Chapter 4.2.1 and 4.2.2

The purpose of this lesson is to:

Look for patterns among definite integrals whose values can be interpreted geometrically, and generalize those patterns with "area functions" that can be used to evaluate the area under the curve where the bounds of integration are a constant and any other value of *x*.

Distinguish between definite and indefinite integrals. Use area functions to integrate numerically.

WARM UP

4-40. Determining slope functions is powerful because they offer the ability to express the rate of change of a function for all values of *x* in the domain. However, what about the area under a function? How can we also write an area function, *A*, that will calculate the area for all values *t* in the domain?

Calculate the areas of the following regions:

a. $A(0)=\int_0^0 5\,dt$

b. $A(1)=\int_0^1 5\,dt$

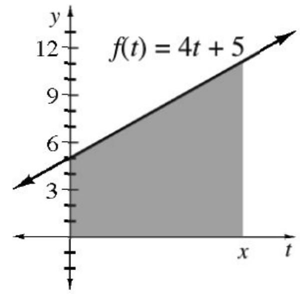
c. $A(2)=\int_0^2 5\,dt$

d. $A(x)=\int_0^x 5\,dt$

e. Generalize your findings for the area under the curve of any constant function.

That is, write an area function, $A(x)$, that is equal to $\int_0^x c\,dt$ where c is a constant.

4-41. What if *f* is not a constant function? Examine the graph of the linear equation *f*(*t*) = 4*t* + 5 at right.



a. What can the equation $A(x)=\int_0^x (4t+5)\,dt$ be used to calculate?

b. Use $A(x)=\int_0^x (4t+5)\,dt$ to evaluate *A*(2) and *A*(9).

c. Use $A(x)=\int_0^x (4t+5)\,dt$ to write an equation for *A*(*x*).

d. Generalize $A(x)=\int_0^x (mt+b)\,dt$. What is the significance of *mt* + *b*?

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NOTES

Definite Integrals with Variable Bounds

__________ __________ can be expressed on bounds ______, where *a* is a _______ and *x* is __________. This type of definite integral results in a function called an ____________, which is often notated with an uppercase letter. Typically this is written as:

The antiderivative $F(x)$ shown above can be interpreted as an _______ ___________ that will determine the area under the graph of $y = f(t)$ from $t = a$ to any value of x. $F(x)$ can also be interpreted as an _____________ __________, which computes the accumulated value of f from $t = a$ to any value of x.

An antiderivative equation depends on the integrand, f, and the lower bound, a. Consequently, changing the lower bound will change the antiderivative by a constant:

________________________, where C is a constant.

There are many algebraic techniques that can be used to write the equation of an antiderivative function. Some of these techniques will be explored in this course. However, it is important to know that not every function has a *closed form* antiderivative. That is, not every antiderivative can be found using operations from arithmetic, algebra, or trigonometry.

4-52. Write a general area function for each integral.

a. $\int_0^x 2dp$

b. $\int_0^x (5r+2)dr$

c. $\int_0^x \left(\frac{3}{2}-k\right)dk$

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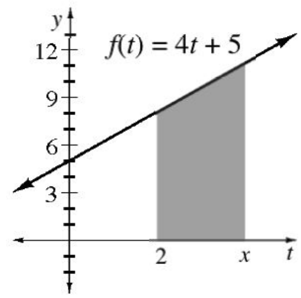
Task Card

4-42. Does it matter what the lower bound of the integral is? What if the lower bound is not 0, but instead is another constant?

a. Discuss with your team how the area under the curve changes as the lower bound changes. Test your conjecture by comparing functions *A* and *B* below.



$$A(x)=\int_0^x (4t+5)\,dt \qquad B(x)=\int_2^x (4t+5)\,dt$$

b. Using a method similar to that used in part (c) of problem 4-29, write an equation for the area function, *B*. Then, compare it to the equation of *A*.

c. Demonstrate algebraically that $B(x)=\int_2^x (4t+5)\,dt = A(x)-A(2)$. Also demonstrate this relationship geometrically using area.

d. Do the two area functions grow at the same rate? Does $A'(x) = B'(x)$? Is this true? Why or why not?

e. Write an equation using $A(x)$ to evaluate $G(x)=\int_c^x (4t+5)\,dt$. Explain geometrically what *G* measures.

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Task Card

4-53. Use your results from problem 4-52 and the formula $\int_c^x f(t)\,dt = A(x) - A(c)$ to evaluate the following definite integrals.

a. $\int_3^8 2\,dx$

b. $\int_{-2}^5 8\,dx$

c. $3\int_2^6 (5x+2)\,dx$

d. $\int_1^3 5x\,dx + \int_1^3 2\,dx$

e. $\int_4^{10} \left(\frac{3}{2} - k\right) dk$

f. $\int_0^{-2} \left(\frac{3}{2} - k\right) dk$

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Homework

4-54. A millipede is moving along the edge of a centimeter ruler with a velocity, v, given by the function $v(t) = 3t - 9$ where $v(t)$ is measured in centimeters per second.

a. Write and evaluate a definite integral that can be used to calculate the displacement of the millipede over the interval $t = 3$ sec to $t = 6$. Hint: Start by sketching a graph of $y = v(t)$.

b. Write and evaluate a definite integral that represents the displacement, $s(t)$, of the millipede from $t = 3$ to any time $t = x$. Then calculate the value of $s(6)$.

c. If the millipede is standing on the 15 centimeter mark at $t = 3$ seconds, then where will it be located at $t = 6$ seconds? How about at $t = 10$ seconds? Show and explain your process.

4-55. Tommy is trying to determine $\int_2^3 f(t)\,dt$. He already knows that $\int_0^x f(t)\,dt = 9x^2 - 2x$. Help him calculate $\int_2^3 f(t)\,dt$.

4-44. What is the general antiderivative, F, for each function, f, below? Homework Help ✎

a. $f(x) = \cos(x)$

b. $f(x) = -\frac{2}{x^2}$

c. $f(x) = -9x^{1/3}$

4-45. Use a Riemann sum with 20 rectangles to approximate the following integrals. Then use the numerical integration feature of your graphing calculator to check your answers. Homework Help ✎

a. $\int_0^4 (2 - 4x^{3/2})\,dx$

b. $\int_1^8 \sqrt{4x+3}\,dx$

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Homework

4-46. Explain why there are an infinite number of antiderivatives for each function. Demonstrate this fact with an example. Homework Help ✎

4-56. Ji Hee is trying to evalute the integrals in the parts (a) through (c) below. She already knows that $\int_0^x g(m)\,dm = \frac{4x+1}{x+2}$. Help her calculate: Homework Help ✎

a. $2\int_0^3 g(m)\,dm$

b. $\int_{-1}^0 g(m)\,dm$

c. $\int_{-1}^5 g(m)\,dm$

4-57. Mateo and Ignacio want to calculate $\int_{-2}^{-5} f(x)\,dx$ where *f* is an even function. They know that $\int_2^5 f(x)\,dx = -3$. Mateo thinks the answer will be –3 while Ignacio thinks the answer will be 3. Who is correct? Explain. Homework Help ✎

4-63. Differentiate. Homework Help ✎

a. $\frac{d}{dx}(7 \cdot \sqrt[3]{x})$

b. $\frac{d}{dm}(3m^{-7} - 7m^3)$

c. $\frac{d}{dk}(k^0)$