Chapter 5.1.3 and 5.1.4

The purpose of this lesson is to:
Understand how f' and f'' can show us critical values
Understand how f' and f'' can show us Minimums and Maximums
Understand how to use the first and second derivative test
Understand how to use endpoints in the f' and f'' test.

WARM UP

5-21. RADIO DAZE

Avantika is very excited. Her favorite radio station, KNRD, is having a contest and she thinks she can win. The grand prize includes unlimited rides on the awesome Rotating Flags roller coaster!

Avantika listens intently as her favorite announcer, Ima Geek, reads the question:

"Our mystery function today has these properties," she starts.

"*f* $f(2) = 5$,

f $f'(2) = 0$,

f $f'(x) < 0$ for all x less than 2, and

f $f'(x) > 0$ for all x greater than 2.

Remember that all of our mystery functions are continuous and defined for all x. To win, you must describe a small portion of the function, including the coordinates of a key point. You must also describe something interesting about the graph at this point. We'll take the twelfth caller, but if your answer is incorrect, you will be barred from all future KNRD contests, so be careful."

Avantika badly needs your team's help with this. A picture is worth a thousand words, so a labeled sketch will be very helpful. Since Avantika will have to tell Ima the answer verbally, your team needs to describe the sketch in words as well.

5-22. Since Avantika answered the first question correctly with your help, she advanced to the second round.

"Our next mystery function today has these properties," Ima says.

"$f''(3) < 0$, $f'(3) = 0$, and $f(3) = 2$.

We'll take the n^{th} caller, where $n = 15$," she said in her nerdiest voice.

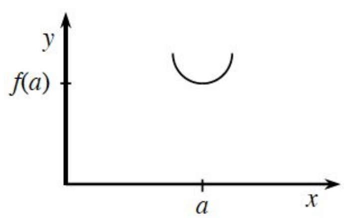
Avantika is really stumped. Can your team come to her rescue yet again?

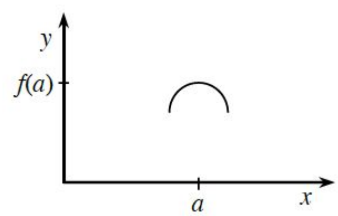
Chapter 5.1.3 and 5.1.4

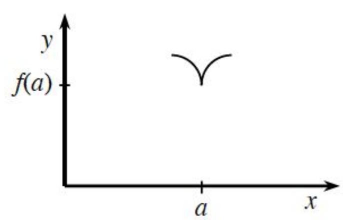
Warm-Up Continued

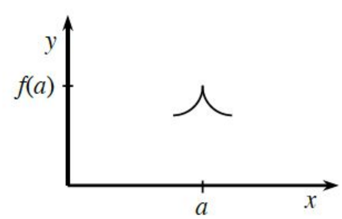
5-23. Ima Geek of KNRD is taking a week vacation and needs to have the rest of the graphs and clues ready for her replacement and father Kit Protector (Ima calls him Pa). She has all of the different graphs that will be used for the week, but she needs to generate a list of properties for each mystery function.

The properties need to describe the characteristics of f' at $x = a$ and for values of x to the left and right of a. The second round will include characteristics of f'' at $x = a$. Kit Protector will announce the list of properties over the radio. For each graph, list as many properties as possible for the contest.









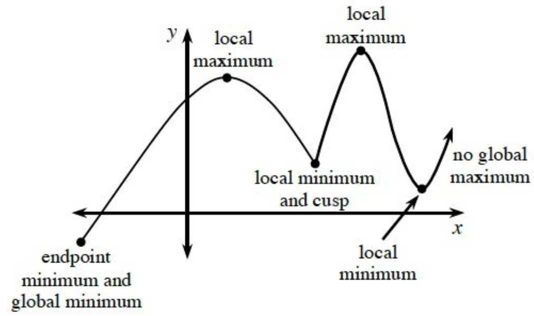
If f has an extrema (local maximum or local minimum) at $x = a$, what do you know about $f'(a)$? Also, what must happen to $f'(x)$ at $x = a$?

Chapter 5.1.3 and 5.1.4

Notes

Extreme Values

Often, we are interested in where a function takes a __________ or a ___________ value.
These values are also called __________ ________ or ____________.



