Lecture #1: Intro to ODEs Date: Mon. 2/6/19

What is meant by the term:

Ordinary Differential Equation

involves only 1 independent variable

something to do w/ derivatives

math expression involving =

If a fcn $f(t)$ satisfies $f'(t) = t$, $f'(t) = \frac{df}{dt}$

What is $f(t)$?

t is the independent variable

f the dependent variable

To find $f(t)$:

$$\int f'(t)\,dt = \int t\,dt$$

By FTOC

$$f(t) = \frac{t^2}{2} + C$$

General Soln of the ODE

Classification of ODEs

An ODE is a math expression that involves derivatives of a function that only depends on one variable.

Finding solns to ODEs will always involve some interpretation. We will need to classify an ODE & ID its general form so that we can detetermine which method to use.

Order

The order of an ODE is the highest order derivative that appears in the eqn.

An ODE of order n can be expressed as

$$f(t, y(t), y'(t), y''(t), \dots, y^{(n)}(t))$$

where $y^{(n)}(t) = \frac{d^n y}{dt^n}$ is the n^{th} derivative of $y(t)$.

Ex. $y''(t) + 2(y(t))^4 = 1$

This ODE is order 2

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In this class we will focus on:

$$y^{(n)}(t) = f(t, y'(t), y''(t), \dots y^{(n-1)}(t))$$

i.e. cases where the expression can always be written explicitly as highest order derivative.

Linearity

An n^{th} order ODE is <u>linear</u> if it can be written as

$$g(t) = a_0(t)\, y^{(n)}(t) + a_1(t)\, y^{(n-1)}(t) + \dots + a_{n-1}(t)\, y'(t) + a_n(t)\, y(t)$$

otherwise, it is <u>nonlinear</u>

Ex. $y''(t) + 2\underline{(y(t))^4} = 1$

Nonlinear

For 2nd order ODE to be linear we need to be able to write it as

$$a_0(t)y''(t) + a_1(t)y'(t) + a_2(t)\cdot y(t) = g(t)$$

$\therefore$ $a_0(t) = 1$
$a_1(t) = 0$
$g(t) = 1$

but we have no $a_2(t)y(t)$ term

$2(y(t))^4$ is the "problem" term

$$y''(t) + \underline{2(y(t))^4} = 1$$

$$1\cdot y''(t) + 2(y(t))^4 = 1$$
$$a_0(t)\, y''(t) + 2(y(t))^4 = g(t)$$

This is known as "Coupling" of dependent variables $\Rightarrow$ Nonlinear

i.e. we have "y's multiplied by other y's"

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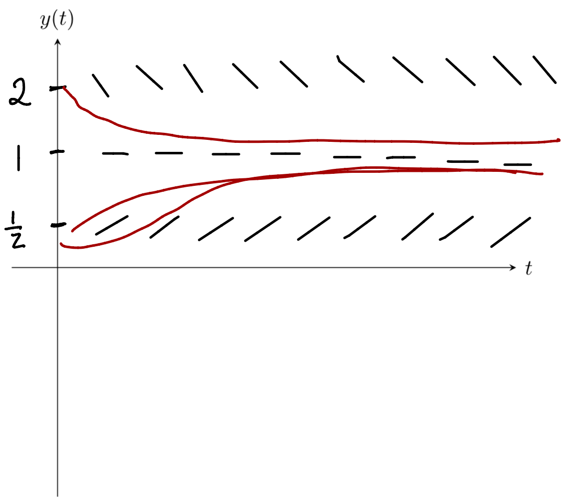
Direction Fields

A way to qualitatively understand a soln's behavior

Ex. $\frac{dy}{dt} + y = 1 \Rightarrow \frac{dy}{dt} = 1 - y$

Plot the tangent lines on t-y axes.



Pick some values of y & plot tangent lines along resulting curve