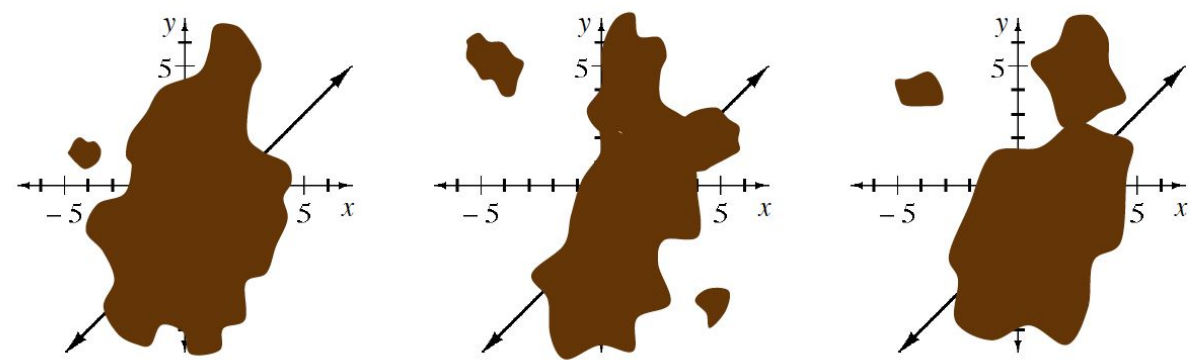
Chapter 1.2.2 and 1.2.3

The purpose of this lesson is to:
Understand polynomial division, end behavior, holes, and asymptotes.

WARM UP

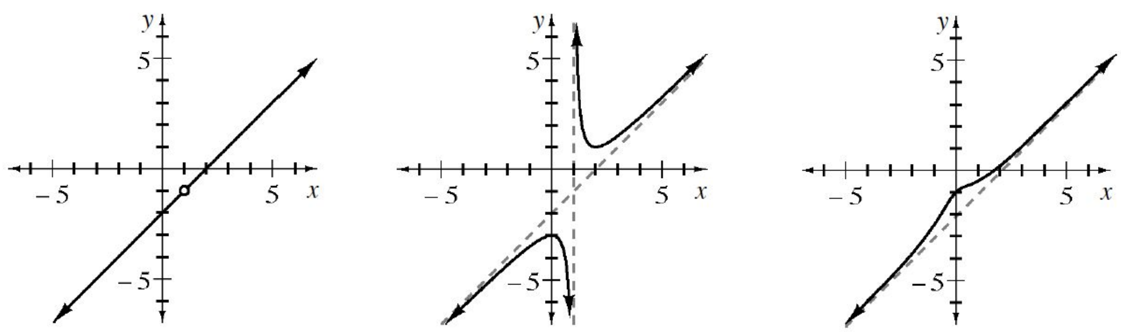
1-29.

Eunsun spilled coffee on her math book, making some of the graphs difficult to read!



a. Help Eunsun determine the equation of each function graphed above.

b. When Eunsun arrived at school, she looked at Rudy's book and discovered that the graphs looked like this:



Compare the graphs in part (a) with the graphs in part (b). Use the words asymptote and hole in your explanation.

c. The actual equations are below, which equation do you think belongs to each graph?

$y = \frac{x^2-3x+2}{x-1}$ $\qquad$ $y = \frac{x^2-3x+3}{x-1}$ $\qquad$ $y = \frac{x^3-2x^2+x-1}{x^2+1}$

Chapter 1.2.2 and 1.2.3

NOTES

Polynomial Division, Holes, and Asymptotes

Given a function like the three from the previous problem, we can simplify through a method called _____________ __________. Let's use one of the functions as an example:

$$y = \frac{x^2-3x+3}{x-1}$$

Set the problem up like _______ _________.
(The Bottom goes on the _________, and the top goes on the __________):

Now solve the problem completely to simplify as far as possible!

Now simplify the other two functions from 1-29 (c) using Polynomial Division!

When Polynomial Division does not cleanly divide, the function has what is called a ___________ ____________.

When Polynomial Division *does* cleanly divide, the function has what is called a _______.

Both the ______ and ___________ ____________ can be found by setting the _______________ of the function equal to ____ and _________.

I will demonstrate by finding the Vertical Asymptote from $y = \frac{x^2-3x+3}{x-1}$

Now you try with the two functions you simplified yourself!

Chapter 1.2.2 and 1.2.3

Task Card PART 1

1-30.

Even though Eunsun's equations were incorrect, to her surprise, she received partial credit on the assignment. The next day, Eunsun's teacher used her homework to teach a new concept, end behavior!

Graph $f(x) = -x + 2 + \frac{2x}{x^2+1}$ on your calculator.

a. Use your calculator to "zoom out" so you can look far left and far right. Write an equation for the end-behavior function of the graph.

b. Describe the connection between the equation of the end-behavior function and the equation of the original function.

c. The linear equation you wrote in part (a) is a slant asymptote for this function. Explain why the name is appropriate.

1-31.

Graph $y = \frac{1}{x^2+1} + 3$. Then look far to the left and far to the right. Determine the equation for the end behavior of this graph. How can this end behavior be predicted from the equation of the original function?

1-44.

