Statistical Models for Causal Analysis

Teppei Yamamoto

Keio University

Introduction to Causal Inference
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Three Modes of Statistical Inference

1. Descriptive Inference: summarizing and exploring data
   - Inferring ideal points from roll-call votes
   - Inferring topics from texts and speeches
   - Inferring social networks from surveys
2. Predictive Inference: forecasting out-of-sample data points
   - Inferring future state failures from past failures
   - Inferring population average turnout from a sample of voters
   - Inferring individual level behavior from aggregate data
3. Causal Inference: predicting *counterfactuals*
   - Inferring the effects of ethnic minority rule on civil war onset
   - Inferring why incumbency status affects election outcomes
   - Inferring whether the lack of war among democracies can be attributed to regime types

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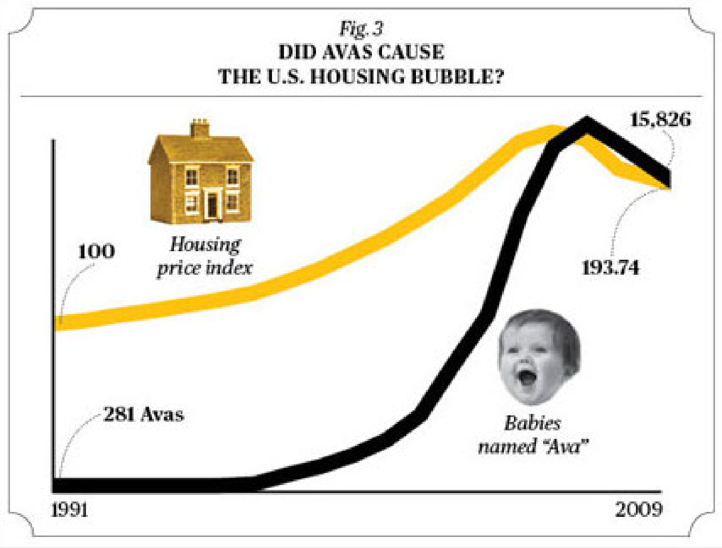
Causal inference is the most difficult of the three.

Correlation ≠ Causation



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Examples:

- Incumbency advantage:
  What *would* have been the election outcome if the candidate had not been an incumbent?
- Democratic peace:
  Would the two countries have fought each other if they had been both autocratic?
- Policy intervention:
  How many more disadvantaged youths *would* get employed under the new job training program?

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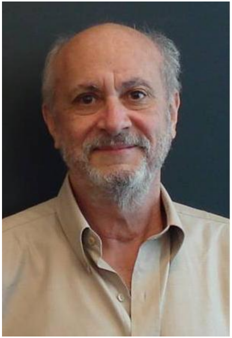
We need a statistical model that can explicitly distinguish factuals and counterfactuals.

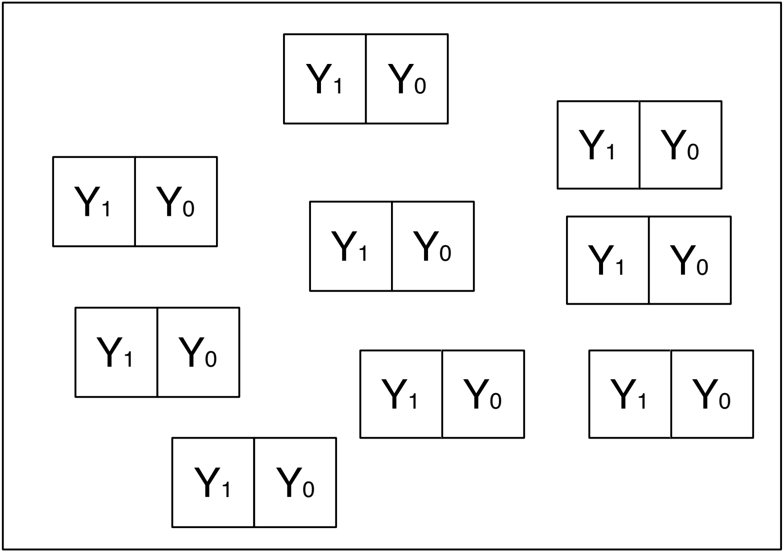
Neyman-Rubin Potential Outcomes Model

Jerzy Neyman (1894–1981)



