Chapter 5.2.1 and 5.2.2

The purpose of this lesson is to:
Understand how to find the derivative of two functions that are multiplied together using the Product Rule
Understand how to find the derivative of a composite function using the Chain Rule

WARM UP

5-47. Today you are going to write the derivative of functions of the form $j(x) = f(x) \cdot g(x)$. Some of these are easy to do using algebra.

a. Write the derivative of:
   i. $j(x) = x(x + 3)$

   ii. $j(x) = (x + 10)(x + 5)$

   iii. $j(x) = (x - 1)^2$

b. Do you notice any patterns?

c. Try some more difficult cases:
   i. $j(x) = (2x + 1)(x + 5)$

   ii. $j(x) = (5x - 1)(x + 10)$

   iii. $j(x) = (3x^2 + 1)(-x^3 + 2)$

d. What do you notice?

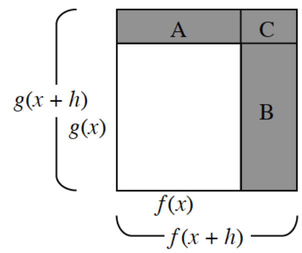
e. Now consider a function that is not easy to expand: $j(x) = x \sin(x)$. It would be nice to develop a method to differentiate the products of two functions. Explain how the functions above can be modeled as $j(x) = f(x) \cdot g(x)$.

Chapter 5.2.1 and 5.2.2

NOTES

5-48. When Hana looked at the function $j(x) = f(x) \cdot g(x)$ she immediately thought of a rectangle whose length and width were $f(x)$ and $g(x)$ and area was $j(x)$. Hana wants to write an equation for j' by using the definition of the derivative as a limit.

a. If $j(x)$ represents the area of a rectangle, what does $j'(x)$ represent?



b. To determine the value of $j'(x)$, she set up the problem as follows:

$$j'(x) = \lim_{h \to 0} \frac{f(x+h) \cdot g(x+h) - f(x) \cdot g(x)}{h}$$

Hana sketches a rectangle in which the shaded area represents the numerator. Examine her diagram and explain why:

$f(x + h) \cdot g(x + h) - f(x) \cdot g(x) =$ (area A) + (area B) + (area C)

The Chain Rule

The Chain Rule allows us to differentiate composite functions.

If $h(x) = f(g(x))$, then ____________________________. Ex: $(3x^2 + 2x + 4)^{-2}$

A careful proof of this result is beyond the scope of this course.

The Product Rule

If $f'(x)$ and $g'(x)$ exist and $j(x) = f(x) \cdot g(x)$, then __________________________________.

Use your results to differentiate $j(x) = (4x - 1)(3x + 1)$. Then verify the derivative using algebra and using the Power Rule. Are your results the same?

5-65. Determine $\frac{d}{dx}((3x + 5)^2)$ in two ways: first by using the Chain Rule and then by using the Product Rule. Verify that your answers match.

Chapter 5.2.1 and 5.2.2

Task Card

5-49. The derivative technique you summarized in part (g) of problem 5-48 is called the **Product Rule**. It seems to work for simple binomials like $f(x) = (4x - 1)(3x + 1)$, but what about other kinds of functions?

a. Differentiate $y = x \sin(x)$ by using the Product Rule.
b. In this case, you cannot check your derivative the same way you did for the previous problems. Find another way to verify your result.
c. Repeat this process for $y = (x - 2)^2(x + 5)^3$.

5-50. SNEAKY PRODUCT RULE PROBLEMS

Write the derivative of each function below by first rewriting each as a product of two differentiable functions.

a. $f(x) = (5x^2 + 8)^2$

b. $f(x) = \frac{5}{(x-8)^2}$

c. $f(x) = \frac{x^3}{(x-8)^2}$

d. $f(x) = \sin^2(x)$

5-67. Differentiate the following functions using the Chain Rule and verify your result with your graphing calculator. You do not need to simplify your answers.

$f(x) = \sin(2x + 3)$ $\quad$ $f(x) = \sqrt{16 - x^2}$ $\quad$ $f(x) = \cos(\sin(x))$

Chapter 5.2.1 and 5.2.2

Homework

5-51. Evaluate the following integrals without a calculator. Homework Help ✎

a. $\int_1^4 \left(3\sqrt{u} + \frac{1}{\sqrt{u}}\right) du$

b. $\int_0^2 6(x-4)^2\, dx$

c. $\int \frac{d}{dx}\left[g(x)\right] dx$

d. $\int_0^1 k^{5/3}\, dk$

e. $\int_1^2 \frac{4}{n^3}\, dn$

5-52. Differentiate each function if possible. If the function is one you cannot differentiate yet, say so and explain how you know. Be sure to use the appropriate notation.
Homework Help ✎

a. $y = x^{100} + 100x$

b. $H(t) = t^4 - 6\sqrt{t}$

c. $y = x \cdot 2^x$

d. $y = x^8 \cos(x)$

e. $f(x) = x^5(x^3 + \pi)$

f. $y = \frac{7}{x} - \frac{x}{7}$

Chapter 5.2.1 and 5.2.2

Homework

5-73. Calculate the area of the region bounded by $y = \sqrt{x+1}$ and $y = x^2 - 2x - 3$.

5-74. Chris found the derivative of $(2x + 5)^2$ as shown below. Homework Help ✎

$$\frac{d}{dx}((2x + 5)^2) = \frac{d}{dx}(4x^2 + 20x + 25) = 8x + 20$$

To check his solution, he tried applying the Power Rule:

$$\frac{d}{dx}((2x + 5)^2) = 2(2x + 5) = 4x + 10$$

5-75. Use your derivative tools to determine $\frac{dh}{dx}$ for each function below. Homework Help ✎

a. $h(x) = \sqrt[3]{\sin(x)}$

b. $h(x) = 6x^{2/3}$

c. $h(x) = \frac{1}{2}x^5\sqrt{x-1}$

d. $h(x) = 2\sin(\frac{1}{x})$

e. $h(x) = -4\cos(x)\sin(x)$

f. $h(x) = \cos(x^2)$