There are two types of extrema: _________ (also called ___________), _________ (also called __________).
Examine the graph at right for examples of local and global extrema.

Endpoint extrema occur in problems where the domain is limited, either by instruction or by the conditions of an application problem. When choosing a global maximum (or minimum), it is necessary to compare the highest (or lowest) local extrema value with any endpoints or jump discontinuities that might exist in the specified domain.

Extreme Value Theorem: If *f* is continuous over a ____________ __________ [*a*, *b*] then *f* has both a global minimum and a global maximum in the interval.

5-33. Karen is determined to locate a maximum for a function *f*. She writes the derivative of *f* and sets it equal to 0. Kirt comments that although her method can locate a maximum, it can also locate a minimum!

Find an example of a function that supports Karen's process for finding the location of a local minimum. Then, find a counterexample that supports Kirt's claim.

Revise Karen's process by adding a second step that guarantees she will find a local maximum (not a local minimum).

Ms. Mathilde pipes in, *"Your two-step process is still not perfect! I can think of a function whose local maximum will NOT be detected using those two steps!"* She challenges Karen and Kirt to sketch a function whose local maximum exists at a point where *f'* is not zero. Help Karen and Kirt sketch this function.

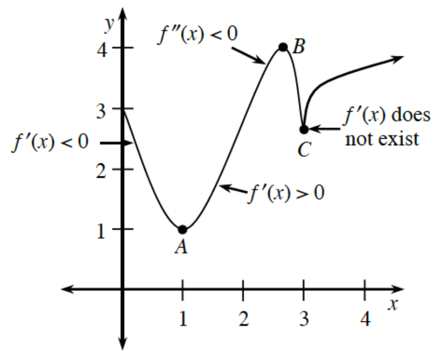
Revise Karen's process again to include Ms. Mathilde's idea.

“This makes things much more complicated,” cries Karen! *“At least I know that there will be either a local maximum or a local minimum where the first derivative equals zero.”*

“I can think of a counterexample to that conjecture as well,” chortled Ms. Mathilde. *“There are some functions that have points of inflection at the location where its first derivative equals zero!”* Find a counterexample that Ms. Mathilde could be thinking of.

Critical Points and the Derivative Tests for Extrema

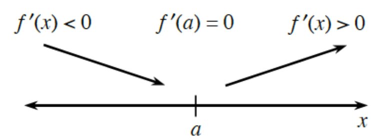
Although the condition $f'(x) = 0$ is often useful for locating relative extrema (points *A* and *B* in the diagram), local extrema can also occur at points where $f'(x)$ does not exist (point *C*). Interior points (not endpoints) where maxima and minima *might* occur are called critical points. To test for global extrema, the endpoints must also be considered.



__________ ________ are points where the first derivative is either zero or does not exist.

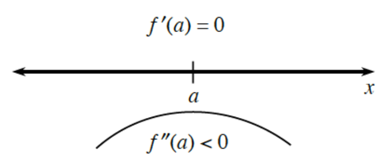
To test whether a critical point is a local minimum or maximum, use either the _________ ___________ _____ or the __________ ____________ ______ (or both).

First Derivative Test



First Derivative Test: If the first derivative changes from __________ (on the left) to __________ (on the right) on either side of a critical point, then the point is a local ____________. Points *A* and *C* demonstrate this situation in the diagrams above and at right. (There is a similar test for a local maximum.)

Second Derivative Test



Second Derivative Test: If the first derivative is ______ and the second derivative is ____________ at a critical point, then the point is a local maximum. Point *B* demonstrates this situation. (There is a similar test for a local minimum.

Chapter 5.1.3 and 5.1.4

Task Card

5-34. Remembering all the information in the Math Notes box is not as hard as it may seem. Drawing diagrams is very useful. You need to remember that a positive *first* derivative means an increasing *function* (positive slope) and a positive *second* derivative means increasing *slope* (concave up).

a. What does a negative first derivative mean?

b. What does a negative second derivative mean?

c. In your own words, state the first derivative test for a relative maximum.

d. In your own words, state the second derivative test for a relative minimum.

