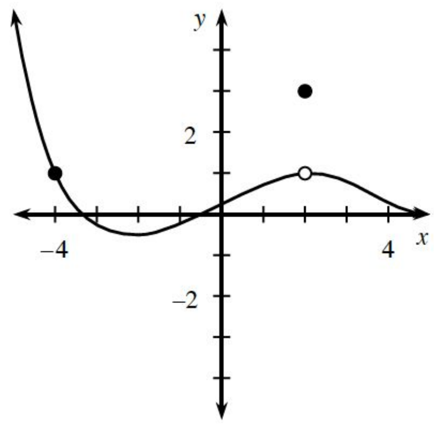
Chapter 2.2.2 and 2.2.3

The purpose of this lesson is to:

- Use limits to determine continuity.
- Explain why a particular graph is or is not continuous at a particular x-value using the three-part definition of continuity and sketch graphs for a variety of situations that fail each part of the definition of continuity.
- Learn the Intermediate Value Theorem.

Warm-Up

2-57. Examine the graph of $y = f(x)$ at right. Evaluate:



a. $\lim_{x \to -\infty} f(x)$

b. $\lim_{x \to \infty} f(x)$

c. $\lim_{x \to -4} f(x)$

d. $\lim_{x \to 2} f(x)$

e. One of the limit statements above determines the horizontal asymptote of *f*. Which one? Explain.

f. Sketch a graph of $y = \arctan(x)$. Describe the end behavior of the graph by writing two limit statements, one for each of its horizontal asymptotes.

2-58. On your graphing calculator, graph $f(x) = \frac{1}{x^2}$.

g. Describe what happens to *f* as *x* approaches 0 on each side? (i.e. Evaluate $\lim_{x \to 0^-} f(x)$ and $\lim_{x \to 0^+} f(x)$.)

h. Do both sides (the left-hand and right-hand limits) agree?

i. What is $\lim_{x \to 0} f(x)$?

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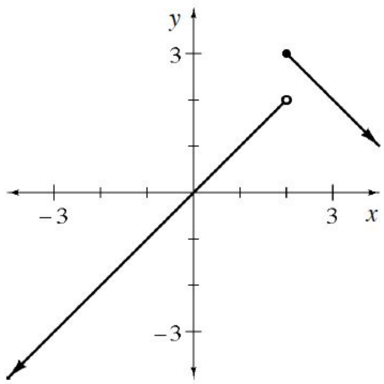
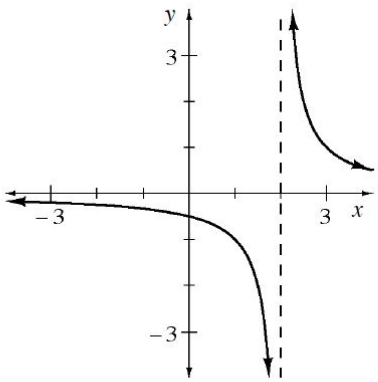
Warm-Up

2-59. For each function below, explain why the limit does not exist at x = 2.

a. 

b. 

2-60. Sketch $f(x) = \frac{|x|}{x}$. Evaluate $\lim_{x \to 0} f(x)$ or explain why it does not exist.

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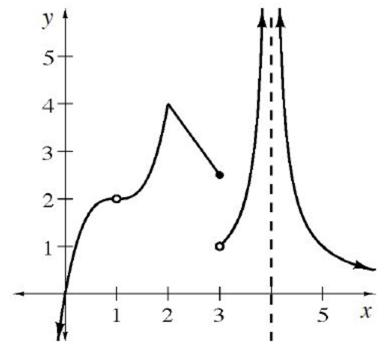
NOTES

2-61. Now let's look at the relationship between limits and continuity. Examine the graph at right and use it to complete the table below.

If a limit does not exist, enter "DNE" into your table.

a	$\lim\limits_{x \to a^-} f(x)$	$\lim\limits_{x \to a^+} f(x)$	$\lim\limits_{x \to a} f(x)$	*f*(*a*)
1				
2				
3				
4				



2-62. Look at your results from problem 2-61 and consider the relationship between limits and continuity by analyzing the following conditions. Justify each response with sketches that show examples and/or counterexamples.

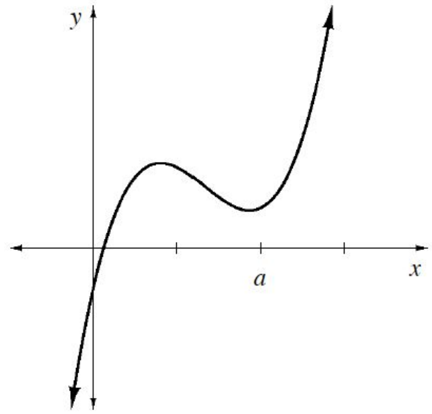
a. Is a function continuous at $x = a$ $\lim\limits_{x \to a} f(x)$ if does not exist?

b. Is a function continuous at $x = a$ if $f(a)$ does not exist?

c. Is a function continuous if $\lim\limits_{x \to a} f(x)$ both and *f*(*a*) exist?

d. Use your answers from parts (a) through (c) to explain when a function is continuous at a point.

