Assignment 4 - CS 4071 - Spring 2018

Due: 2018-02-19

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1. Exercise 6.5

Problem: Solve the following instance of the knapsack problem for capacity C=30.

i	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**
v_i	60	50	40	30	20	10	5	1
w_i	30	100	10	10	8	8	1	1

To solve this knapsack problem, we want to greedily select objects of decreasing densities, i.e. the ratio of value to weight, v_i/w_i, until the knapsack is full. So first we compute the densities for each object.

i	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**
$d_i = v_i/w_i$	2	0.5	4	3	2.5	1.25	5	1

Then we repeatedly select available objects with the next highest density until the knapsack is full (or we run out of items).

0. Starting capacity: 30
1. Select: 6 ($d = 5, w = 1$), fraction: 1, remaining capacity: 29
2. Select: 2 ($d = 4, w = 10$), fraction: 1, remaining capacity: 19
3. Select: 3 ($d = 3, w = 10$), fraction: 1, remaining capacity: 9
4. Select: 4 ($d = 2.5, w = 8$), fraction: 1, remaining capacity: 1
5. Select: 0 ($d = 2, w = 30$), fraction: 1/30, remaining capacity: 0

The fractions, f_i, of each item that yield the optimal value for the knapsack are provided in the table below.

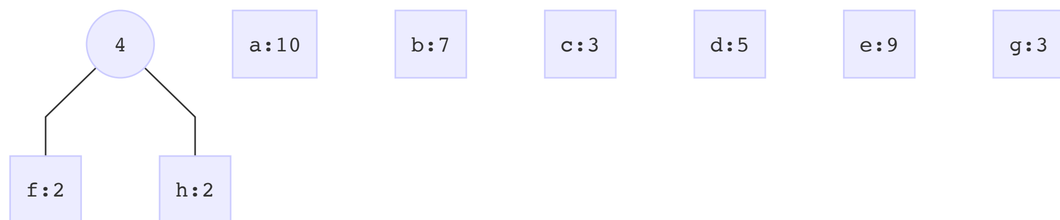
i	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**Total**
f_i	1/30	0	1	1	1	0	1	0	
$f_i v_i$	2	0	40	30	20	0	5	0	97
$f_i w_i$	1	0	10	10	8	0	1	0	30

2. Exercise 6.9

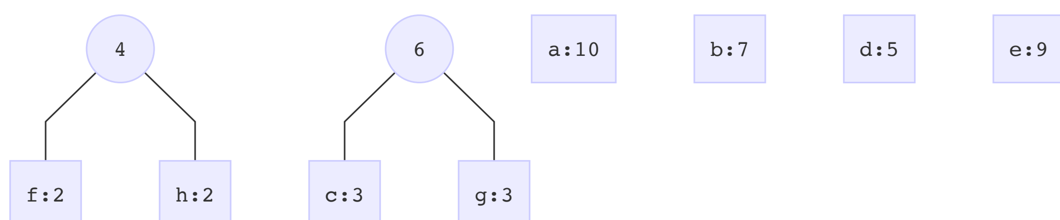
Problem: Trace the action of *HuffmanCode* for the letters *a, b, c, d, e, f, g, h* occuring with frequencies 10, 7, 3, 5, 9, 2, 3, 2.

