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22C:019 Homework 4

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4. a. $P(1) = 1^3 = [(1(n+1))/2]^2$
b. base case: $1^3 = [(1(1+1))/2]^2 = 1$
c. inductive hypothesis P(k) is true
$1^3 + 2^3 ... + k^3 = [(k(k+1))/2]^2$
d. P(k) implies P(k+1)
$1^3 + 2^3 ... + k^3 + (k+1)^3 = [(k+1(k+2))/2]^2$
e. $[(k(k+1))/2]^2 + (k+1) = [(k+1(k+2))/2]^2$

6. base case : P(1) is true $1x1! = (1+1)!-1$
$= 1$
inductive step: p(k) is true
$1x1! + 2x2!+....+kxk! = (k+1)!-1$
show P(k+1) is true
$1x1! + 2x2!+....+kxk! + (k+1)(k+1)! = (k+1)!-1+(k+1)(k+1)!$
$= (k+2)!-1$
P(k+1) is true
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16. $P(n) = 1x2x3+2x3x4 + n(n+1)(n+2) = (n(n+1)(n+2)(n+3))/4$
base case: P(1) is true $1x2x3 = (1(1+1)(1+2)(1+3))/4$
inductive step : P(k) is true
$1x2x3+2x3x4 + k(k+1)(k+2) = (k(k+1)(k+2)(k+3))/4$
p(k+1) is true
$1x2x3+2x3x4 + k(k+1)(k+2)+ (k+1)(k+2)(k+3)$
$=((k+1)(k+2)(k+3)(k+4))/4$

20. $P(n) = 3^n < n$ for n is an int greater than 6
base step P(7)
$3^7 = 2187$ and $7! = 5040$; $3^< 7!$
inductive step: p(k) is ture for k>6
$3^k < k!$ prove p(k+1) is true
$P(k+1) = 3^{(k+1)}$
$= 3(3^k) <3(k!) <(k+1)!$

32. $3 / n^3+2n$ when n is positive
base case: $P(1) = 1^3+2(1) = 3$
inductive step: assume p(k) is true
$k^3+2k$ is divisible by 3
p(k+1) is true

$(k+1)^3 + 2(k+1) = k^3+1^3+3k^2+3k+2k+2$
$=(k^3+2K) +3(k^2+k+1)$ is / by 3
therefore $p(k+1)$ is true

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4.a. base case:
P(18) = one 4 cent stamp and two 7 cent stamps
P(19) = three 4 cent stamps and one 7 cent stamp
P(20) = five 4 cent stamps
P(21) = three 7 cent stamps
b.we can form m stamps for all $18 <= j <= k$ where $k >= 21$
c. k+1 cent stamps using just 4 cent and 7 cent stamps
d. P(k-3) is true; k-3 stamps. Put one more 4 cent stamp on the envelope and formed k+1 cent postage
e. principle of strong induction is true whenever $n >= 18$

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2b. $f(n+1) = 3f(n)$
$f(1) = 3f(1) = 3x1 = 3$
$f(2) = 3f(2) = 3x3 = 9$
$f(3) = 3f(3) = 3x9 = 27$
$f(4) = 3f(4) = 3x27 = 81$

2d. $f(n+1) = f(n)^2 + f(n) + 1$
$f(1) = f(0)^2 + f(0) + 1 = 1+1+1 = 3$
$f(2) = f(1)^2 + f(1) + 1 = 9 + 3 + 1 = 13$
$f(3) = f(2)^2 + f(2) + 1 = 169 + 13 + 1 = 183$
$f(4) = f(3)^2 + f(3) + 1 = 33489 + 183 + 1 = 33673$
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8a. $a_n = 4n-2$
$a_1 = 4(1) – 2 = 2$
$a_(n+1) = 4(n+1) -2 = 4n-2+4 = 4_n+4$
8d. $a_n = 1+(-1)^n$
$(-1)^n = a_n-1$
$a_1 = 1+(-1) = 0$
$a_n+1 = 1+ (-1)^n+1$
$= 2-a_n$
$a_1 = 0$ and $a_n+1 = 2-a_n$

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4. 72 types
8. 156000 ways