Chapter 2.1.3 and 2.2.1

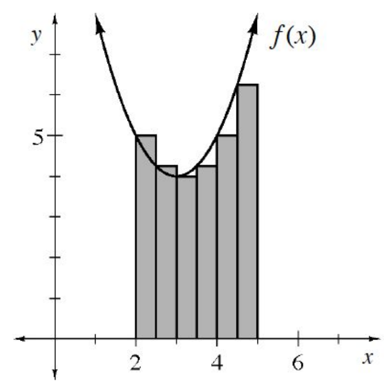
The purpose of this lesson is to:

- Approximate area under a curve using Riemann sums and write Riemann sums using sigma notation.

- Investigate the informal definition of a limit as a prediction and understand what it means for a limit to exist (informally).

Notes

2-17. This problem will help you develop a shortcut for writing and evaluating the summation of areas when approximating the area under a curve.

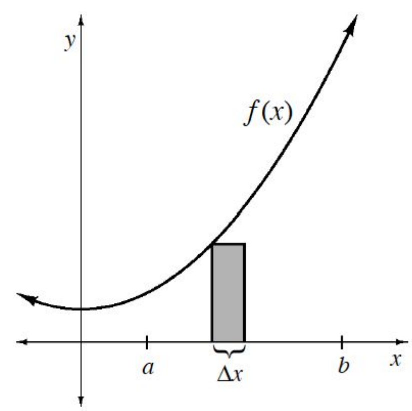


a. Given $f(x) = x^2 - 6x + 13$, approximate the area under the curve for $2 \le x \le 5$ using six left endpoint rectangles as shown in the diagram at right.

b. Write and evaluate the summation in Sigma Notation

Approximating Area Using Left Endpoint Rectangles

The area under the _____________ __ from ______ to _____ can be approximated by ____________the areas of __________________________. In the diagram at right, the shaded rectangle is a typical rectangle, one that represents all rectangles across the region.



If each rectangle has a width of ____, then the summation can be written in _______________ as follows:

$$A \approx \sum_{i=0}^{n-1} \left[\Delta x \cdot f\left(a + \Delta x \cdot i\right) \right]$$

Using rectangles to approximate area under the curve is generally known as a ______________, named in honor of Georg Friedrich Bernhard Riemann (1826 – 1866).

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Notes

2-30. Use summation notation to write an expression that will approximate the area under the curve for a function *f* over each interval below, using the specified number of left endpoint rectangles.

a. $3 \le x \le 10$; 21 rectangles

b. $-2 \le x \le 6$; 10 rectangles

2-31. Rewrite your summation expression from part (a) of problem 2-30 so that right endpoint rectangles are used to approximate the area instead.

2-32. Write a general expression using summation notation that can be used to approximate area under a curve using right endpoint rectangles.

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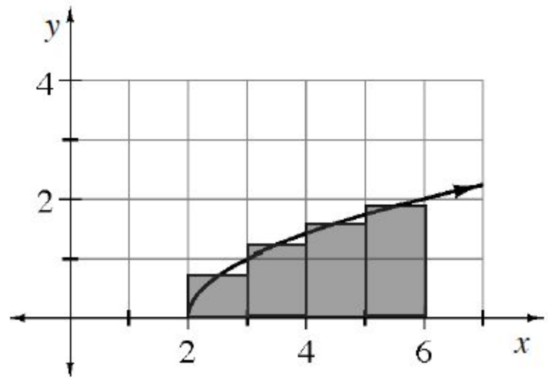
Task Card

2-34. The estimation of the area under the curve for $2 \le x \le 6$ where $f(x) = \sqrt{x-2}$ is shown at right using four midpoint rectangles.

a. Use sigma notation to write a Riemann sum that describes the given situation.



b. If the rectangles used in part (a) are rotated about the vertical line x=2, we could use the resulting figure to estimate the volume of the rotated flag. Describe the resulting three-dimensional shape. Include a sketch.

c. Estimate the volume of this rotated region by calculating the volume of each rotated rectangle. How reasonable is this result?

The rate that people are entering a local office is given below in people/hour. Use the table to answer questions 1-3.

Time (hours)	0	1	3	4	7
r ‘ (t) ppl/hr	12	7	3	5	8

1. Use a left Riemann sum with 4 subintervals to approximate the total number of people entering the office over the interval .

