Chapter 3.3.2 and 3.3.3

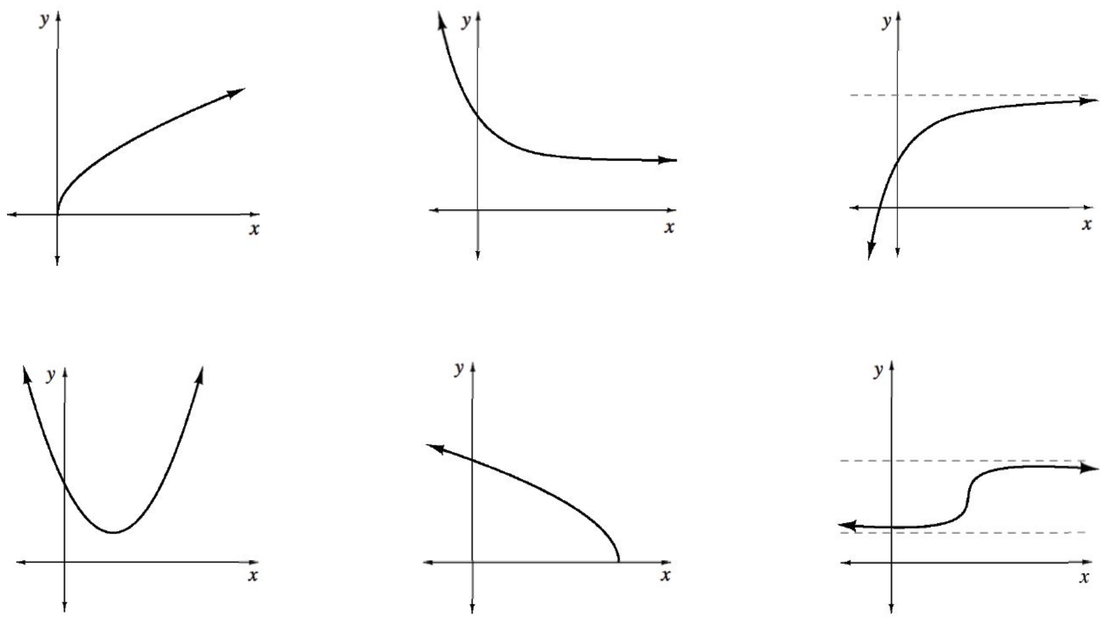
The purpose of this lesson is to:

- **Discuss the effect that increasing and decreasing slopes have on the shape of the curve and develop vocabulary about concavity.**
- **Sketch first and second derivatives of a given function.**

WARM UP

3-96. For each graph below, answer the following two questions using complete sentences. Remember that for some graphs, the conditions change as *x* increases.

- As *x* increases, does *f* increase or decrease?
- As *x* increases, does the slope of *f* increase or decrease?



Chapter 3.3.2 and 3.3.3

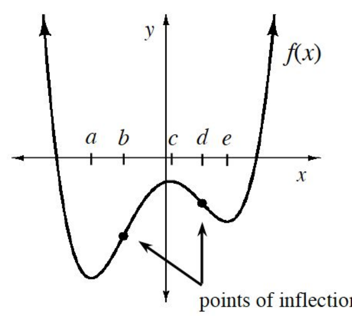
NOTES

Curve Analysis

A function is __________ on an interval if *y* increases as *x* increases. Likewise, a function is ___________ on an interval if *y* decreases as *x* increases.
One way to determine if a function is increasing or decreasing is to determine the domain on which the slope of its tangent lines, $\frac{dy}{dx}$, is positive or negative.

$f(x)$ is increasing because (a, c) $\cup$ (e, ∞) because $\frac{dy}{dx}$ > 0 and decreasing for (−∞, a) $\cup$ (c, e) because $\frac{dy}{dx}$ < 0.

