Lecture #17: The Convolution Integral

Date: Mon 4/15/19

Suppose that $H(s) = F(s) \cdot G(s)$

We do not have multiplication with the Laplace Transform

i.e: $\mathcal{L}^{-1}\{H(s)\} \neq \mathcal{L}^{-1}\{F(s)\}\mathcal{L}^{-1}\{G(s)\}$

Convolution will help us make sense of the idea of a product. We will denote this as "$*$"

Def If $F(s) = \mathcal{L}\{f(t)\}$ & $G(s) = \mathcal{L}\{g(t)\}$, then if $H(s) = F(s) \cdot G(s)$ We have

$$h(t) = \mathcal{L}^{-1}\{H(s)\} = \int_0^t f(\tau)\, g(t-\tau)\, d\tau$$

$$= \int_0^t f(t-\tau)\, g(\tau)\, d\tau$$

$$= (f*g)(\tau)$$

Ex. $H(s) = \dfrac{1}{s(s^2+1)}$

Want to find $h(t) = \mathcal{L}^{-1}\{H(s)\}$

Let $F(s) = \dfrac{1}{s}$ and $G(s) = \dfrac{1}{s^2+1}$

$\Rightarrow f(t) = \mathcal{L}^{-1}\left\{\frac{1}{s}\right\} = 1$ $\qquad \Rightarrow g(t) = \mathcal{L}^{-1}\left\{\frac{1}{s^2+1}\right\} = \sin(t)$

By Convolution:

$$h(t) = \mathcal{L}^{-1}\{F \cdot G\} = (f*g)(\tau)$$

$$= \int_0^t f(t-\tau)\, g(\tau)\, d\tau = \int_0^t 1 \cdot \sin(\tau)\, d\tau$$

$$= -\cos(\tau)\Big|_0^t = 1 - \cos(t)$$

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Properties of Convolution

$f * g = g * f$	Commutative
$f*(g_1+g_2) = f*g_1 + f*g_2$	Distributive
$(f*g)*h = f*(g*h)$	associative
$f*0 = 0*f = 0$	(note 0 is fcn 0 not the # zero!

Note that it is not generally true that $f*1=f$ or $f*f$ is non-negative. See text book for example