Chapter 5.2.3 and 5.2.4

The purpose of this lesson is to:
Continue practicing the Chain Rule and Product Rule
Learn how the Product Rule can be used with the Chain Rule to solve a quotient of two functions. (The Quotient Rule!)

WARM UP

5-76. Differentiate each of the following functions. (No need to simplify!) Use your calculator to check your solutions.

a. $f(x) = (2x^4 - 1)^3$

b. $f(x) = \sin(2x^4 - 1)$

c. $f(x) = (\sqrt{x} + 3)^2$

d. $f(x) = (x^2 - 5x + 2)^4$

e. $f(x) = (x - 3)^2 + (x^2 + 1)^2$

5-77. The Chain Rule can be combined with other rules, such as the Product Rule, the rule for sums, and the rule for multiplying by constants. Differentiate each of the following functions.

a. $f(x) = 5(2x^4 - 1)^3$

b. $f(x) = (2x^4 - 1)^3 + \sin(2x^4 - 1)$

c. $f(x) = x(2x^4 - 1)^3$

d. $f(x) = 3x \sin(2x^4 - 1)$

e. $f(x) = \sqrt{(x-3)^2 + (x^2+1)^2}$

5-78. THE BUCKLED FREEWAY RETURNS!

The buckled freeway is an example of a situation that can be modeled by using a composite function: the pavement's length depends on temperature, and temperature depends on the time of day.

a. Assuming that $t = 0$ is at midnight and t is measured in hours, the temperature F, in degrees Fahrenheit, at time t is given by $F(t) = 20\cos(0.25t - 4) + 85$, while the length of the portion of freeway (in meters) for temperature F is given by $L(F) = 0.0008F^2 + 238.9$. Write an equation relating the length L with time t.

b. What is $\frac{dL}{dt}$? What does $\frac{dL}{dt}$ represent?

c. During the day, when did this portion of the road reach its maximum length? What was the length?

5-87. So far you have learned to differentiate *powers* and *products*. In this problem, you will discover how to use these techniques on *quotients*.

- Explain why $\frac{1}{2x+1} = (2x+1)^{-1}$. Then use this fact to find $\frac{d}{dx}\left[\frac{1}{2x+1}\right]$.

- Now find $\frac{d}{dx}\left[\frac{x}{2x+1}\right]$ by first rewriting $\frac{x}{2x+1}$ as a product. Simplify your result.

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NOTES

5-89. Since many functions are written in the form *j*(*x*) = $\frac{f(x)}{g(x)}$, it will be convenient to derive a general formula for the derivative of a quotient. Below are two demonstrations of how the **Quotient Rule** can be derived. Work through each demonstration and then compare the results. Are they the same?

Leslie's Demonstration	Tom's Demonstration
1. Rewrite $\frac{f(x)}{g(x)}$ as a product. 2. Use the Product Rule and the Chain Rule to find $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right)$. 3. Simplify as much as possible.	1. Let $j(x)=\frac{f(x)}{g(x)}$ and solve for $f(x)$. 2. Use the Product Rule to find $f'(x)$. 3. Solve for $j'(x)$. 4. Simplify as much as possible.

The Quotient Rule

If $f'(x)$ and $g'(x)$ exist and $h(x) = \frac{f(x)}{g(x)}$ where $g(x) \neq 0$, then ______________________________
.

5-92. THE CONTRIVED FUNCTION COMPANY

The contrived function company has just opened and wants to keep track of their resource-cost ratio. Resources are determined by the amount of money available. According to company projections, the anticipated resources function is $r(x) = (x - 3)^2 + 8$, where x is time in months and $r(x)$ is in thousands of dollars. The cost function is $c(x) = 2x + 1$.

Calculate when the company will have its lowest resource-cost ratio, $\frac{r(x)}{c(x)}$.

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Task Card

5-90. Use your Quotient Rule to differentiate each function below.

a. $y = \frac{\sin(x)}{\sqrt{x}}$

b. $y = \frac{3x^2-2x+1}{5x-1}$

c. $y = \frac{5-6x^2+2x^3}{x-x^2}$

d. $y = \frac{1}{x^3} + \frac{1}{x^4}$

5-88. Rewrite each of the functions below as a product and then write the derivative or use the Quotient Rule to find the derivative.

- $y = \frac{\sqrt{16-x^2}}{3}$

- $f(x) = \frac{5x}{(2x+1)^3}$

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Homework

5-79. Sometimes people make mistakes when differentiating using the Chain Rule. Explain the mistakes in the work below. Then write the correct derivatives. Homework Help ✎

a. $f(x) = -5(6x^4 - 1)^{10} + \pi$
   $f'(x) = -50(24x^3 - 1)^9$

b. $f(x) = \sin(x^2 - x)$
   $f'(x) = -\cos(x^2 - x)$

5-80. For parts (a) and (b) below, explain why obtaining the derivative requires the use of the Chain Rule and the Product Rule. Then write the derivative. Homework Help ✎

a. $\frac{d}{dx}\sqrt{\sin(x)\cos(x)}$

b. $\frac{d}{dx}\left(\sqrt{x}\,\sin(x^3)\right)$

5-81. If the height (in meters) of an object traveling vertically is represented by the function $h(t) = -4.9t^2 + 21t + 3$ where t is measured in seconds, determine its: Homework Help ✎

a. Starting velocity

b. Starting position

c. Maximum height

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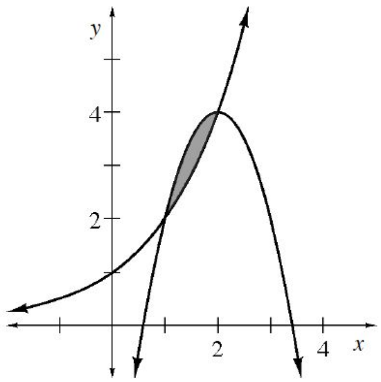
Homework

5-85. Use your calculator to estimate the shaded area shown between the graphs of $y = 4 - 2(x - 2)^2$ and $y = 2^x$. Homework Help ✎



5-94. Use your derivative tools to differentiate each expression below. Homework Help ✎

a. $\frac{d}{dx}\left(\sin \sqrt[3]{x+3}\right)$

b. $\frac{d}{dx}\left(\frac{4-x^2}{x^3-2}\right)$

c. $\frac{d}{dx}\left(\frac{1}{\cos(x)}\right)$

d. $\frac{d}{dx}$ $((6x - 1)(\sin(2x))$

e. $\frac{d}{dx}\left(\frac{\sin(3x)}{2x^3}\right)$