Full Waveform Inversion - Adjoint Tomography

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Equation of motion

$$\rho \delta_t^2 \mathbf{s} = \nabla \cdot \mathbf{T} + \mathbf{f} \ ,$$

where ρ denotes the distribution of density, $\mathbf{s}$ is the displacement, $\mathbf{T}$ is the stress tensor, $\mathbf{f}$ is the source term (i.e. earthquake).

$$\mathbf{T} = \mathbf{c}^U : \nabla \mathbf{s} - \sum_{\ell=1}^{L} \mathbf{R}^\ell,$$

where $\mathbf{c}^U$ is the unrelaxed elastic tensor, $\nabla \mathbf{s}$ is the displacement gradient, $\mathbf{R}^\ell$ represent the linear solids (Tromp, 2005).

Equation of motion

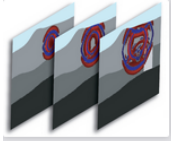
For the sake of completeness we note that the adjoint momentum equation for a rotating, self-gravitating Earth model is given by

$$\rho\left(\partial_t^2 \mathbf{s}^\dagger - 2\boldsymbol{\Omega} \times \partial_t \mathbf{s}^\dagger\right) = \boldsymbol{\nabla} \cdot \boldsymbol{T}^\dagger + \boldsymbol{\nabla}(\rho \mathbf{s}^\dagger \cdot \mathbf{g}) - \rho \boldsymbol{\nabla} \phi^\dagger - \boldsymbol{\nabla} \cdot (\rho \mathbf{s}^\dagger)\mathbf{g} + \mathbf{f}^\dagger$$

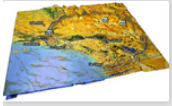
where $\boldsymbol{\Omega}$ denotes the angular velocity of the earth model and $\mathbf{g}$ the equilibrium gravitational acceleration.

(Tromp, 2005)

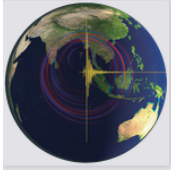
Numerical Solvers



SPECFEM2D simulates forward and adjoint seismic wave propagation in two-dimensional acoustic, (an)elastic, poroelastic or coupled acoustic-(an)elastic-poroelastic media, with Convolution PML absorbing conditions.

