Chapter 3.4.4 and Review

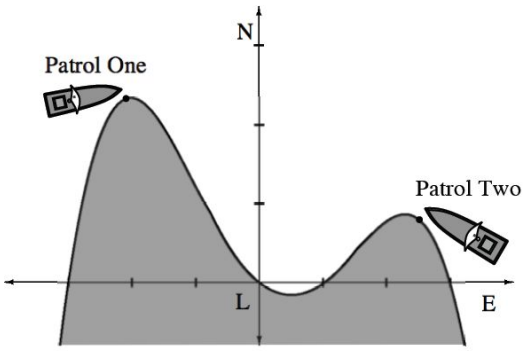
The purpose of this lesson is to:

- **Verify that a graphing calculator can, for some functions, give misleading results. Analyze several unusual functions.**
- **Apply knowledge of tangent lines and derivatives to solve a problem**

WARM UP

3-182. TROUBLE OFF PYTHAGORAS BAY

The zoo boat Kra is stranded off the coast of Pythagoras Bay, whose coastline can be represented by the curve at right. The captain has escaped on a lifeboat with a penguin and a bobcat, but a giraffe remains on board, hoping to be saved. Two patrol boats are heading towards the lighthouse along the coastline, one from the east and the other from the west. Both boats have strong searchlights that are fixed and mounted to the front of each boat. Below are the reports called in from each of the boats.



"Patrol One calling in. We are currently 2.10 miles west and 2.32 miles north of the lighthouse on the coastline and we spot the zoo boat in our direct line of sight."
"Patrol Two here. We have the giraffe spotted in our direct line of site and our location is 2.50 miles east and slightly north of the lighthouse on the coastline."
You are aboard a freight boat large enough to rescue a giant giraffe, and currently located at the lighthouse at (0, 0). It is your job to give the captain of the freight boat the information needed to locate the scared animal.
A sketch of the portion of the coastline that is being patrolled by the two boats is shown in the graph above. The scale for the graph is one mile for each unit.

a. You can hear the giraffe bellowing in the distance. Using the graph and the information provided from the patrol boats, write the equation of a polynomial function that can be used to model the coastline.

b. Help! The giraffe's neck is entirely submerged! Write the equations of the lines that represent the direct lines of sites from each patrol boat to the bellowing beast.

c. It is known that giraffes can swim, but not very well. You need to act fast. What is the position of the giraffe and how far is it from the freight boat?

Chapter 3.4.4 and Review

Task Card

3-183. Determine algebraically whether $f(x) = 2x^3 - 5x^2 + 1$ is concave up or concave down at $x = 0$ and at $x = 2$. Then, verify your solution with a graph of $y = f''(x)$. Homework Help ✎

3-184. Write the antiderivative for each function below. Test your solution by verifying that $F'(x) = f(x)$. Homework Help ✎

a. $f(x) = 4x^{-3} + \frac{1}{2}x - 3$

b. $f(x) = \frac{6}{x^2}$

3-185. If $f(x) = 3 - x$ and $g(x) = \frac{1}{3-x}$, write and simplify $h(x) = f(g(x))$ and $j(x) = g(f(x))$. Does $h(x) = j(x)$? What does this mean? Justify your answer completely. Homework Help ✎

3-186. Write a Riemann sum to estimate the area under the curve for $5 \le x \le 13$ using n left endpoint rectangles given $f(x) = 4\sqrt[3]{x-5}$. Then calculate the areas for $n = 8$, 20, and 50 rectangles. Which approximation is most accurate and why? Homework Help ✎

Chapter 3.4.4 and Review

Task Card

3-187. HANAH STRIKES AGAIN!

To calculate the slope of a line tangent to f at $x = a$, most graphing calculators use Hanah's method from problem 3-35, formally called the **symmetric difference quotient**, shown below.

$$\lim_{h \to 0} \frac{f(x+h) - f(x-h)}{2h}$$

a. Use the symmetric difference quotient to determine $f\,'$ if $f(x) = 3x - 2$.

b. Use this symmetric difference in your graphing calculator to graph $f\,'$ for $f(x) = \sin(x)$ for $h = 0.001$.

3-188. ANOTHER PROBLEM FOR HANAH Homework Help ✎

a. Graph $f(x) = x^{2/3}$ on graph paper. Then without a calculator, sketch $y = f\,'(x)$ on the same set of axes.

b. Describe the graphs of $y = f(x)$ and $y = f\,'(x)$ at $x = 0$.

c. Use your graphing calculator to determine the slope of $f(x) = x^{2/3}$ at the vertex.

d. Calculators are not always accurate. Some graphing calculators incorrectly determine slopes at a vertex, as well as other cusps, because they use the symmetric difference quotient to calculate the slope of a tangent line.

e. For $f(x) = x^{2/3}$, use $\frac{f(x+h)-f(x-h)}{2h}$, to calculate $f'(0)$ for $h = 0.1$, -0.1, and 0.01. What do you notice? What leads a calculator to give a false derivative of $f(x) = x^{2/3}$ at $x = 0$?

3-189. Without a calculator, write the equation of the line tangent to $g(z) = \frac{z^7+5z^6-z^3}{z^2}$ at $z = 0$. Homework Help ✎

3-190. Determine algebraically where $y = x^3 + \frac{3}{2}x^2 - 6x + 2$ is decreasing. Then, check your answer by graphing the function on your graphing calculator. How did a derivative help you solve this problem? Homework Help ✎