0. Initial forest

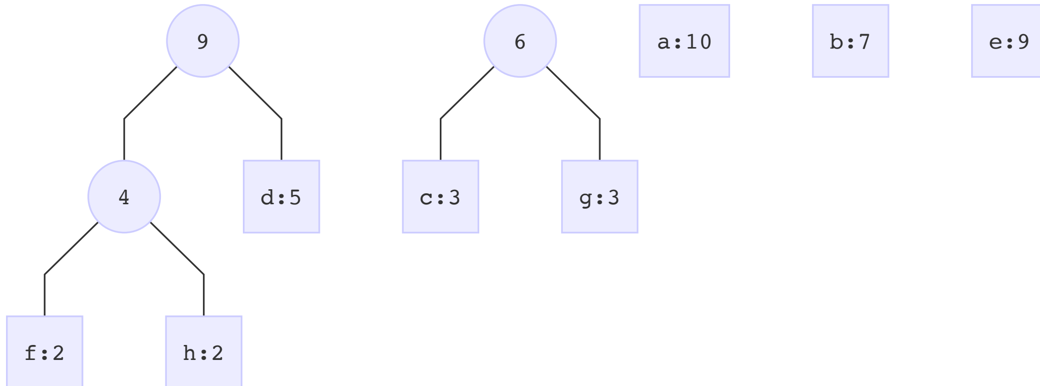
1. Stage 1



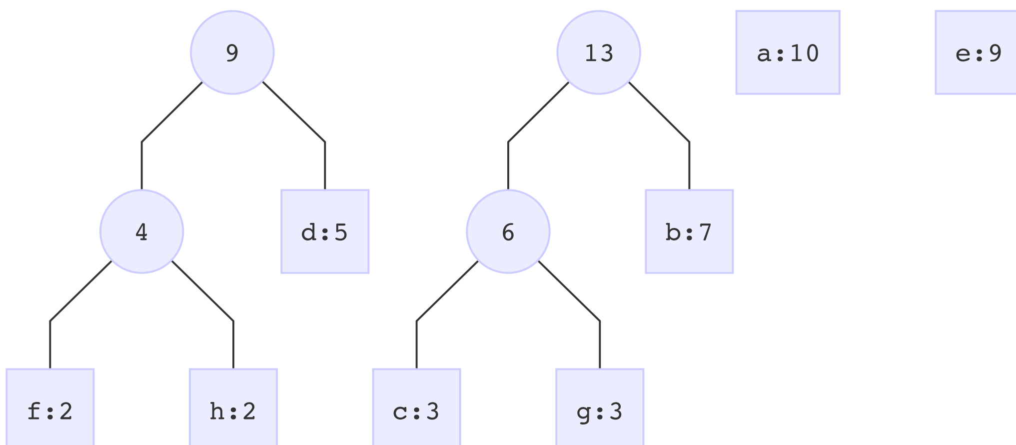
2. Stage 2



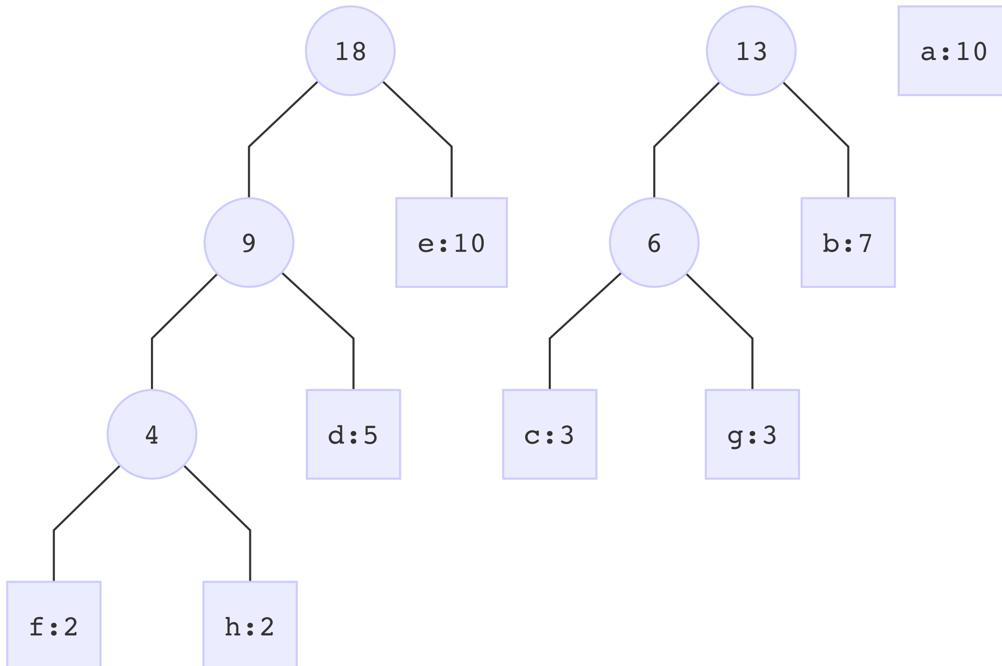
3. Stage 3



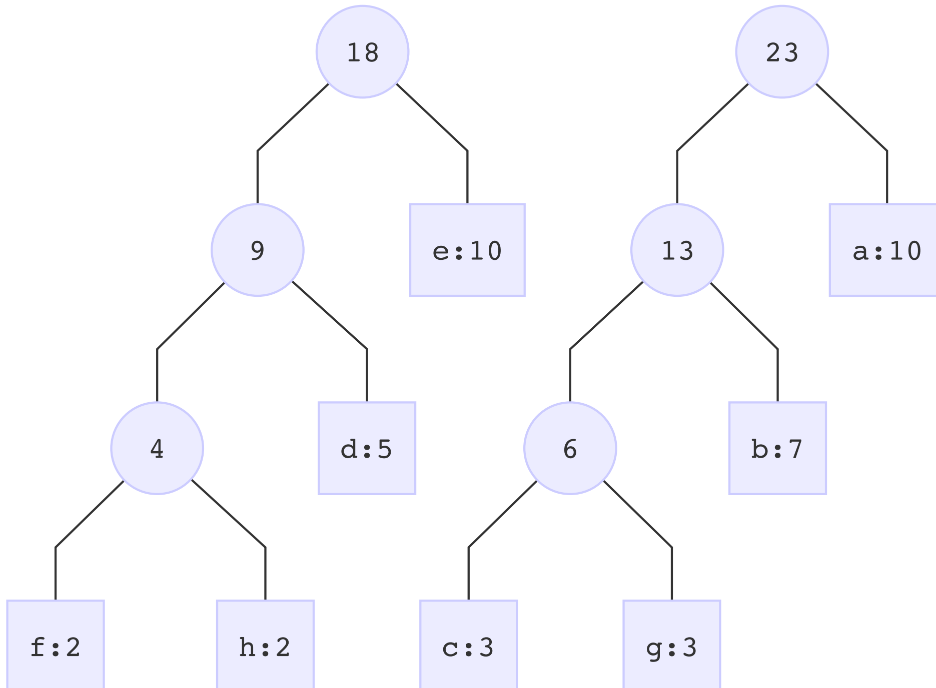
4. Stage 4



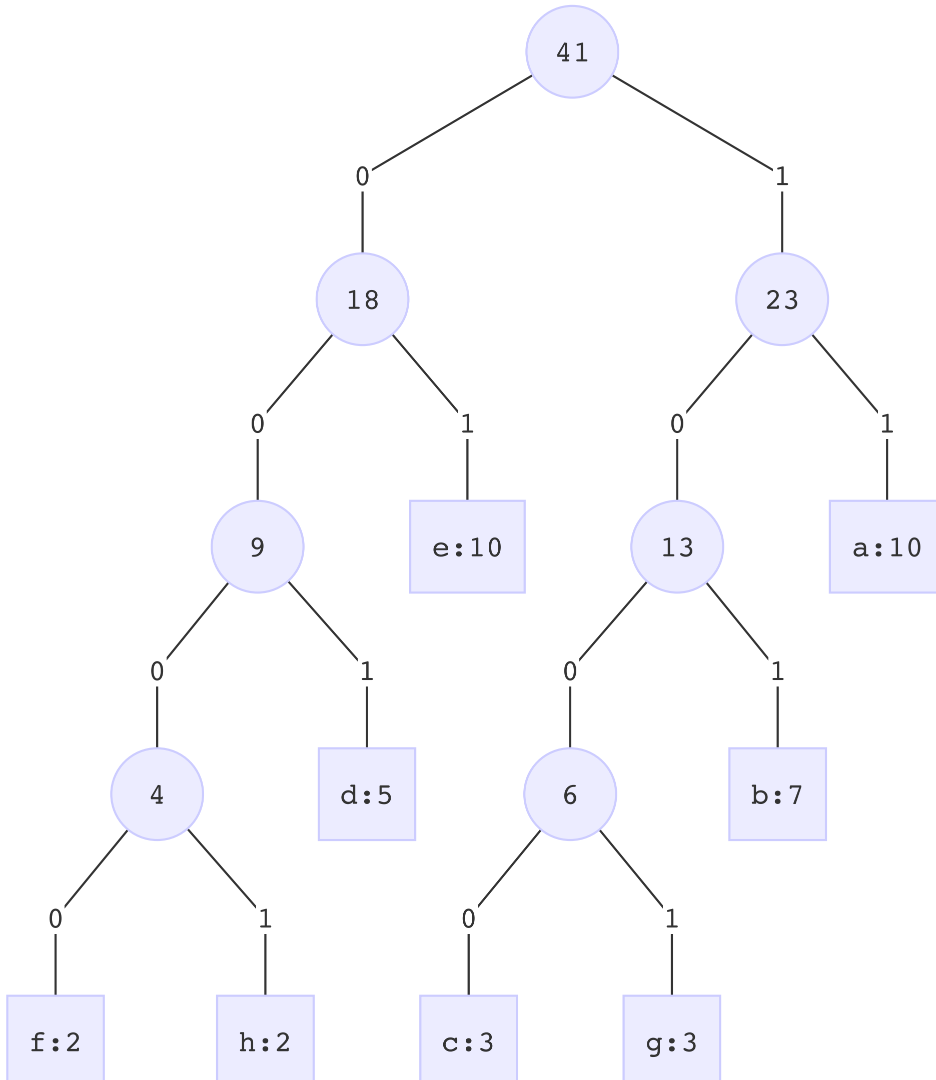
5. Stage 5



6. Stage 6



7. Stage 7: Huffman Tree


41
0
1
18
23
0
1
0
1
9
e:10
13
a:10
0
1
0
1
4
d:5
6
b:7
0
1
0
1
f:2
h:2
c:3
g:3

letter	frequency	encoding
a	10	`11`
b	7	`101`
c	3	`1000`
d	5	`001`
e	9	`01`
f	2	`0000`
g	3	`1001`
h	2	`0001`