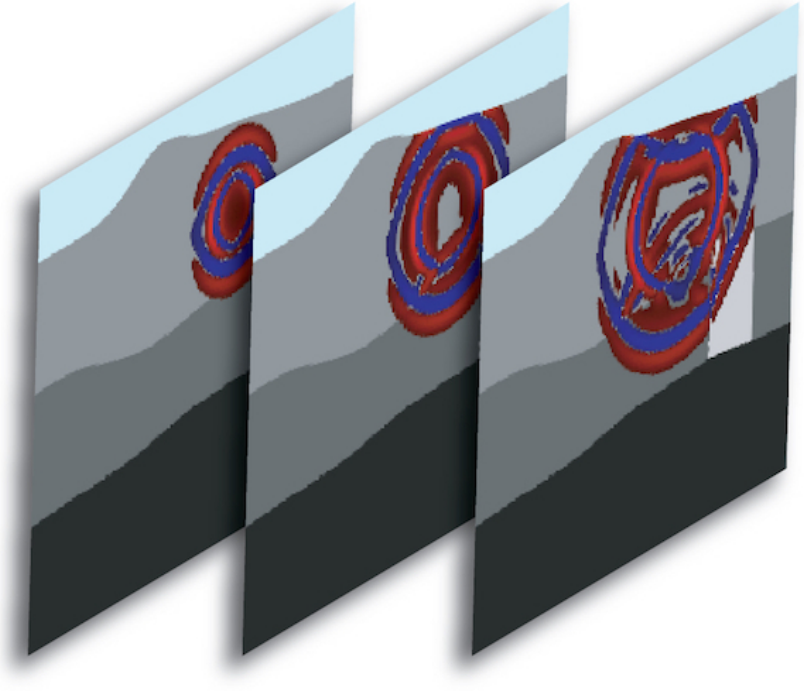


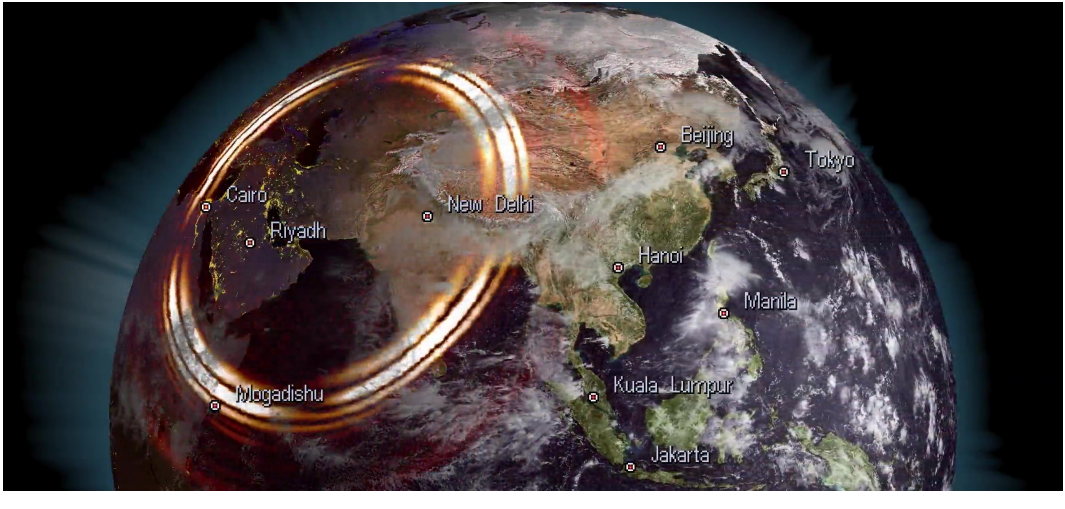
SPECFEM3D_Cartesian simulates acoustic (fluid), elastic (solid), coupled acoustic/elastic, poroelastic or seismic wave propagation in any type of conforming mesh of hexahedra (structured or not).



SPECFEM3D_GLOBE simulates global and regional (continental-scale) seismic wave propagation.

Komatitsch and Tromp, 2002ab.

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Adjoint Method

Adjoint method enables us efficiently compute sensitivity kernels using data residuals.

1. Perform a forward simulation
2. Compute a data misfit with a defined function.
3. Compute a secondary wavefield (adjoint wavefield) with the associated adjoint source.
4. Interaction between these wavefields produces the sensitivity kernels.

