Lecture # 21: Complex Eigenvalues

Date: Wed. 5/1/19

Thm (7.4.5) If

$$\vec{x} = u(t) + iv(t)$$

is a complex soln of the system

$$\frac{d\vec{x}}{dt} = \vec{p}(t)\vec{x}$$

then its real part $u(t)$ & its imaginary part $v(t)$ are also solns of the system.

The gen. form of our system of ODEs is

$$x' = Ax$$

For complex e-vals $r_1 = a+ib$, $r_2 = a-ib$ these are complex conjugates i.e. $r_1 = \overline{r_2}$

Then their corr. e-vecs will also be complex conjugates i.e. $\vec{\xi}^{(1)} = \overline{\vec{\xi}^{(2)}}$

Then solutions to the system will be

$$\vec{x}^{(1)}(t) = \vec{\xi}^{(1)} e^{r_1 t} \quad , \quad \vec{x}^{(2)}(t) = \overline{\vec{\xi}^{(1)}}\, e^{\overline{r_1} t}$$

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Let $\vec{\xi}^{(1)} = \vec{\alpha} + i\vec{\beta}$, $\vec{\alpha}$, $\vec{\beta}$ real valued vectors

then

$$\vec{X}^{(1)}(t) = (\vec{\alpha} + i\vec{\beta})\, e^{(a+ib)t}$$

$$= (\vec{\alpha} + i\vec{\beta})\, e^{at}(\cos(bt) + i\sin(bt))$$

$$= e^{at}(\vec{\alpha}\cos(bt) - \vec{\beta}\sin(bt)) + ie^{at}(\vec{\alpha}\sin(bt) + \vec{\beta}\cos(bt))$$

Writing in form $\vec{X}^{(1)}(t) = \vec{u}(t) + i\vec{v}(t)$

$$\Rightarrow u(t) = e^{at}(\vec{\alpha}\cos(bt) - \vec{\beta}\sin(bt))$$

$$v(t) = e^{at}(\vec{\alpha}\sin(bt) + \vec{\beta}\cos(bt))$$

Then gen. soln to system will be

$$\vec{X}(t) = C_1\vec{u}(t) + C_2\vec{v}(t) \qquad \text{(for 2x2 system)}$$