Chapter 2.2.4 and 2.3.1

The purpose of this lesson is to:

- Calculate limits algebraically and use limit statements to describe the behavior of a function.
- Use slopes of secant lines to estimate velocity at a point.

WARM UP

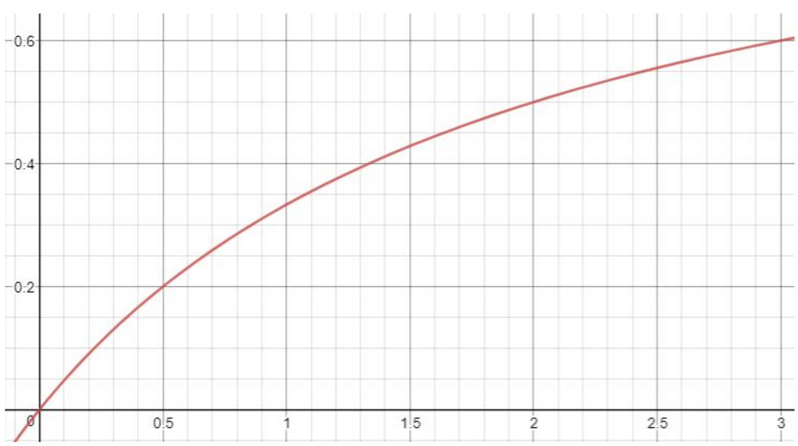
So far you have looked at limits on graphs and in terms of continuity. Today you will apply algebraic computations to determine limits and better understand graphs.

2-89. A NEW REASON TO SIMPLIFY

a. Yashar was trying to determine the following limit: $\lim_{x \to 2} \frac{x^2-2x}{x^2-4}$. When he substituted in x = 2 he got $\frac{0}{0}$. Explain why Yashar cannot determine this limit it its current form.

b. Meanwhile Hripsime rewrote the limit as $\lim_{x \to 2} \frac{x}{x+2}$. Explain what is happening on the graph at x = 2.

c. Determine the value of the limit in part (b) using the graph below.



d. Explain what Hripsime did to move from Yashar's function to his own.

e. Explain why Hripsime's method is useful.

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Notes

There are two main methods for solving limits algebraically:

1. We can ___________ the top and or bottom of the function. This let's us _________ parts of the function so that we no longer have __________.

2. We can also ____________ a value in for x that is ___________ to the value we're _____________.

2-90. Given the limits below, state the algebraic method(s) that can help you evaluate the limit.

a. $\lim_{x \to 3} \frac{x^2-9}{x-3}$

b. $\lim_{x \to 3^+} \frac{1}{x-3}$

c. $\lim_{x \to 9} \frac{x-9}{\sqrt{x}-3}$

d. Which problem(s) above have limits that exists and which problem(s) have limits that do not exist? Explain the graphical significance of your answer.

To find horizontal asymptotes:

- If the degree (the largest exponent) of the ______________ is bigger than the degree of the ______________, the horizontal asymptote is the ___________________________.

- If the degree of the _________________ is bigger than the __________________, there is no horizontal asymptote.

- If the degrees of the numerator and denominator are the ___________, the horizontal asymptote equals the leading _____________________ of the numerator divided by the leading ______________________ of the denominator

To find vertical asymptotes:

Find wherever the __________________ is equal to __________ after holes have been removed. To do this, simply ____ and solve.

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Task Card

2-91. Evaluate the following limits.

a. $\lim\limits_{x \to 1} \frac{2x^2-2}{x-1}$

b. $\lim\limits_{x \to \infty} \frac{2x^2-7}{x^2+4x-1}$

c. $\lim\limits_{x \to 0} \frac{2x^3-7}{3x^3+4x-5}$

d. $\lim\limits_{x \to 1} \frac{\sqrt{x}-1}{x-1}$

e. $\lim\limits_{x \to 4^-} \frac{5}{x-4}$

f. $\lim\limits_{x \to -\infty} \frac{x^2-1}{x+2}$

2-92. For each function below, complete the following tasks:

a. List all horizontal asymptotes, if any, then determine $\lim\limits_{x \to -\infty} f(x)$ and $\lim\limits_{x \to -\infty} f(x)$.

b. List all vertical asymptotes and holes, if any, then determine $\lim\limits_{x \to \text{V.A.}^+} f(x)$, $\lim\limits_{x \to \text{V.A.}^-} f(x)$, and $\lim\limits_{x \to \text{holes}} f(x)$.

c. $f(x) = \frac{2(x-3)}{3x-15}$

d. $f(x) = \arctan(x)$

e. $f(x) = \frac{4(x-2)(x-3)}{x-2}$

2-93. In CPM *Precalculus*, limits were used to compare the magnitudes of various functions as they "raced" to infinity. It was discovered that as exponential, power, and logarithmic functions approached infinity an order of domination emerged in the race. The results are summarized below showing the order of domination.