The coordinate point where a function changes from increasing to decreasing or vice versa is called an __________. This extreme point is either a __________ or a __________. The function graphed at right has extrema at $x = a$, $x = c$, and $x = e$. Note: If a function is increasing or decreasing over its entire domain, the function is ____________.



When the *slopes of the tangent lines* increase on an interval, the graph is _________ ___ because it curves up. However, when the slopes decrease on an interval, the graph is __________ _______ because it curves down.

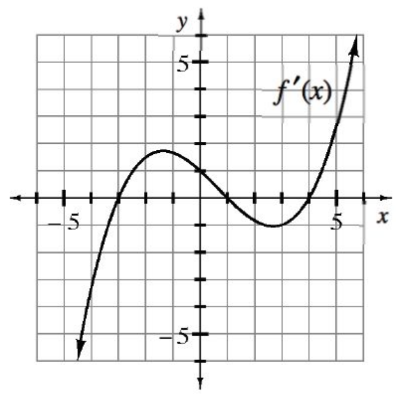
The graph of $y = f(x)$, shown above, is concave up for (−∞, b) $\cup$ (d, ∞) and concave down for (b, d).
A coordinate point where the concavity changes is called a ______ ____ ____________. At this point, the curve changes from concave up to concave down or vice versa.

Chapter 3.3.2 and 3.3.3

Task Card

3-98. Review the graphs from problem 3-96. Which of the curves are concave up, which are concave down, and which are sometimes concave up and sometimes concave down?

3-99. The graph at right is the *slope function* of *f*. Examine the graph carefully as you complete the parts below.



a. At what *x*-values is $f'(x) = 0$? What happens to *f* at these *x*-values?

b. Identify the part(s) of the domain on which *f* is increasing. Explain which graphical clues you used to determine this.

c. At what *x*-value(s) does *f* have a local minimum (i.e. the lowest point on a local region of a curve)? How can you tell? Explain which graphical clues you used to determine this.

d. Approximate the interval(s) over which f' is increasing. What happens to *f* at these *x*-values?

3-100. Describe the difference between stating, "*f* is increasing" and "f' is increasing." Which of the two statements indicates that f' is positive?

3-111. CURVE ANALYSIS

a. On the Lesson 3.3.3 Resource Page (or a large sheet of graph paper), set up three sets of axes—one above the other so that the *y*-axes are vertically aligned. On each set of axes, use a scale of –5 to 5 for the *x*-axis. On the top set of axes, sketch the graph of a function $y = f(x)$ such that:
  - *f* is a continuous, smooth function on $(-\infty, \infty)$
  - *f* has zeros at $x = -4, -\frac{3}{2}$, 2, and 5
  - *f* has local maxima at $(-3, 4)$ and $(2, 0)$
  - *f* has local minima at $(0, -2)$ and $(4, -3)$
  - *f* has points of inflection at $(-2, 3)$, $(1, -1)$, and $(3, -1)$

- f is increasing on $(-\infty, -3) \cup (0, 2) \cup (4, \infty)$
- f is decreasing on $(-3, 0) \cup (2, 4)$

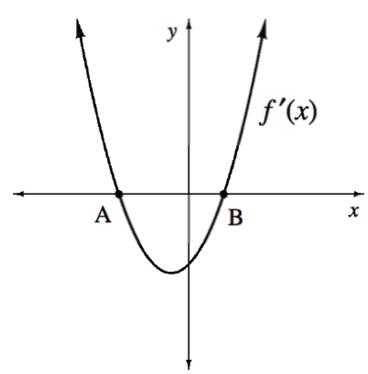
b. Obtain a sheet of stickers or colored markers. Choose one color to represent local maxima another color to represent local minima, and a third color to represent points of inflection. On the x-axis, use these colors to draw dots at the x-values where the maxima, minima, and points of inflection occur.

c. Use a straightedge to draw tangent lines to your function at every integer value from $-5 \le x \le 5$. Approximate the slopes of the tangent lines and record these slopes in a table of data: x-value vs. slope.

