Assignment 1 - CS 4071 - Spring 2018

Due: 2018-01-26

1. Exercise 1.1

1. x^{123}

   $123 = 1111011_2$

   $x \to x \times (x)^2 = x^3 \to x \times (x^3)^2 = x^7 \to x \times (x^7)^2$

   $= x^{15} \to (x^{15})^2 = x^{30} \to x \times (x^{30})^2 = x^{61} \to x \times (x^{61})^2 = x^{123}$

2. x^{64}

   $64 = 1000000_2$

   $x \to (x)^2 = x^2 \to (x^2)^2 = x^4 \to (x^4)^2$

   $= x^8 \to (x^8)^2 = x^{16} \to (x^{16})^2 = x^{32} \to (x^{32})^2 = x^{64}$

3. x^{65}

   $65 = 1000001_2$

   $x \to (x)^2 = x^2 \to (x^2)^2 = x^4 \to (x^4)^2$

   $= x^8 \to (x^8)^2 = x^{16} \to (x^{16})^2 = x^{32} \to x \times (x^{32})^2 = x^{65}$

4. x^{711}

   $711 = 1011000111_2$

   $x \to (x)^2 = x^2 \to x \times (x^2)^2 = x^5 \to x \times (x^5)^2 = x^{11} \to (x^{11})^2 = x^{22} \to (x^{22})^2$

   $= x^{44} \to (x^{44})^2 = x^{88} \to x \times (x^{88})^2 = x^{177} \to x \times (x^{177})^2 = x^{355} \to x \times (x^{355})^2 = x^{711}$

2. Exercise 1.7

1. NaiveGCD(24, 108)
   = gcd(24, 84) = gcd(24, 60) = gcd(24, 36)
   = gcd(24, 12) = gcd(12,12) = 12
2. NaiveGCD(23, 108)
   = gcd(23, 85) = gcd(23, 62) = gcd(23, 39)
   = gcd(23, 16) = gcd(7, 16) = gcd(7, 9) = gcd(7, 2)
   = gcd(5, 2) = gcd(3, 2) = gcd(1, 2) = gcd(1, 1) = 1
3. NaiveGCD(89, 144)
   = gcd(89, 55) = gcd(34, 55) = gcd(34, 21)
   = gcd(13, 21) = gcd(13, 8) = gcd(5, 8) = gcd(5, 3)
   = gcd(2, 3) = gcd(2, 1) = gcd(1, 1) = 1

4. NaiveGCD(1953, 1937)
   = gcd(16, 1937) = gcd(16, 1921) = gcd(16, 1905)
   = gcd(16, 1889) = ... = gcd(16, 49) = gcd(16, 33) = gcd(16, 17) = gcd(16, 1)
   = gcd(15, 1) = gcd(14, 1) = ... = gcd(3, 1) = gcd(2, 1) = gcd(1, 1) = 1

3. Exercise 1.9

lcm(*a*, *b*) = *a* * *b* / gcd(*a*, *b*)

4. Exercise 1.17

Horner's rule for 5th degree polynomial:

$$((((a_5 \times x + a_4) \times x + a_3) \times x + a_2) \times x + a_1) \times x + a_0$$

For the polynomial $7x^5 - 3x^3 + 2x^2 + x - 5$ the coefficients are:

$$a_5 = 7, a_4 = 0, a_3 = -3, a_2 = 2, a_1 = 1, a_0 = -5$$

So Horner's rule is:

$$((((7 \times x + 0) \times x + -3) \times x + 2) \times x + 1) \times x + -5$$

For $x = 7$:

$$((((7 \times 7 + 0) \times 7 + -3) \times 7 + 2) \times 7 + 1) \times 7 + -5$$

$$= ((((49) \times 7 + -3) \times 7 + 2) \times 7 + 1) \times 7 + -5$$

$$= (((340) \times 7 + 2) \times 7 + 1) \times 7 + -5$$

$$= ((2,382) \times 7 + 1) \times 7 + -5$$

$$= (16,675) \times 7 + -5$$

$$= 116,720$$

8. Exercise 1.19

function `ModifiedHornerEval(a[0:n], v)`

Input: `a[0:n]` (an array of real numbers), `v` (a real number)

Output: the values of polynomial $P(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$ at $x = v$ and $x = -v$

```
v2 = v*v
if n is even:
    EvenSum <= a[n]
    OddSum <= 0
```

```
else:
    EvenSum <= 0
    OddSum <= a[n]
endif
for i <= n-1 downto 0 do:
    if i is even:
        EvenSum <= EvenSum * v2 + a[i]
    else:
        OddSum <= OddSum * v2 + a[i]
    endif
endfor
OddSum = OddSum*v
return(P(v) is EvenSum+OddSum and P(-v) is EvenSum-OddSum)
```