Chapter 3.2.1 and 3.2.2

The purpose of this lesson is to:

1. **Use limits to write three different variations of the definition of the derivative as a limit.**

2. **Choose an appropriate method to write a slope function.**

WARM UP

How does a graphing calculator determine the slope of a tangent line? Or, how did mathematicians determine slopes before technology was available? In this lesson, we will look at slopes of secant lines and tangent lines by examining three different methods to determine the exact slope of a tangent line at its point of tangency.

To start, we will revisit the use of secant lines by studying the different slopes when $x = 4$ in the Ramp Lab. Later, we will use secant lines to help us determine the slopes of tangent lines with precision.

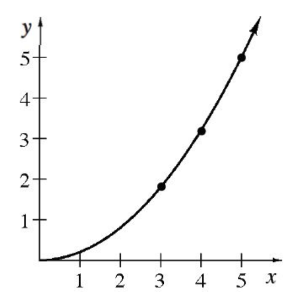
3-35. Each of the following students used a different method to estimate the velocity of the ball at $t = 4$ seconds.

x (seconds)	1	2	3	4	5	6
f(x) (meters)	0.2	0.8	1.8	3.2	5.0	7.2

Hana estimated $f'(4) \approx m \approx \frac{f(5)-f(4)}{5-4} = \frac{5.0-3.2}{5-4} = 1.8$.

Anah estimated the slope as $f'(4) \approx m \approx \frac{f(4)-f(3)}{4-3} = \frac{3.2-1.8}{4-3} = 1.4$.

Hanah estimated the slope as $f'(4) \approx m \approx \frac{f(5)-f(3)}{5-3} = \frac{5.0-1.8}{5-3} = 1.6$.



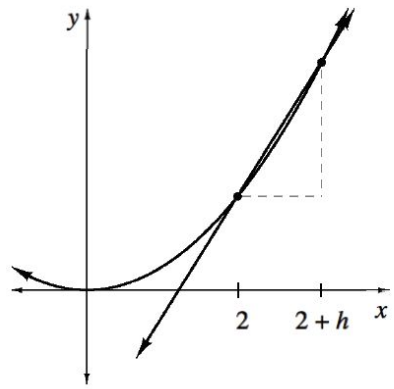
a. What are the units of the slopes found by each student? Explain why.

b. Compare and contrast each students method.

c. Use the graph at the right of $y = f(x)$ on your paper and sketch the secant lines that Hana, Anah, and Hanah used to approximate the slope at $t = 4$.

d. Is Hanah's slope the average of Hana and Anah's? Does this mean that Hanah's method gives the best approximation?

3-36. To approximate the slope of the curve $f(x) = 0.25x^2$ at $x = 2$, Devin picked another point on the curve, a small distance of h to the right of $x = 2$.



a. If the second point is located at the point $(2 + h, f(2 + h))$, write and simplify an expression that represents the slope of this secant line.

b. Whose method did Devin use from problem 3-35: Hana's, Anah's, or Hanah's?

c. To get the *exact* slope of the tangent line at $x = 2$, what should be done with h? Write an expression that represents the true slope at $x = 2$.

d. Use your expression from part (c) to determine the slope of the curve at $x = 2$.

e. Does it matter if h is positive? Why or why not?

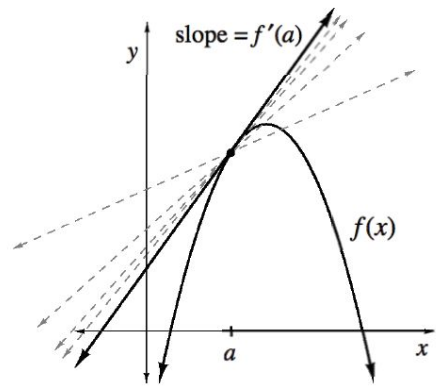
Chapter 3.2.1 and 3.2.2

NOTES

The Derivative

The slope of a line tangent to at any point x is called the ___________ of f at x. It is found by taking a limit of the slope of a secant line as $h \to 0$. The standard form of this type of limit is:

$f'(x) =$



If the slope (or instantaneous rate of change) at a particular x-value is desired, such as at $x = a$, then the notation used is $f'(a)$. This slope can be found by replacing x with a.

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

3-39. The Math Notes box in this lesson features the definition of the derivative using Hana's method. Write the definition of the derivative using Anah's method and the definition of the derivative at a point ($x = a$) using Hanah's method.

3-40. Use the definition of the derivative to write a slope function, for $f(x) = 4x^2 - 3$. Then use your slope function to calculate $f'(11)$ and $f'(1000)$.

3-41. Lulu used the limit below to write the derivative of a function f.

$$f'(x) = \lim_{h \to 0} \frac{2(x+h)^3 - 2x^3}{h}$$

What is the equation of f?

What is the equation of f'?

Chapter 3.2.1 and 3.2.2

Task Card

3-52. USING MULTIPLE STRATEGIES TO WRITE f'

Let $f(x) = x^2 + 4x + 2$. You will write an equation for f' using three different methods outlined below. Each method should produce the same result.

a. Use the definition of the derivative.

b. Use the Power Rule.

c. Use your graphing calculator to graph the equation $f'(x) = \frac{f(x+h)-f(x)}{h}$ for $h = 0.01$. Examine the graph and write an approximate equation for f'.

3-53. Revisit the Power Rule from problem 3-6. Will the Power Rule work for $f(x) = x^n$ if $n = 0$? When n is negative? What about for non-integer values of n? Investigate these conditions with your team and summarize your results.

