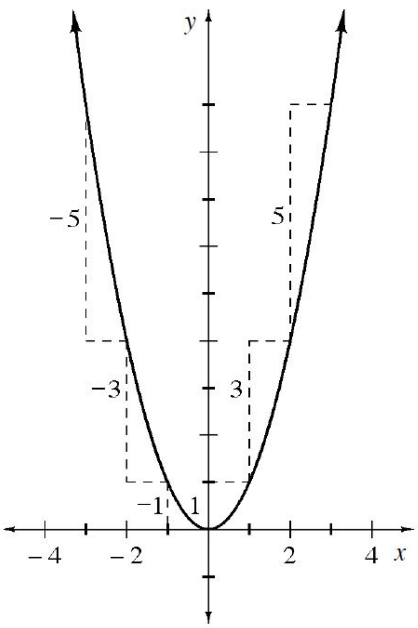
Chapter 1.3

The purpose of this lesson is to:
Understand how a table of values connects to a function
Understand how to find a finite difference from a function
Understand what finite differences tell you about a function
Understand how to make predictions using finite differences

WARM UP

1-96.



HOW DOES IT CHANGE?

A large focus in calculus is on how functions change. Whether a function is increasing or decreasing, how the change is occurring can be measured. Consider the graph of $f(x) = x^2$. Review the graph and note how the function values are changing. Slope triangles are shown for $-3 \le x \le 3$.

The table below shows the finite differences, Δy , which are the differences between consecutive y-values.

x	−3	−2	−1	0	1	2	3
$f(x)$	9	4	1	0	1	4	9

Δy **−5** **−3** **−1** **+1** **+3** **+5**

Patterns among the finite differences reveal information about the way a function is changing. According to the finite differences in the table above, how are the function values changing as x increases? Do you see a pattern?

1-97.

Does this pattern hold true for other quadratic functions? Using a table with finite differences labeled, describe how Δy changes as x increases for the functions listed below. Look for patterns within the table as well as between these different quadratic functions. Record your findings.

a. $f(x) = 2x^2 - 3x + 1$

b. $f(x) = -3x^2 + 6$

1-98.

Based on your results from problem 1-97, how do quadratic functions change? Summarize your findings by using the general equation: $f(x)=ax^2+bx+c$. In other words, when the graph of a function is a parabola, what will the graph of its finite differences look like?

1-99.

Now consider the functions below. Describe how each of function changes as *x* increases. Be sure to try a variety of examples to verify your observations.

Constant Functions	Linear Functions	Cubic Functions
$f(x) = a$	$f(x) = ax + b$	$f(x) =ax^3+bx^2+cx+d$

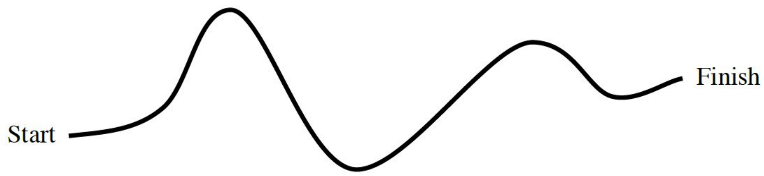
1-100.

Make a prediction about how the graph of $f(x) = x^n$ changes.

Chapter 1.3

1-110.

The path of the roller coaster is shown below.



a. Describe the path of the roller coaster so that someone who has not seen it can draw it. Be sure to include words that will help to describe the steepness of the curve as well as its direction.
b. When writing a slope statement, it is reasonable to start at the left of the graph and move right—just like you read a sentence in English. Make a list of words that are useful when describing the path of a graph.

1-111.

The following two slope statements describe the *same* graph. Read both statements. Then sketch a graph of the function described.

"The graph starts off flat at the left and starts to increase at $x = -3$ until the graph flattens out at $x = 0$. Then the values decreases until the graph flattens out around $x = 2$ and continues to stay flat."

"The graph starts off flat at the left but slowly gets steeper. The slope starts getting really steep at $x = -2$, but at $x = -1$, the slope becomes less steep. At $x = 0$, the slope is flat for an instant and then gets steeper but negative. At $x = 1$, the slope starts to become less steep again, eventually getting closer and closer to zero slope."

Chapter 1.3

Task Card

1-112.

Finite differences can be used to analyze the slope of a graph at various x-values. Some graphs have predictable slope patterns. For example, in Lesson 1.3.1, you found patterns in the way polynomial functions change. For example, cubic functions change with a quadratic pattern, quadratic functions change with a linear pattern, and linear functions change with a constant pattern. What about other functions?

Your team will be assigned one of the function groups listed below to investigate. For each of the two functions in your function group, complete the following tasks:

- Graph the function.
- State the domain and range using appropriate notation.
- Analyze the finite differences.
- Write a slope statement.

Function Group	Equation (a)	Equation (b)
Rational	$f(x) = \frac{1}{x}$	$f(x) = \frac{1}{x^2}$
Trigonometric	$f(x) = \sin(x)$	$f(x) = \cos(x)$
Exponential	$f(x) = (0.5)^x$	$f(x) = 2^x$
Logarithmic	$f(x) = \log(x)$	$f(x) = \log_2(x)$
Radical	$f(x) = \sqrt{x}$	$f(x) = \sqrt[3]{x}$

Chapter 1.3

Homework

1-102.

State the domain of each of the following functions. Homework Help ✎

a. $f(x) = \frac{x-2}{x^2+4}$

b. $g(x) = \frac{\sqrt{x+2}}{x^2+x}$

1-103.

Multiple Choice: The values of x for which the graphs of $y = x + 3$ and $y^2 = 6x$ intersect are:

A. −3 and 3

B. −3

C. 3

D. 0

E. None of these

1-105.

Given $f(x) = 2x^2 - 3$: Homework Help ✎

a. Evaluate $f(2)$.

b. Without writing the equation of the inverse, determine $f^{-1}(5)$. Explain your process.

c. Solve for x if $f(x + 2) - f(x - 2) = 64$.

Chapter 1.3

Homework

Write each of the following items below 10 times. Yes. That's your homework. Do it. Yes, I will have the TAs actually count. It is of note that 9 is not 10. So 10 times each. **Deal with it.**

Pro Tip: Legitimately say out loud each thing you're writing as you write it. Do NOT mindlessly copy, it won't help you remember these.

Common Trigonometric Identities

Reciprocal

$\sec(\theta) = \frac{1}{\cos(\theta)}$ $\csc(\theta) = \frac{1}{\sin(\theta)}$

$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$ $\cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)}$

Pythagorean

$\sin^2(\theta) + \cos^2(\theta) = 1$

$\tan^2(\theta) + 1 = \sec^2(\theta)$

$1 + \cot^2(\theta) = \csc^2(\theta)$

Angle Sum

$\sin(a \pm b) = \sin(a)\cos(b) \pm \cos(a)\sin(b)$

$\cos(a \pm b) = \cos(a)\cos(b) \mp \sin(a)\sin(b)$

Double Angle

$\sin(2a) = 2\sin(a)\cos(a)$

$$\cos(2a) = \begin{cases} \cos^2(a) - \sin^2(a) \\ 2\cos^2(a) - 1 \\ 1 - 2\sin^2(a) \end{cases}$$