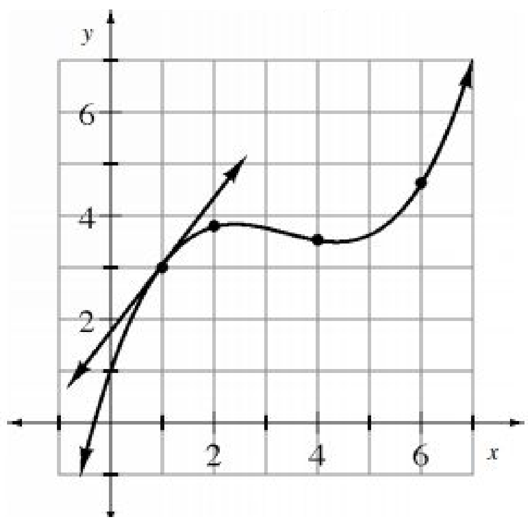
Chapter 3.1.1 and 3.1.2

The purpose of this lesson is to:
Understand Average Rate of Change (AROC)
Understand the Instantaneous Rate of Change (IROC)

WARM UP

3-1. Notice that a tangent line to the graph has been drawn at x = 1. Carefully draw tangent lines to the curve at x = 2, x = 4, and x = 6.



a. Write a slope statement for *f*.

b. Draw a **secant line (a line passing through two points of a curve)** and use it to estimate the slope at x=2. Be sure to extend the secant line so that you can use the grid lines to approximate the slope.

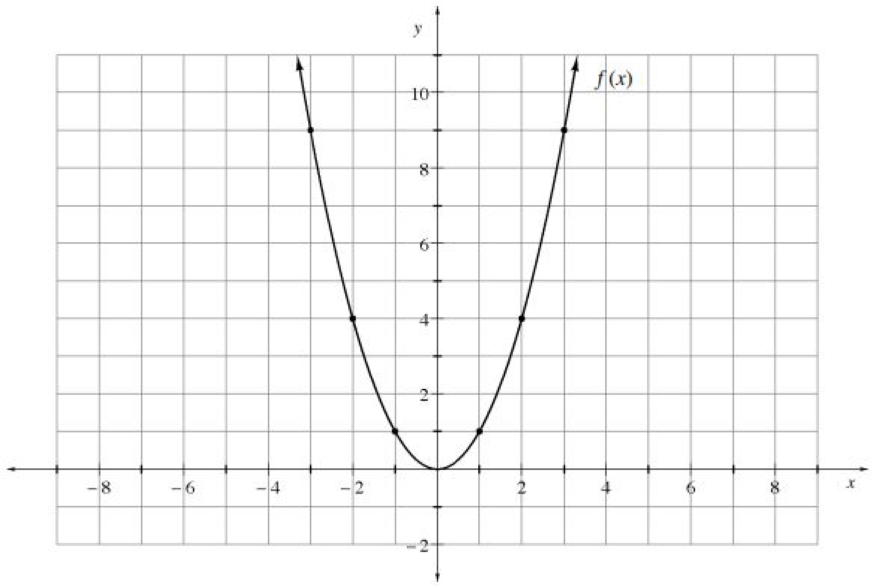
c. If this graph represents the position of a roller coaster ride during its first six seconds, where is it moving the fastest? (What range of time, roughly)

d. Where is it moving the slowest? How did you determine your answers?

e. How can you relate the tangent lines you drew to your answers for (c) and (d)?

3-2. Using the graph of $f(x) = x^2$. Accurately draw tangent lines for $x = -3, -2, -1, 0, 1, 2, 3$.