2. Use a right Riemann sum with 4 subintervals to approximate the total number of people entering the office over the interval .

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Notes

2-42. FOR SALE, Part One

Jacinda has a 1988 Rustang that she wants to sell. Travis is interested in buying her car, but they have not decided on a price. Travis offers $1000 for the car stating that this is what the car is worth according to its Blue Book value. Jacinda states, *"My car is worth more than $1000! If I wanted the Blue Book value, I would have traded it in when I bought my new car. If you look at the used cars advertised in the classified ads, you will see it is worth a lot more than $1000."*

Taking on the challenge, Travis agrees to look at similar Rustangs in the classified section of the newspaper. Below are all the Rustangs that Travis finds advertised.

Year	1978	1980	1981	1983	1984	1986
Asking Price	$900	$1220	$1380	$1700	$1860	$2180

a. From the data, can you make a prediction about the asking price for a 1988 Rustang? How reliable is this prediction?

b. Jacinda decides to do her own investigation using a local paper. Below is her data. According to her research, what price do you predict for a 1988 Rustang?

Year	1990	1991	1993	1994	1996
Asking Price	$4450	$5125	$6475	$7150	$8500

c. Based on this information, will Travis and Jacinda agree on the price?

d. Jacinda and Travis decide that additional research is necessary. They grab another paper and find a 1987 Rustang for sale for $2340 and a 1989 Rustang for sale for $3775. Will this new information help them to make a decision about the fair price of the car?

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Notes

2-43. FOR SALE, Part Two

By trying to predict the price for the 1988 Rustang, we are seeking a "**________**," or a final prediction of the price as the year approaches 1988. This can be written:

$$\lim_{t \to 1988^-} (\text{asking price}) = \qquad \text{and} \qquad \lim_{t \to 1988^+} (\text{asking price}) =$$

The left-hand limit is read *"As the year approaches 1988 from the left, the asking price approaches $__________."*

Translate the right-hand limit into a sentence:

$\lim_{t \to 1988} (\text{asking price})$ uses *both* sides of 1988 to estimate a value.

Since $\lim_{t \to 1988^-} (\text{asking price}) \neq \lim_{t \to 1988^+} (\text{asking price})$ we state that the $\lim_{t \to 1988} (\text{asking price})$ ___________ _______ __________ because the two sides _______ ______ _________.

What must be true about the left- and right-hand limits for the $\lim_{t \to 1988} (\text{asking price})$ to exist?

An Intuitive Definition of Limit

When you graph a function $y = f(x)$, most of the time you can guess what the value of, say, $f(3)$ is by knowing the values of $f(x)$ when x is ______ ______ to 3. One way to think about this is to assume you have the graph $y = f(x)$ for $2 < x < 4$, except at $x = 3$. Can you make a reasonable accurate guess as to the value of $f(3)$? If so, and this value is L, we say that the ________ of $f(x)$ exists at _____ and use the notation $\lim_{x \to 3} f(x) = L$.

For example, if $g(3.01) = 4.02$, $g(3.001) = 4.005$, and $g(2.999) = 3.997$, it is reasonable to guess that $g(3) = 4$ and therefore $\lim_{x \to 3} g(x) = 4$.

You can also take one-sided limits using numbers less than a (the notation is $\lim_{x \to a^-} f(x)$) or greater than a (the notation is $\lim_{x \to a^+} f(x)$).

An important point is that $\lim_{x \to a} f(x)$ does not need to equal $f(a)$.

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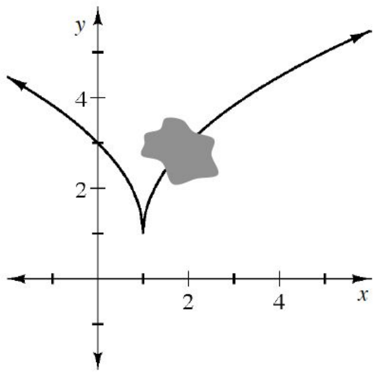
Task Card

2-44. STICKY LIMITS

Holly is trying to predict the value of *y* when *x* = 2. Unfortunately, her brother Max stuck gum on the area she was trying to look at! Can she still make a good prediction? Estimate $\lim_{x\to 2} f(x)$.



