Chapter 2.1.1 and 2.1.2

The purpose of this lesson is to:

- Approximate **area under a curve using trapezoids** and determine distance from velocity information.
- Approximate **area under a curve using rectangles** and use calculators to compute area approximations.
- Understand how to read and use Summation Notation to solve **Area Under a Curve**

NOTES

2-1. Elena and J. T. are cruising at 65 mph (95 feet per second) on their way to Calculus Camp when their car runs out of gas. Elena knows that there is a gas station at the entrance to Calculus Camp in 1 mile. She quickly decides to determine how far the car will travel by taking velocity measurements as the car decelerates. The velocity of the car at selected points in time is recorded in the table below.

Time (seconds)	0	5	8	13	23	33	38	48	63	73	83	93	102
Velocity (ft/seconds)	95	85	81	70	62	48	44	35	25	19	12	4	0

a. Sketch a velocity graph using the data shown in the table above.

b. Describe how the velocity is changing. When is the velocity changing the fastest? The slowest?

c. Approximately how far did the car travel in the first 5 seconds? Use a trapezoid to approximate the distance traveled for $0 \le t \le 5$. **Area of a trapezoid:** $\frac{1}{2} * w * (b1 + b2)$

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Notes

The __________t__ _______ can be used to approximate the area under a curve.

In general, this looks like:

However, when the change in x is ___________ or _____ ________, we can use a shortcut:

Time (seconds)	0	5	8	13	23	33	38	48	63	73	83	93	102
Velocity (ft/seconds)	95	85	81	70	62	48	44	35	25	19	12	4	0

d. With the data provided, use trapezoids to approximate the total distance traveled by the car after it ran out of gas. It is important to show your work in a systematic and organized way.

e. Did the car reach the gas station before stopping? If not, how far did Elena and J. T. have to walk?

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Task Card

2-2. Sketch a scatterplot of the data below.

x	0	3	6	9	12	15
$f(x)$	5	10	11	15	13	6

a. "Connect" your scatterplot so that f becomes a continuous function. Is there more than one way to do this? Explain.

b. Use five trapezoids to approximate the area under f on $0 \le x \le 15$. Organize your steps systematically.

Tristin organized his work like this:

$$\frac{3}{2}(5 + 10) + \frac{3}{2}(10 + 11) + \frac{3}{2}(11 + 15) + \frac{3}{2}(15 + 13) + \frac{3}{2}(13 + 6) = A$$

c. Why does the fraction $\frac{3}{2}$ keep recurring throughout this equation?

d. Simplify Tristin's equation by "factoring out" the $\frac{3}{2}$:

e. Describe any new patterns you see:

f. Use the Trapezoid Rule from the notes box to set up and compute an approximation for the area under $y = g(x)$ on $2 \le x \le 10$ using four trapezoids of equal height.

2-6. Hooree is learning to hula-hoop. Using a tachometer, her coach keeps a record of her hula-hooping rate (rotations per minute) at select times.

Time (minutes)	2	4	9	15	19	26	30
Rate (rotations per minute)	14	18	22	16	10	8	6

a. Sketch a graph of Hooree's rate.

b. Find a way to approximate the number of rotations she makes during the 30-minute period. It is important to show your method in a systematic, organized way.

c. Do you think your approximation was over or under the actual number? Explain.

a. What appears to be happening to Hooree when $t > 9$?

Chapter 2.1.1

Notes

Summation Notation

The capital Greek letter sigma, Σ, (equivalent to “S” in English) is used in mathematics as a compact way to indicate a _____. The notation is used as a shorter way to write a long list of numbers ________ together. For example:

$$\sum_{k=1}^{3} k^2 =$$

Translated, this expression means the sum from _____ to _____ of the expression __ equals __.

Note: We call *k* the _____ and k^2 the __________ of the summation. The values of the index are __________________ ____________ __________ only, so the values of *k* in this example are __, __, and __.

2-3. In previous problems, approximating the area under a curve required adding the areas of many rectangles or trapezoids together. Imagine the work required to write down the expression if 100 rectangles were used! To reduce the amount of writing, mathematicians developed summation notation, which is explained in the following Math Notes box. Use the definition of summation notation to write out each sum in expanded form.

a. $\sum_{j=1}^{10} j$

b. $\sum_{m=1}^{10} m$

c. $\sum_{n=0}^{5} (4n-3)$

d. $\sum_{k=1}^{4} k(3k-1)^2$

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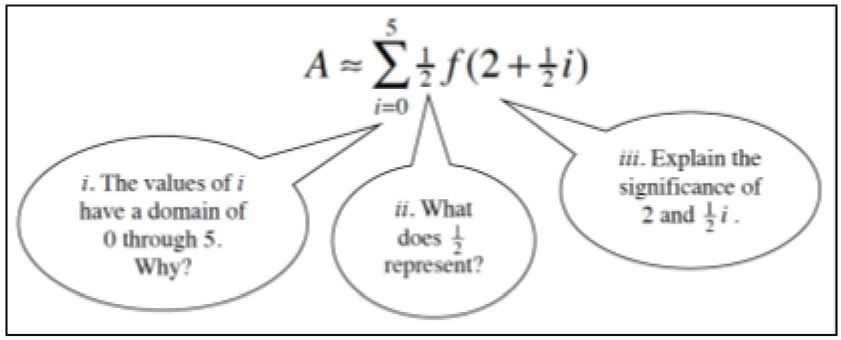
Notes

