Chapter 2.3.3 and 2.4.1

The purpose of this lesson is to:

- You will apply your knowledge of rates of change to develop a method to approximate the velocity of an object at an instant. You will explore local linearity concepts and analyze the proofs of

- You will complete the development of Riemann sums and use a graphing calculator to investigate using left endpoint, right endpoint, and midpoint rectangles to approximate area under a curve.

WARM UP

2-122. Investigate the local behavior of $y = \frac{1}{x}$ at $x = 0.5$ by graphing the function on your calculator and zooming in.

a. What does the graph look like? Sketch the graph before and after you zoom in.

b. Since the graph of $f(x) = \frac{1}{x}$ resembles a line in a small "local" region, we say the function is **locally linear**. Is $f(x) = \frac{1}{x}$ truly linear close to (0.5, 2)? Why or why not?

2-123. Study the list of basic functions below. Although the graphs vary widely by shape, some look exactly the same when you zoom in very close. What does each function look like when you zoom in at $x = -2$, $x = 0$, and $x = 4$? Record your observations.

a. $y = x^2$

b. $y = \sin(x)$

c. $y = 1 - \cos(x)$

d. $y = x^3$

e. $y = |x|$

f. $y = \frac{x}{x+1}$

g. You probably noticed that $y = |x|$ does not have a local linearization at $x = 0$ because at $x = 0$ it has a **cusp**. What do you think the term "cusp" means and why do you think a function cannot be linearized at a cusp?

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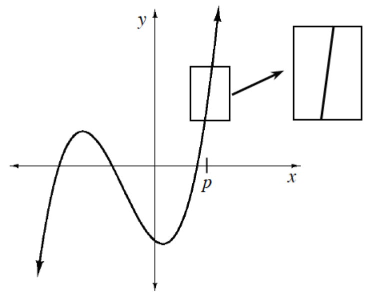
NOTES

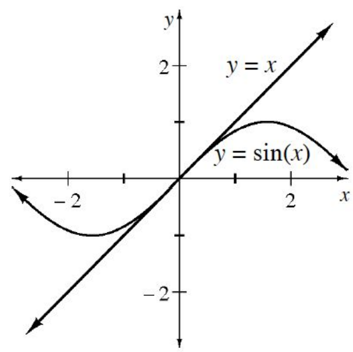
Local Linearity

When the graph of a function, *f*, appears __________ near a point, $x = p$, that line can be used to approximate or model the function's behavior near that point. This line is called the _______________ of *f* at $x = p$.

While zooming in on a particular _______ of a function, often the curvature of the function becomes less noticeable as seen in the example at right.



2-124. Examine the linearization of $y = \sin(x)$ at (0, 0). Compare the values of $y = \sin(x)$ and the line $y = x$ at values close to $x = 0$.



a. Complete the table below for various *x*-values.

x	–1	–0.1	–0.01	0	0.01	0.1	1
***y* = sin(*x*)**							
y* = *x							

a. For what values of *x* is the line a good approximation of $y = \sin(x)$? Write a statement summarizing your findings.

b. On what domain is $y = x$ an overestimate? On what domain is it an underestimate? You should be able to *see* this information on both the graph and the table.

c. Estimate $\lim_{x\to0} \frac{\sin(x)}{x}$.

d. Now estimate $\lim_{x\to0} \frac{1-\cos(x)}{x}$.

