

1. Plot the direction field and solution curves for several values of y_0 .

(a) $y' = -1 - 2y$ $y(0) = y_0$

Solution:

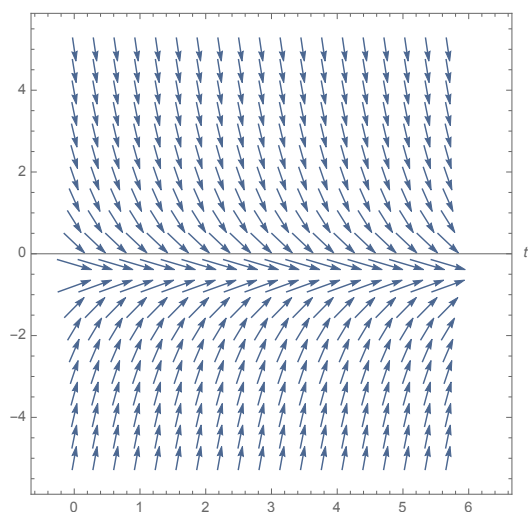


Figure 1: Direction Field for 2(b)

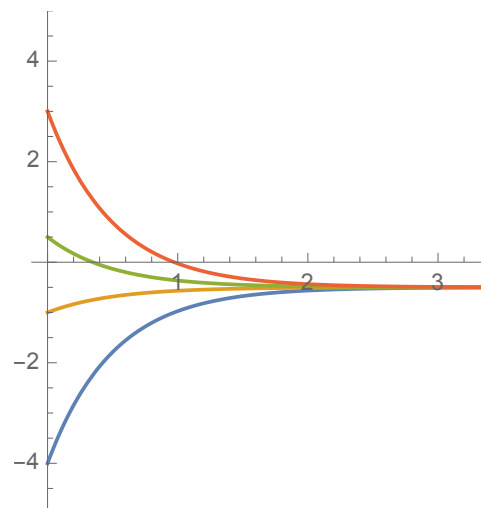


Figure 2: Some Solution Curves for 2(b)

(b) $y' = 2y - 10$; $y(0) = y_0$

Solution:

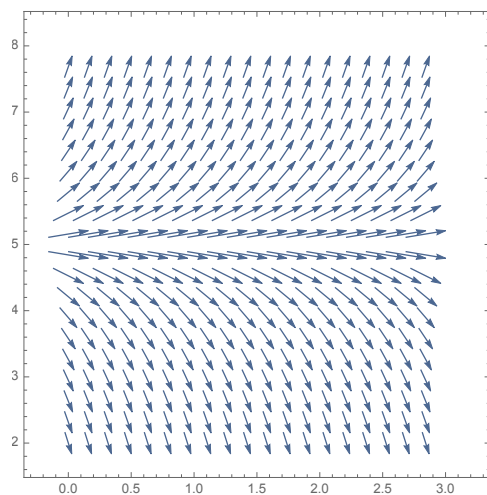


Figure 3: Direction Field for 2(b)

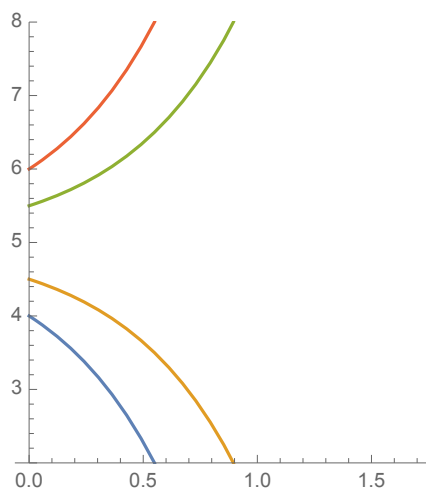


Figure 4: Some Solution Curves for 2(b)

2. Verify that equation

$$t^2 y'' + 5ty' + 4y = 0, \quad t > 0$$

has solutions

$$y_1(t) = t^{-2}; \quad y_2(t) = t^{-2} \ln(t)$$

Solution: Recall that to verify if a given equation is a valid solution to our ODE we simply calculate the needed derivatives, plug them into our ODE and verify that it results in a truthful statement.

My respective derivatives are

$$\begin{aligned} y_1(t) &= t^{-2} & y_2(t) &= t^{-2} \ln(t) \\ y_1'(t) &= -2t^{-3} & y_2'(t) &= -2t^{-3} \ln(t) + t^{-3} \\ y_1''(t) &= 6t^{-4} & y_2''(t) &= 6t^{-4} \ln(t) - 5t^{-4} \end{aligned}$$

Now we simply plug and chug to find that $y_1(t)$ and $y_2(t)$ are both solutions to the ODE.

For each of the given differential equations, determine its order and whether it is linear or nonlinear.

(a) $t^2 \frac{d^2 y}{dt^2} + t \frac{dy}{dt} + 2y = \sin(t)$

Solution: This is 2nd order and nonlinear. I always like to “translate” the $\frac{dy}{dt}$ notation to simple prime notation so feel free to do so as well. It can make patterns easier to see in my opinion.

$$t^2 y'' + ty' + 2y = \sin(t)$$

The highest derivative is the 2nd derivative. There are no dependent variables multiplied by one another (i.e. no y 's multiplied by another y) so this equation is linear.

(b) $\frac{dy}{dt} + ty^2 = 0$

Solution: 1st order, nonlinear. Again “translating” the notation we have

$$y' + ty^2 = 0$$

We see that the highest derivative is the first derivative. In this case we have a y^2 term. In other words, we have $y \cdot y$. So this equation is nonlinear.

(c) $\frac{d^3 y}{dt^3} + t \frac{d^2 y}{dt^2} + (\cos^2(t))y = t^3$

Solution: 3rd order, linear. Again “translating” the notation we have

$$y''' + ty'' + (\cos^2(t))y = t^3$$

Our highest derivative is the 3rd derivative. Even though we have the $\cos(t)$ term it only involves our independent variable t . Thus, this is a linear equation.

(d) $\frac{d^3 y}{dt^3} + t \frac{d^2 y}{dt^2} + \sin(t + y) = t^3$

Solution: 3rd order, nonlinear. Again “translating” the notation we have

$$y''' + ty'' + \sin(t + y)y = t^3$$

Again we have the 3rd derivative. The term $\sin(t + y)$ involves our independent and our dependent variable. In other words we cannot write our differential equation explicitly in terms of the form $p(t) \cdot y^{(n)}$. Since we have a y “trapped” inside the sin function this equation is nonlinear.