1. For the given matrices determine whether the given vector is in the column space of the matrix. If so, express the vector as a linear combination of the column vectors of the matrix. Use your calculator to perform any matrix calculations.

(a)

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 1 & 3 \end{bmatrix}; \qquad \mathbf{b} = \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}$$

Solution: To determine if $\mathbf{b}$ is in the column space of A we must solve the system given by

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & -1 \\ 1 & 0 & 1 & 0 \\ 2 & 1 & 3 & 2 \end{array}\right] \underset{\substack{\text{Row} \\ \text{Reduce}}}{=} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

Since this system is inconsistent (i.e. does not have a solution) then $\mathbf{b}$ is not in the column space of A.

(b)

$$B = \begin{bmatrix} 1 & -1 & 1 \\ 9 & 3 & 1 \\ 1 & 1 & 1 \end{bmatrix}; \qquad \mathbf{c} = \begin{bmatrix} 5 \\ 1 \\ -1 \end{bmatrix}$$

Solution: To determine if $\mathbf{c}$ is in the column space of B we must solve the system given by

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 5 \\ 9 & 3 & 1 & 1 \\ 1 & 1 & 1 & -1 \end{array}\right] \underset{\substack{\text{Row} \\ \text{Reduce}}}{=} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

Since this system has at least one solution then $\mathbf{c}$ is in the column space of B.

2. Find Null(A) for the following matrices. Use your calculator to perform any matrix calculations.

   (a) $A = \begin{bmatrix} 1 & -6 & 4 & 0 \\ 0 & 0 & 2 & 0 \end{bmatrix}$

Solution: To determine Null(A) we must find all vectors that satisfy the equation $A\mathbf{x} = \mathbf{0}$.

$$\left[\begin{array}{cccc|c} 1 & -6 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{array}\right]$$

This system has the solution

$$\mathbf{x} = \begin{bmatrix} -6x_2 \\ x_2 \\ 0 \\ x_4 \end{bmatrix} = x_2 \begin{bmatrix} -6 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

So the spanning set for Null(A) is

$$\left\{ \begin{bmatrix} -6 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$$

(b) $B = \begin{bmatrix} 1 & 5 & -4 & -3 & 1 \\ 0 & 1 & -2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

Solution: To determine Null(B) we must find all vectors that satisfy the equation $B\mathbf{x} = \mathbf{0}$. Since

$$\left[\begin{array}{ccccc:c} 1 & 5 & -4 & -3 & 1 & 0 \\ 0 & 1 & -2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \underset{\substack{\text{Row}\\\text{Reduce}}}{=} \left[\begin{array}{ccccc:c} 1 & 0 & 6 & -8 & 1 & 0 \\ 0 & 1 & -2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

This system has the solution

$$\mathbf{x} = \begin{bmatrix} -6x_3 + 8x_4 - x_5 \\ 2x_3 - x_4 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = x_2 \begin{bmatrix} -6 \\ 2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 8 \\ -1 \\ 0 \\ 1 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

So the spanning set for Null(A) is

$$\left\{ \begin{bmatrix} -6 \\ 2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 8 \\ -1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$$