For $f(x) = \frac{x^2+5x+4}{x+4}$ and $g(x) = \frac{x^2+5x+3}{x+4}$:

a. Draw a careful sketch of each function. Use a dashed line for an asymptote and an open circle for a "hole" (a single point which the graph appears to go through, but where it is actually undefined).

b. For both f and g, write the equations of all asymptotes and the coordinates of any holes.

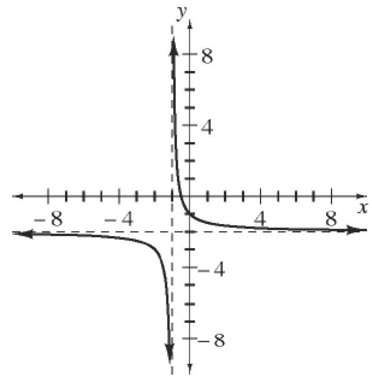
c. State the domains and ranges of f and g.

Chapter 1.2.2 and 1.2.3

Task Card PART 2

1-47.

Examine the graph of $y = \frac{1}{x+1} - 2$ at right. Use the graph to answer the questions below.



a. What does y approach as $x \to \infty$?
b. What does y approach as $x \to -\infty$?
c. What does y approach as $x \to -1^-$ (the symbol "$\#^-$" means approaching from the negative direction, or from the left)?
d. What does y approach as $x \to -1^+$ (from the positive direction, or from the right)?
e. Name all horizontal and vertical asymptotes.

1-48.

Sketch a graph of a function that has a vertical asymptote at $x = 2$, a hole at $x = -4$, and as $x \to \infty$, $y \to 3$.

1-50.

For the following functions, when 2 is substituted for x, the fraction has the indeterminate form (or undefined form) $\frac{0}{0}$. State whether the graphs of the following functions have holes, asymptotes, or neither at $x = 2$. Explain your answer.

a. $f(x) = \frac{x-2}{x-2}$

b. $g(x) = \frac{(x-2)^2}{x-2}$

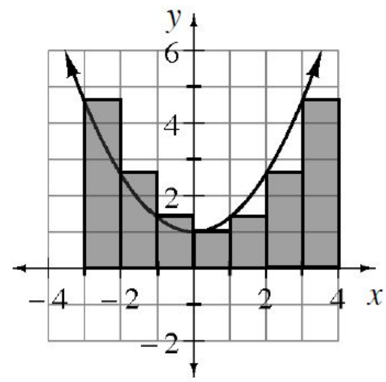
c. $h(x) = \frac{x-2}{(x-2)^2}$

d. Sketch a graph and write the equation of a function that looks like $y = x - 2$ with a hole at $x = 4$.

Chapter 1.2.2 and 1.2.3

Homework

1-36. The graph of $j(x) = \frac{2}{5}x^2 + 1$ is shown at right.



a. Approximate the area under the curve for $-3 \le x \le 3$ by calculating the sum of the areas of the six left endpoint rectangles as shown. (The height of a left endpoint rectangle is determined by the function's value at the left x-value.)

b. Is the approximation in part (a) too high or too low? How can you tell?

c. Now, sketch this function with six right endpoint rectangles and compute the approximate area.

d. You should have obtained the same answers using right and left endpoint rectangles. Will this be true for all functions? If so, explain why. If not, explain what was special about this case that made the area estimates equal. Give an example of a case where the area estimates will be different.

1-41.

Wei Kit knows that radical expressions can be rewritten using rational exponents. Study his examples below.

Examples: $\sqrt{x} = x^{1/2}$ $(\sqrt[5]{z})^2 = z^{2/5}$ $\sqrt[3]{m^2} = m^{2/3}$

Rewrite the following radicals expressions with rational exponents. Homework Help ✎

a. $\sqrt{k^7}$

b. $\sqrt[3]{t^4}$

c. $(\sqrt{n})^4$

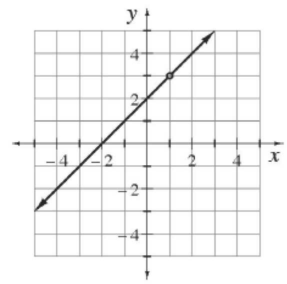
d. $\sqrt[5]{b^{31}}$

Chapter 1.2.2 and 1.2.3

Homework

1-51.

Analyze the graph of $y = \frac{(x+2)(x-1)}{x-1}$ at the right.



a. What does y approach as $x \to \infty$? What does y approach as $x \to -\infty$? Describe how your answer can be predicted from the given equation.

b. What does y approach as $x \to 1^-$ (1 from the left)? What does y approach as $x \to 1^+$ (1 from the right)? Describe how your answer can be predicted from the given equation.

1-53.

Convert the following domain and range from interval notation to set notation. Then sketch a possible function with the given domain and range.

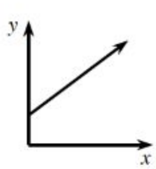
$D = (-\infty, 2) \cup (2, \infty)$

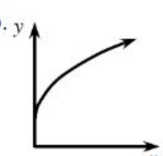
$R = (-\infty, -1) \cup (-1, \infty)$

1-59.

Each of the continuous functions in the table below is increasing, but each increases differently. Match each graph below with the function that grows in a similar fashion in the table.

x	1	2	3	4	5	6	7	8	9
f(x)	64	68.8	74.6	81.5	89.8	99.7	111.7	126	143.2
g(x)	38	52	66	80	94	108	122	136	150
h(x)	22	42.9	57.3	68.5	77.6	85.3	92	97.9	103.1

a.


b.


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