Stability Analysis of 1st Order ODEs

Math 2310-360: Differential Equations Spring 2019

Sketching $f(y)$ vs. y:

When plotting $f(y)$ vs y we're plotting our y' vs y. I think it's helpful to think of it as plotting a function $f(x)$ with respect to x. So think of $\frac{dy}{dt} = y^2(1-y)^2$ as $f(x) = x^2(1-x)^2$.

To find equilibrium points:

We solve the equation $y'(t) = 0$. In other words the roots to our $y'(t)$ are our equilibrium points. (In calculus 1 we knew these as critical points.)

To classify equilibrium points:

1. First we must determine what is actually going on with $f(y)$ near our equilibrium points. We determine the intervals established by our equilibrium points. Then we apply some calculus 1 knowledge:
   - When $f(y) > 0$ our $y(t)$ is increasing. We represent this on our $f(y)$ vs y sketch with right arrows: $\rightarrow\rightarrow\rightarrow$
   - When $f(y) < 0$ our $y(t)$ is decreasing. We represent this on our $f(y)$ vs y sketch with left arrows: $\leftarrow\leftarrow\leftarrow$
2. Next we determine the behavior of each equilibrium point by looking at our arrows
   - Stable if $\rightarrow \bullet \leftarrow$
   - Unstable if $\leftarrow \bullet \rightarrow$
   - Semi-stable if $\rightarrow \bullet \rightarrow$ or $\leftarrow \bullet \leftarrow$

Inflection Points:

Sometimes on an interval our solutions will change direction. This occurs at inflection points which are found when $y''(t) = 0$.

Lecture #5: Stability Analysis 1st order ODEs

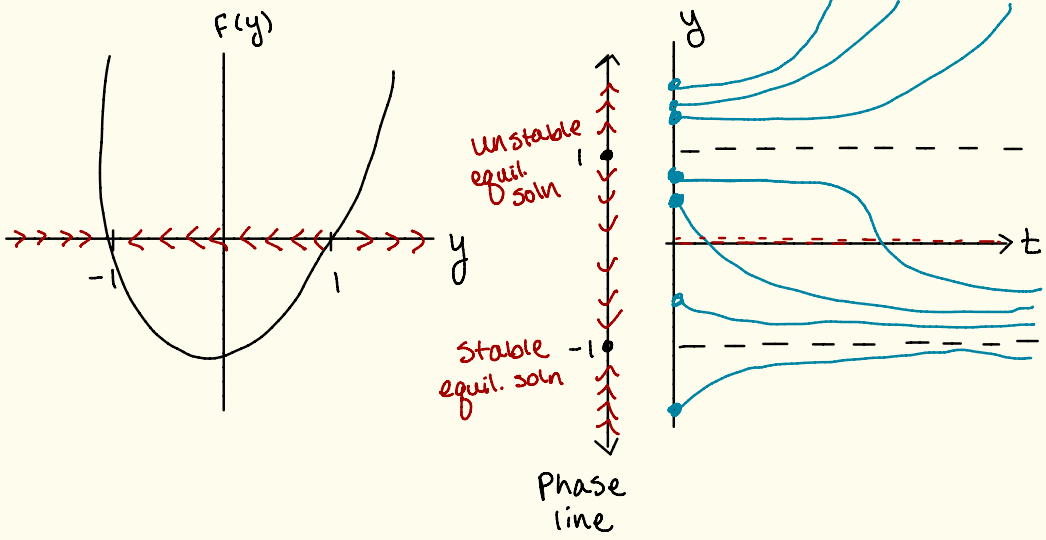
Date: Wed 2/20/19

Ex. $\frac{dy}{dt} = y^2 - 1$

For $f(y) = \frac{dy}{dt}$ we sketch $f(y)$ vs. y.

$$f(y) = y^2 - 1$$
$$= (y+1)(y-1)$$

zeros @ $y = 1$ & $y = -1$ ← Found equilibrium pts (i.e. critical pts of y)



Inflection pts: $f(y) = y^2 - 1$

$f'(y) = y'' = 2y \Rightarrow$ inflection pt @ $y = 0$