Lecture # 3: Integrating Factors

Date: Wed. 2/13/19

Integrating Factors

General form

$$\frac{dy}{dt} + p(t)y = g(t)$$

$$\Rightarrow \quad y' + p(t)y = g(t)$$

Integrating Factor defined as

$$\mu(t) = e^{\int p(t)\,dt}$$

Greek letter mu

$$= \exp\left(\int p(t)\,dt\right)$$

Where does this come from?

$$\mu(t)\left[y' + p(t)y\right] = \mu(t)\,g(t)$$

$$\Rightarrow \mu(t)y' + \mu(t)p(t)y = \mu(t)g(t)$$

Want the LHS to represent (or be condensed down to) the product rule of diff.

Recall: $\frac{d}{dt}[fg] = f'g + fg'$

So we need $\dfrac{d\mu(t)}{dt} = \mu(t)p(t)$

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$$\frac{d\mu}{dt} = \mu \, p(t)$$

This is a sep. eqn. Rewriting:

$$\frac{1}{\mu} \, d\mu = p(t) \, dt$$

Integrating

$$\int \frac{1}{\mu} \, d\mu = \int p(t) \, dt$$

$$\Rightarrow \ln(\mu) = \int p(t) \, dt$$

Exponentiate both sides

$$e^{\ln(\mu)} = e^{\int p(t) \, dt}$$

$$\Rightarrow \mu(t) = e^{\int p(t) \, dt}$$