Phase Planes & Portraits

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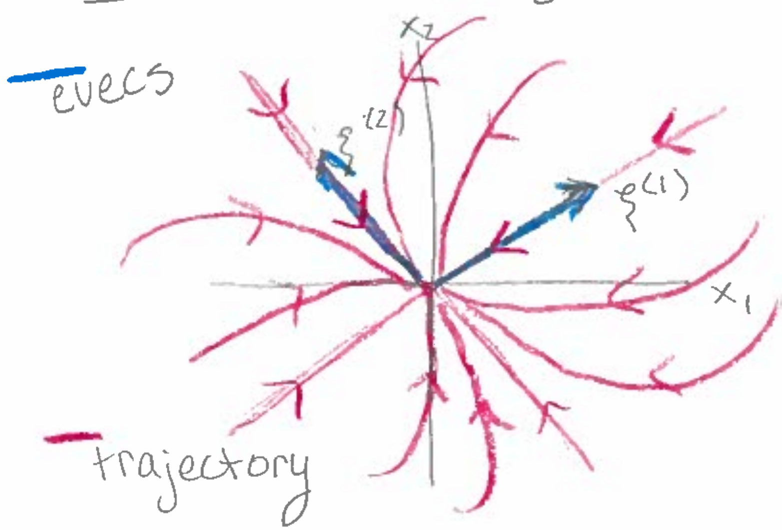
Real (and different) ε-vals

$0 < r_1 < r_2$	Both positive	$r_1 < r_2$	Node
$0 < r_2 < r_1$	" "	$r_2 < r_1$	
$r_2 < r_1 < 0$	Both negative	$r_2 < r_1$	
$r_1 < r_2 < 0$	" "	$r_2 > r_1$	

Ex for $r_1 < r_2 < 0$



$r_2 > r_1$ so trajectories move along line defined by $\xi^{(2)}$, the evec assoc. w/ r_2

This is a nodal sink

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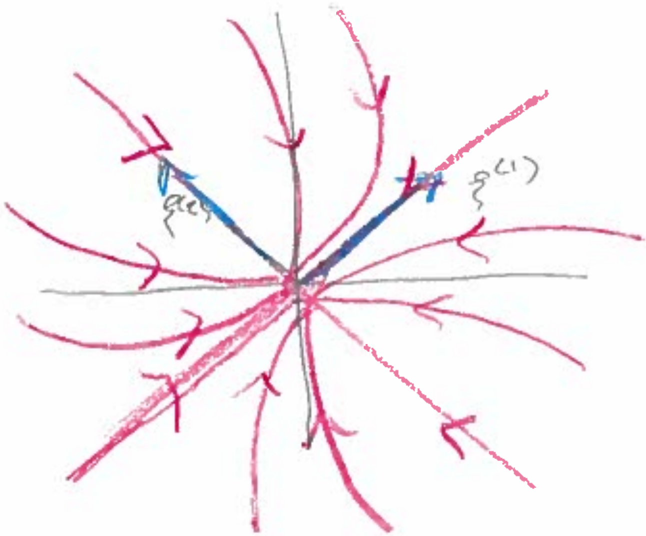
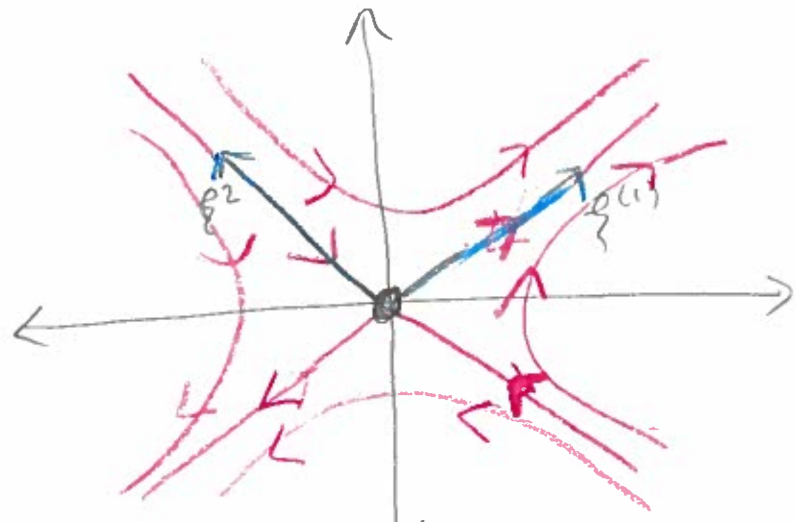
For $r_2 < r_1 < 0$