x	-3	-2	-1	0	1	2	3
m	-6	-4	-2	0	2	4	6

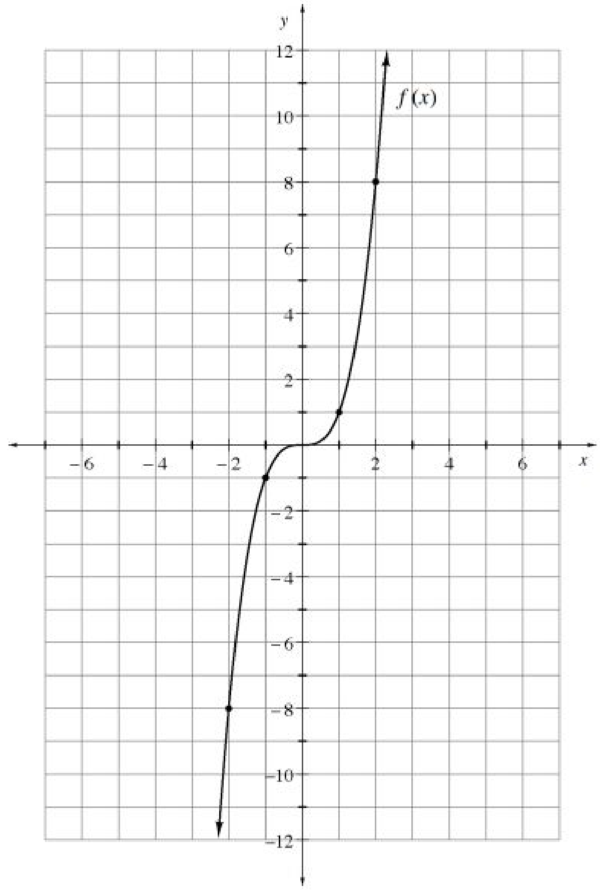


a. Graph the data from the table above onto the graph. Graph the points as (x,*m*).

b. Use the table and the graph to write a **slope function**, *f* ′, a function that gives the slope of the line tangent to *f* for any *x*. What type of function is *f* ′? Your solution should be a **function**, not a description. (i.e: $f'(x)=3x^2+5$, note this is NOT the solution)

3-3. Using the graph of $f(x) = x^3$. Accurately draw tangent lines for $x = -2, -1, 0, 1, 2$

a. Using the same method you used in problem 3-1, determine the slope of the tangent line for each x-value and enter it into the table below:



x	-2	-1	0	1	2
m	12	3	0	3	12

b. Use the table and the graph to write a **slope function**, f', a function that gives the slope of the line tangent to f for any x. What type of function is f'? (Hint: Graph it).

3-6. SLOPE FUNCTIONS FOR $f(x) = x^n$

Write a general slope function f' for $f(x) = x^n$ when n is any positive integer. Show that your slope function works for more than one n-value.

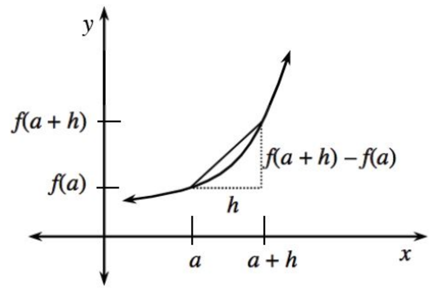
Chapter 3.1.1 and 3.1.2

NOTES

AROC (Average Rate of Change) and IROC (Instantaneous Rate of Change)

The _________ rate of change for *f* over the interval [*a*, *a* + *h*] is:

AROC = $\frac{f(a+h)-f(a)}{(a+h)-a} = \frac{f(a+h)-f(a)}{h}$



The ______________ rate of change for *f* at *a* where *h* represents the change from *a* to *a* + *h* is:

IROC = $\lim_{h \to 0} \frac{f(a+h)-f(a)}{(a+h)-a} = \lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$

Solve for the slope equation of the following functions using the limit definition shown above:

1) $y = -2x + 5$

2) $f(x) = -4x - 2$

3) $y = 4x^2 + 1$

4) $f(x) = -3x^2 + 4$

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NOTES

Power Rule

Earlier you found the **Power Rule** to write slope functions of polynomial functions of the form $f(x) = x^n$ where n is a positive integer. However, the Power Rule extends to all real values of n.

If $f(x) = ax^n$, then $f'(x) = nax^{n-1}$ for all real values of n.

3-19. For which of the functions below can we apply the Power Rule from problem 3-5? Solve.

a. $y = \frac{1}{x}$

b. $y = x^5$

c. $y = \sqrt{x}$

d. $y = 2^x$

e. $y = 4x^0$

3-20. Use the results from problems 3-5, 3-7, and 3-8 to write an equation for $g'(x)$ for each function below.

a. $g(x) = 2x^3$

b. $g(x) = x^8 - x^2$

c. $g(x) = -4x^3 - 2x + 5$

d. $g(x) = 6(x + 2)^4$

e. $g(x) = (x + 7)^{10} - 12x^5$

f. $g(x) = 2(x - 3)^3 + 4(x + l)^2$

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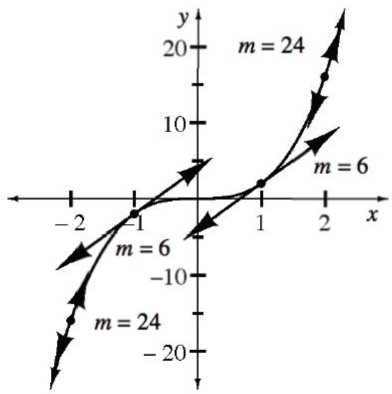
Task Card

3-11. Below is a graph of the function $f(x) = 2x^3$ with tangent lines drawn at $x = -2, -1, 1,$ and 2. Use the slopes provided in the graph to determine the slope function f'. Notice that $f'(0) = 0$. It might be helpful to make a table of data relating x to m. Homework Help ✎