5-36. ENERGY CRISIS

In January 2001, energy prices in California began to rise dramatically. The average cost per megawatt hour could be modeled by the function

$$C(d) = 0.1d^3 - 4.2d^2 + 54d + 10$$

where d is the day in January and $C(d)$ is the average cost per megawatt.

a. On which day in January was the average megawatt cost the highest? Be sure to justify your answer by using the appropriate test.

b. Using your results from part (a), set an appropriate window that contains the function during the month of January. Graph the function and verify your answer to part (a). Alter your answer if necessary. Explain what happened.

Chapter 5.1.3 and 5.1.4

Homework

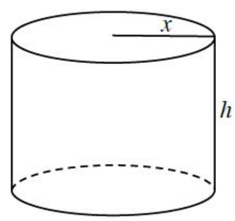
5-37. PI DAY PIES

Every year, to celebrate Pi Day, the Math Club sells pies to students (Pican pies, of course). From past experience, they know that if they sell the pies for $5.75 each, 314 pies will be sold. However, for every 25¢ they raise the price, they will sell approximately 10 fewer pies.

a. How much should the Math Club charge per pie so that they make the most money? Solve this problem any way you want, but be sure to include an equation and graph in your solution.

b. If you have not done so already, determine how to use derivatives to help you answer this question. Explain completely.

5-38. The Tasty Tuna Company needs to design a can that will have a volume of 20 in^3. If the base of the can is x inches and the height is h inches:



a. Write an equation for the total surface area of the can using both x and h.

b. Write an equation for the volume of the can using x and h.

c. Use your results from part (b) to write an equation for the surface area in terms of x.

d. What dimensions that will minimize the surface area?

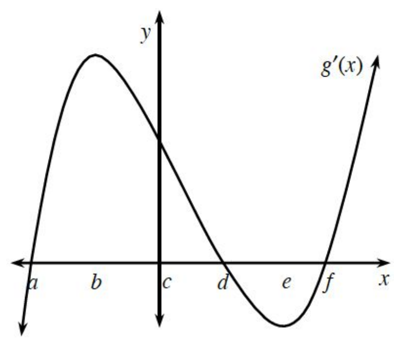
5-41. Sketch the graph of a continuous, differentiable function with only one absolute maximum and two relative minima. [Homework Help ✎]

5-42. The Second Derivative Test is easy to apply once you know the second derivative, but unfortunately it does not always work. The First Derivative Test works more often, but is harder to apply. Compare the two tests by investigating the graphs of $y = x^2$, $y = x^3$, and $y = x^4$ at the origin. Homework Help ✎

b. Use the First Derivative Test to investigate the behavior of $y = x^2$, $y = x^3$, and $y = x^4$ at the origin.

c. Try to confirm your results from part (a) by using the Second Derivative Test. When does the test work? What can you conclude if you know that the second derivative equals zero at the origin?

5-43. Using the graph of the slope function at right, determine where the following situations occur for g : Homework Help ✎



d. Relative minima.
e. Relative maxima.
f. Interval(s) over which g is increasing.

g. Inflection points.
h. Interval(s) over which g is concave up.

i. Interval(s) over which g is concave down.

A. **5-45.** Evaluate each of the integrals below. Assume that a and b are constants. Homework Help ✎

a. $\int (a^2)\, dx$

b. $\int (bu^2 - a)\, du$

c. $\int (am - b)\, dm$

Chapter 5.1.3 and 5.1.4

Homework

5-27. Review Math Notes box in this lesson. Let $f(x) = -\frac{1}{4}x^4 + 3x^3 - 10x^2 + 40$. Homework Help ✎

Identify the local maxima and local minima of *f*.

Identify the global maximum and gloabal minimum of *f*.

Identify the global maximum and global minimum values of *f* over the interval [2, 5].

Identify the global maximum and global minimum values of *f* over the interval [–2, 1].

Explain how your answers to parts (a) through (d) demonstrate the Extreme Value Theorem.

5-46. Let $f(x) = x^2 + 3x$ and $g(x) = \sin(x)$. Match the expressions in the left-hand column with their simplifications in the right-hand column. Homework Help ✎

$f'(g(x))$	$\sin^2(x) + 3\sin(x)$
$f(g(x))$	$\cos(x^2 + 3x)$
$g(f(x))$	$2\sin(x) + 3$
$g'(f(x))$	$\sin(x^2 + 3x)$