Worksheet 8
Vector Spaces

1. Let H be the set of all vectors of the form $\begin{bmatrix} s + 3t \\ s - t \\ 2s - t \\ 4t \end{bmatrix}$. Show that H is a subspace of $\mathbb{R}^4$.

2. Let

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 4 \\ 2 \\ 6 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$$

   (a) Is $\mathbf{w} \in \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$? How many vectors are there in $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$?

   (b) How many vectors are in Span $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$?

3. Again consider the set of vectors

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 4 \\ 2 \\ 6 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$$

(a) Is $\mathbf{w}$ in the subpsace spanned by $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$? Why or why not?

(b) Let $\mathbf{u} = \begin{bmatrix} 8 \\ 4 \\ 7 \end{bmatrix}$. Is $\mathbf{u}$ in the subpsace spanned by $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$? Why or why not?