3-54. Expand the function $f(x) = (2x - 3)(x^2 + 2)$. Then, use that expansion and the Power Rule to write an equation for $f'(x)$. Finally, use your equation for $f'(x)$ to write the equation of the line tangent to the curve $f(x) = (2x - 3)(x^2 + 2)$ at $x = 3$. Write your equation in point-slope form.

3-55. Lazy Lulu wants to determine the derivative of $f(x)$ at $x = a$. She used Hana's method to set up the definition of the derivative:

$$\lim_{h \to 0} \frac{((9+h)^2-1)-80}{h}$$

Lulu is lazy and does not want to do algebraic computations. Help Lulu *undo* the definition of the derivative so she can use the Power Rule instead.

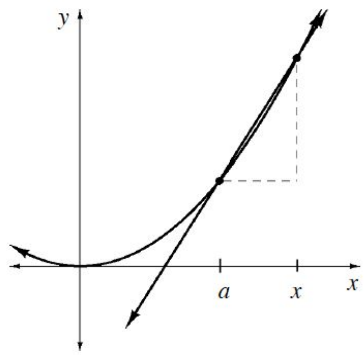
a. What is $f(x)$?

b. What is the value of a?

c. Avoid the algebra! Use the Power Rule to write an equation for f'.

d. What is $f'(a)$?
e. Write the equation of the line tangent to $f(x)$ at $x = a$.

3-56. ANOTHER DEFINTION OF THE DERIVATIVE: INTRODUCING ANA



Hana, Anah, and Hanah have a stepsister named Ana. Ana also found a method to determine the derivative at a point. Her method is a little different from the rest.

Ana's Method: $\lim\limits_{x \to a} \frac{f(x)-f(a)}{x-a}$

a. Use Ana's method to confirm that the *derivative* of $f(x) = x^2$ at $x = 2$ is 4. Does Ana's method work?

b. Use Ana's method to write the *derivative function* of $f(x) = x^2$.

c. What is special about Ana's name?

3-57. Graph the function $h(x) = x\sin(2x)$.

a. Can you use the Power Rule to determine the derivative function? Why or why not?

b. Choose a coordinate on $y = h(x)$ that is very close to $\left(\frac{\pi}{6}, h\left(\frac{\pi}{6}\right)\right)$. Then use your calculator to approximate $h'\left(\frac{\pi}{6}\right)$ using the slope of the secant line containing the point you chose and $\left(\frac{\pi}{6}, h\left(\frac{\pi}{6}\right)\right)$.

c. Use your approximation from part (b) to write the equation of the line tangent to the curve at $x = \frac{\pi}{6}$.

Chapter 3.2.1 and 3.2.2

Homework

The use of $\frac{dy}{dx}$ comes from $\frac{\Delta y}{\Delta x}$, which is an expression for slope read as “the change in y over the change in x”. We use Δ to represent change. When the change gets smaller and smaller until it is infinitely small (infinitesimal) we use the symbol d.

It is useful to think of change when working with derivatives. For example $\frac{dh}{dt}$ ccan represent the change in the height of an object with respect to time. Create expressions using the symbol d that represent the following instantaneous change statements. Homework Help ✎

a. The change in the velocity, v, with respect to time.

b. The change in volume, V, with respect to the radius, r, of a cone.

c. The change in area, A, of a circle with respect to the perimeter, p.

3-60. Write an equation for f ′(x) for each function below. Homework Help ✎

a. f(x) = $\frac{9}{x}$
b. f(x) = $-3x^7 - 6x$
c. f(x) = $5x^{-4}$
d. f(x) = m

3-61. Sketch a graph of a function that has the properties listed below. Describe anything special about this function. 3-61 HW eTool (Desmos) Homework Help ✎.

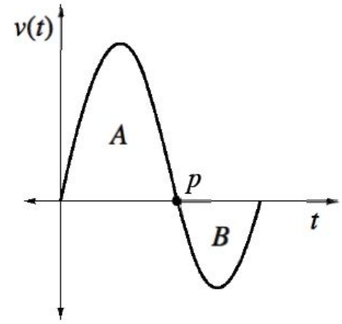
a. $\lim_{x \to -\infty} f(x) = 4$
b. f(1) = −1
c. f(−x) = −f(x)

3-62. Hanah wrote this derivative function: $f'(x) = \lim_{h \to 0} \frac{((x+h)^2-3)-((x-h)^2-3)}{2h}$

Homework Help ✎

a. What is the equation of f(x)?
b. What is the equation of f ′(x)? (Note: Avoid the algebra by using the Power Rule.)
c. Use your slope function to calculate f ′(0) and f ′(1).

3-63. Answer the following questions using the graph at right, which shows the velocity of a runner over time. The letters A and B represent the areas of the two regions in the diagram. Homework Help ✎



a. If the runner was at the starting line at t = 0, describe her direction during the interval illustrated on the graph at right.

b. What is the significance of point p?

c. What does region A represent in this situation? That is, what does it tell you about the runner? What about region B?

d. If A = 30 meters and B = −5 meters, what does A + B represent in this situation? Why is B negative?

3-64. Using the definition of the derivative as a limit, show that the derivative of f(x) = $\frac{1}{x^2}$ is $f'(x) = -\frac{2}{x^3}$. That is, show algebraically that the following limit statement is true: Homework Help ✎

$$\lim_{h\to 0} \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h} = -\frac{2}{x^3}$$