3. Exercise 6.10

Problem: Given the Huffman Code Tree in Figure 6.6, decode the string `1001111100001101111001`

> **Figure 6.6**
>
> | letter | frequency | encoding |
> |---|---|---|
> | *a* | 9 | `01` |
> | *b* | 8 | `00` |
> | *c* | 5 | `101` |
> | *d* | 3 | `1001` |
> | *e* | 15 | `11` |
> | *f* | 2 | `1000` |

We read binary characters from the encoded string one at a time. Because the Huffman Code is a prefix code, when the binary substring we've read matches one of the letter encodings from Figure 6.6 we can unambiguously determine which letter is decoded.

Reading the string from the front:

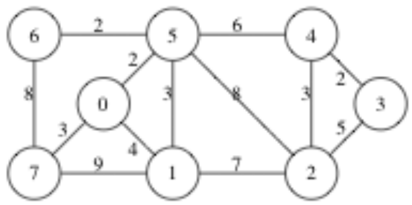
1. `1001` = *d*

2. `11` = *e*
3. `1000` = *f*
4. `01` = *a*
5. `101` = *c*
6. `11` = *e*
7. `1001` = *d*

Then `10011110000110111100 1` decoded is "defaced".

4.

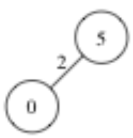
Consider the following weighted graph G.:



a)

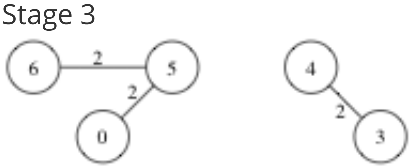
Problem: Trace the action of procedure `Kruskal` for G.

1. Stage 1

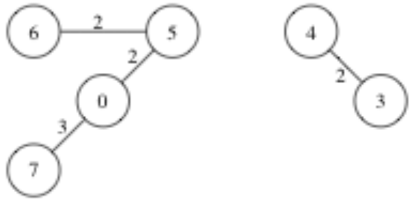


2. Stage 2

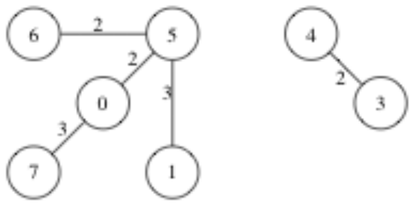
3. Stage 3



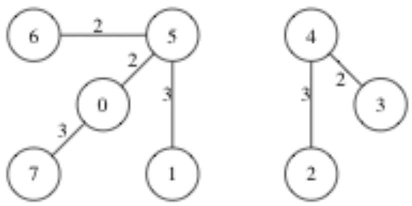
4. Stage 4



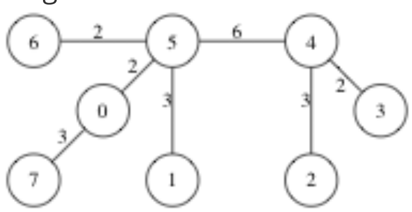
5. Stage 5



6. Stage 6



7. Stage 7



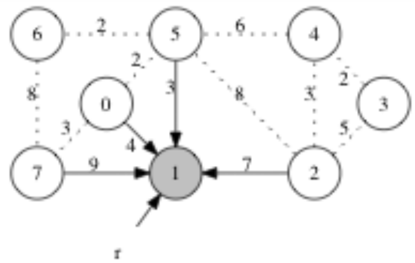
b)

Problem: Trace the action of procedure `Prim` for G, with r = 1.

