Chapter 4.2.3 and 4.2.4

The purpose of this lesson is to:
Demonstrate both parts of the Fundamental Theorem of Calculus.
Use both parts of the Fundamental Theorem of Calculus to simplify integral expressions.

WARM UP

4-65. Anita is calmly sketching horizontal lines in her notebook when she notices a pattern among the area functions under horizontal lines. For example, the area function under the line $y = 2$ can be found with the definite integral $A(x) = \int_a^x 2\,dt$. Sketch a graph of $y = 2$ and use it to investigate how the value of the lower bound, *a,* affects the equation of the area function.

a. If $a = 0$, write the equation for $A(x)$.

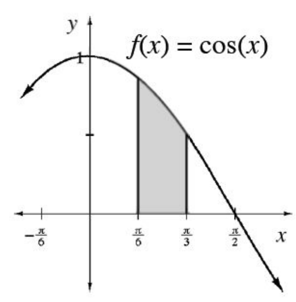
b. If $a = 1$, write the equation for $A(x)$.

c. If $a = 10$, write the equation for $A(x)$.

d. If $a = –5$, write the equation for $A(x)$
.

e. Examine the area functions your wrote in parts (a) through (d). How are they the same? How are they different? In particular, how are they all related to the original function $y= 2$?

4-66. Anita's teammate Tommy is stuck. He wants to evaluate $\int_{\pi/6}^{\pi/3} \cos(x)dx$, but he does not have a calculator.



a. Tommy knows that this integral can be evaluated with $A\left(\frac{\pi}{3}\right) - A\left(\frac{\pi}{6}\right)$. Unfortunately, he does not know the equation for $A(x)$. Thinking about her answer to part (e) of problem 4-65, Anita suggests finding some function whose derivative is equal to $\cos(x)$.

Tommy objects. *"I can think of more than one function like that!"*

What is Tommy talking about? List four functions whose derivative is $\cos(x)$. In other words, list four different **antiderivatives** of cos(*x*).

b. "*Don't worry, Tommy,"* says Anita in a calm, comforting voice. "*You can use any of those antiderivatives!*"

Try it. Each member of your team should choose a different antiderivative for *y* = cos(*x*) and use it to evaluate $\int_{\pi/6}^{\pi/3} \cos(x)dx = A\left(\frac{\pi}{3}\right) - A\left(\frac{\pi}{6}\right) = \_\_\_$. Compare your results.

c. Tommy and Anita are delighted by their *fundamental* discovery. They now have a procedure to calculate the exact area under the curve of any function! "*From now on*," announces Tommy, "*I will not worry about the constant when evaluating a definite integral. I will write my antiderivative with a +C instead."*

Explain why all area functions lead to the same result when evaluating a definite integral.

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NOTES

Indefinite Integrals

Unlike definite integrals, which are defined between two bounds, indefinite integrals have _____ ___________ and are written in the form:

.

The _________ ________ $\int f(x)dx$ represents *all* of the antiderivatives, $F(x) = \int_a^x f(t)\,dt$, that are associated with every possible value of *a.* For that reason, an indefinite integral has __________ many antiderivatives, each differing by a __________ that we represent with __.

$$\int f(x)dx = F(x) + C$$

Example: $\int (2x+3)\,dx = x^2 + 3x + C$ because $\frac{d}{dx}(x^2 + 3x + C) =$ ___________________ for any *C*.

The Fundamental Theorem of Calculus

Part 1: If *f* is continuous on ______ and $F(x) = \int_a^x f(t)dt$, then ____ = ____ for all *x* in (*a, b*).

Therefore,

Part 2: If *F* is any function such that the derivative *F ′* exists for all *x* in an open interval containing

[*a, b*] then, $\int_a^b f(x)\,dx =$.

4-79. When applying the Fundamental Theorem of Calculus to evaluate a definite integral like $\int_a^b f(x)dx$, a common way to show your steps is:

$$\int_a^b f(x)dx$$
$$= F(x)\big|_a^b$$
$$= F(b) - F(a)$$
$$= \_\_\_\_\_$$

Evaluate $\int_4^9 (2x+5)dx$, showing your steps as outlined above.

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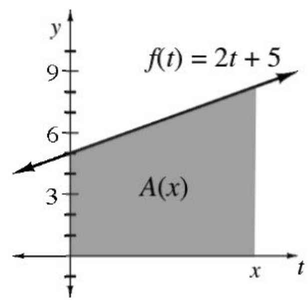
Task Card

4-67. Use Tommy and Anita's technique to evaluate $\int_1^2 (6x^2+7)dx$. Test your results using a graphing calculator.

4-68. THE FUNDAMENTAL THEOREM OF CALCULUS



a. Use geometry to write an equation for the area of the shaded region in the graph at right. That is, what is $\int_0^x (2t+5)dt$

b. What is $A'(x)$? Compare it to the original function.

c. Part (b) shows that the derivative of an area function is the original function. Test this idea on a general linear function, $f(x) = mx + b$, by determining $A(x)$ and $A'(x)$. Does the same result happen? (In other words, find the integral of $f(x) = mx + b$ then find your answer's derivative).