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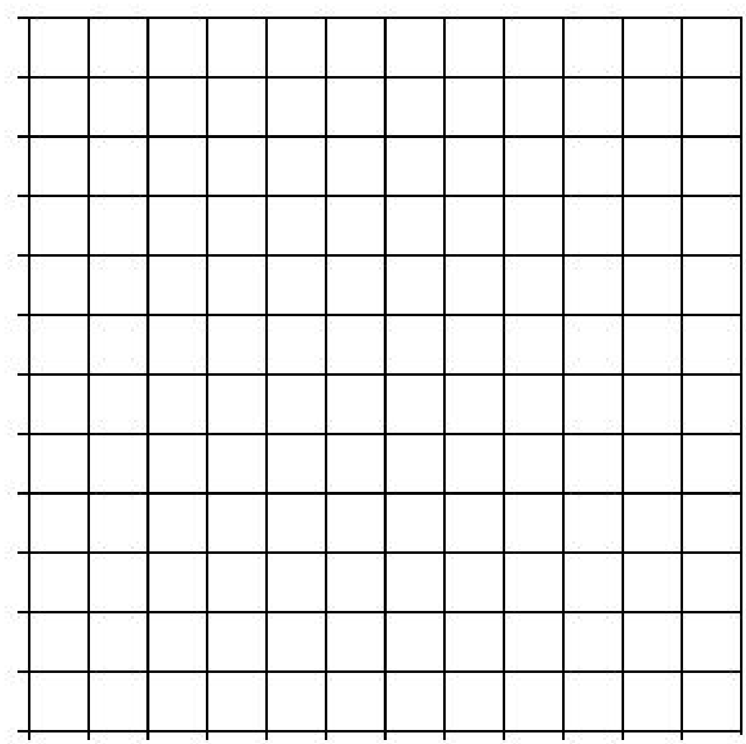
Task Card

2-132. Sketch a sine wave on 0 < *x* < *π*. Determine the domain on which:



a. The left endpoint rectangles are an overestimate of the actual area.

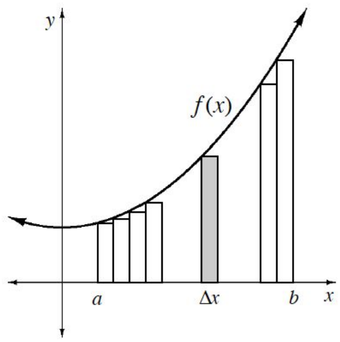
b. The left endpoint rectangles are an underestimate of the actual area.

c. The right endpoint rectangles are an overestimate of the actual area.

d. The right endpoint rectangles are an underestimate of the actual area.

e. The left endpoint rectangles and right endpoint rectangles yield equal areas.

2-133. Examine the graph below. Use the graph to complete parts (a) and (b) below.



a. Assume that an area is being estimated using *n* rectangles of equal width. Write an expression for Δ*x* using *a*, *b*, and *n*.

b. As the number of rectangles increases, what happens to the width of each rectangle?

2-134. Rewrite the following Riemann sum by substituting your expression for Δ*x* from part (a) of problem 2-133 into the summation statement.

$$A \approx \sum_{i=0}^{n-1}\left[\Delta x \cdot f\left(a+\Delta x \cdot i\right)\right]$$

Changing the value of *n* varies the number of rectangles that are used in the approximation.

2-135. OUT OF GAS, Part Two

After arriving at Calculus Camp, J.T. and Elena realized that they could have made a much better estimate of the distance traveled after running out of gas than they had in problem 2-1. Elena suggested using a function to approximate the data, thereby allowing them to use a Riemann sum.

a. Elena discovered that $y = 0.00538x^2 - 1.43x + 91.7$ represents a curve of best fit for the data in problem 2-1. Sketch this curve on your calculator, making sure you can see a complete picture of Elena and J.T.'s road trip.

b. Use sigma notation to write two Riemann sums that approximate the distance traveled. Write one expression for the left endpoint and one expression for the right endpoint rectangles. Assume that the area is divided into n rectangles.

c. Consider the shape of the graph. Will left endpoint rectangles generate an over or and underestimate of the actual area? Justify your answer.

d. Have each member of your team choose a different number of rectangles (n = 10, n = 20, etc.) and calculate the corresponding distances using both right and left endpoint rectangles. Share this information with your team—keep a detailed record of everyone's findings. Use your calculator so you have time to record many different values for n. What happens as n increases?

e. How can Elena get the best estimate? How many rectangles should she use? Should they be left or right endpoint rectangles?

