1. Row reduce the matrix

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 4 & 5 & 6 & 7 \\ 6 & 7 & 8 & 9 \end{bmatrix}$$

to reduced echelon form. Circle the pivot positions in the final matrix.

Solution: Write the system in matrix form and then get in row-echelon form

$$\begin{bmatrix} 1 & 3 & 1 & 0 \\ -4 & -9 & 2 & 0 \\ 0 & -3 & -6 & 0 \end{bmatrix} \underset{\substack{\text{Row} \\ \text{Reduce}}}{=} \begin{bmatrix} 1 & 0 & -5 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

where the pivot positions in the row reduced matrix are circled.

2. Write a vector equation that is equivalent to the system of equations.

$$\begin{array}{rcrcrcr} & & x_2 & + & 5x_3 & = & 9 \\ 4x_1 & + & 6x_2 & - & x_3 & = & 7 \\ -x_1 & + & 3x_2 & - & -8x_3 & = & -2 \end{array} \tag{1}$$

Solution: Writing the system of equations as a vector equation we have

$$x_1 \begin{bmatrix} 0 \\ 4 \\ -1 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ 6 \\ 3 \end{bmatrix} + x_3 \begin{bmatrix} 5 \\ -1 \\ -8 \end{bmatrix} = \begin{bmatrix} 9 \\ 7 \\ -2 \end{bmatrix}$$

3. Write a system of equations that is equivalent to the vector equation.

$$v_1 \begin{bmatrix} 6 \\ -1 \\ 5 \end{bmatrix} + v_2 \begin{bmatrix} -3 \\ 4 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ -7 \\ -5 \end{bmatrix}$$

Solution: Writing the vector equation as a system of equations we have

$$\begin{array}{rcrcr} 6v_1 & - & 3v_2 & = & 1 \\ -v_1 & - & 4v_2 & = & -7 \\ 5v_1 & & & = & -5 \end{array}$$

4. Determine if $\mathbf{b}$ is a linear combination of $\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}$ where

$$\mathbf{v_1} = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}; \quad \mathbf{v_2} = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}; \quad \mathbf{v_3} = \begin{bmatrix} 5 \\ -6 \\ 8 \end{bmatrix}; \quad \mathbf{b} = \begin{bmatrix} 2 \\ -1 \\ 6 \end{bmatrix}$$

You may use your calculator or other tool to perform row reduction.

Solution: To determine if $\mathbf{b}$ is a linear combination of $\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}$ we seek constants c_1, c_2, c_3 such that

$$c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + c_3\mathbf{v}_3 = \mathbf{b}$$

Write the system in matrix form and then get into row-echelon form

$$\left[\begin{array}{rrr:r} 1 & 0 & 5 & 2 \\ -2 & 1 & -6 & -1 \\ 0 & 2 & 8 & 6 \end{array}\right] \underset{\substack{\text{Row} \\ \text{Reduce}}}{=} \left[\begin{array}{rrr:r} 1 & 0 & 5 & 2 \\ 0 & 1 & 4 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Putting back into system form we have

$$\begin{array}{ccccccc} c_1 & & & + & 5c_3 & = & 2 \\ & & c_2 & + & 4c_3 & = & 3 \end{array}$$

Since the system has at least one solution, then $\mathbf{b}$ is a linear combination of $\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}$.