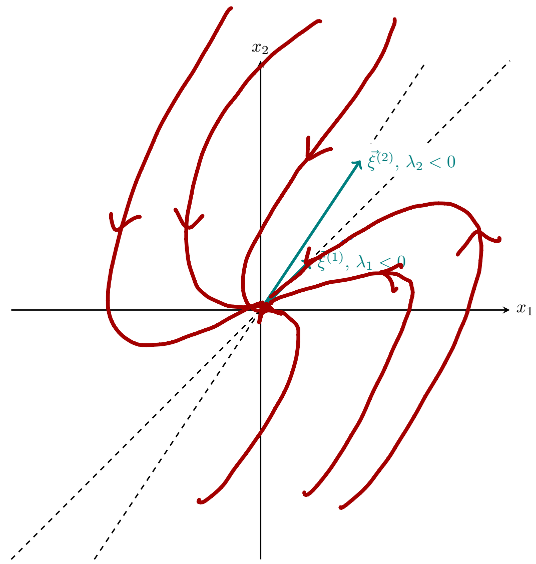
1. Find the general solution of the system of equations

$$\vec{x}' = \begin{bmatrix} 1 & -2 \\ 3 & -4 \end{bmatrix} \vec{x}$$

and draw a phase portrait for the system on the axes provided to the right.



Solution: The eigenvalues and associated eigenvectors are as follows

$$\lambda_1 = -1 : \quad \vec{\xi}^{(1)} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$
$$\lambda_2 = -2 : \quad \vec{\xi}^{(2)} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

Since

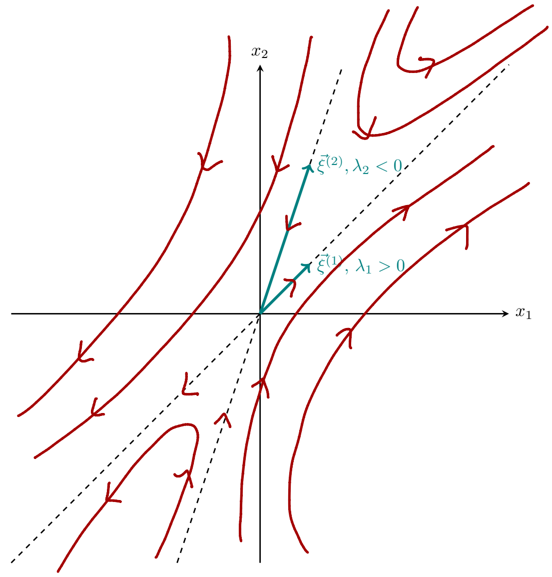
- $\lambda_1 < 0$ trajectories along this eigenvector will point towards the origin.
- $\lambda_2 < 0$ trajectories along this eigenvector will also point towards the origin.

This means that the origin is a nodal sink.

2. Find the general solution of the system of equations

$$\vec{x}' = \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix} \vec{x}$$

and draw a phase portrait for the system on the axes provided to the right.



Solution: The eigenvalues and associated eigenvectors are as follows

$$\lambda_1 = 1 : \quad \vec{\xi}^{(1)} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$
$$\lambda_2 = -1 : \quad \vec{\xi}^{(2)} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

Since

- $\lambda_1 > 0$ trajectories along this eigenvector will point away from the origin.
- $\lambda_2 < 0$ trajectories along this eigenvector will point towards the origin.

This means that the origin is a saddle point.