CMPSCI 403: Introduction to Robotics: Perception, Mechanics, Dynamics, and Control

HW 06

Due Online, Friday, Oct. 29th, 11:50 PM

Using the example given in class as a guide, derive the equations of motion (in MATLAB) for the double pendulum with parameter definitions as in the figure below. A torque τ_1 with vector $\tau_1\hat{k}$ (where $\hat{k} = \hat{\imath} \times \hat{\jmath}$) acts between the base and body 1, and a torque τ_2 with vector $\tau_2\hat{k}$ acts between body 1 and body 2. Assume gravity $g = 9.81\,\mathrm{m/s}^2$ in the $-\hat{\jmath}$ direction. Write a function to simulate the double pendulum. Provide a copy of your working code.

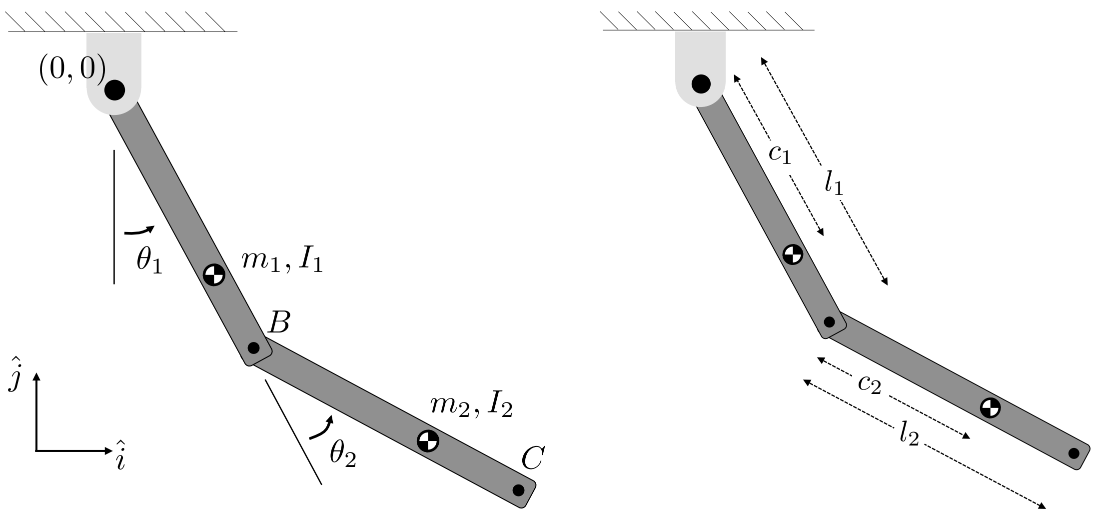

Figure 1: Double pendulum and parameter definitions.

1. With $\tau_1 = \tau_2 = 0$, solve the initial boundary value problem from the initial condition

$$\theta_1 = 3\,\mathrm{rad},\ \theta_2 = 0\,\mathrm{rad},\ \dot{\theta}_1 = \dot{\theta}_2 = 0\,\mathrm{rad/s}$$

   on the time interval $t = [0\,\mathrm{s},\ 7\,\mathrm{s}]$. Use parameters

$$m_1 = m_2 = 1\,\mathrm{kg},\ I_1 = I_2 = 0.05\,\mathrm{kg{\cdot}m}^2,\ l_1 = 1\,\mathrm{m},\ l_2 = 0.5\,\mathrm{m},\ c_1 = 0.5\,\mathrm{m},\ c_2 = .25\,\mathrm{m}.$$

   Plot $\theta_1(t)$ and $\theta_2(t)$. Does the solution display any repetitive and predictable patterns?

2. Plot the total system energy $(T + V)$ over the same interval. Verify energy conservation (You will see almost constant energy having a small drift).

3. (Optional) Derive the equations again considering the addition of three springs with potential energies

$$V_{e1} = \frac{1}{2}\kappa_1(\theta_1 - \theta_{1,0})^2 \qquad V_{e2} = \frac{1}{2}\kappa_2(\theta_2 - \theta_{2,0})^2$$

$$V_{e3} = \frac{1}{2}k_3(\|\mathbf{r}_C - \mathbf{r}_0\| - l_0)^2$$

   The vector $\mathbf{r}_0 = [r_x\ r_y]^T$ represents the attachment point for the last spring.

4. (Optional) After verifying energy conservation of your equations, simulate the system with

$$\kappa_1 = \kappa_2 = 10\,\mathrm{Nm/rad},\ k_3 = 50\,\mathrm{N/m}$$

$$\theta_{1,0} = \theta_{2,0} = 0,\ l_0 = 0,\ r_x = 0\,\mathrm{m},\ r_y = 0.5\,\mathrm{m}$$

   Apply $\tau_1 = -\dot{\theta}_1$ and $\tau_2 = -\dot{\theta}_2$ and use the same initial state as in step 2.