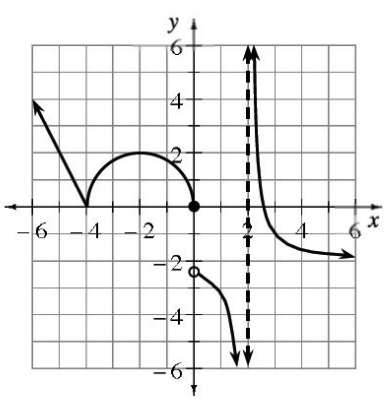
3-12. Without your calculator, evaluate each limit. Homework Help ✎

a. $\lim\limits_{h\to 0}\frac{(2(x+h)-3)-(2x-3)}{h}$

b. $\lim\limits_{h\to 0}\frac{((x+h)^2+(x+h))-(x^2+x)}{h}$

3-13. Is the function graphed below continuous at the following values of x? If not, explain which conditions of continuity fail. Homework Help ✎

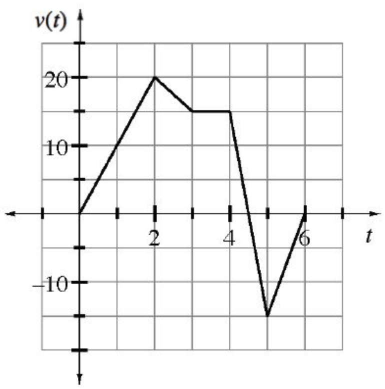
$x = -4, -2, 0,$ and 2



3-14. For the graph in problem 3-13, state the domain and range using interval notation. Homework Help ✎

3-15. Recall the conjecture you developed in problem 3-5 and use it to determine the slope function, $f\,'$, for each of the following functions. Homework Help ✎

a. $f(x) = x^9$
b. $f(x) = x^{13}$
c. $f(x) = 2x$
d. $f(x) = 6$



3-16. After class, Stevie travels in a straight hallway with a velocity shown in the graph above, where t is measured in minutes and $v(t)$ is measured in feet per minute. Homework Help ✎

a. Explain what is happening when $t > 4.5$ minutes.
b. Calculate the total distance Stevie traveled.
c. If Stevie only travels in the straight hallway, how far does he end up from his original starting place?
d. What is Stevie's acceleration at $t = 1$?
e. When is Stevie's acceleration equal to zero?

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Homework

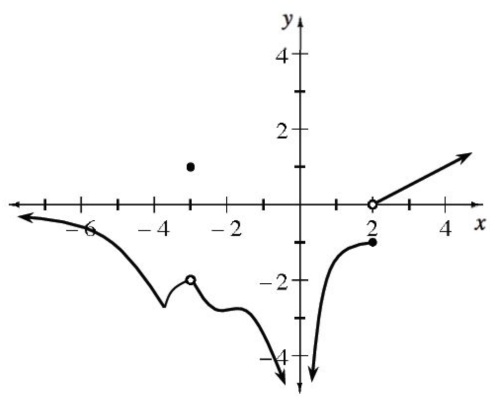
3-27. If $f(15) = -3$ and $f(20) = 4$, *must* f have a root between $x = 15$ and $x = 20$? Explain why or why not. Be sure to include sketches that support your reasoning. Homework Help ✎

3-28. Write slope functions for the following functions: Homework Help ✎

a. $f(x) = 7x^2$
b. $f(x) = \pi^2$ (Careful!)
c. $f(x) = 2(x - 2)^4 + 18x$
d. $f(x) = \frac{1}{3}x^6 + 2x^4 - 3$

3-29. Is the function graphed below continuous at the following values of x? If not, explain which conditions of continuity fail. Homework Help ✎

$x = -4, -3, 0,$ and 2



3-30. For the graph in problem 3-29, state the domain and range using interval notation. Homework Help ✎

3-31. Evaluate the following limits. Homework Help ✎

a. $\lim_{x\to 4^-} (3x^2)$

b. $\lim_{x\to 3^+} (6 - 2x)$

c. $\lim_{x\to\infty} (\sqrt{x})$

d. $\lim_{x \to \infty} \left(\frac{1}{x^2}\right)$

3-32. Jasmin rolled a ball down a very steep ramp and got the distance function $s(t) = 2.3t^2$, where t is measured in seconds and $s(t)$ is measured in feet. Sketch a graph of her distance function on your paper. Then, carefully approximate the speed of the ball at $t = 3$ seconds. 3-32 HW eTool (Desmos) Homework Help ✎.

3-34. In many textbooks the derivative (or IROC) is described in terms of x and Δx (delta x) instead of x and h. Homework Help ✎

e. Explain why h and Δx are equivalent.

f. Use the diagram at right and the definition of IROC to write the slope of the tangent line in terms of x and Δx.