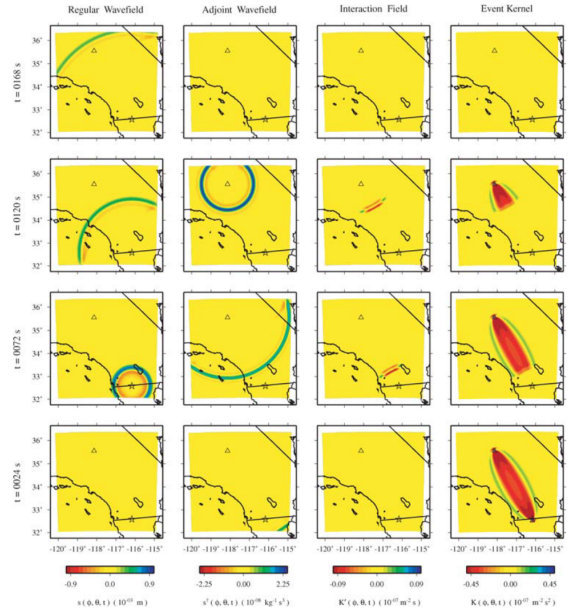


Figure 1: Tape et al., 2007

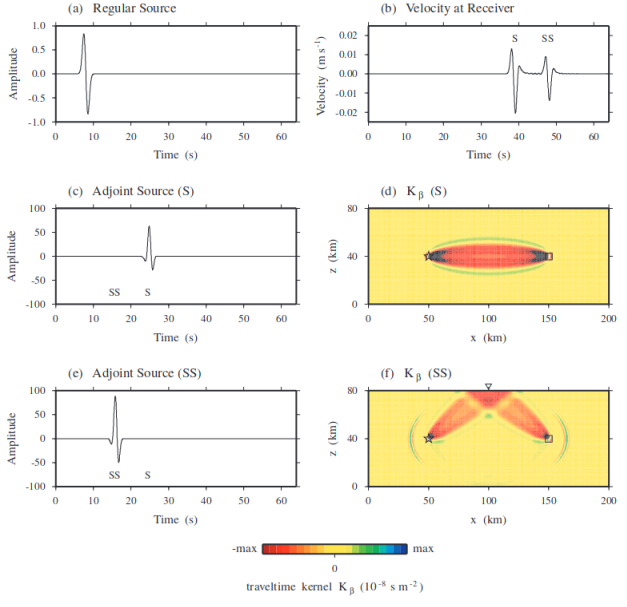


Figure 2: 2D kernels (Tromp et al., 2005)

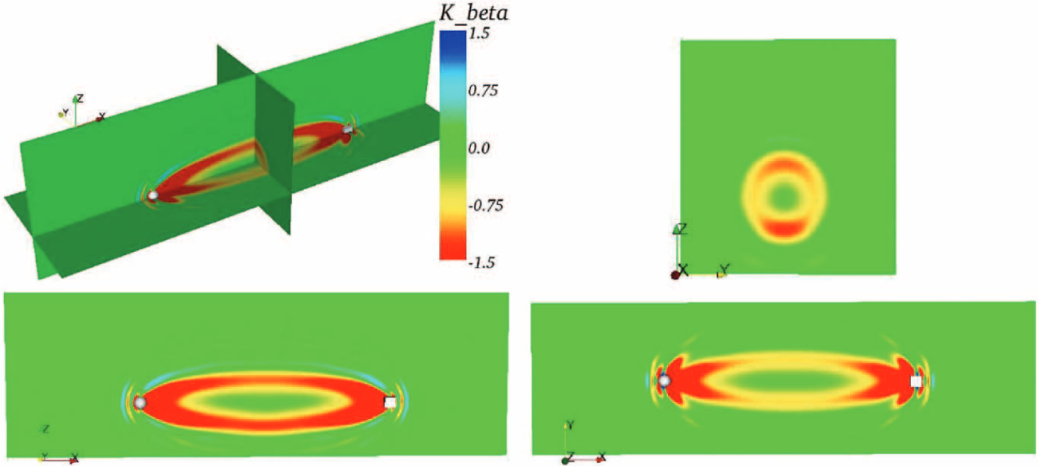


Figure 3: 3D S Kernel (Liu & Tromp, 2006)