For $r_1 > 0$, $r_2 < 0$

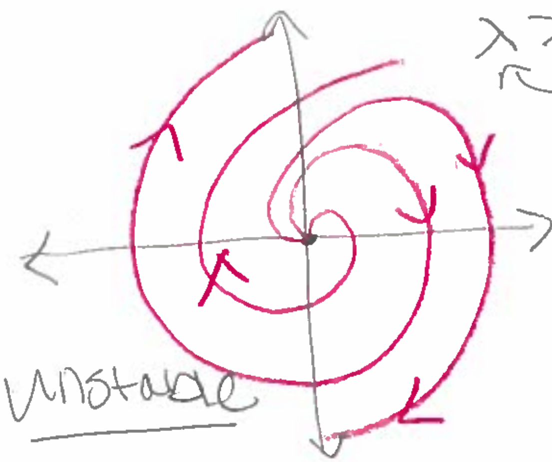


Saddle Point

<u>complex e-vals</u>

$r_1 = \lambda + i\mu$ & $r_2 = \lambda - i\mu$

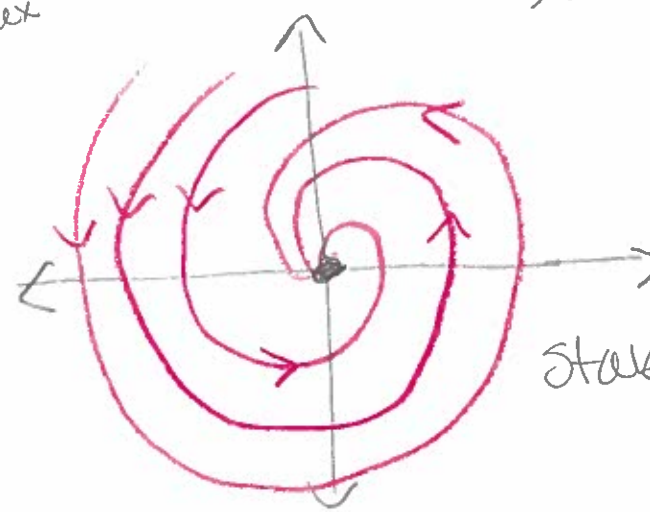
$\lambda > 0$ real part of complex #



<u>unstable</u>

<u>Spiral Points</u>

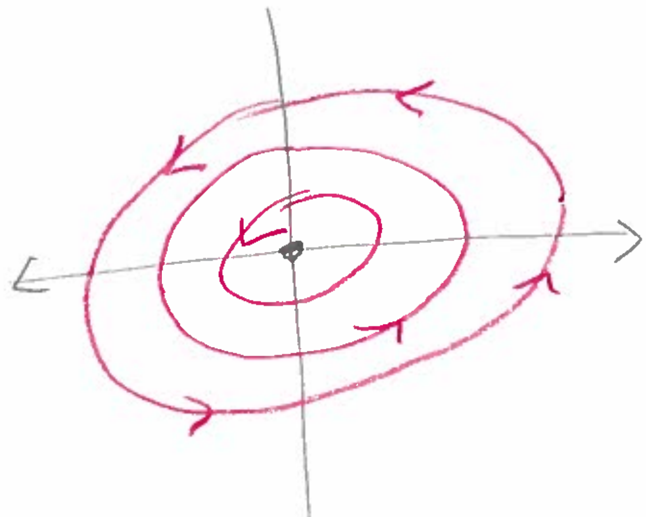
$\lambda < 0$



stable

$r_1 = +i\mu$, $r_2 = -i\mu$

i.e. evals are purely imaginary



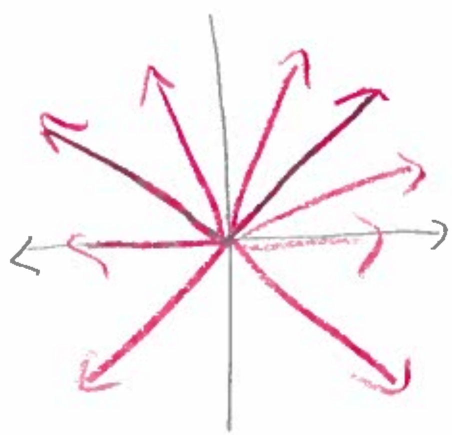
Center

moves clockwise for $\mu > 0$

& counter clockwise for $\mu < 0$

Repeated Evals

$r_1 = r_2$ w/ 2 L.I. evecs

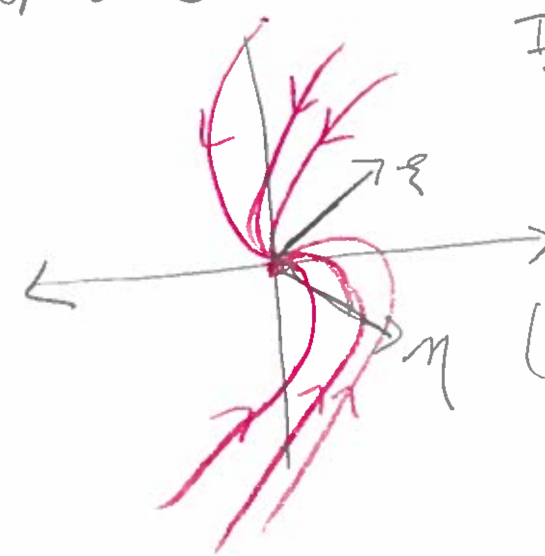


Proper Node

or

Star Point

w/ one evec



Improper Node

(w/ $r_1 = r_2 < 0$)

Locally Linear Systems

Autonomous Systems

Similar to Autonomous Eqns seen in 2.5

Systems of Form

$$\frac{dx}{dt} = F(x,y)$$
$$\frac{dy}{dt} = G(x,y)$$

$\Longrightarrow$ also written as $\vec{x}\,' = \vec{F}(\vec{x})$

Often too difficult to actually solve & only a qualitative picture of the soln is possible.

This is an extension of what we learned for linear systems.

Essentially, we want to find critical points of the system. This are the equilibrium or constant solutions to the system.

Goal: 1) Find critical points

2) Approximate the nonlinear system near the critical points using a linear system. i.e. we are examining where the system is <u>locally linear</u>.

3) Analyze each of the locally linear systems w/ techniques from linear systems.

4) Draw a phase portrait

To approx a nonlinear system locally we use

$$\frac{d}{dt}\begin{bmatrix} u \\ v \end{bmatrix} = \underbrace{\begin{bmatrix} F_x & F_y \\ G_x & G_y \end{bmatrix}_{(x_0, y_0)}}_{\text{The } \underline{\text{Jacobian}} \text{ matrix evaluated at our critical point } (x_0, y_0)} \begin{bmatrix} u \\ v \end{bmatrix}$$

We assume that $\det(J) \neq 0$