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Homework

2-131. Evaluate the following limits. Homework Help ✎

a. $\lim_{x\to\infty} \frac{\sqrt{9x^2-3x+1}}{4x^2-1}$

b. $\lim_{x\to-1} \frac{x+1}{x^2+5x+6}$

c. $\lim_{x\to-\infty} \frac{2-3x-4x^2}{(1-3x)^2}$

d. $\lim_{x\to 2} \frac{|x^2-4|}{x-2}$

2-136. Sketch a graph of the region bounded by the functions $f(x) = x^2$, $g(x) = -2x + 8$, and the x-axis. Homework Help ✎

a. How can you estimate the area in this region?

b. Using your method, estimate the area of the region.

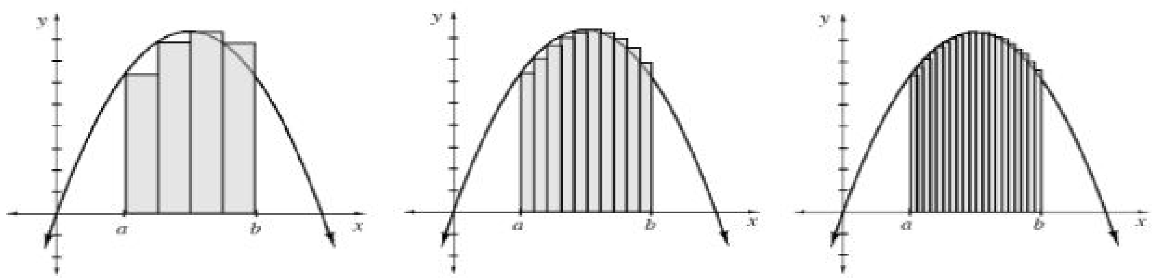
2-137. Write a single expression using the Trapezoid Rule, which will approximate the area under $f(x) = 2x^2 - 4x + 3$ over the interval $0 \le x \le 1$ using five trapezoids of equal height. Do not evaluate the expression. Homework Help ✎

2-140. WHICH IS BETTER? Part Three

Below is a comparison of using different numbers of rectangles to approximate the *same* area under a curve for *f*. Decide which situation will best approximate the area under the curve for $a \leq x \leq b$. Explain why. Homework Help ✎

If, in each situation, the rectangles all have equal widths, write expressions to approximate the areas under the curves.

a. 4 rectangles　　b. 10 rectangles　　c. 20 rectangles



2-142. Determine the values of the following limits. If the limit does not exist, indicate why not. Homework Help ✎

a. $\lim\limits_{x\to 4^+}\left(\sqrt{x-4}-5\right)$

b. $\lim\limits_{x\to -2}\frac{x^2-4x-12}{x+2}$

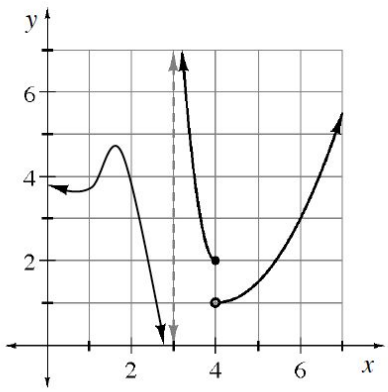
c. $\lim\limits_{x\to 6}\frac{x^2-4x-12}{x+2}$

d. $\lim\limits_{x\to \infty}\frac{1}{x-1}$

e. $\lim\limits_{x\to -\infty}\frac{x^2+6x-7}{x}$

f. $\lim\limits_{x\to \infty}\frac{x^2-7x-10}{x^2}$

2-143. Review the three conditions of continuity. Then, examine the graph below and determine at which values of *x* the function is *not* continuous. Explain which condition the function fails at each discontinuity. Homework Help ✎



2-144. Use the graph from problem 2-143 to complete the following table: Homework Help ✎

a	$\lim\limits_{x\to a} f(x)$	f (a)	Continuous at x = a?
1			
2			
3			
4			

2-145. Given the function *f*(*x*) = $2x^2 - x + 3$, calculate the following values. Homework Help ✎

a. $\dfrac{f(3)-f(2)}{1}$

b. $\dfrac{f(2.1)-f(2)}{0.1}$

c. $\dfrac{f(2.01)-f(2)}{0.01}$

d. Estimate $\lim\limits_{x\to 2} \dfrac{f(x)-f(2)}{x-2}$.