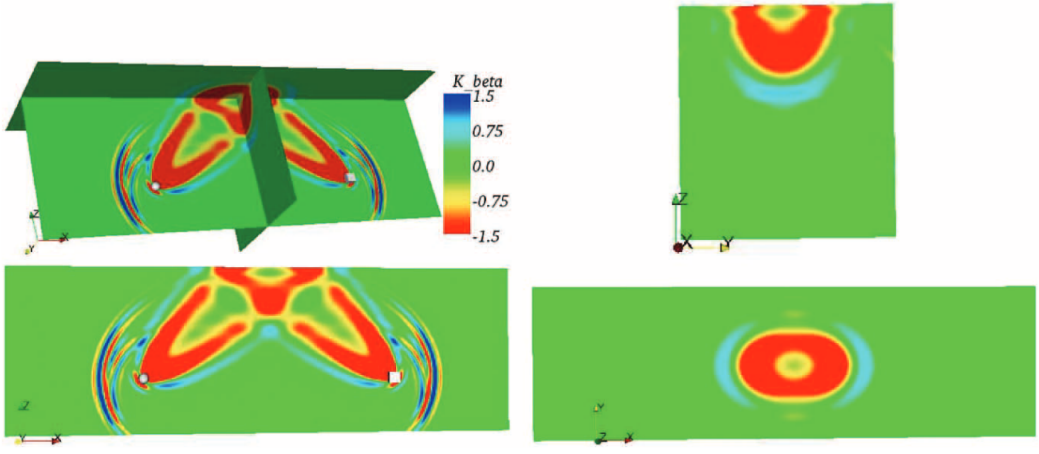


Figure 4: 3D SS kernel (Liu & Tromp, 2006)

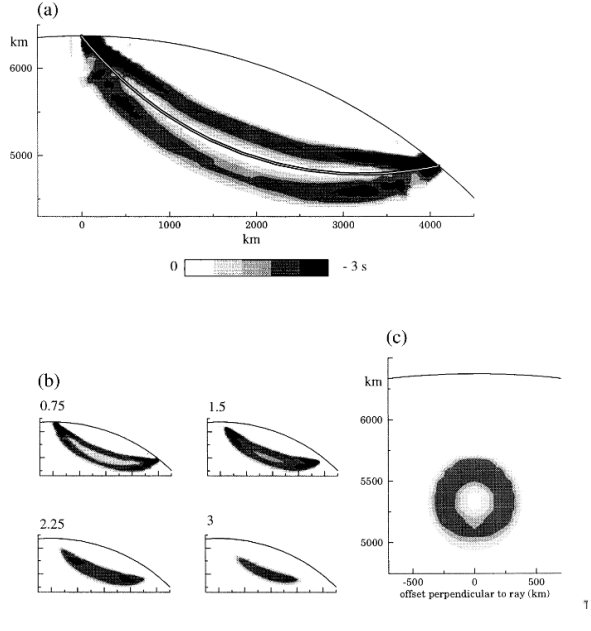


Figure 5: Cross sections of 3D sensitivity kernels (Marquering et al., 1999)

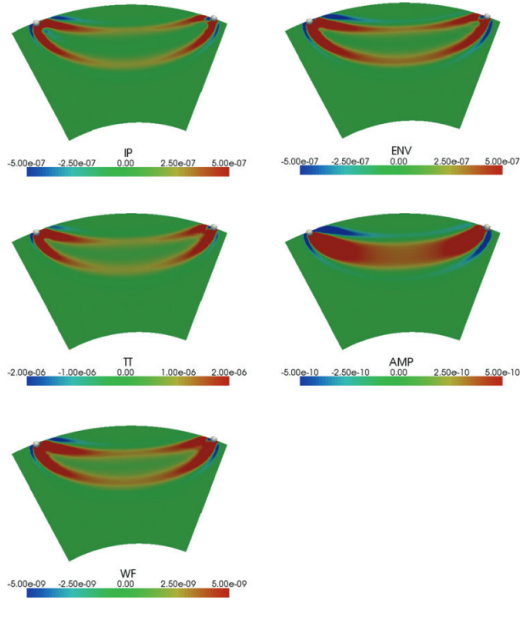


Figure 6: Kernels for different misfit functions (Bozdağ et al., 2011)

