Worksheet 3 Solutions

Matrix Equations & Solution Sets

MATH 2250, Fall 2018

1. Write the soluton of the homogeneous system below in parametric vector form.

$$\begin{array}{rcrcrcl} x_1 & + & 3x_2 & + & x_3 & = & 0 \\ -4x_1 & - & 9x_2 & + & 2x_3 & = & 0 \\ & - & 3x_2 & - & 6x_3 & = & 0 \end{array} \tag{1}$$

Solution: Write the system in matrix form and then get into row-echelon form

$$\left[\begin{array}{rrr:r} 1 & 3 & 1 & 0 \\ -4 & -9 & 2 & 0 \\ 0 & -3 & -6 & 0 \end{array}\right] \underset{\substack{\text{Row}\\\text{Reduce}}}{=} \left[\begin{array}{rrr:r} 1 & 0 & -5 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

This system has a free variable. Re-writing in system form we have

$$\begin{array}{rcrcrcl} x_1 & + & & - & 5x_3 & = & 0 \\ & - & x_2 & + & 2x_3 & = & 0 \end{array}$$

Solving for each variable we have

$$\begin{aligned} x_1 - 5x_3 = 0 &\implies x_1 = 5x_3 \\ -x_2 + 2x_3 = 0 &\implies x_2 = 2x_3 \end{aligned}$$

Choosing $x_3 = t$ we can write the solution in parametric vector form as

$$\mathbf{x} = \begin{bmatrix} 5t \\ 2t \\ t \end{bmatrix} = t \begin{bmatrix} 5 \\ 2 \\ 1 \end{bmatrix}$$

2. Determine if the solution has a non-trivial solution. If the solution involves a free variable write your solution in parametric vector form.

$$\begin{array}{rcrcrcl} 2x_1 & - & 5x_2 & + & 8x_3 & = & 0 \\ -2x_1 & - & 7x_2 & - & x_3 & = & 0 \\ 4x_1 & + & 2x_2 & - & 7x_3 & = & 0 \end{array} \tag{2}$$

Solution: Write the system in matrix form and then get in row-echelon form

$$\left[\begin{array}{rrr:r} 2 & -5 & 8 & 0 \\ -2 & -7 & -1 & 0 \\ 4 & 2 & -7 & 0 \end{array}\right] \underset{\substack{\text{Row}\\\text{Reduce}}}{=} \left[\begin{array}{rrr:r} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

This means that the system has only the trivial solution.