0. Stage 0

i	0	1	2	3	4	5	6	7
nearest[i]	4	0	7	inf	inf	3	inf	9
parent[i]	1	-1	1	-	-	1	-	1
inTree[i]	F	T	F	F	F	F	F	F

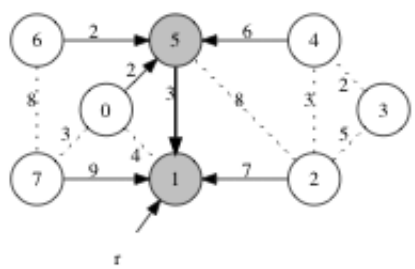
Weight = 0



1. Stage 1

i	0	1	2	3	4	5	6	7
nearest[i]	2	0	7	inf	6	3	2	9
parent[i]	5	-1	1	-	5	1	5	1
inTree[i]	F	T	F	F	F	T	F	F

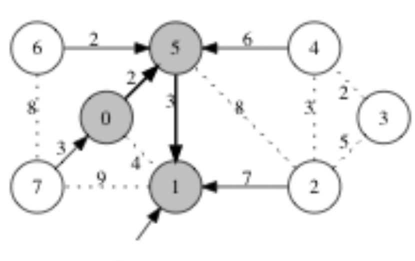
Weight = 3



2. Stage 2

i	0	1	2	3	4	5	6	7
nearest[i]	2	0	7	inf	6	3	2	3
parent[i]	5	-1	1	-	5	1	5	0
inTree[i]	T	T	F	F	F	T	F	F

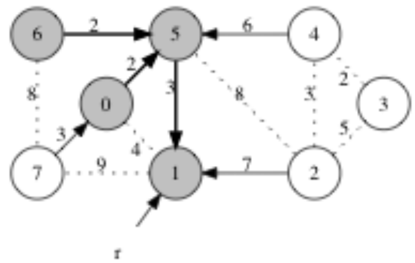
Weight = 5



3. Stage 3

i	0	1	2	3	4	5	6	7
nearest[i]	2	0	7	inf	6	3	2	3
parent[i]	5	-1	1	-	5	1	5	0
inTree[i]	T	T	F	F	F	T	T	F

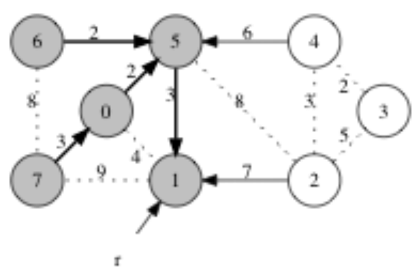
Weight = 7



4. Stage 4

i	0	1	2	3	4	5	6	7
nearest[i]	2	0	7	inf	6	3	2	3
parent[i]	5	-1	1	-	5	1	5	0
inTree[i]	T	T	F	F	F	T	T	T

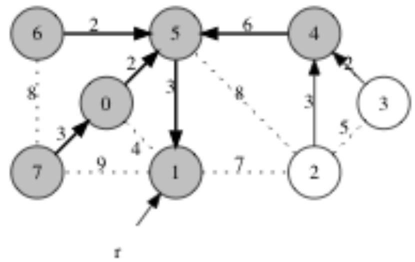
Weight = 10



5. Stage 5

i	0	1	2	3	4	5	6	7
nearest[i]	2	0	3	2	6	3	2	3
parent[i]	5	-1	4	4	5	1	5	0
inTree[i]	T	T	F	F	T	T	T	T

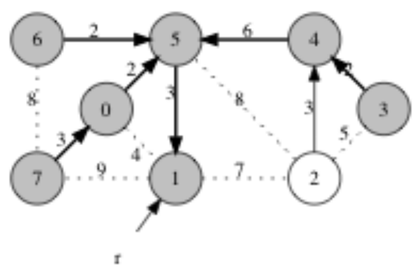
Weight = 16



6. Stage 6

i	0	1	2	3	4	5	6	7
nearest[i]	2	0	3	2	6	3	2	3
parent[i]	5	-1	4	4	5	1	5	0
inTree[i]	T	T	F	T	T	T	T	T

Weight = 18

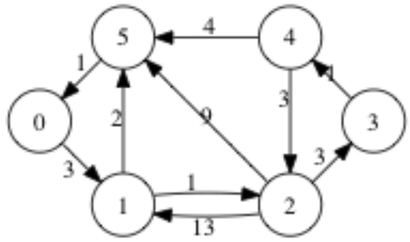


7. Stage 7

i	0	1	2	3	4	5	6	7
nearest[i]	2	0	3	2	6	3	2	3
parent[i]	5	-1	4	4	5	1	5	0
inTree[i]	T	T	T	T	T	T	T	T

Weight = 21

5.

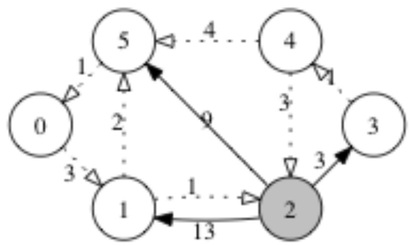


a)

Problem: Trace the action of procedure Dijkstra for the digraph with initial vertex $r = 2$.

