Class Title

The purpose of this lesson is to:
Use analysis to find the derivative of sin(x) and cos(x).
Understand what we learn from the second derivative.
Understand how to find 2nd, 3rd, and nth derivatives.

Notes

3-69. We will use the definition of the derivative to write equations for $f'(x)$ and $g'(x)$ if $f(x) = \sin(x)$ and $g(x) = \cos(x)$. You will need to use the trigonometric identities to simplify your expressions.

Note the following statements are true: $\sin(x+h)=\sin(x)\cos(h)+\sin(h)\cos(x)$

$$\lim_{h\to 0}\frac{\sin(h)}{h} = 1 \text{ and } \lim_{h\to 0}\frac{\cos(h)-1}{h} = 0$$

3-70. DERIVATIVES OF SINE AND COSINE GRAPHICALLY

It appears that the derivatives of sine and cosine are related. What about the second derivatives?

a. Set up five sets of axes, making sure the y-axes of the graphs are vertically aligned. Each x-axis should have domain $-2\pi \le x \le 2\pi$, scaled by $\frac{\pi}{6}$. Each y-axis should have range $-2 \le y \le 2$, scaled by $\frac{1}{2}$.

b. As accurately as you can, sketch $f(x) = \sin(x)$ on the first set of axes. Draw bold dots on all maximum and minimum points.

c. On the second set of axes, sketch $y = f'(x)$ as accurately as you can. Compare the graph of $y = f(x)$ with $y = f'(x)$. What does f look like when $f'(x) = 0$?

Repeat the process for f'', f''', and $f^{(4)}$, the second, third, and fourth derivatives of f. As you work, you might discover shortcuts that will expedite this process. What do you notice about the fourth derivative?

> Notation: We cannot simply put more and more tick marks for, say, the 7th derivative. Instead, the 7th derivative of f is written $f^{(7)}(x)$. The nth derivative of a function is written $f^{(n)}(x)$.

d. Predict $f^{(20)}(x)$ and $f^{(101)}(x)$.

3-71. Rewrite $y = \frac{1}{x}$ using exponents.

a. Write the slope function y', or $\frac{dy}{dx}$, algebraically by using the definition of the derivative.

b. Use the Power Rule to confirm your answer to part (a).

3-72. The graph of the equation $y = x^3 - 9x^2 - 16x + 1$ has a slope of 5 at exactly two points. What are the coordinates of these points? Describe your process.

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NOTES

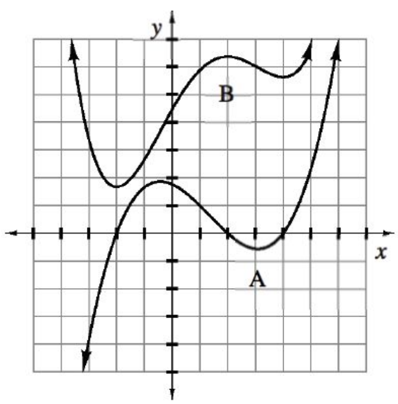
3-82. Knowing if a function increases or decreases tells us something, but not everything, about its possible shape.

a. Draw an example of a function that is increasing everywhere. What type of function behaves like this? Is there more than one possible shape?

b. Draw an example of a function that is decreasing, then increasing, then decreasing again. What type of function behaves like this?

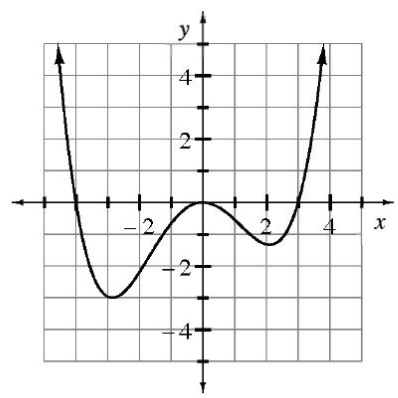
c. What type of function infinitely alternates between increasing and decreasing?

3-83. One of the functions below is *f* and the other is its slope function. Can you determine which function, A or B, is the slope function of the other? How do you know?



3-84. Use the graph of $y = f(x)$ at right to complete the parts below.

