Summary of Modeling Problem Types

Math 2310-360: Differential Equations Spring 2019

Tank Problems

In your textbook it is given that

$$\frac{dQ}{dt} = (\text{rate in}) - (\text{rate out})$$

Where Q is the amount of substance at any time t. This version is referring only to the rate of substance in and out. When reading an actual problem however, we are given the rate and concentration of the *solution* in and out of the tank. It is helpful to instead think about this equation as follows

$$\frac{dQ}{dt} = (\text{rate of soln. in}) \cdot (\text{ conc. of soln in }) - (\text{rate of soln. out}) \cdot (\text{ conc. of soln. out })$$

Compounding Interest

If interest is **compounded continuously**

ODE	Solution
$\frac{dS}{dt} = rS(t)$	$S(t) = S_0 e^{rt}$

When you have **deposits or withdrawls** at some constant rate k:

ODE	Solution
$\frac{ds}{dt} = rS(t) + k$	$S(t) = S_0 e^{rt} + \left(\frac{k}{r}\right)(e^{rt} - 1)$

$S(t)$: Amount in the account at time t
r: Interest rate
S_0: Initial amount in the account.

If interest is **compounded m times per year**

ODE	Solution
	$S(t) = S_0 \left(1 + \frac{r}{m}\right)$

Newton's Law of Cooling

$$\frac{dT}{dt} = k(T(t) - A)$$

k: Negative constant of proportionality
$T(t)$: Temperature of warm object at time t
A: Ambient temperature (temperature of environment)

Exponential Growth

ODE	**Solution**
$\dfrac{dP}{dt} = rP(t)$	$P(t) = P_0 e^{rt}$

r: Rate of growth or decline
$P(t)$: Population at time t
P_0: Initial population

Logistic Growth

ODE	**Solution**
$\dfrac{dy}{dt} = ry\left(1 - \dfrac{y}{K}\right)$	$y(t) = \dfrac{y_0 K}{y_0 + (K - y_0)e^{-rt}}$

r: Rate of growth or decline
$y(t)$: Population at time t
y_0: Initial population
K: Carrying capacity