Chapter 4.4.2 and 4.2.3

The purpose of this lesson is to:
Understand how to find the area between two curves.

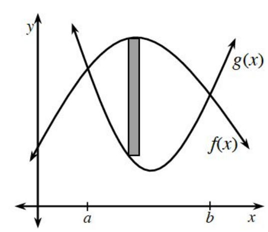
WARM UP

4-142. Calculate the area of the regions described below. Review problem 4-132 for a description of a complete solution.

a. The area between $y = -2(x^2 - 1)$ and $y = -x^2 + 1$.

b. The area between $y = \sin(x)$ and $y = \frac{3}{4}x - 7$ for $\pi \le x \le 2\pi$.

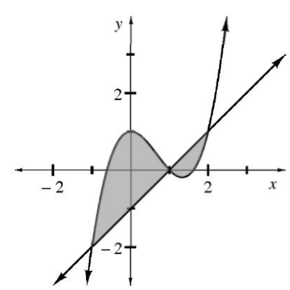
4-143. What will the result be if $\int_a^b (g(x) - f(x))\,dx$ is calculated instead of $\int_a^b (f(x) - g(x))\,dx$ for the functions f and g shown below? Explain your thinking.



4-145. Examine the area bounded by the graphs of the functions $f(x) = x - 1$ and $g(x) = x^3 - 2x^2 + 1$, shown at right.

a. Explain why this region requires two integrals.

b. Write and evaluate a set of integrals to calculate the area between the curves. Check your answer with your graphing calculator.



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NOTES

Area Under a Curve vs. Area Between Curves

Definite integrals are often thought of as representing the area of a region on a graph. However, it is important to contrast ______ __________ ________ with _____ ________ ____ ________.

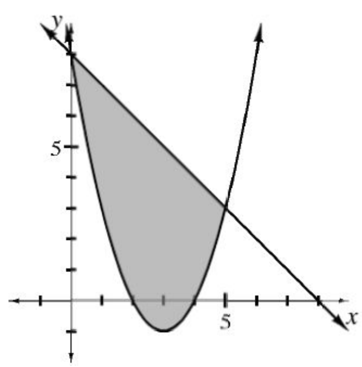
Area Under a Curve: When evaluating a definite integral, shading the *area under a curve* is a convenient way to visualize what you are doing. But, in application, definite integrals are rarely used to measure geometric area. Instead, definite integrals are used to calculate the *accumulated value* of a function.

For example $\int_a^b \left(\frac{\text{words}}{\text{hour}}\right) dt$ = total words read over the interval _____.

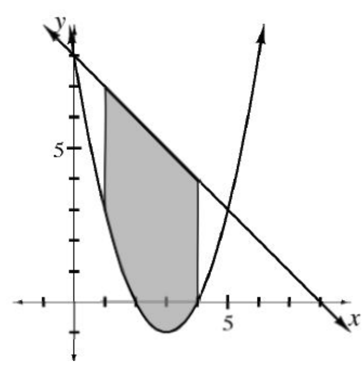
Area Between Curves: However, *area between curves* always represents a __________ area, measured in square units. For that reason, the integrand must involve the ______________ between two functions that bound (or contain) a region, and must be set up to guarantee a positive result.

For example: Both definite integrals below represent the geometric area of the region *R* which is bounded by $y = f(x)$ and $y = g(x)$ on the interval $[a, b]$ where $f(x) \geq g(x)$.

$\int_a^b \left(f(x) - g(x)\right) dx$ = area of *R* or $\int_a^b \left|g(x) - f(x)\right| dx$ = area of *R*



$\int_0^5 \left(f(x) - g(x)\right) dx$



$\int_1^4 \left(f(x) - g(x)\right) dx$

As shown on the graphs above, the bounds, ________ and ________, can be the intersection of $y = f(x)$ and $y = g(x)$ or (vertical) boundary lines of the region.

Note: Whether you are analyzing the area under a curve or computing the area between curves, definite integrals can be set up with reference to the *x*- or *y*-axis.

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Task Card

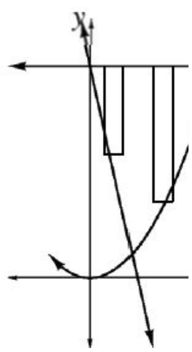
4-155. Adam, Becky, and Cathy are each working on calculating the area of the region bounded by the curves $y = x^2$, $y = 9$, and $y = -8x + 9$. Each person is approaching the problem using a different method, as shown below.

[Lesson 4.4.3 Resource Page]

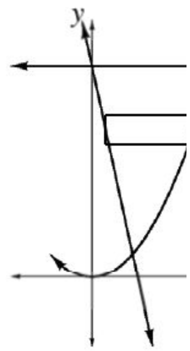
a. Label the dimensions of a typical rectangle in each diagram below.

b. Describe the technique each student is using. Decide if the method is valid. Then compute each integral to determine if each gives the same area.