logarithmic < power < exponential

Within each function family there exists an order of domination as well. For example, as x approaches infinity:

a. Exponential: $1.1^x < 2^x < 8^x$

b. Power: $\sqrt{x} < x^2 < x^{10}$

c. Logarithmic: $\log_{10}(x) < \log_5(x) < \log_2(x)$

Use the idea of dominant terms to evaluate the following limits:

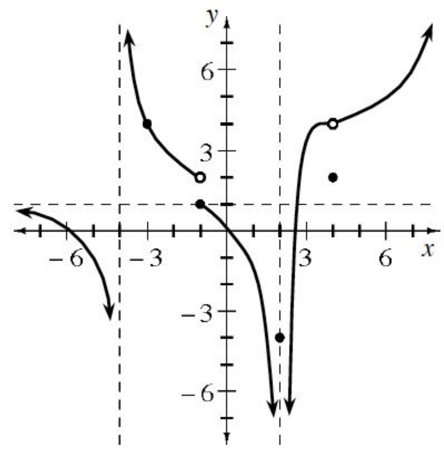
a. $\lim\limits_{x\to\infty} \frac{2^x+x^3}{x^5+x}$

b. $\lim\limits_{x\to\infty} \frac{2x^2-8x}{5x^2+x}$

c. $\lim\limits_{x\to\infty} \frac{2x^2-8x}{5x^2+3^x}$

d. Evaluate each of the limits in parts (a), (b), and (c) again, but this time let $x \to -\infty$.

2-94. Use the graph of $y = f(x)$ at right to determine the following values. If the limit does not exist, explain why.



a. $f(-4)$

b. $\lim_{x \to -4} f(x)$

c. $f(-1)$

d. $\lim_{x \to -1^-} f(x)$

e. $\lim_{x \to -1^+} f(x)$

f. $\lim_{x \to -1} f(x)$

g. $f(2)$

h. $\lim_{x \to 2} f(x)$

i. $f(4)$

j. $\lim_{x \to 4} f(x)$

k. Is the function continuous at $x = 4$? Explain your reasoning using the formal definition of continuity.

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Homework

2-96. Suppose *f* and *g* are both discontinuous at $x = 3$. Using the table below, for which of the functions does the limit as x approaches 3 *appear* to exist? Justify your answer. Homework Help ✎

x	2.8	2.9	2.99	3	3.01	3.1	3.2
$f(x)$	6.97	6.98	6.99	?	7.01	7.02	7.03
$g(x)$	6.97	6.98	6.99	?	7.99	7.98	7.97

2-97. Let *f* be an even function such that $f(2) = 4$ and $f(10) = 20$. Which of the following statements must be true? Could be true? Must be false? Homework Help ✎

I. $f(-10) = 20$ II. $f(-2) = -4$ III. $f(0) = 0$

2-98. If $1 < a < b$, which of the following logarithmic expressions represents a value that is negative? Between 0 and 1? Equal to 1? Greater than 1? Homework Help ✎

I. $\log_a(b)$ II. $\log_b \frac{1}{a}$ III. $\log_b(a)$ IV. $\log_a(a)$

2-99. What are the *x*- and *y*-intercepts of the graph of $x + 3 = 3^{3(y+1)}$? Homework Help ✎

2-100. Let $f(x) = x^2 - 9$, and $g(x) = 2x^2 - 12x + 18$. State all horizontal asymptotes, vertical asymptotes, and holes (if any) for $y = \frac{f(x)}{g(x)}$ and $y = \frac{g(x)}{f(x)}$. Homework Help ✎

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Homework

2-109. The sigma notation expressions below represents Riemann sums that calculate the area under the curve of a function, f, for a ≤ x ≤ b using n rectangles of equal width. For each summation, determine the values of a, b, and n. Homework Help ✎

a. $\sum_{i=0}^{17} \frac{1}{3} f\left(-6+\frac{1}{3} i\right)$

b. $\sum_{i=0}^{9} \frac{1}{10} f\left(4+\frac{1}{10} i\right)$

2-110. Lena loves limits because they help her visualize the graphs of complicated looking functions. For example, by evaluating the limit as $x \to \infty$, she can determine if a rational function such as $f(x) = \frac{p(x)}{r(x)}$ will have a horizontal asymptote or not. Homework Help ✎

Evaluate each limit below and explain to Lena how she can determine $\lim_{x \to \infty} \frac{p(x)}{r(x)}$ without graphing. (Be careful! The expressions below are all slightly different.)

a. $\lim_{x \to \infty} \left(\frac{x^2-7x+6}{x^3+9x-2}\right)$

b. $\lim_{x \to \infty} \left(\frac{x^2-7x+6}{x^2+9x-2}\right)$

c. $\lim_{x \to \infty} \left(\frac{x^3-7x+6}{x^2+9x-2}\right)$