Chapter 5.2.5 and 5.3.3

The purpose of this lesson is to:
Find the Derivatives of other trig functions using the Quotient Rule
Learn to Maximize area and Volume

WARM UP

5-100. Which of the expressions below are equivalent? Which are not? Explain.

a. $\sin^{-1}(x)$
b. $\csc(x)$
c. $\arcsin(x)$

5-101. MORE TRIGONOMETRIC DERIVATIVES

Now that you know the Quotient Rule, you can differentiate the other trigonometric functions:

$y = \tan(x)$ $\quad$ $y = \cot(x)$ $\quad$ $y = \sec(x)$ $\quad$ $y = \csc(x)$

To do this, you will need to use the trigonometric identities.

Rewrite $\tan(x)$ as a ratio of sine and cosine. Then find $\frac{d}{dx}(\tan(x))$.

a. Prove that $\frac{d}{dx}(\sec(x)) = \sec(x)\tan(x)$.

b. Write similar formulas for $\frac{d}{dx}$ (csc(x)) and $\frac{d}{dx}$ (cot(x)).

c. Organize your derivatives into a chart like the one below.

Trig Function	Derivative
$y = \sin(x)$	
$y = \tan(x)$	
$y = \sec(x)$	

Trig "Co-Function"	Derivative
$y = \cos(x)$	
$y = \cot(x)$	
$y = \csc(x)$	

d. Look for a pattern that will help you remember these derivatives.

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NOTES

To find the minimum or maximum area first _________ ___ _________. Label your pieces and fill them into your ________________ for the area or volume. Finally, take the _____ _________ and set it equal to ________. Solve.

5-128. Shyrley makes pendants. She starts with a 10 cm length of wire and cuts it in two pieces, which she then forms into two geometric shapes.

a. Suppose one piece of wire is bent into the perimeter of a square pendant and the other piece of wire is bent into the perimeter of an equilateral triangle pendant. Where should she cut the wire in order to minimize the combined areas of the two pendants?

b. Now suppose one piece is bent into the circumference of a circular pendant, and the other into the perimeter of a square pendant. Now how should she cut the wire in order to minimize the combined area of the two pendants?

c. Suppose Shyrley wants to maximize the combined area of the circle and square. Now how should she cut the wire?

5-129. MR. WAHMAN'S WHOPPER WAFFLE CONES!

Mr. Wahman loves to eat ice cream. He especially loves waffle cones, but he can never get the ice cream shop to make them the way he likes them. He wants cones which hold as much ice cream as possible, but without ice cream sticking over the edge of the cone (too messy). He decides to design his own waffle cone. His waffle iron makes circular waffles, which he can twist into a cone by first making one cut along a radius. He needs to know what size cone to make. What are the dimensions of the cone that holds the most ice cream (level to the top)?

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Task Card

5-102. A MIXTURE OF DERIVATIVE PROBLEMS

Using the properties you have learned, differentiate each of the following functions.

a. $f(x) = \frac{\sin(x)}{x}$

b. $f(x) = x^2 \cos(\sqrt{x})$

c. $f(x) = \tan(x^2)$

d. $f(x) = \sqrt{\frac{x^2+1}{x}}$

e. $f(x) = \frac{1}{1+\frac{1}{x}}$

f. $f(x) = \sec(x)\csc(x)$

g. $f(x) = \cos^3\left(\frac{x}{x+1}\right)$

h. $f(x) = \cot\left(\sqrt{\cos(x+1)}\right)$

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Homework

5-106. Use your derivative tools to write the second derivative, $\frac{d^2y}{dx^2}$, of each function below. Homework Help ✎

a. $y = \frac{\sin(x)}{x}$

b. $y = \csc^2(x) - \cot^2(x)$

c. $y = \sqrt{\frac{1}{x}}$

5-132. For each part below, what can you conclude (if anything) about f if you know the given information? (Note: Each part is different function.) Homework Help ✎

a. $f'(-2) = 0$ and $f''(-2) > 0$
b. $f'(x) < 0$ for $x > 3$, $f'(x) > 0$ for $x < 3$, and $f'(3) = 0$
c. $f''(3) > 0$
d. $f''(3) < 0$
e. $f''(3) = 0$
f. f is continuous at $x = 3$, but not differentiable there.
g. f is defined and continuous everywhere, and has just one critical point at $x = 2$, which is a local maximum.
h. $f''(3)$ does not exist and $f''(x) < 0$ for $x < 3$ and $f''(x) > 0$ for $x > 3$

5-137. $\frac{d}{dx}\int_0^{\pi/3} \sin(x)\,dx =$ Homework Help