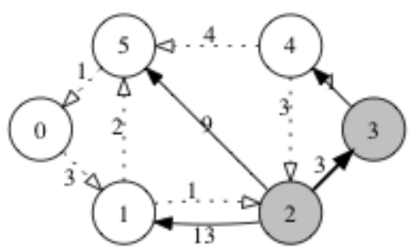
0. Stage 0

i	0	1	2	3	4	5
dist[i]	inf	13	0	3	inf	9
parent[i]	-	2	-1	2	-	2
inTree[i]	F	F	T	F	F	F



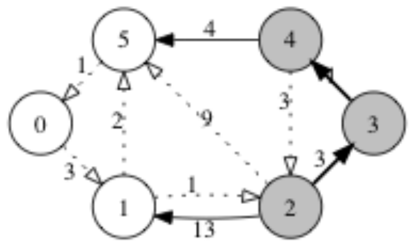
1. Stage 1

i	0	1	2	3	4	5
dist[i]	inf	13	0	3	4	9
parent[i]	-	2	-1	2	3	2
inTree[i]	F	F	T	T	F	F



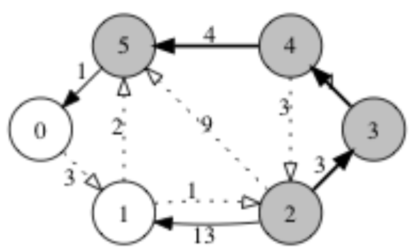
2. Stage 2

i	0	1	2	3	4	5
dist[i]	inf	13	0	3	4	8
parent[i]	-	2	-1	2	3	4
inTree[i]	F	F	T	T	T	F



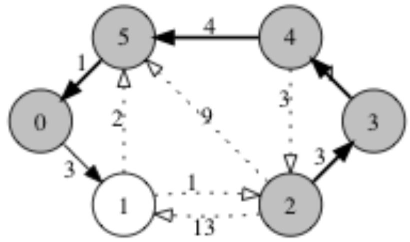
3. Stage 3

i	0	1	2	3	4	5
dist[i]	9	13	0	3	4	8
parent[i]	5	2	-1	2	3	4
inTree[i]	F	F	T	T	T	T



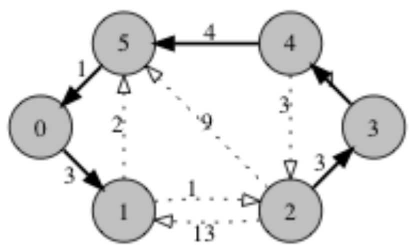
4. Stage 4

i	0	1	2	3	4	5
dist[i]	9	12	0	3	4	8
parent[i]	5	0	-1	2	3	4
inTree[i]	T	F	T	T	T	T

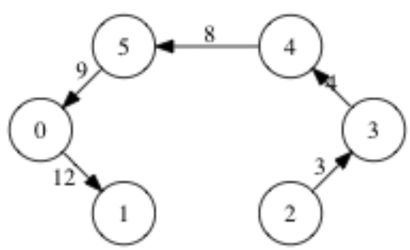


5. Stage 5

i	0	1	2	3	4	5
dist[i]	9	12	0	3	4	8
parent[i]	5	0	-1	2	3	4
inTree[i]	T	T	T	T	T	T



Minimum spanning path:

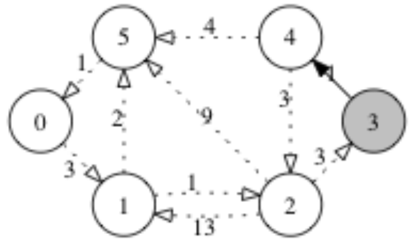


b)

Problem: Repeat for $r = 3$.

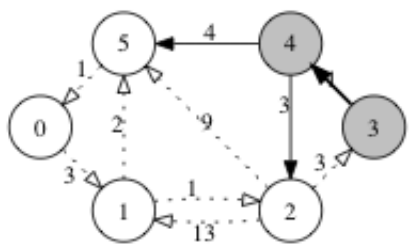
0. Stage 0

i	0	1	2	3	4	5
dist[i]	inf	inf	inf	0	1	inf
parent[i]	-	-	-	-1	3	-
inTree[i]	F	F	F	T	F	F