4-69. The relationship between derivatives and integrals is fundamental to calculus. Given what you've learned about that relationship, how can you simplify the derivative of an integral:

$$\frac{d}{dx}\int_a^x f(t)\,dt =$$

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Task Card

4-80. Evaluate the following definite integrals by applying the Fundamental Theorem of Calculus. Check your answers by using the integration function on your calculator.

a. $\int_1^8 2x^{-3}dx$

b. $\int_{\pi/4}^{\pi/2} \sin(x)\,dx$

c. $\int_4^9 3\sqrt{x}\,dx$

d. $\int_0^2 (3x^2-6x+2)\,dx$

e. $2\int_1^3 \frac{3}{x^2}\,dx$

f. $\int_{-1}^8 x^{1/3}\,dx$

g. $\int_3^9 (2x-20)\,dx + 2\int_{-2}^3 (x-10)\,dx$

h. $\int_1^{27} (9m^{-2/3}-2m^{-3})\,dm$

4-81. Use the Fundamental Theorem of Calculus to evaluate each integral expression.

a. $\frac{d}{dx}\int_3^x (3t-5)\,dt$

b. $\int_3^x \left(\frac{d}{dt}(3t-5)\right)dt$

c. $\frac{d}{dx}\int_3^5 (3x-5)\,dx$

d. $\int \left(\frac{d}{dx}(3x-5)\right)dx$

e. $\frac{d}{dx}\int \cos(x^2)\,dx$

f. $\int_1^4 \left(\frac{d}{dx}\cos(x^2)\right)dx$

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Homework

4-83. Chang Young is attempting to evaluate the following integral: $\int_1^5 (5x+2)\,dx$. Homework Help ✎

He writes the following steps:

$$\frac{5}{2}5^2+2(5)+C-\frac{5}{2}1^2+2(1)+C$$
$$\frac{5}{2}\cdot 25+10+C-\frac{5}{2}+2+C$$
$$\frac{125}{2}-\frac{5}{2}+12+2C$$
$$\frac{120}{2}+12+2C$$
$$60+12+2C$$
$$72+2C$$

He knows that this is a definite integral and there should not be any *C*s in his answer. Also, the answer key says the answer is 68. He needs your help to find his error and figure out how to eliminate his + 2*C*.

4-84. Evaluate each of the following integrals. Homework Help ✎

a. $\int (6x^3-2x+5)\,dx$

b. $\int_2^4 (6x^3-2x+5)\,dx$

c. $\int (9t^2-1)\,dt$

d. $\int_{-2}^2 (9t^2-1)\,dt$

e. $\int \left(\sin(m)+\frac{1}{3}m^2\right)dm$

f. $\int_{-\pi}^{\pi} \left(\sin(m)+\frac{1}{3}m^2\right)dm$

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Homework

4-85. Rewrite each of the following integral expressions as single integrals. Homework Help ✎

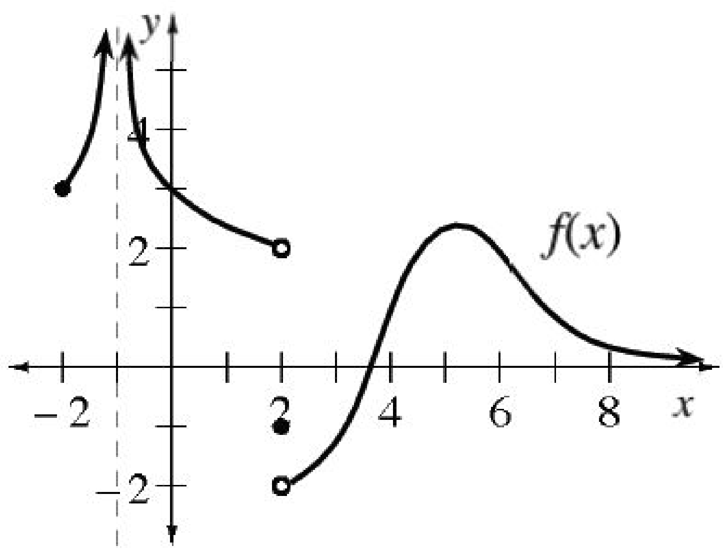
a. $\int_{-3}^{-5} f(x)\,dx + \int_{-5}^{-3} g(x)\,dx$

b. $3\int_{1}^{6} f(x)\,dx + 5\int_{1}^{6} g(x)\,dx$

c. $\int_{6}^{11} f(x)\,dx + \int_{11}^{6} f(x)\,dx$

d. $\int_{7}^{10} f(t)\,dt - \int_{7}^{9} f(t)\,dt$

4-72. The graph of a function $y = f(x)$ is shown at the right. Use the graph to evaluate the following limits. Homework Help ✎



a. $\lim_{x \to -1} f(x)$

b. $\lim_{x \to 2} f(x)$

c. $\lim_{x \to 2^-} f(x)$

d. $\lim_{x \to 2^+} f(x)$

e. $\lim_{x \to 5} f(x)$

f. $\lim_{x \to \infty} f(x)$

g. Where (if anywhere) does the derivative of *f* not exist?