Chapter 3.3.4 and 3.4.1

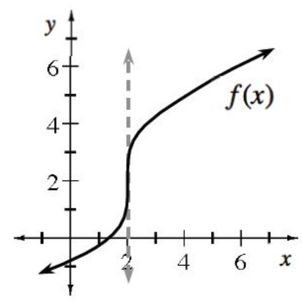
The purpose of this lesson is to:

Connect concavity with the sign of the second derivative.

Explore situations where the derivative does not exist at a particular point. Examine tangent lines and investigate local linearity. Determine an antiderivative of a polynomial.

WARM UP

3-141. Review the definition of a derivative. Explain why a limit is needed to calculate the slope of a tangent line. Be clear in your explanation and use a diagram to help demonstrate the limit.



3-142. A line tangent to f at $x = 2$ is shown at right. Does the slope of the tangent exist? Does $f'(2)$ exist? Why or why not?

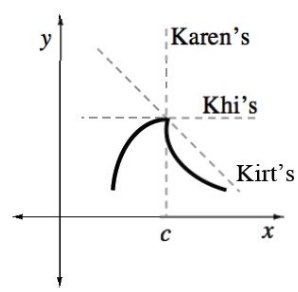
3-143. On your graphing calculator, graph $f(x) = \sqrt[3]{x}$.

a. Describe what happens to the slope of f at $x = 0$.

b. Use the Power Rule to write an equation for $f'(x)$ and use it to calculate $f'(0)$. Explain what happened. Does this confirm what you see in the graph?

3-144. HELP!

Koy's team needs your help to settle a dispute. For the function shown in bold at right, each of her team members drew a different tangent at $x = c$.



Khi reasoned that if you follow x as it approaches c from the left side, then his tangent makes the most sense. Karen says hers makes sense because hers is tangent to the right side of the curve at $x = c$. Kirt argues that his is the best solution for both sides.

Koy does not think any of them are correct because the slope of a tangent is a limit, and a limit cannot exist if both sides do not agree.

Which team member is correct and why?

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NOTES

Derivative Notations

In calculus, there is not a single correct notation for expressing a derivative. Instead, several different notations have been used by different mathematicians. The usefulness of each notation varies in a given context. The most common derivative notations are listed below. Derivative functions describe the ________________ _______ ____ _________ of another function at any value of x. Since a derivative describes a rate, it is often expressed as a fraction.

First derivative notation:

$\frac{dy}{dx}$ read as "____, _____" or "___ ________ ___""

$f'(x)$ read as "f prime of x"

Second derivative notation:

$\frac{d^2y}{dx^2}$ (or $\frac{d(dy/dx)}{dx}$) read as "the _________ derivative of __ with respect to __"

$f''(x)$ read as "__ __________ _______ ___ __"

Higher derivatives are expressed as:

$\frac{d^ny}{dx^n}$ read as "the ____ derivative of __ with respect to __"

$f^{(n)}(x)$ read as "the ____ derivative of __ with respect to __"

Antiderivatives

An ________________ of a function f is a function F whose derivative is f. That is ____________ The antiderivative of f' is f. However, for the antiderivative of a function f, we use a capital letter F. For example, we write the antiderivative of g as ___.

Since there are an infinite number of antiderivatives that are different only by a _________ term, called a "family," we add a constant "C" to represent all of them. This is known as the ____________ _____________ (or simply the antiderivative).

For example: If $F(x) = 5x^3 + 3x^2 + C$, then $F(x) = 5x^3 + 3x^2 + 5$ and $F(x) = 5x^3 + 3x^2 - 9$ are both antiderivatives of f. This is why we write $F(x) = 5x^3 + 3x^2 + C$.

DO NOT FORGET YOUR __________

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Task Card

3-126. THE SECOND DERIVATIVE IN MOTION PROBLEMS

a. If f' represents the rate of change of f, then what does f'' represent? If f' represents velocity, then what does f'' represent?

b. Since concavity depends on how the slope is *changing*, concavity must depend on the *slope of the slope*. What does this mean? Explain this in your own words.

c. Examine the curves below and complete the table with the signs (positive or negative) of f' and f''. The first entry has been done for you.

***f*(*x*)**	**Increasing or** **Decreasing?** **Walking away from the** **motion detector? or** **Walking towards the** **motion detector?**	**Concave Up or** **Concave Down?** **Getting faster?** **or** **Getting slower?**	**$f'(x)$**	**$f''(x)$**
	*Increasing* *Away from motion detector*	*Concave up* *Getting faster*	*positive*	*positive*
				

				
				

3-146. ANTIDERIVATIVES

a. If $f'(x) = 8x^3 - 10x - 5$, write a possible equation for f. The equation for f is called an **antiderivative** of f'.

b. Explain why there is always more than one antiderivative.

