Chapter 5.4.1 and 5.5.2

The purpose of this lesson is to:
Understand how the Chain Rule is used in the FtoC
Find limits of functions in Indeterminate Form using L'Hopital's Rule

NOTES Part 1

In Chapter 4 you learned the Fundamental Theorem of Calculus, which related derivatives to integrals. You now have the skills to apply your derivative rules to the Fundamental Theorem of Calculus.

5-138. The Fundamental Theorem of Calculus states that derivatives and antiderivatives *undo* each other: $\frac{d}{dx}\int_a^x f(t)dt = f(x)$

a. Examine the expression $\frac{d}{dx}\int_a^x f(t)dt$. List the steps needed to show that this expression is equivalent to *f*(*x*).

b. For the definite integral shown in part (a), the upper bound, *x*, is a function. Let's differentiate a definite integral with a more complicated upper bound, $\frac{d}{dx}\int_a^{g(x)} f(t)dt$, by completing the parts below.

i. $\frac{d}{dx}\int_a^{g(x)} f(t)dt$ How is this definite integral different than the one in part (a)?

ii. $\frac{d}{dx}(F(g(x))-F(a))$ What was done to get this?

iii. $f(g(x)) \cdot g'(x) - 0$ What derivative rule was used?
Why did the second part become 0?

In general, $\frac{d}{dx}\int_a^{g(x)} f(t)\,dt =$ _____________.

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Task Card 1

5-139. Using the steps outlined in the previous problem, what is $\frac{d}{dx}\int_{h(x)}^{g(x)} f(t)\,dt$?

5-140. Differentiate each of the following definite integrals.

a. $\frac{d}{dx}\int_2^{x^2} \sin(t^5)\,dt$

b. $\frac{d}{dx}\int_3^{x^2} \sin(\csc(t))\,dt$

c. $\frac{d}{dx}\int_{\tan(x)}^{4} \cot(t^2)\,dt$

d. $\frac{d}{dx}\int_{\sin x}^{\cot x} \sqrt{1+t^3}\,dt$

e. $\frac{d}{dx}\int_{x^3}^{3\sin(x)} \sqrt{t^2-1}\,dt$

f. $\frac{d}{dx}\int_{\csc(x)}^{x^3} \frac{t-3}{t+1}\,dt$

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NOTES Part 2

Indeterminate Forms And l'Hôpital's Rule

If we have a limit of the form $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{0}{0}$ (or $\frac{\infty}{\infty}$) it is in an ____________ ______ and there is a powerful tool called __________ _____ which states:

If ____________________________, then _________________________________ .

In other words, if the function is _________ or __________ after the value a is plugged in, we can say the function is in _____________ ________. So take the ___________ of the top and bottom, and then __________ ___________ again.

5-162. Try using l'Hôpital's Rule to evaluate the following limits, which you originally saw in Chapter 1. Remember to use l'Hôpital's only when the limit is in an indeterminate form! If the limit is not in an indeterminate form, use another method.

a. $\lim_{x \to 0} \frac{\sin(x)}{x}$

b. $\lim_{x \to 0} \frac{\sin^2(x)}{x}$

c. $\lim_{x \to 0} \frac{\sin(x)}{x^2}$

d. $\lim_{x \to 0} \frac{\cos(x)}{x}$

e. $\lim_{x \to 0} \frac{1-\cos(x)}{x-1}$

f. $\lim_{x \to 0} \frac{1-\cos(x)}{x}$

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TASK CARD Part 2

5-163. Repeated Use of l'Hôpital's Rule

a. Aram is trying to use l'Hôpital's Rule on the following limit and gets another indeterminate form. He is not sure what to do. Verify that Aram applied l'Hôpital's Rule correctly.

b. $\lim\limits_{x \to 0} \frac{2x^4 - 1.5x^2}{x^3 + 2x^2} = \lim\limits_{x \to 0} \frac{8x^3 - 3x}{3x^2 + 4x}$

c. Mara thinks she can help Aram by applying l'Hôpital's Rule again. Test Mara's theory and confirm your answer by graphing.

d. Use Mara's method to evaluate each of the following limits:

   i. $\lim\limits_{x \to \infty} \frac{(2x-5)^2}{2x - x^3}$

   ii. $\lim\limits_{x \to 0} \frac{1 - \cos(x)}{x^2}$

5-164. Evaluate each of the following limits.

a. $\lim\limits_{x \to 0} (x^2 \cdot \csc^2(x))$

b. $\lim\limits_{x \to 0} \left(\frac{1}{\sin(x)} - \cot(x) \right)$

5-165. Find the error in the following solution. Then evaluate the limit correctly.

$$\lim_{x \to 1} \frac{x^3 - x^2 + x - 1}{x^3 - x^2} = \lim_{x \to 1} \frac{3x^2 - 2x + 1}{3x^2 - 2x} = \lim_{x \to 1} \frac{6x - 2}{6x - 2} = 1$$

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Homework

5-168. Evaluate each of the following limits. For each part, describe your method. Homework Help ✎

a. $\lim_{x \to 1} \frac{x-1}{x^2+1}$

b. $\lim_{x \to 2} \frac{x^3-8}{x^2-4}$

c. $\lim_{x \to \infty} \frac{\sqrt{x}}{3x+4}$

d. $\lim_{x \to 0} \frac{\sin^2(x)}{\sin(x^2)}$

5-170. Write the antiderivative of each of the following functions. Homework Help ✎

a. $f(x) = \sin(x)$

b. $f(x) = \frac{2}{(x+3)^3}$

c. $f(x) = \sec^2(x)$

d. $f(x) = \frac{3}{x^3} + \frac{2}{x^4}$

5-172. Write the derivative of each of the following functions. Homework Help ✎

a. $f(x) = (2x + 1)^2(3x - 1)^3$

b. $g(x) = \dfrac{1}{\sin(x^2)}$

c. $f(x) = x\tan(x^2)$

d. $g(x) = \cos\left(x\sqrt{x+1}\right)$