## More about Inverses & Intro to Determinants

1. Is the matrix

$$A = \left[ \begin{array}{rrr} 5 & 0 & 0 \\ -3 & -7 & 0 \\ 8 & 5 & -1 \end{array} \right]$$

invertible? Use as few calculations as possible.

**Solution:** This matrix is in lower triangular form. If we perform EROs to obtain the reduced row echelon form we have

$$\begin{bmatrix} 5 & 0 & 0 \\ -3 & -7 & 0 \\ 8 & 5 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

We see that A has 3 pivot positions. Since A is  $3 \times 3$  then by the Invertible Matrix Theorem part (c), A must be invertible.

2. Find the determinant of

$$B = \left[ \begin{array}{rrr} 4 & 3 & 0 \\ 6 & 5 & 2 \\ 9 & 7 & 3 \end{array} \right]$$

by hand using a cofactor expansion. Verify your calculation with your calculator.

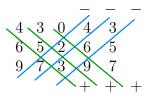
**Solution:** Since there is a zero in the first row, we will use a cofactor expansion with this row. The determinant of B is then

$$\det(B) = \begin{bmatrix} 4 & 3 & 0 \\ 6 & 5 & 2 \\ 9 & 7 & 3 \end{bmatrix} = 4 \begin{vmatrix} 5 & 2 \\ 7 & 3 \end{vmatrix} - 3 \begin{vmatrix} 6 & 2 \\ 9 & 3 \end{vmatrix} + 0 \begin{vmatrix} 6 & 5 \\ 9 & 7 \end{vmatrix}$$
$$= 4 [(5)(3) - (7)(2)] - 3 [(6)(3) - (2)(9)] + 0$$
$$= 4(1) - 3(0)$$
$$= 4$$

3. Calculate the determinant of B in the previous problem by multiplying by diagonals.

**Solution:** We begin by copying the first two columns of B:

Then by multiplying the diagonals in the following way



we have

$$det(B) = (4)(5)(3) + (3)(2)(9) + (0)(6)(7) - (9)(5)(0) - (7)(2)(4) - (3)(6)(3)$$

$$= 60 + 63 + 0 - 0 - 56 - 63$$

$$= 4$$