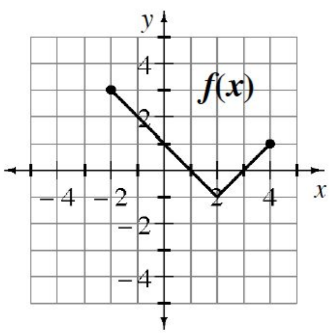
Chapter 1.2.4 and 1.2.5

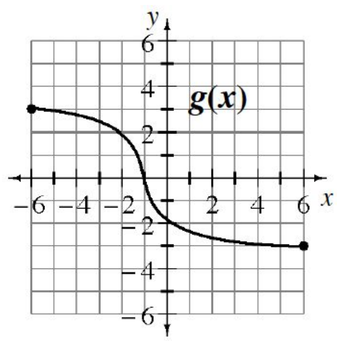
The purpose of this lesson is to:
Understand what a composite function is and what it looks like
Understand how to find the inverse of a function
Understand the attributes of even and odd functions

WARM UP

1-61.

Given the two functions *f* and *g* graphed below:





a. State domain and range of *f*.
b. State domain and range of *g*.
c. What is $f(g(-2))$?
d. What is $g(f(-2))$?
e. What is $f(f(3))$?
f. Why is $f(g(5))$ undefined?

1-62.

If $f(x) = x^2$, $g(x) = x + 1$, and $h(x) = \frac{1}{x}$, express each given function as a composition of *f*, *g*, and/or *h*.

For example, $k(x) = (x + 1)^2$ can be expressed as $k(x) = f(g(x))$.

a. $j(x) = \frac{1}{x^2}$

b. $m(x) = \frac{1}{x} + 1$

c. $p(x) = x^4$

d. $n(x) = \frac{1}{x^2+1}$

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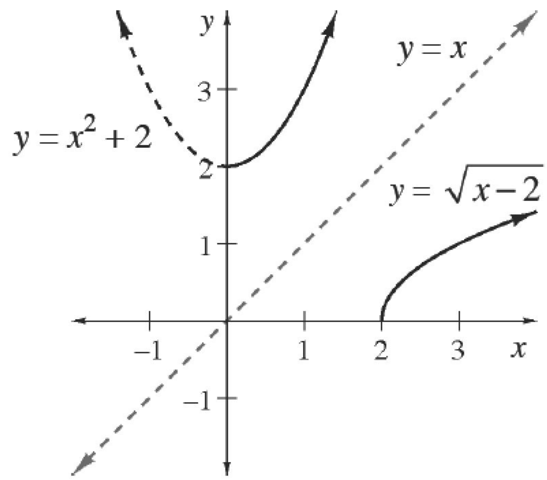
NOTES

Composite Function: A function whose values are found from _____ _________ functions where one is used as the _____________ variable in the other. This means the ___________ of one function are used as the __________ of the other. I.e: f(g(x)).

Inverse Functions

We say that f and g are _________ __________ if ________ for all x in the domain of g and _________ for all x in the domain of f. We write ___ for the inverse of f. So $g(x) = f^{-1}(x)$ and $f(x) = g^{-1}(x)$.

Note: The domains of some functions must be restricted in order for the inverses to be inverse functions of each other.



For example $f(x) = x^2 + 2$ for $x \geq 0$ and $g(x) = \sqrt{x-2}$ for $x \geq 2$.

If we graph $y = f(x)$ and $y = f^{-1}(x)$ on the same set of axes then their graphs are ____________ across the line $y = x$.

If a function satisfies the __________ _______ ______, then an inverse function exists.

$y = \frac{x+5}{2x-1}$, find y^{-1}

Even and Odd Functions

A function f is an ______ function if, for all x in its domain, ____________.

A function f is an ______ function if, for all x in its domain, ___________.

Example: If $f(x) = 2x^3 + \sin(x)$, then $f(-x) =$

$=$

$=$

Therefore $f(-x) =$

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Task Card

1-64.

INVERSE FUNCTIONS

Let $h(x) = 3x + 2$ and $j(x) = \frac{x-2}{3}$

a. Write an equation for $h(j(x))$. What do you notice?

b. Functions such that $f(g(x)) = g(f(x)) = x$ are called inverse functions. Explain why this notation shows that f and g are inverse functions.

c. If $f(x) = e^x + 2$, write the equation of a function g such that $f(g(x)) = x$.

1-65.