2-16. Discuss with your team how to use the summation feature of your graphing calculator to obtain the value of the summation below, where the function is $f(x) = 2\cos(\pi x)$. Then evaluate the sum.

$$\sum_{i=4}^{12} 2f(3+0.25i)$$



2-15. Evaluate: $\displaystyle\sum_{j=2}^{20} \left(2j^2-1\right)$

2-18. Using the same function $f(x) = x^2 - 6x + 13$, write an expression using sigma notation to approximate the area under the curve for $4 \le x \le 12$ using:

a. Two left endpoint rectangles of equal width.

b. Twenty-four left endpoint rectangles of equal width.

c. Nine left endpoint rectangles of equal width.

d. Explain why sigma notation requires that the rectangles have equal widths.

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Task Card

2-4. HELP!

a. Your teammate wrote the expansion for the sum below. Explain what was done incorrectly in this expansion.

$$\sum_{p=3}^{5} p^2 = 3^2 + 3.5^2 + 4^2 + 4.5^2 + 5^2 = 82.5$$

b. Realizing that their first expansion was incorrect, your teammate tried again. Explain what was done incorrectly this time.

$$\sum_{p=3}^{5} p^2 = (3+4+5)^2 = 144$$

c. Demonstrate how to correctly expand and simplify this sum.

2-5. Rewrite each of the following sums using summation notation.

a. $3 \cdot 4^1 + 3 \cdot 4^2 + 3 \cdot 4^3 + 3 \cdot 4^4$

b. 9 +16 + 25 + 36

c. $\frac{1}{2} f(0) + \frac{1}{2} f(2) + \frac{1}{2} f(4) + \frac{1}{2} f(6) + \frac{1}{2} f(8)$

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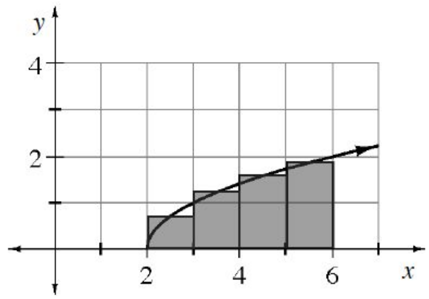
Homework

2-9. For $f(x) = \sqrt{x-2}$, he estimation of the area under the curve for $2 \le x \le 6$ is shown below using four midpoint rectangles. Approximate the area using these rectangles. How reasonable is your result? Homework Help ✎



2-10. Recall that the area between a function and the *x*-axis is defined as negative if the region is below the *x*-axis. Therefore, given $f(x) = \frac{1}{2}x - 6$ what is the area under the curve for $2 \le x \le 12$? 2-10 HW eTool (Desmos) Homework Help ✎.

2-11. Expand and evaluate each of the following sums. Homework Help ✎

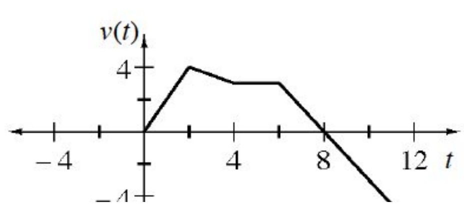
a. $\sum_{n=-4}^{4} n^2$

b. $\sum_{k=-4}^{4} k^3$

c. $\sum_{j=-3}^{3} 2^j$

d. $\sum_{i=-5}^{5} \sin(i)$

2-13. A bug is walking on your graph paper along the x-axis. The bug's velocity (in feet per second) is shown on the graph at right. Homework Help ✎



a. When did the bug turn around?
b. When was the bug's speed the greatest?
c. After 12 seconds, how far is the bug from its starting position?
d. Remember that acceleration is the rate of change of velocity. Calculate the acceleration of the bug at the following times.
    i. 1 second
    ii. 5 seconds
    iii. 10 seconds

2-20. What is the difference, if any, between the values of $\sum_{j=3}^{8} j^2$ and $\sum_{j=2}^{7} (j+1)^2$?
Homework Help ✎

2-22. Rewrite the summation notation $\sum_{i=6}^{11} f(i)$ so that the index goes from *i* = 10 to *i* = 15 and the result will be the same for any given function. Homework Help ✎

2-25. Write a complete set of approach statements for $y = \frac{(3x-1)(x+2)}{3x-1}$. Homework Help ✎
Note: A complete set of approach statements will include information about what happens when $x \to \infty$ and $x \to -\infty$. (This will tell us about the location of horizontal asymptotes, if any.) Also, a complete set of approach statements will describe what happens on each side of a hole or vertical asymptote. So the first step will be to determine the location of holes and vertical asymptotes.

2-26. If $f(x) = 2x + 3$, calculate the area under the curve $5 \le y \le 7$. 2-26 HW eTool (Desmos)
Homework Help ✎.