Ryan Dewsnap
32000408
CS403 Homework 2

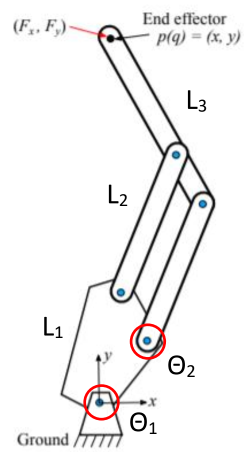
1. How many degrees of freedom (n) does this mechanism possess?

N = 5 links (including ground)
J = 5 joints
m = 3 (for planar mechanism)

$$\begin{aligned} n &= 3(5-1-5)+5 \\ &= 3(-1)+5 \\ &= -3+5 \\ n &= 2\ degrees\ of\ freedom \end{aligned}$$

2. Where would you attach motors to control the position of the end effector in the plane?

Since the mechanism has 2 degrees of freedom, 2 motors are needed for maximum control over the end effector. The motors would need to be placed in the locations indicated by the **red circles**. I believe the motor at the parallel linkage could also be placed on the adjacent joint since the other link is restricting its movement. Θ_1 describes the angle of the first motor and Θ_2 describes the angle of the second motor.



3. Based on your choice, define joint position vectors q ∈ R^n and derive forward kinematic equations that relate the inputs (motor angles) to the outputs (end effector position in Cartesian coordinate, i.e. p = (x, y)).

Due to the parallel linkage, the link with the end effector will always be 90 degrees relative to the first link. We will define the 3 link lengths $L_1 = 1$, $L_2 = 2$, $L_3 = 2$.

$$\begin{pmatrix} x_{ee} \\ y_{ee} \end{pmatrix} = \begin{pmatrix} 1 * cos(\Theta_1) + 2 * cos(\Theta_1 + \Theta_2) + 2 * cos(\Theta_1 + 90) \\ 1 * sin(\Theta_1) + 2 * sin(\Theta_1 + \Theta_2) + 2 * sin(\Theta_1 + 90) \end{pmatrix} = p$$

$$q = \begin{pmatrix} \Theta_1 \\ \Theta_2 \\ 90 - \Theta_2 \end{pmatrix}$$

4. Obtain the Jacobian matrix $J(q) = \frac{dp}{dq}$ symbolically in Matlab.

CS403_HW2.m

```matlab
clc
close all

l1 = 1; l2 = 2; l3 = 2;

syms theta1 theta2 theta3;

p = [l1*cos(theta1)+l2*cos(theta1+theta2)+l3*cos(theta1+90)];
q = [theta1; theta2; theta3];

jacobian = jacobian(p, q)
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Command Window

```
jacobian =

[ - 2*sin(theta1 + 90) - 2*sin(theta1 + theta2) - sin(theta1), -2*sin(theta1 + theta2), 0]

>>
```

5. Use this Jacobian to relate output forces (F_x, F_y) at the tip of the arm to statically equivalent input torques (τ_1, τ_2).