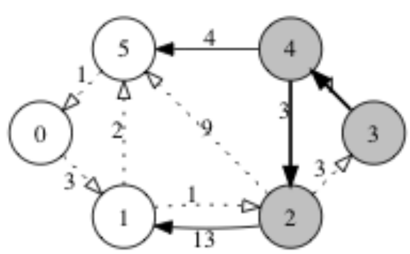
1. Stage 1

i	0	1	2	3	4	5
dist[i]	inf	inf	4	0	1	5
parent[i]	-	-	4	-1	3	4
inTree[i]	F	F	F	T	T	F



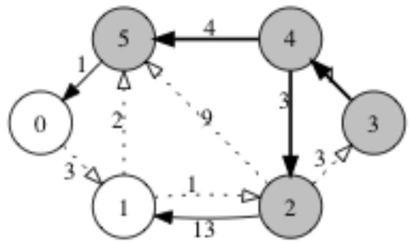
2. Stage 2

i	0	1	2	3	4	5
dist[i]	inf	17	4	0	1	5
parent[i]	-	2	4	-1	3	4
inTree[i]	F	F	T	T	T	F



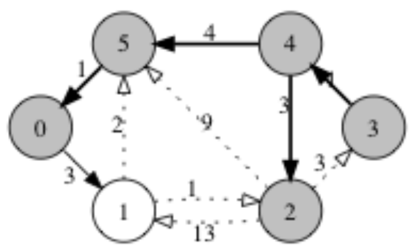
3. Stage 3

i	0	1	2	3	4	5
dist[i]	6	17	4	0	1	5
parent[i]	5	2	4	-1	3	4
inTree[i]	F	F	T	T	T	T



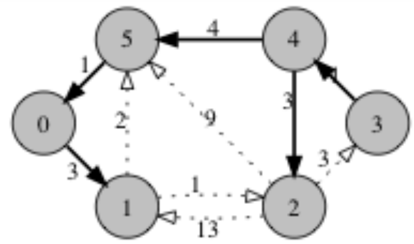
4. Stage 4

i	0	1	2	3	4	5
dist[i]	6	9	4	0	1	5
parent[i]	5	0	4	-1	3	4
inTree[i]	T	F	T	T	T	T

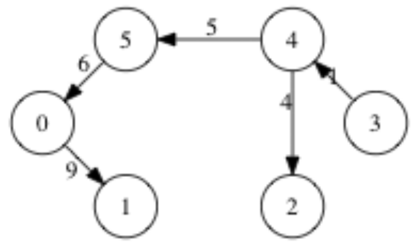


5. Stage 5

i	0	1	2	3	4	5
dist[i]	6	9	4	0	1	5
parent[i]	5	0	4	-1	3	4
inTree[i]	T	T	T	T	T	T



Minimum spanning path:



6. Exercise 7.13

Problem: Verify formula (7.4.4)

$$AB = \begin{bmatrix} a_{00} & a_{01} \\ a_{10} & a_{11} \end{bmatrix} \begin{bmatrix} b_{00} & b_{01} \\ b_{10} & b_{11} \end{bmatrix} = \begin{bmatrix} a_{00}b_{00} + a_{01}b_{10} & a_{00}b_{01} + a_{01}b_{11} \\ a_{10}b_{00} + a_{11}b_{10} & a_{10}b_{01} + a_{11}b_{11} \end{bmatrix}$$

$$AB \stackrel{?}{=} \begin{bmatrix} m_1 + m_4 - m_5 + m_7 & m_3 + m_5 \\ m_2 + m_4 & m_1 + m_3 - m_2 + m_6 \end{bmatrix}$$

$$\Longrightarrow \begin{aligned} &(1)\ a_{00}b_{00} + a_{01}b_{10} \stackrel{?}{=} m_1 + m_4 - m_5 + m_7 \\ &(2)\ a_{00}b_{01} + a_{01}b_{11} \stackrel{?}{=} m_3 + m_5 \\ &(3)\ a_{10}b_{00} + a_{11}b_{10} \stackrel{?}{=} m_2 + m_4 \\ &(4)\ a_{10}b_{01} + a_{11}b_{11} \stackrel{?}{=} m_1 + m_3 - m_2 + m_6 \end{aligned}$$