The Three Conditions of Continuity



Continuity is an important concept in calculus because many important theorems of calculus require functions to be continuous over the _________ ___ ________. Simply stating that you can trace a graph without lifting your pencil is neither a complete nor a formal way to justify the continuity of a function at a point.

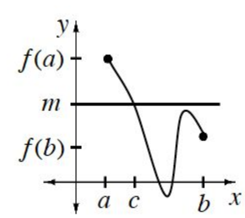
In order to justify that a function *f* is ____________ at the point _______, you must demonstrate that *f* meets all three conditions listed below.

1. $\lim_{x \to a} f(x)$ exists
2. $f(a)$ exists
3. $\lim_{x \to a} f(x) = f(a)$

Recall that this means:
$\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$

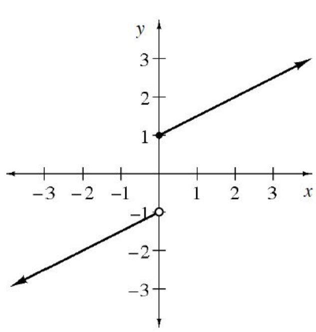
A function is ____________ _______ ____ ____________ if it is continuous at each point in the interval.

Intermediate Value Theorem



Let *f* be a function continuous on the closed interval ________. Then for every value *m* between *f* (*a*) and *f* (*b*) there exists a ___________ ___, ______________, such that ________.

2-79. Examine the function below. Notice that *f* (–2) = –2 and *f* (2) = 2, yet there is no root between *x* = –2 and 2. Why does this not contradict the Intermediate Value Theorem?



2-80. For some continuous function f, $f(-3) = 5$ and $f(2) = -3$. What is the minimum number of values possible for a that satisfy $f(a) = 1$?

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Task Card

2-63. Given the graph at right, determine the following values.

a. $\lim\limits_{x\to-\infty} f(x)$

b. $\lim\limits_{x\to\infty} f(x)$

c. $f(-2)$

d. $\lim\limits_{x\to-2} f(x)$

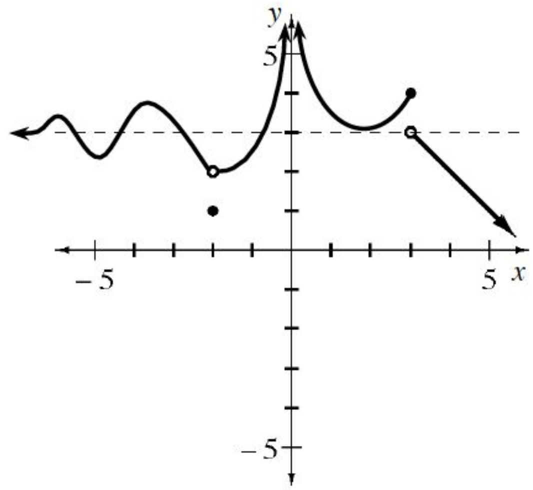
e. $f(0)$

f. $\lim\limits_{x\to0} f(x)$

g. $f(3)$

h. $\lim\limits_{x\to3} f(x)$

i. Is the function continuous at $x = 0$? Explain.



A function is **continuous over an interval** if it is continuous at each point in the interval.

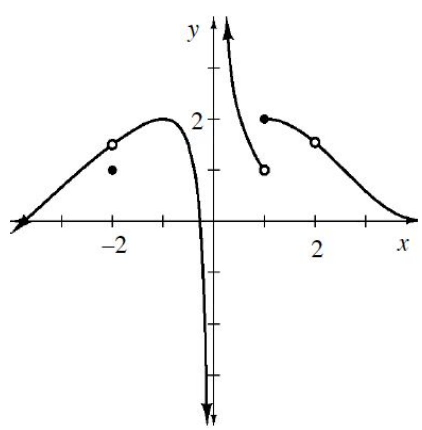
2-73. Examine the conditions of continuity given in the Math Notes and summarize them with your team. Then demonstrate your understanding of continuity by sketching functions for parts (a) through (c).

a. Sketch a function that satisfies condition 1, but not condition 2 (and therefore not condition 3).

b. Sketch a function that satisfies condition 2, but not conditions 1 or 3.

c. Sketch a function that satisfies conditions 1 and 2, but not condition 3.

2-74. Examine the graph at the right. Identify four values of *x* where a discontinuity exists. At each of these values, state the condition(s) of continuity that fail(s).

