Worksheet 5 Solutions

Linear Transformations

MATH 2250, Fall 2018

1. Let

$$A = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

and define $T : \mathbb{R}^2 \to \mathbb{R}^2$ by $T(\mathbf{x}) = A\mathbf{x}$. Find the images under T of

$$\mathbf{u} = \begin{bmatrix} 1 \\ -3 \end{bmatrix} \qquad \text{and} \qquad \mathbf{v} = \begin{bmatrix} a \\ b \end{bmatrix}$$

Solution:

$$T(\mathbf{u}) = A\mathbf{u} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ -3 \end{bmatrix} = \begin{bmatrix} 2 \\ -6 \end{bmatrix}, \; T(\mathbf{v}) = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 2a \\ 2b \end{bmatrix}$$

The image of $\mathbf{u}$ under T is

$$T(\mathbf{u}) = A\mathbf{u} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ -3 \end{bmatrix} = \begin{bmatrix} 2 \\ -6 \end{bmatrix}$$

The image of $\mathbf{v}$ under T is

$$T(\mathbf{v}) = A\mathbf{v} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 2a \\ 2b \end{bmatrix}$$

2. Given the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^4$ and

$$T(\mathbf{e}_1) = (3, 1, 3, 1)$$
$$T(\mathbf{e}_2) = (-5, 2, 0, 0)$$

where

$$\mathbf{e}_1 = (1, 0) \qquad \text{and} \qquad \mathbf{e}_2 = (0, 1)$$

find the standard matrix of T.

Solution: The standard matrix of a linear transformation T is given by the matrix

$$\begin{aligned} A &= \begin{bmatrix} T(\mathbf{e}_1) & T(\mathbf{e}_2) \end{bmatrix} \\ &= \begin{bmatrix} 3 & -5 \\ 1 & 2 \\ 3 & 0 \\ 1 & 0 \end{bmatrix} \end{aligned}$$