To verify 7.4.4, we need to verify each of the four equations about, where:

$$\begin{aligned} m_1 &= (a_{00} + a_{11})(b_{00} + b_{11}) && = a_{00}b_{00} + a_{00}b_{11} + a_{11}b_{00} + a_{11}b_{11} \\ m_2 &= (a_{10} + a_{11})(b_{00}) && = a_{10}b_{00} + a_{11}b_{00} \\ m_3 &= a_{00}(b_{01} - b_{11}) && = a_{00}b_{01} - a_{00}b_{11} \\ m_4 &= a_{11}(b_{10} - b_{00}) && = a_{11}b_{10} - a_{11}b_{00} \\ m_5 &= (a_{00} + a_{01})(b_{11}) && = a_{00}b_{11} + a_{01}b_{11} \\ m_6 &= (a_{10} - a_{00})(b_{00} + b_{10}) && = a_{10}b_{00} + a_{10}b_{10} - a_{00}b_{00} - a_{00}b_{10} \\ m_7 &= (a_{01} - a_{11})(b_{10} + b_{11}) && = a_{01}b_{10} + a_{01}b_{11} - a_{11}b_{10} - a_{11}b_{11} \end{aligned}$$

1.

$$
\begin{aligned}
&m_1 + m_4 - m_5 + m_7 \\
&= (a_{00}b_{00} + a_{00}b_{11} + a_{11}b_{00} + a_{11}b_{11}) + (a_{11}b_{10} - a_{11}b_{00}) - \dots \\
&\quad (a_{00}b_{11} + a_{01}b_{11}) + (a_{01}b_{10} + a_{01}b_{11} - a_{11}b_{10} - a_{11}b_{11}) \\
&= a_{00}b_{00} + \cancel{a_{00}b_{11}}^{1} + \cancel{a_{11}b_{00}}^{2} + \cancel{a_{11}b_{11}}^{3} + \cancel{a_{11}b_{10}}^{4} - \cancel{a_{11}b_{00}}^{2} + \dots \\
&\quad - \cancel{a_{00}b_{11}}^{1} - \cancel{a_{01}b_{11}}^{5} + a_{01}b_{10} + \cancel{a_{01}b_{11}}^{5} - \cancel{a_{11}b_{10}}^{4} - \cancel{a_{11}b_{11}}^{3} \\
&= a_{00}b_{00} + a_{01}b_{10}
\end{aligned}
$$

2.

$$
\begin{aligned}
m_3 + m_5 &= (a_{00}b_{01} - a_{00}b_{11}) + (a_{00}b_{11} + a_{01}b_{11}) \\
&= a_{00}b_{01} - \cancel{a_{00}b_{11}} + \cancel{a_{00}b_{11}} + a_{01}b_{11} \\
&= a_{00}b_{01} + a_{01}b_{11}
\end{aligned}
$$

3.

$$
\begin{aligned}
m_2 + m_4 &= (a_{10}b_{00} + a_{11}b_{00}) + (a_{11}b_{10} - a_{11}b_{00}) \\
&= a_{10}b_{00} + \cancel{a_{11}b_{00}} + a_{11}b_{10} - \cancel{a_{11}b_{00}} \\
&= a_{10}b_{00} + a_{11}b_{10}
\end{aligned}
$$

4.

$$
\begin{aligned}
&m_1 + m_3 - m_2 + m_6 \\
&= (a_{00}b_{00} + a_{00}b_{11} + a_{11}b_{00} + a_{11}b_{11}) + (a_{00}b_{01} - a_{00}b_{11}) + \dots \\
&\quad -(a_{10}b_{00} + a_{11}b_{00}) + (a_{10}b_{00} + a_{10}b_{10} - a_{00}b_{00} - a_{00}b_{10}) \\
&= \cancel{a_{00}b_{00}}^{1} + \cancel{a_{00}b_{11}}^{2} + \cancel{a_{11}b_{00}}^{3} + a_{11}b_{11} + \cancel{a_{00}b_{01}}^{4} - \cancel{a_{00}b_{11}}^{2} + \dots \\
&\quad - \cancel{a_{10}b_{00}}^{5} - \cancel{a_{11}b_{00}}^{3} + \cancel{a_{10}b_{00}}^{5} + a_{10}b_{10} - \cancel{a_{00}b_{00}}^{1} - \cancel{a_{00}b_{10}}^{4} \\
&= a_{10}b_{01} + a_{11}b_{11}
\end{aligned}
$$

Since each of the four equations are true, formula 7.4.4 is verified.

7.

Problem: Design a greedy algorithm to solve the optimal merge pattern problem. In this problem, we have n sorted files of lengths $f_0, f_1, \dots, f_{n-1}$, and we wish to merge them into a single file by a sequence of merges of pairs of files. To merge two files of lengths m_1 and m_2 takes $m_1 + m_2$ operations. Describe your algorithm in general, and illustrate it for files of lengths 10,7,3,5,9,2,3,2. (Can you make a connection with Huffman codes and Exercise 1 of this assignment?)