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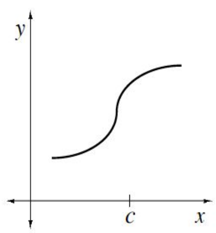
Task Card

3-145. If a function *f* is continuous at $x = c$ and exactly one tangent line can be drawn in at $x = c$ (i.e. $\lim_{x \to c} f'(x)$ exists), then the function is **differentiable** at that point (c, $f(c)$). The graphs below show examples of functions that are not differentiable at $x = c$, each for a different reason.

i.



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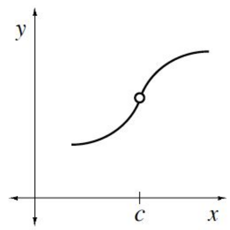

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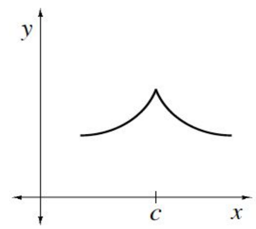

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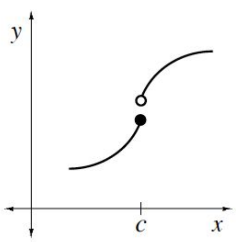
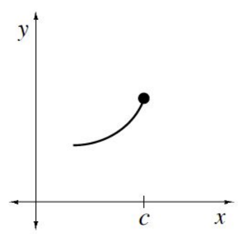


v.



a. With your team, discuss why the slope of a tangent line does not exist at $x = c$ in each case.

b. Sketch $y = f'(x)$ for each function. Pay close attention for what happens at $x = c$.

c. In your own words, describe when a function, *f*, is differentiable and when it is non-differentiable at a point, $x = c$.

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Homework

3-132. Without your calculator, write the equation of the line tangent to $g(z) = \frac{z^7 + 5z^6 - z^3}{z^2}$ at $z = 1$. Homework Help ✎

3-133. If $f'(x) = -6x^{1/2} - \sin(x)$, write a possible function for f. Then write another possible equation. Homework Help ✎

3-134. For each of the following functions, write the *second derivative* with respect to x.
Homework Help ✎

a. $y = 8x^{99}$

b. $y = -3\sqrt{x}$

c. $f(x) = \frac{2}{3}x - 6x^{-2}$

d. $f(x) = 7 - 2\cos(x)$

3-147. Write the general antiderivative F for each function below. Test your solution by verifying that $F'(x) = f(x)$. Homework Help ✎

a. $f(x) = -6x^5 + 12x^2$

b. $f(x) = 3\cos(x)$

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Homework

3-150. Use the definition of a derivative to write an equation for y' if $y = \frac{3}{4}x^2 - 11x + 34$. Confirm your answer with the Power Rule. Homework Help ✎

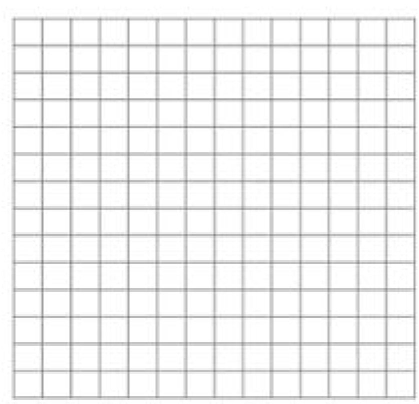
3-151. State the domain and the range of each function below. Homework Help ✎

a. $y = \frac{x^2 - x - 6}{x + 1}$

b. $y = \frac{x^2 - x - 6}{x + 2}$

3-152. Write an equation for z' if $z(x) = 3x^2 + 5x + 1$. Then, write the equation of the tangent line in point slope form at $x = -2$. Homework Help ✎

3-153. Sketch a continuous curve which meets all of the following criteria: Homework Help ✎



a. $f'(x) > 0$ for all x

b. f is concave down.

c. $f(2) = 1$

d. $f'(2) = \frac{1}{2}$

e. How many roots does f have?

f. What can you say about the location of the root(s)?

g. What is $\lim_{x \to -\infty} f(x)$?

h. Is it possible that $f'(1) = 1$? $f'(1) = \frac{1}{4}$? For each case, explain why or why not.