d. On the middle set of axes, plot the points from your table of data. Connect these points to create a smooth, continuous curve. Be sure to consider what happens as x approaches positive infinity and negative infinity. This graph represents the derivative, f', of your function. Mark the local maxima and local minima values with the same colors you used before. Then choose a fourth color and label the zeros of f'. Compare these points to their corresponding x-values on the graph of $y = f(x)$. What appears to be significant about these points?

e. Sketch tangent lines on f' and approximate their slopes. Record these slopes in a table of data. Plot this data on the third set of axes and connect the points in a smooth, continuous curve. Again, be sure to consider what happens as x approaches positive infinity and negative infinity. This graph represents f'', the second derivative of f. Mark all of the zeros with the same color you used on f'. What do you notice about the graphs of $y = f(x)$ and $y' = f'(x)$ where f'' is zero?

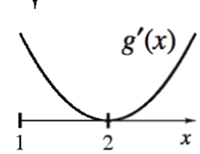
3-112. As shown on the diagram at right, the graph of $y = f'(x)$, the first derivative of $y = f(x)$, has roots at $x = $ A and $x = $ B.



a. With your team, find two different ways to justify that $f(x)$ has a local maximum at $x = $ A. One method should involve $f'(x)$ and the other should involve $f''(x)$. Be prepared to share your justifications with your class.

b. How many local maxima, local minima and inflection points does f have? Justify your answer.

c. The graph of $y = g'(x)$, the first derivative of $y = g(x)$, has a root at $x = 2$, as shown at right. Based on this graph, does $g(x)$ have local maximum, local



minimum, or a point of inflection at $x = 2$? Justify your answer using $g'(x)$ or $g''(x)$.

Chapter 3.3.2 and 3.3.3

Homework

3-101. Draw the following function, given its slope statement below.

The slope starts close to zero. When $x = -5$, the slope increases quickly. Then at $x = 0$ the slope decreases quickly until $x = 5$ when the slope is close to zero again. Homework Help ✎

3-102. Write the slope function for each of the following functions. Homework Help ✎

a. $f(x) = \frac{2}{3} * (x - 5)^3 + x^2$

b. $f(x) = \sqrt[3]{x}$

c. $f(x) = \sin\left(\frac{\pi}{4}\right)$

d. $f(x) = \frac{1}{(x+1)^2}$

3-105. If $f'(x) = \cos(x)$, write two different possible equations for f. How many equations are possible? Homework Help ✎

3-106. Examine the Riemann sum below for the area under f. Homework Help ✎

$$\sum_{i=0}^{11} \frac{6-3}{12} f\left(3 + \frac{6-3}{12} \cdot i\right)$$

a. How many rectangles were used?
b. If the area being approximated is over the interval $a \leq x \leq b$, what are the values of a and b?

3-109. Curves can be labeled with descriptors such as "concave down" and "increasing." On graph paper (or sketch the graphs in the space below), graph each function and label its respective parts. Use different colors to represent concavity. Homework Help ✎

a. $f(x) = 2x^2 + x - 15$
b. $g(x) = x^3 - 12x - 1$

3-113. Use your observations from problem 3-98 to algebraically verify that $y = x^3 + \frac{3}{2}x^2 - 6x + 2$ is concave up when $x = 0$. Homework Help ✎

3-116. For each function below, write and evaluate a Riemann sum to calculate the area under the curve for $-2 \le x \le 1$ using 24 left endpoint rectangles. Homework Help ✎

a. $f(x) = 2^x$
b. $f(x) = \sqrt{x+2}$

3-118. Multiple Choice: If $f'(x) = 6x^2 - 4$, then which of the following could be f? Homework Help ✎

A. $f(x) = 2x^3 - 4x + 1$
B. $f(x) = 2x^3 - 4x - 4$
C. $f(x) = 2x^3 - 4x + 4$
D. $f(x) = 2x(x^2 - 2)$
E. all of these