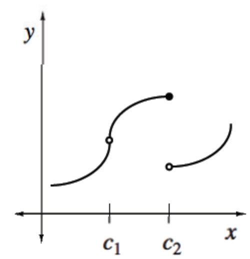
Chapter 3.4.2 and 3.4.3

The purpose of this lesson is to:

- Connect concavity with the sign of the second derivative.
- Explore situations where the derivative does not exist at a particular point. Examine tangent lines and investigate local linearity. Determine an antiderivative of a polynomial.

WARM UP

3-156. Explain why a function must be continuous at $x = c$ to be differentiable at $x = c$. The graph at right may help you.



3-157. FUNKY FUNCTIONS, Part One

a. Graph $f(x) = 2 + (0.1 - |x|)^2$ and rewrite f as a piecewise-defined function.

b. Zoom in at $x = 0$ on your graphing calculator and carefully examine the shape of the graph at $x = 0$. Does f appear differentiable at $x = 0$? Why or why not?

c. To confirm whether or not $f(x) = 2 + (0.1 - |x|)^2$ is differentiable at $x = 0$, we need to examine f'. Use your piecewise-defined function from part (a) to demonstate which condition of differentiablity fails at $x = 0$.

d. Analyze $\lim\limits_{h\to 0^-} \frac{f(x+h)-f(x)}{h}$ and $\lim\limits_{h\to 0^+} \frac{f(x+h)-f(x)}{h}$. Do they agree? Explain.

3-158. Use the *definition of the derivative as a limit* to write the slope function for $f(x) = 4x^2 - 3$. Then use your slope function to calculate $f'(11)$ and $f'(1000)$.

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NOTES

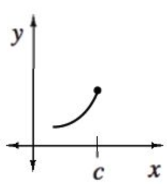
Differentiability

A function, *f*, is **differentiable** at *x* = *c* if:

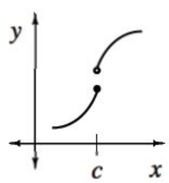
- *f* is ____________ at *x* = *c* and
- $\lim_{x \to c} f'(x)$ ________

When a function is differentiable at all values of *x* in its domain, then it is said to be differentiable everywhere. A function that is **twice differentiable** is both differentiable everywhere and $\lim_{x \to c} f''(x)$ exists everywhere.

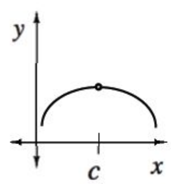
The graphs below illustrate functions that are not differentiable at *x* = *c*. They each fail at least one of the conditions of differentiability listed above.



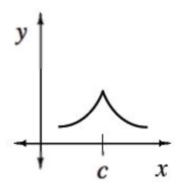
Graph A



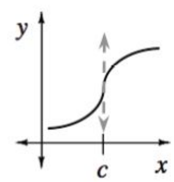
Graph B



Graph C



Graph D



Graph E

The “sharp point” in Graph D is often called a __________.

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Task Card

3-159. THE ABSOLUTE VALUE FUNCTION

a. Graph $f(x) = |x|$ on graph paper and without a calculator, sketch $y = f'(x)$.

b. What happens to $y = f'(x)$ at the vertex of $f(x) = |x|$? Verify your observations by examining the slopes on both sides of the vertex.

c. Use your graphing calculator to determine the slope of $f(x) = |x|$ at the vertex. What happened?

d. Part of the reason most graphing calculators incorrectly determine slopes at the vertex of an absolute value graph, as well as other cusps, is because they use the **symmetric difference quotient** (Hanah's Method) to calculate the slope of a tangent.

e. For $f(x) = |x|$, use $f'(x) = \frac{f(x+h)-f(x-h)}{2h}$ to calculate $f'(0)$ for $h = 0.1$, -0.1, and 0.01. What do you notice? For functions like of $f(x) = |x|$, some calculators falsely calculate the derivative at the cusp as 0. Why do you think this happens?

3-170. FUNKY FUNCTIONS, Part Two

One of the reasons we need to analyze functions analytically is because graphs can be misleading. When viewed with a standard window, the graph of $f(x) = 2 + (0.1 - |x|)^2$ can *look* differentiable at $x = 0$ when it is not! Examine the graphs of the following "funky functions" and their equations to determine if they are differentiable at $x = c$.

a. $f(x) = \frac{\sin(x)}{x}$, $c = 0$

b. $f(x) = \begin{cases} |x|^x & \text{for } x \neq 0 \\ 1 & \text{for } x = 0 \end{cases}$, $c = 0$

c. $f(x) = |x^3 + 0.125|$, $c = -0.5$

3-171. Graph the function defined by $g(r)=\begin{cases} 0 & \text{for } -2 \le r \le 2 \\ r^3+2r^2-4r-8 & \text{otherwise} \end{cases}$.

a. Is g continuous at $r = 2$? Explain your answer.

b. Is g is differentiable at $r = 2$? Explain your answer.

