Chapter 5.1.2 and 5.1.2

The purpose of this lesson is to:

Understand how to model velocity, and acceleration from position functions and vice versa.

Maximize volume, and find the connection between maximization and derivative functions.

WARM UP

5-2. Often, the coefficients of a position function give direct information about the movement of an object. Study the coefficients of a function for a football kicked straight up in the air. Its height h (in feet) at time t (in seconds) is represented by the function $h(t) = -16t^2 + 30t + 2$.

a. Why is a quadratic function appropriate for this situation?

b. What is the ball's starting position? Why is the starting position not zero?

c. Write a function to represent the ball's velocity, $v(t)$, at time t. Then, use the velocity function to determine the ball's initial velocity (when $t = 0$).

d. How is the initial velocity represented in the *height* function?

e. Write a function to represent the ball's acceleration, $a(t)$, at time t. What are the units of acceleration? What is the significance of your result?

f. If the height function had been written using *meters* instead of feet, what should $a(t)$ be? What would the units for $a(t)$ be?

5-3. A baseball is thrown vertically with an initial velocity of 40 ft/sec and an initial height of 6 feet off the ground.

a. Assuming the only force on the baseball after it is thrown is gravity, what is $a(t)$?

b. Use $a(t)$ to write functions for $v(t)$ and $h(t)$. State the units for each.

c. How high was the ball when $t = 2$ seconds?

d. When is the ball at its maximum height? Describe your method.

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Warm Up

5-4. On Earth, objects under the force of gravity have a position function of the form $h(t) = -16t^2 + bt + c$, where b and c are constants, h is the height of the object in feet, and t is time in seconds.

a. Based on this form of a position function, write equations for the general functions of $v(t)$ and $a(t)$.

b. Study all three equations: $h(t)$, $v(t)$, and $a(t)$. Where does the initial velocity appear in $h(t)$? Where does the initial position appear in $h(t)$?

c. Study $a(t)$. What type of function is this? What does this tell you about the acceleration of a falling object on Earth?

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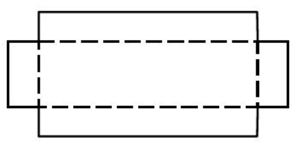
Task Card

5-12. WRAP IT UP, Part One

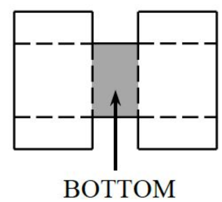
Jaycee's Department store is preparing for the upcoming holiday season. Each year, customers line up to get their gifts wrapped and sometimes those gifts do not fit inside the boxes the company provides. This year, Jaycee's wants to redesign each of its boxes in order to maximize the volumes. Lesson 5.1.2 Resource Page

Jaycee's uses four basic box designs, shown below, with each outer rectangle dimensions measuring 7" by 4". Your task is to determine the dimensions of each box design that will maximize the volume.

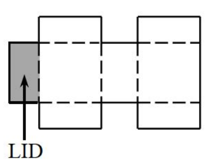
Open Box



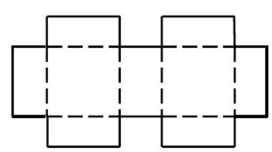
Strong Open Box



Closed Box



Strong Closed Box



Each box uses multiple congruent square cutouts. For each box:

a. Write a function for the volume, V, for any sized square cutout where x is the edge length of the square cutout.

b. State the physical limitations for the domain, x.

c. Using a graph of your volume function, determine the dimensions of the box that generates the maximum volume.

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Task Card

5-13. WRAP IT UP, Part Two

Explore a way to determine the maximum volume without a graph:

a. What is the value of $V'(x)$ when the box has its maximum volume? Is this always true?

b. Since the derivative of a cubic function is quadratic, there are potentially two values of x at which $V'(x) = 0$. Determine both of these values without your calculator.

c. Describe a method that will determine the maximum and minimum values of a function.

d. If setting the derivative equal to zero gives multiple answers, how can you decide which represents a maximimum and which represents a minimimum? Looking at the graph can help, but a graphing calculator may not show "funky functions" accurately. With your team, determine a method that will allow you to distinguish between a maximum and a minimum. How can you use calculus to *prove* whether a curve has reached its highest point or its lowest point?

e. Sketch the graph of $f(x) = x^{2/3}$ on your calculator and observe that the minimum value occurs when $x = 0$. What happens when you set $f'(x) = 0$? Adjust your description from part (c) to account for this situation.

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Homework

5-14. A rectangular piece of sheet metal with a perimeter of 50 cm is rolled into a cylinder with two open ends. Homework Help ✎

a. Write equations for the radius and height of the cylinder in terms of x.

b. Express the volume of the cylinder as a function of x.

c. Determine the value of x that will maximize the volume and calclate the maximum value.

5-15. Write the second derivative with respect to x for each function. Homework Help ✎

d. $y = (x - 10)^2$

e. $g(x) = -5x + 10$

f. $y = 5x^8 - 3x^{-8}$

5-7. You know that the first derivative, f', tells us the slope and the rate of change of f. Homework Help ✎

a. What does the *second derivative*, f'', tell you about f'? What does f'' tell you about f?

b. Write equations for $f'(x)$ and $f''(x)$ if $f(x) = x^3 + 3x^2 - 9x + 2$.

c. Is f getting steeper or less steep at $x = 1$? At $x = -2$? Use your derivatives from part (b) to explain your reasoning.

d. The values of $f''(1)$ and $f''(-2)$ can be used to determine concavity at $x = 1$ and $x = -2$. Where is f concave up? Where is f concave down

5-9. The Math Booster Club shot a rocket from the ground straight up into the air to celebrate Pi Day. The launcher shot the rocket with a starting velocity of 182 ft/sec. Write a function, s, for the height of the rocket at time, t. Then determine the rocket's maximum height and the amount of time it was in the air.