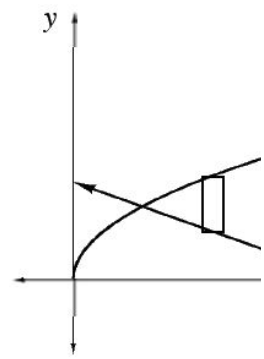
a. Adam's Method



b. Becky's Method



c. Cathy's Method



d. Area =

$\int_0^1 (9-(-8x+$

$+\int_1^3 (9-x^2)d$

e. Area =

$\int_1^9 \left(\sqrt{y}-\frac{9-y}{8}\right)$

f. Area =

$\int_1^9 \left(\sqrt{x}-\frac{9-x}{8}\right)dx$

4-156. Use two different strategies to calculate the area in the first quadrant bounded by $y = \sqrt{x}$ and $y = 4$. Be prepared to present your solution and describe your method to the class.

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Task Card

4-157. Calculate the area bound by the curves $y = 2\sqrt{x}$ and $y = x$ in the first quadrant by setting up and evaluating an integral expression in terms of y.

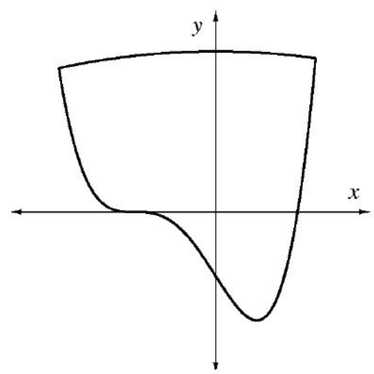
4-158. Calculate the total area enclosed by the functions $y = \sin(x)$ and $y = -\sin(x)$ for $0 \le x \le 2\pi$. Be prepared to present your solution and describe your method to the class.

4-159. FUNKY DESK

The Funky Furniture Company has designed a new desk for schools. The desktop is formed by the region bounded by the functions:

$$f(x) = \frac{1}{512}x^4 + \frac{1}{32}x^3 - 2x - 8$$

$$g(x) = -\frac{2}{225}x^2 + 20$$



a. The elbowroom is the distance from the x-axis to the lowest point on the curve. How much elbowroom is available on the desk if x and y are measured in inches?

b. Sketch the region on your paper. Draw and label a typical rectangle that can be used to calculate the area of the desk.

c. Set up and evaluate an integral to calculate the area of the desktop.

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Homework

4-160. Examine the following integrals. Consider the multiple tools available for evaluating integrals and use the best strategy for each part. Evaluate each integral and briefly describe your method. Homework Help ✎

a. $\int_{-1}^{1} \sqrt{x^2}\, dx$

b. $\int \left(8x^3 - \frac{1}{2}x\right) dx$

c. $\int_{1}^{5} \frac{3x^2-5x-2}{3x+1}\, dx$

d. $\int \left[\frac{d}{dx}(y)\right] dx$

4-161. Define possible functions *f* and *g* so that $h(x) = f(g(x))$. (Note: $f(x) \neq x$ and $g(x) \neq x$) Homework Help ✎

e. $h(x) = \sqrt[5]{\cos(x)}$

f. $h(x) = (3x\cos(x^2))^3$

g. $h(x) = 1$

h. $h(x) = x$

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Homework

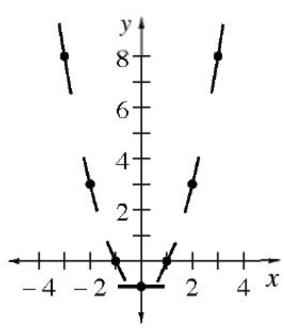
4-163. Determine the linearization of $y = 4 - 2x^2$ at (1, 2). Then use it to approximate the value of y when $x = 1.15$. Homework Help ✎

4-164. Set up an integral and compute the area of the region bounded by the graphs of the given functions. You may use a graphing calculator. Homework Help ✎

a. The area between $y = \sin(x)$ and $y = (x - 1)^4 - 1$.

b. The area between $y = x(x - 3)$ and $y = \sqrt{x}$.

4-165. Theresa loves tangents! This time, she has drawn several tangents and erased her original function. What is the equation of her original function? Homework Help ✎



4-166. For each of the following functions, write an equation for the end-behavior function. Explain your method. Homework Help ✎

a. $y = \frac{6}{x} + 2x^2$

b. $y = \frac{\sin(x)}{x}$

c. $y = \frac{x^2 - 3x - 10}{x^2 + 1}$