3-172. Sketch the graph of a function defined for all real numbers that satisfies *all* of the following properties. (There are many possible answers.)

a. $f(0) = -1$

b. $f(x)$ is not differentiable at $x = 2$.

c. $f(x)$ is decreasing for all $x \neq 2$.

3-173. Compare how distance and velocity are related with the scenarios in parts (a) and (b).

a. A ball is rolled down a ramp so that the distance it travels in feet at time t is $d(t) = 6t^2 + 2t$. Without your calculator, determine the velocity, $d'(t)$, at $t = 1$, 3, and 10 seconds. Explain what concepts of calculus you applied in order to solve this problem.

b. When a football is kicked from the ground straight up into the air its velocity, measured in feet per second, is $v(t) = -32t + 80$. On one set of axes, sketch a graph of the height function and a graph of the velocity function. Calculate the maximum height obtained the ball. Explain what calculus concepts you applied to solve this problem.

c. Both (a) and (b) involve distance and velocity. However, each part required a different solution method or approach. Describe the relationship between distance and velocity, as well as the derivative and area under a curve.

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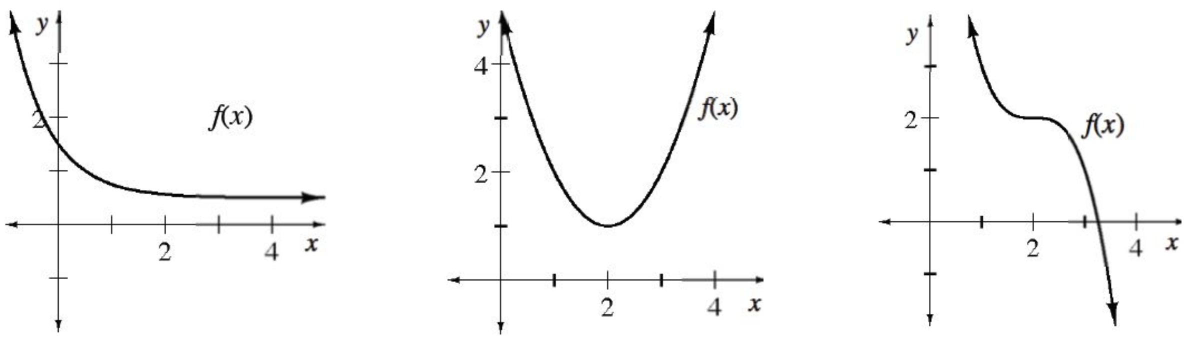
Homework

3-161. Write the equations of the lines tangent to the curve $y = x^3 - 4x$ at both $x = 0$ and $x = 2$. Then, determine the point of intersection for these two tangent lines. 3-161 HW eTool(Desmos) Homework Help ✎.

3-162. For each graph below: Homework Help ✎

Write a slope statement for *f*.
Sketch the graph of $y = f'(x)$ using a different color.



3-165. What is $\frac{dy}{dx}$ for each of the following functions? You will need to rewrite each equation first. Homework Help ✎

a. $y = \sqrt[3]{\frac{1}{x^2}}$

b. $y = x\sqrt{x}$

c. $y = \sin^2(x) + \cos^2(x)$

d. $y = \frac{x+2}{x}$

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Homework

3-147. Write the general antiderivative F for each function below. Test your solution by verifying that $F'(x) = f(x)$. Homework Help ✎

a. $f(x) = -6x^5 + 12x^2$

b. $f(x) = 3\cos(x)$

3-166. What is the general antiderivative, F, for each function below? Test your solution by verifying that $F'(x) = f(x)$. Homework Help ✎

a. $f(x) = 3x^{1/2} - 7x$

b. $f(x) = \cos(x) + 2\sin(x)$

3-169. *Use the definition of a derivative as a limit to write an equation for* f' *if* $f(x) = 2x + 9$. *Use the Power Rule to confirm your answer.* *Homework Help ✎*

3-152. Write an equation for z' if $z(x) = 3x^2 + 5x + 1$. Then, write the equation of the tangent line in point slope form at $x = -2$. Homework Help ✎