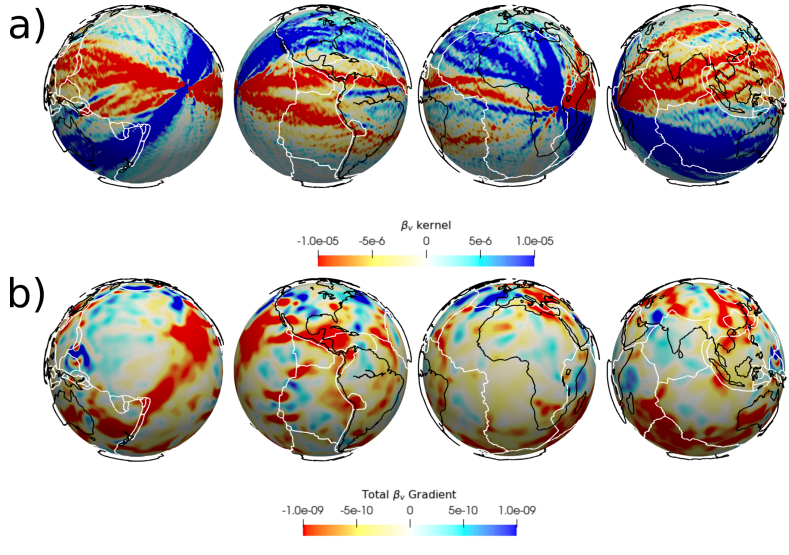


Figure 7: a) Event b) Total S-Wave Kernels (Örsvuran, 2021)

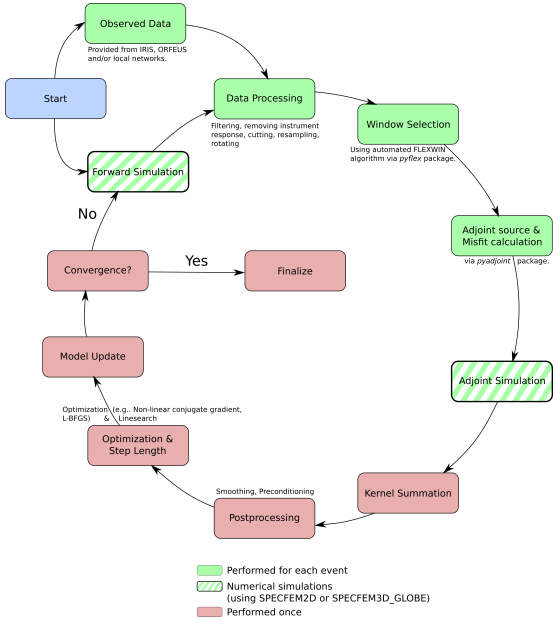
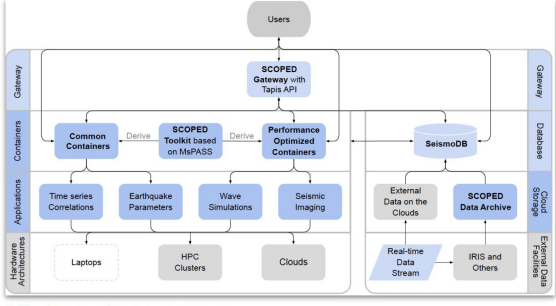


Figure 8: Adjoint Tomography Workflow (Örsvuran, 2021)



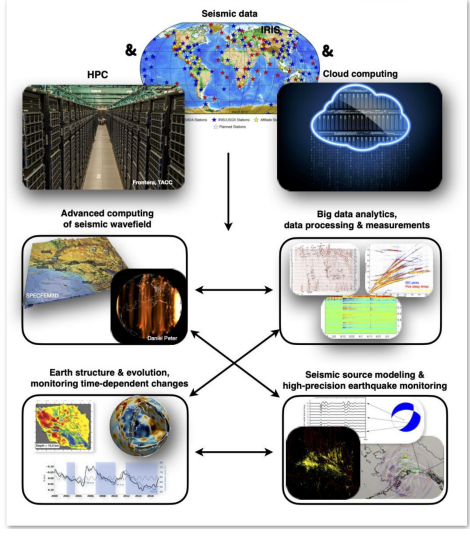


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Figure 9: https://seisscoped.org

https://github.com/rdno/asc2024course