Exercise 1.13 Prove that:
$Fib(n)\ is\ the\ closest\ integer\ to\ \frac{\phi^n}{\sqrt{5}},\ where\ \phi\ =\ \frac{(1+\sqrt{5})}{2}$

$Let\ \psi\ =\ \frac{(1-\sqrt{5})}{2}$
$Prove\ Fib(n) = \frac{(\phi^n-\psi^n)}{\sqrt{5}}\ (hint\ from\ book)$

$Proof\ by\ Induction:$
$Base\ Case:$
$n = 0:\ Fib(0)\ =\ \frac{(\phi^0-\psi^0)}{\sqrt{5}}\ =\ 0\ (Fib(0)\ =\ 0)$

$Induction\ Step:$
$Assume\ Fib(k)\ =\ \frac{(\phi^k-\psi^k)}{\sqrt{5}}$
$Prove\ Fib(k+1) = \frac{(\phi^{k+1}-\psi^{k+1})}{\sqrt{5}}$
$Fib(k+1) = Fib(k) + Fib(k-1)\ (By\ definition\ of\ Fibonnaci\ series)$
$Fib(k+1) = \frac{(\phi^k-\psi^k)}{\sqrt{5}} + \frac{(\phi^{k-1}-\psi^{k-1})}{\sqrt{5}}$
$Fib(k+1) = \frac{(\phi^{k-1}\phi-\psi^{k-1}\psi)}{\sqrt{5}} + \frac{(\phi^{k-1}-\psi^{k-1})}{\sqrt{5}}$
$Fib(k+1) = \frac{\phi^{k-1}(\phi+1)-\psi^{k-1}(\psi+1)}{\sqrt{5}}$
$Fib(k+1) = \frac{\phi^{k-1}\phi^2-\psi^{k-1}\psi^2}{\sqrt{5}}\ (\phi^2 = \phi + 1\ and\ \psi^2 = \psi + 1)$
$Fib(k+1) = \frac{(\phi^{k+1}-\psi^{k+1})}{\sqrt{5}}$

$We\ have\ shown\ that\ Fib(n) = \frac{\phi^n}{\sqrt{5}}\ minus\ a\ factor\ of\ \frac{\psi^n}{\sqrt{5}}$
$We\ will\ now\ show\ that\ |\frac{\psi^n}{\sqrt{5}}| < 0.5\ which\ would\ mean\ that\ Fib(n)\ is\ the\ closest\ integer\ to \frac{\phi^n}{\sqrt{5}}$

$Proof\ by\ Induction:$
$Base\ Case:$
$n = 0:\ |\frac{\psi^0}{\sqrt{5}}| = |\frac{1}{\sqrt{5}}| < 0.5$

$Induction\ Step:$
$Assume|\frac{\psi^k}{\sqrt{5}}| < 0.5$
$Prove|\frac{\psi^{k+1}}{\sqrt{5}}| < 0.5$
$|\frac{\psi^{k+1}}{\sqrt{5}}| = |\psi||\frac{\psi^k}{\sqrt{5}}| < |\psi| * 0.5\ (from\ induction\ assumption)$
$|\psi| * 0.5 < 0.5$
$|\psi| < 1$
$|\frac{1-\sqrt{5}}{2}| \approx 0.618 < 1$