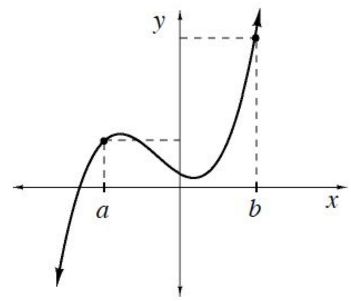


2-75. If *g* is continuous for all real numbers, such that $g(-4) = -10$ and $g(-1) = 3$, explain why *g must* have a root (*x*-intercept) for an *x*-value in the interval $(-4, -1)$. Include a sketch of a possible function.

$|x|$

2-76. Use the three conditions of continuity to justify why $f(x)$ = is continuous at $x = 0$.

2-78. Explain why a function that is continuous for all x-values on $[a, b]$ must pass through every y-value between $f(a)$ and $f(b)$ *at least once* in that interval. This is called the ________________ _______ __________ for continuous functions.



Chapter 2.2.2 and 2.2.3

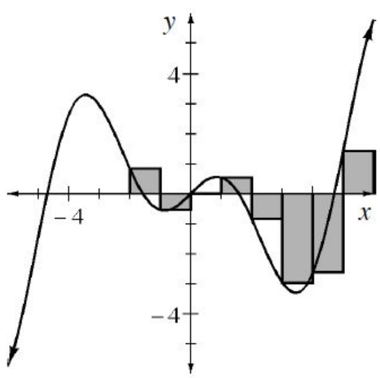
Homework

2-64. Kimberly is always complaining that it is either too hot or too cold. As a matter of fact, she is so picky that she is only happy when it is exactly 72ºF. At 8:00 a.m. it is 65°F. By 3:00 p.m. it is 90º. Homework Help ✎

a. Is there a time when Kimberly is happy?

b. If at 6:00 p.m. the temp is 70°F, what is the minimum number of times Kimberly was happy today?

2-69. Using sigma notation, write a Riemann sum to estimate the area under the function $f(x) = x\cos(x)$ for $-2 \le x \le 6$ with eight left endpoint rectangles of equal width. Then use the summation feature of your graphing calculator to calculate the estimated area. Homework Help ✎



2-71. If $f(x) = \frac{x-3}{x+5}$, evaluate: Homework Help ✎

a. $\lim\limits_{x\to\infty} f(x)$

b. $\lim\limits_{x\to-\infty} f(x)$

c. $\lim\limits_{x\to-5} f(x)$

d. $f(x-5)$

e. $f(2m+3)$

f. $f(x+h)$

g. For parts (a) and (b), explain the graphical significance of $\lim\limits_{x\to\infty} f(x)$ and $\lim\limits_{x\to-\infty} f(x)$.

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Homework

2-81. A helium balloon is released from the ground and floats upward. The height of the balloon is shown at the following times: Homework Help ✎

Time (s)	0	1	2	3	4	5	6	7	8	9	10
Height (feet)	0	50	98	144	188	230	270	308	344	378	410

a. What is the average velocity over the first 10 seconds of the balloon's flight? Over the first 5 seconds?
b. Calculate the finite differences for the heights. How is the velocity changing? Explore this using the 2-81 HW eTool (Desmos).
c. What do the finite differences tell you about the height function for the balloon?

2-82. Examine the expanded sums below and write the equivalent sigma notation. Homework Help ✎

a. $\frac{2}{3}f(-2+\frac{2}{3}\cdot 0)+\frac{2}{3}f(-2+\frac{2}{3}\cdot 1)+\frac{2}{3}f(-2+\frac{2}{3}\cdot 2)+\frac{2}{3}f(-2+\frac{2}{3}\cdot 3)$

b. $\frac{1}{2}f(6+\frac{1}{2}\cdot 0)+\frac{1}{2}f(6+\frac{1}{2}\cdot 1)+\frac{1}{2}f(6+\frac{1}{2}\cdot 2)+\frac{1}{2}f(6+\frac{1}{2}\cdot 3)+\frac{1}{2}f(6+\frac{1}{2}\cdot 4)$

2-83. The Intermediate Value Theorem is sometimes used to prove that roots exist. For example, $f(x) = 5\sqrt[3]{x-2} - 4$ is a continuous function. Given $f(2) = -4$ and $f(3) = 1$, does f have a root somewhere between $x = 2$ and $x = 3$? Why or why not? 2-83 HW eTool (Desmos) Homework Help ✎.

2-85. Jamal wrote the following Riemann sum to estimate the area under $f(x) = 3x^2 - 2$. 2-85 HW eTool (Desmos) Homework Help ✎.

$$\sum_{i=0}^{9}\frac{1}{2}f\left(-3+\frac{1}{2}i\right)$$

a. Draw a sketch of the region. How many rectangles did he use?
b. For what domain of f did Jamal estimate the area?
c. Use the summation feature of your calculator to approximate the area using Jamal's Riemann sum.