2-45. Since a limit is a prediction based on a pattern of *y*-values on a continuous graph, will the limit from problem 2-44 change if you found out that *f* (2) = 8? Why or why not?

2-46. Express the following limit statements as approach statements using complete sentences. Then draw graphs that can represent each limit.

a. $\lim_{x\to 5} f(x) = 6$

b. $\lim_{h\to 3^+} g(h) = -\infty$

2-47. Without a calculator, sketch $y = 3\sqrt{x-1} - 2$.

b. Write a complete set of approach statements for this function. Include $x \to 1^+$.

b. Approach statements describe what *y* is approaching as *x* approaches some value. This is the same as a limit. For example, one approach statement for $y = 3\sqrt{x-1} - 2$ can be rewritten using limits as:

$$\lim_{x\to 1^+} (3\sqrt{x-1} - 2) = -2$$

c. Use your approach statement from part (a) to rewrite $x \to 1^-$ and $x \to \infty$ as limit statements.

2-48. Determine if each of the following conjectures is *always* true, *sometimes* true, or *never* true. Then provide examples and/or counterexamples to support your claim.

Conjecture 1: When a limit exists at a certain x-value, the function is defined for that x-value.
Conjecture 2: If a function is defined at a certain x-value, then the limit exists at that x-value.

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Homework

2-49. Translate the following limit equations using a complete sentence. Then draw a graph to represent each situation. Homework Help ✎

a. $\lim_{x \to -1^+} \left(\sqrt{x+1}+3\right) = 3$

b. $\lim_{\text{time} \to \infty}$ (a soda's temperature) = room tempature

2-51. Sketch a graph of each of the functions below. Compare the equations and their graphs. Then write a complete set of approach statements for each. Homework Help ✎

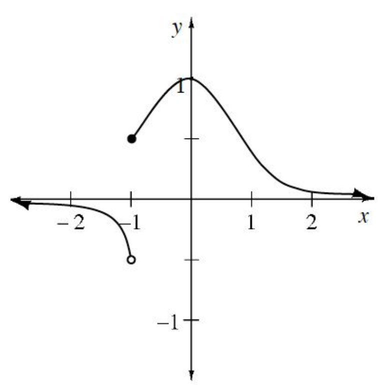
a. y = $\frac{(x+6)(x-1)}{x-1}$

b. y = $\frac{(x+6)(x-1)}{x-2}$

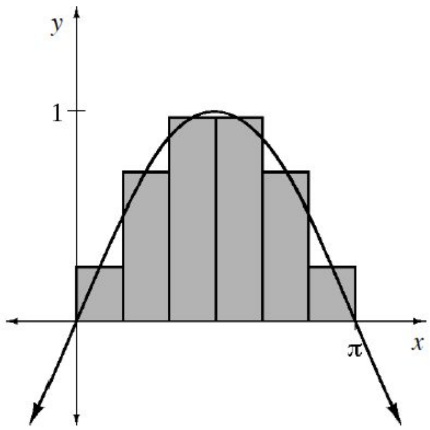
c. Explain why one graph has a hole while the other has a vertical asymptote.

d. What is the end behavior of each function?

2-52. Write as many limit statements as you can about the function graphed below as $x \to -1$ and $x \to \pm\infty$. Homework Help ✎



2-55. For $f(x) = \sin(x)$, an estimation of the area under the curve for $0 \le x \le \pi$ is shown below using six midpoint rectangles of equal width. Homework Help ✎



a. Estimate the area using these rectangles.
b. If the shaded region is rotated about the x-axis, then each of these rectangles becomes what shape? Sketch a picture representing this situation.
c. Estimate the volume of this rotated region by calculating the volume of each of the rotated rectangles.

2-41. Rewrite each of the following sums using summation notation. Homework Help ✎

a. $5 + 7 + 9 + 11 + 13$

b. $2\cos(2\pi) + 3\cos(3\pi) + 4\cos(4\pi) + 5\cos(5\pi)$

c. $\frac{1}{5}f(-2)+\frac{1}{5}f(-1)+\frac{1}{5}f(0)+\frac{1}{5}f(1)+\frac{1}{5}f(2)$