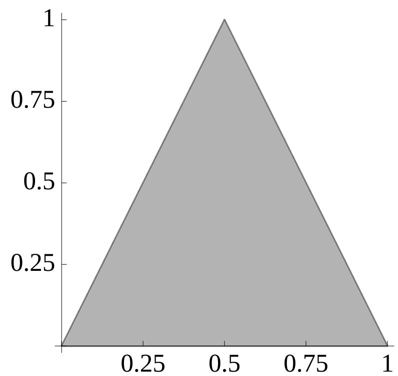
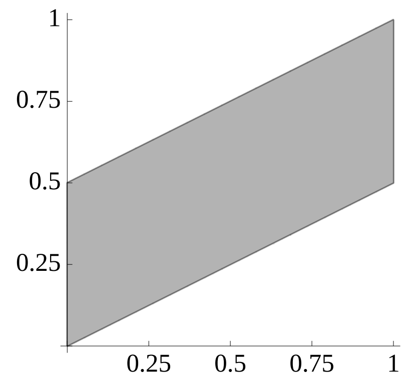


$\boxed{\text{C}}$



$\boxed{\text{B}}$



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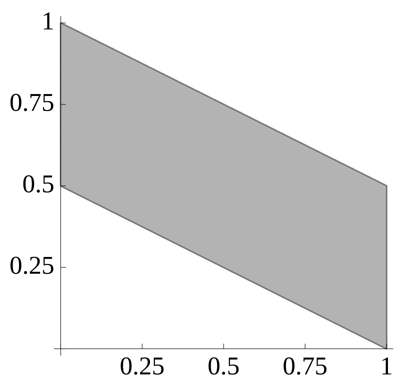

$\boxed{\text{A}}$