1. Is the matrix
$$A = \begin{bmatrix} 5 & 0 & 0 \\ -3 & -7 & 0 \\ 8 & 5 & -1 \end{bmatrix}$$
invertible? Use as few calculations as possible.

Solution: This matrix is in lower triangular form. If we perform EROs to obtain the reduced row echelon form we have

$$\begin{bmatrix} 5 & 0 & 0 \\ -3 & -7 & 0 \\ 8 & 5 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

We see that A has 3 pivot positions. Since A is 3×3 then by the Invertible Matrix Theorem part (c), A must be invertible.

2. Find the determinant of
$$B = \begin{bmatrix} 4 & 3 & 0 \\ 6 & 5 & 2 \\ 9 & 7 & 3 \end{bmatrix}$$
by hand using a cofactor expansion. Verify your calculation with your calculator.

Solution: Since there is a zero in the first row, we will use a cofactor expansion with this row. The determinant of B is then

$$\begin{aligned} \det(B) = \begin{bmatrix} 4 & 3 & 0 \\ 6 & 5 & 2 \\ 9 & 7 & 3 \end{bmatrix} &= 4 \begin{vmatrix} 5 & 2 \\ 7 & 3 \end{vmatrix} - 3 \begin{vmatrix} 6 & 2 \\ 9 & 3 \end{vmatrix} + 0 \begin{vmatrix} 6 & 5 \\ 9 & 7 \end{vmatrix} \\ &= 4\Big[(5)(3) - (7)(2)\Big] - 3\Big[(6)(3) - (2)(9)\Big] + 0 \\ &= 4(1) - 3(0) \\ &= 4 \end{aligned}$$

3. Calculate the determinant of B in the previous problem by multiplying by diagonals.

Solution: We begin by copying the first two columns of B:

$$\begin{array}{ccc:cc} 4 & 3 & 0 & 4 & 3 \\ 6 & 5 & 2 & 6 & 5 \\ 9 & 7 & 3 & 9 & 7 \end{array}$$

Then by multiplying the diagonals in the following way

$$\begin{array}{cccccc} & & & - & - & - \\ 4 & 3 & 0 & 4 & 3 & \\ 6 & 5 & 2 & 6 & 5 & \\ 9 & 7 & 3 & 9 & 7 & \\ & & & + & + & + \end{array}$$

we have

$$\begin{aligned} \det(B) &= (4)(5)(3) + (3)(2)(9) + (0)(6)(7) - (9)(5)(0) - (7)(2)(4) - (3)(6)(3) \\ &= 60 + 63 + 0 - 0 - 56 - 63 \\ &= 4 \end{aligned}$$