An inverse function undoes what a function does. For example, $sin(\pi/6) = 1/2$, which means the sine function takes the angle $\frac{\pi}{6}$ and returns the ratio $\frac{1}{2}$. Therefore the *inverse sine* function takes the ratio $\frac{1}{2}$ and returns the angle $\pi/6$. The notation for inverse functions can be confusing; the inverse of f is written f^{-1}. The inverse sine function is written $\sin^{-1}(x)$. $\sin^{-1}(x)$ is also referred to as arcsin(x). Note: $\sin^{-1}(x) \neq \frac{1}{sin(x)}$.

Write each of the statements below entirely in symbols.

a. The inverse sine of ½ is $\frac{\pi}{6}$.

b. When the inverse of a function g is applied to 7, the result is 5.

1-66.

In parts (a) and (b), solve for x.

a. $f(x) = 2^x$

b. $g(x) = \frac{x+1}{x}$

c. Now write the inverse equations of f and g. What do you notice?

1-67.

a. Study the table for the functions f and g at right. f does not have an inverse function. Explain why not.

x	$f(x)$	$g(x)$
-2	5	-3
-1	8	-1
0	9	0
1	8	2
2	5	3

b. Evaluate:

i. $g^{-1}(2)$

ii. $f(g^{-1}(2))$

iii. $g^{-1}(g(2))$

c. If $h(3)=4$ and $j(x)=h^{-1}(x)$, what is $j(4)$?

1-78. Decide if each of the following functions is even, odd, or neither. Use the formal definitions to prove your answers, and then describe the type of symmetry each function has.

a. $f(x) = x^2$

b. $g(x) = 2x^3$

c. $h(x) = 2 + x^4$

d. $j(x) = 2 + x^5$

e. $k(x) = \sin(2x)$

f. $j(x) = \arctan(x)$

1-79.

Robert wonders about even and odd functions. He wants to know what happens when you combine even functions with odd functions. If f is even and g is odd, determine if the following combinations of f and g are even, odd, or neither. Be sure to explain your decision.

a. $h(x) = f(x) + g(x)$

b. $j(x) = f(x) \cdot g(x)$

c. $k(x) = f(g(x))$

d. $l(x) = g(f(x))$

e. $m(x) = |f(x)|$

f. $n(x) = |g(x)|$

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Homework

1-70.

Selected values of a continuous *even* function are shown below. 1-70 HW eTool (Desmos). Homework Help ✎

x	0	1	2	3
$f(x)$	0	2	4	6

a. What are $f(-1)$, $f(-2)$, and $f(-3)$?

b. Sketch a possible graph of the function on the domain $-3 \le x \le 3$.

c. Sketch another possible graph of the function on the domain $-3 \le x \le 3$.

d. Can the graph be a quadratic function? If so, write a possible equation for the function. If not, explain why not.

1-71.

State the domain of each of the following functions. Homework Help ✎

a. $f(x) = \sqrt{x+2}$

b. $g(x) = \frac{1}{x-4} + 3$

c. $h(x) = \log(x - 4)$

d. $j(x) = \sqrt{\frac{2-x}{x}}$

1-72.

Helen thinks $\sqrt{x^2} = x$. Felicia does not agree. Homework Help ✎

a. Use various values of x to check whether or not Helen is correct.

b. Write an accurate expression for $\sqrt{x^2}$.

1-82.

Are the following functions even, odd, or neither? Homework Help ✎

a. $y = x^{1/3}$

b. $y = -x^2 + 4$

c. $y = x^3 + x^2 + 1$

1-83.

If $f(x) = x^2 + 5x$ and $g(x) = x + 3$, evaluate each of the following expressions. Homework Help ✎

a. $f(-2)$

b. $g(-2)$

c. $f(g(-2))$

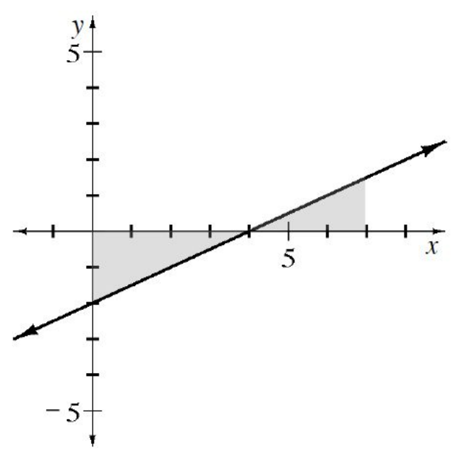
d. $g(f(-2))$

e. $f(f(-2))$

f. $g(g(-2))$

1-84.

Examine the graph of $f(x) = 0.5x - 2$ at right.



a. Calculate the area of the shaded region using geometry. (Recall that area below the x-axis is considered negative.)

b. What is the value of k if the area under the curve for $0 \le x \le k$ is 10? How did you obtain your solution?