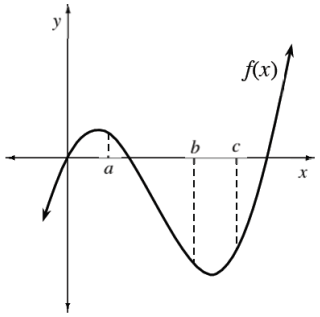
Chapter 4.1.3

The purpose of this lesson is to:
Understand the different properties of definite integrals

WARM UP

4-27. PROPERTIES OF DEFINITE INTEGRALS

Consider the integral expressions below. For each expression, draw and shade the region for a generic function. Simplify each integral expression and summarize each case on your paper. (In other words, rewrite them as one expression).



a. $\int_a^a f(x)\,dx$

b. $\int_a^b f(x)\,dx + \int_b^c f(x)\,dx$

c. $\int_b^a f(x)\,dx + \int_a^b f(x)\,dx$

d. $\int_b^a k \cdot f(x)\,dx$, where *k* is a constant

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Task Card

4-28. PROPERTIES OF DEFINITE INTEGRALS, CONTINUED

You have developed methods of simplifying integral expressions with a single function. What happens when we combine definite integrals with two different functions? Investigate the following relationship:

$$\int_a^b f(x)\,dx + \int_a^b g(x)\,dx$$

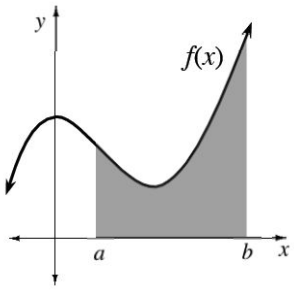
a. Evaluate $\int_0^2 x\,dx + \int_0^2 3x\,dx$.

b. Evaluate $\int_0^2 (4x)\,dx$

c. Rewrite the expression $\int_a^b f(x)\,dx + \int_a^b g(x)\,dx$ into a simplified form.

4-29. TRANSLATIONS OF FUNCTIONS

Examine what happens to the area of a region when a function is translated. Some cases to consider are listed below, but do not feel restricted to them. When finished, summarize your findings clearly.



a. Does $\int_a^b [f(x)+k]\,dx = \int_a^b f(x)\,dx + k$? Explain why or why not.

b. Does $\int_a^b f(x)\,dx = \int_{a+c}^{b+c} f(x)\,dx$? Explain why or why not.

c. Does $\int_a^b f(x)\,dx = \int_{a+c}^{b+c} f(x-c)\,dx$? Explain why or why not.

d. Does $\int_a^b f(x)\,dx = \int_a^b f(x+c)\,dx$? Explain why or why not.

e. Summarize the definite integral properties that are correct on your paper.

4-30. With your team, write general formulas for all the properties of definite integrals you discovered today.

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Homework

4-31. Differentiate the following equations with respect to x. That is, what is $\frac{dy}{dx}$? Homework Help ✎

a. $y = \cos(x) + \sin(x)$

b. $y = x \cdot \sqrt[3]{x^2}$

c. $y = (6 - 5x)(1 - 2x)$

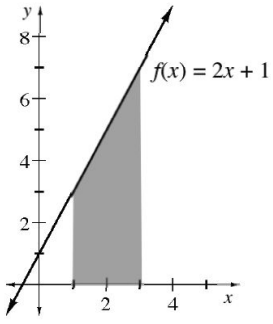
4-32. Evaluate the following definite integrals without a calculator. Then write a statement about the connection between them. Check your answers with a calculator. Homework Help ✎

d. $\int_2^9 8x\,dx$

e. $\int_2^9 (8x+5)\,dx$

f. $\int_2^9 5\,dx$

4-33. Given the graph at right of $f(x) = 2x + 1$, evaluate: Homework Help ✎



g. $\int_1^3 (2x+1)\,dx$

h. $\int_1^3 (2t+1)\,dt$

i. What is the difference between the expressions in parts (a) and (b)?

4-34. 4-34 HW eTool (Desmos) Homework Help ✎.

j. Write the equations of the two lines tangent to the curve $f(x) = x^3 - x^2 + x + 1$ that have a slope of 2.

k. Determine the equations of the lines perpendicular to the tangent lines from part (a) at their points of tangency to *f*.

4-35. Given $f(x) = \sin(x)$, $g(x) = x^2$ and $h(x) = \frac{1}{x}$, use compositions of functions to express each of the following functions. Homework Help ✎

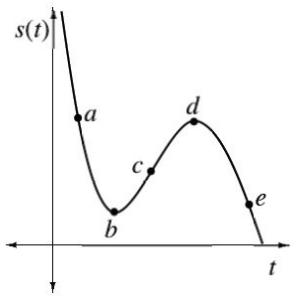
l. $y = \sin(x^2)$

m. $y = \sin^2(x)$

n. $y = \csc(x)$

o. $y = \csc^2\left(\frac{1}{x}\right)$

4-36. Using the distance vs. time graph at right, determine if the velocity is positive, negative, or zero at each labeled point on the graph. Homework Help ✎



4-37. Sketch a graph of $f(x) = x^3 - 2x^2$. At what point(s) will the line tangent to f be parallel to the secant line through $(0, f(0))$ and $(2, f(2))$? 4-37 HW eTool(Desmos) Homework Help ✎.

4-38. Sketch a graph of $f(x) = x^3 + 3x^2 - 45x + 8$. Homework Help ✎

p. Calculate the slope of the line tangent to the curve at $x = -2$.

q. Determine the point on the curve where the slope is the smallest (steepest negative slope). What is the name of this point?

4-39. Let $f(x) = \begin{cases} 2x^2 - 4 \text{ for } x \le 3 \\ -2x - 5 \text{ for } x > 3 \end{cases}$. Homework Help ✎

r. What is $\lim_{x \to 3^+} f(x)$?

s. What is $\lim_{x \to 3^-} f(x)$?

t. What do your results from parts (a) and (b) tell you about f?