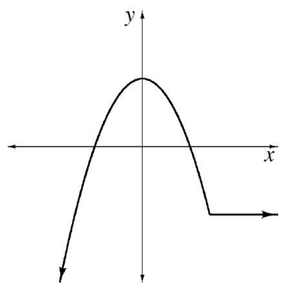
a. At what values of x does f change from increasing to decreasing or decreasing to increasing? What is $f'(x)$ at these points?
b. State the intervals where f is increasing. What is true about f' over these intervals?
c. Using your answers from parts (a) and (b), sketch the graph of $y = f'(x)$.

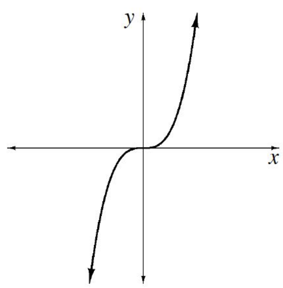


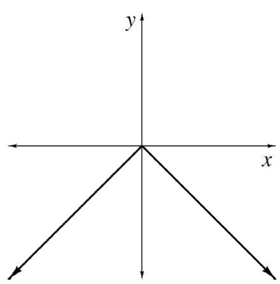
Class Title

Task Card

3-85. Sketch the slope function for each function below.







What happens to the slope at a corner (called a cusp)?

Class Title

Homework

3-87. Using the statement below, identify where the slope of the function is positive, negative, and zero.

"The function increases from negative infinity until it reaches a peak at $x = -4$ then decreases until $x = 1$ where the graph turns and then increases until positive infinity." Homework Help ✎

3-88. Given each function below, write its slope function. Homework Help ✎

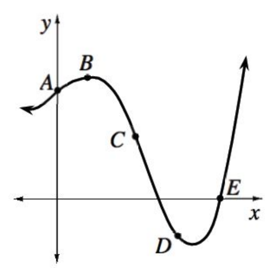
a. $f(x) = -x$

b. $f(x) = 0$

c. $f(x) = \frac{1}{6}(x-2)^3$

d. $f(x) = 9x + \sin(x)$

3-89. Name all point(s) on the graph at right that meet the given criteria. 3-89 HW eTool (Desmos) Homework Help ✎.

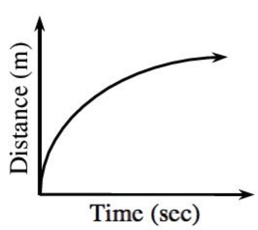


a. The slope of the tangent line is most positive.

b. The slope of the tangent line is negative.

c. The slope of the tangent line is the most negative.

3-90. The graph below shows the distance from a fixed point traveled by a toy car. Use the graph to sketch the velocity of the car. Homework Help ✎



3-91. Graph the function $f(x) = -2x^2 + 8x$. Estimating Area Under a Curve (Desmos) Homework Help ✎.

a. Approximate the area under the curve for $0 \le x \le 4$ using four trapezoids.

b. Write a Riemann sum to approximate the area under the curve for $0 \le x \le 4$ using four left endpoint rectangles. Then, use the summation feature of your graphing calculator to evaluate the sum. Compare the accuracy of the trapezoids and the rectangles.

c. Will the approximation with trapezoids always equal the approximation with rectangles for all functions? Why or why not?

3-92. While using the definition of a derivative, Lulu used the following limit. For what function was she determining the derivative? How can you tell? Without simplifying this expression, determine the limit. (Use the Power Rule)

$$\lim_{h\to 0} \frac{(6(x+h)^2-5(x+h)+3)-(6x^2-5x+3)}{h}$$

3-93. If $f'(x) = 3x^2 + 2x - 5$, write a possible equation for f. Then write another possible function. Homework Help ✎

3-95. Evaluate each limit. If the limit does not exist due to a vertical asymptote, then add an approach statement stating if y is approaching negative or positive infinity. Homework Help ✎

a. $\lim\limits_{x\to 0} \frac{x^2+3x-10}{x-2}$

b. $\lim\limits_{x\to -5} \frac{x^2+3x-10}{x-2}$

c. $\lim\limits_{x\to 2} \frac{x^2+3x-10}{x-2}$

d. $\lim\limits_{x\to \infty} \frac{x^2+3x-10}{x-2}$

Use the limits above to describe the shape of the graph of $y = \frac{x^2+3x-10}{x-2}$. State all horizontal asymptotes, vertical asymptotes, and holes.