Chapter 1.2

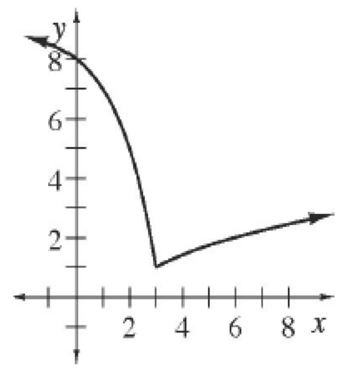
The purpose of this lesson is to:
Understand informal continuity
Understand continuity in the context of piecewise functions
Understand how to test for continuity in piecewise functions

WARM UP/Notes

Talk with your group about what you believe it means for a function to be continuous. Show some examples, and feel free to use pictures.

Intuitive Notion of Continuity

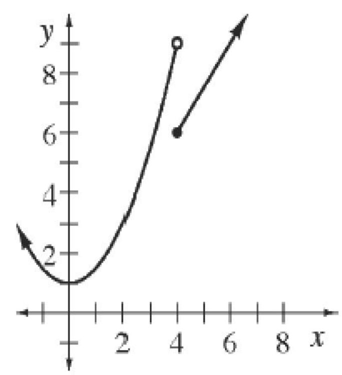
The formal definition of continuity will be given in Chapter 2. For now, we can say that a function is ___________ if the graph of the function can be drawn without ________ _______ __________.



The graph of a continuous function is shown at right. The equation of this piecewise-defined function is:

$$f(x) = \begin{cases} 9 - 2^x & \text{for } x \leq 3 \\ \sqrt{x-2} & \text{for } x > 3 \end{cases}$$

The graph is continuous at x=3 because _________________________. Though the graph is connected at x = 3, notice that the _________ of the individual equations do *not* ________ at x = 3. This ensures that the piecewise-defined equation is a _________.



When the values of a function are not connected as in problem 1-11, we say that the function is _____ continuous. The function $g(x)$ below is not continuous at x = 4 because _____________________________ . This can be seen in the graph at right.

$$g(x) = \begin{cases} 0.5x^2 + 1 \text{ for } x < 4 \\ 1.5x \text{ for } x \geq 4 \end{cases}$$

It is important to note that polynomial, rational, power, exponential, logarithmic, and trigonometric functions are ______________ at all points in their _________.

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Practice/Notes II

1-12.

Examine the graph of the function $h(x) = \begin{cases} 9 - 2^x & \text{for } x \le 3 \\ \sqrt{x-2} & \text{for } x > 3 \end{cases}$ in the preceding Math Notes box.

a. In your own words, explain why h is continuous at $x = 3$.

b. Explain why h can also be defined as $h(x) = \begin{cases} 9 - 2^x & \text{for } x < 3 \\ \sqrt{x-2} & \text{for } x \ge 3 \end{cases}$.

State what it means for a function to fit each of the following categories:

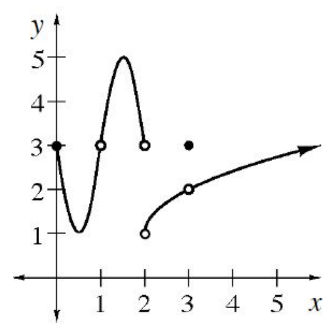
Category I: Continuous for all real values of x.

Category II: Continuous on its domain only.

Category III: Discontinuous on its domain.

Domain and Range Notation

There are two accepted forms of notation for domain and range. Examine the graph at right. The domain and range of the function shown can be noted in either interval notation or in set notation.



__________ __________: $D = [0, 1) \cup (1, \infty)$

$R = [1, \infty)$

_____ _________: $D = \{x\colon x \ge 0 \text{ and } x \ne 1\}$ or $D = \{x\colon 0 \le x < 1 \text{ or } x > 1\}$

$R = \{y\colon y \ge 1\}$

1-13.

Explain why the two forms of set notation for the domain in the preceding math notes are equivalent.

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Task Card

1-14.

Compare the domains of f, g, and h. Explain what aspect of each function limits the domain.

$f(x) = \sqrt{x-25}$ $\quad$ $g(x) = \frac{1}{x-25}$ $\quad$ $h(x) = \log(x-25)$

1-17.

Let $f(x) = \begin{cases} x^2+2 & \text{for } x < 1 \\ -x & \text{for } x \geq 1 \end{cases}$.

a. Sketch the graph of $y = f(x)$.

b. Modify one piece of the function so that it is continuous.

1-18.

Determine values of a and b such that g is a continuous function.

$$g(x) = \begin{cases} \sqrt{x+3} & \text{for } x < 1 \\ a(x-1)^2+b & \text{for } 1 \leq x < 3 \\ -x+2 & \text{for } x \geq 3 \end{cases}$$

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Homework

1-24.

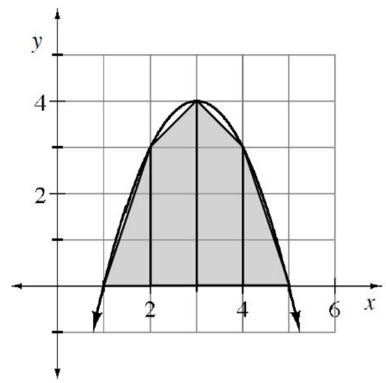
Sketch a graph of the piecewise-defined function $g(x) = \begin{cases} 2^x & \text{for } x \le 2 \\ 3x - 2 & \text{for } x > 2 \end{cases}$

a. State the domain and range of g.

b. Is g continuous at $x = 2$? Explain.

c. Is g continuous for all values of x?

1-25. The parabola $y = -(x - 3)2 + 4$ is graphed at right. Four trapezoids of equal width are inscribed for $1 \le x \le 5$.



a. Use the combined area of these trapezoids to approximate the area under the parabola for $1 \le x \le 5$.

b. Is this area greater or less than the true area under the parabola? Explain.

1-28.

Use polynomial division to rewrite each of the following rational expressions.

a. $\frac{x^3+2x^2-3x+4}{x+3}$

b. $\frac{x^4-5x^2+3x-3}{x-2}$