Assignment 2 - CS 4071 - Spring 2018

Due: 2018-02-19 Group #13: Robert DiMartino (dimartrt), Hayden Schiff (schiffha), Jeremiah Leak (leakjz)

1. Exercise 2.24

Problem: Give pseudocode for interpolation search, and analyze its worst-cast complexity.

2. Exercise 3.6

Problem: Using the Ratio Limit Theorem, prove the following:

$$O(108) \subset O(\ln n) \subset O(n) \subset O(n \ln n) \subset O(n^2) \subset O(n^3) \subset O(2^n) \subset O(3^n)$$

i.

$$\lim_{n \to \infty} \frac{108}{\ln n} = 0$$
$$\therefore O(108) \subset O(\ln n)$$

ii.

$$\lim_{n \to \infty} \frac{\ln n}{n}$$
$$= \lim_{n \to \infty} \frac{\frac{1}{n}}{1}$$
$$= \frac{0}{1} = 0$$
$$\therefore O(\ln n) \subset O(n)$$

iii.

$$\lim_{n \to \infty} \frac{n}{n \ln n}$$
$$= \lim_{n \to \infty} \frac{1}{1 + \ln n}$$
$$= 0$$
$$\therefore O(n) \subset O(n \ln n)$$

iv.

$$\lim_{n\to\infty} \frac{n \ln n}{n^2}$$
$$= \lim_{n\to\infty} \frac{1 + \ln n}{2n}$$
$$= \lim_{n\to\infty} \frac{\frac{1}{n}}{2}$$
$$= \frac{0}{2} = 0$$
$$\therefore O(n \ln n) \subset O(n^2)$$

v.

$$\lim_{n\to\infty} \frac{n^2}{n^3}$$
$$= \lim_{n\to\infty} \frac{1}{n}$$
$$= 0$$
$$\therefore O(n^2) \subset O(n^3)$$

vi.

$$\lim_{n\to\infty} \frac{n^3}{2^n}$$
$$= \lim_{n\to\infty} \frac{3n^2}{2^n \ln 2}$$
$$= \lim_{n\to\infty} \frac{6n}{2^n \ln^2 2}$$
$$= \lim_{n\to\infty} \frac{6}{2^n \ln^3 2}$$
$$= 0$$
$$\therefore O(n^3) \subset O(2^n)$$

vii.

$$\lim_{n\to\infty} \frac{2^n}{3^n}$$
$$= \lim_{n\to\infty} \left(\frac{2}{3}\right)^n$$
$$= 0$$
$$\therefore O(2^n) \subset O(3^n)$$