Donald Rubin (1943–)

Neyman Urn Model



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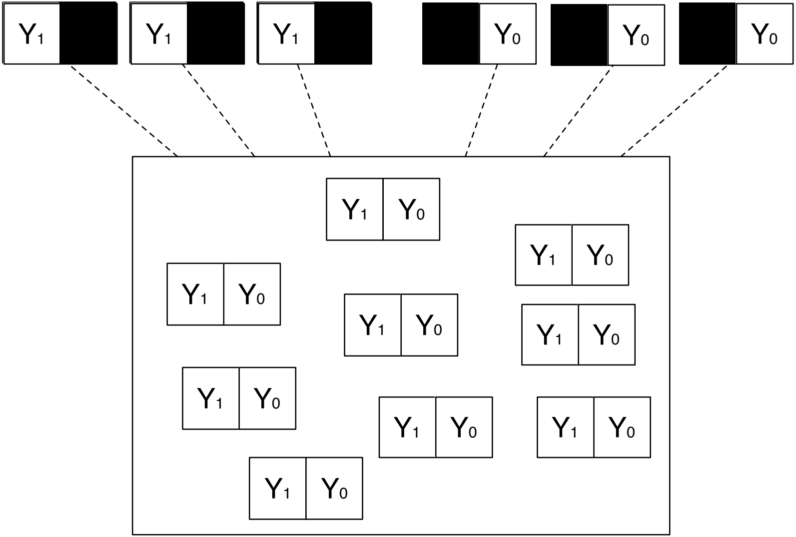

Causality with Potential Outcomes

Definition (Treatment)

D_i: Indicator of treatment intake for unit i, where $i = 1, ..., N$

$$D_i = \begin{cases} 1 & \text{if unit } i \text{ received the treatment} \\ 0 & \text{otherwise} \end{cases}$$

Definition (Observed Outcome)

Y_i: Variable of interest whose value may be affected by the treatment

Definition (Potential Outcomes)

Y_{di}: Value of the outcome that *would* be realized if unit i received the treatment d, where $d = 0$ or 1

$$Y_{di} = \begin{cases} Y_{1i} & \text{Potential outcome for unit } i \text{ with treatment} \\ Y_{0i} & \text{Potential outcome for unit } i \text{ without treatment} \end{cases}$$

Alternative notation: $Y_i(d)$, Y_i^d, etc.

Causality with Potential Outcomes

Definition (Causal Effect, or Unit Treatment Effect)

Causal effect of the treatment on the outcome for unit i is the difference between its two potential outcomes:

$$\tau_i = Y_{1i} - Y_{0i}$$

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Note that observed outcomes are realized from potential outcomes as

$$Y_i = Y_{D_i i} = D_i Y_{1i} + (1 - D_i) Y_{0i} \quad \text{so} \quad Y_i = \begin{cases} Y_{1i} & \text{if } D_i = 1 \\ Y_{0i} & \text{if } D_i = 0 \end{cases}$$

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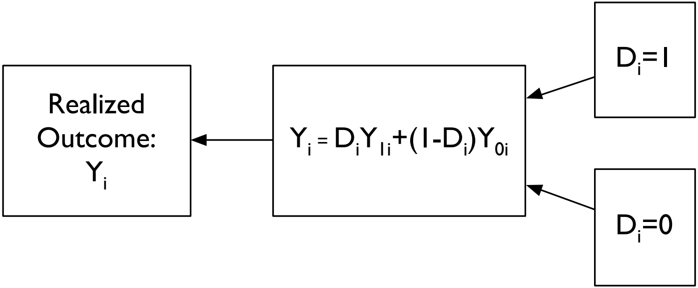
Fundamental Problem of Causal Inference (Holland 1986):

We can never observe both Y_{1i} and Y_{0i} for the same i

This makes τ_i unidentifiable without further assumptions.

Causal Inference as a Missing Data Problem

Problem: Causal Inference is difficult because it involves missing data.
How can we calculate $\tau_i = Y_{1i} - Y_{0i}$?



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- One "solution": Assume unit homogeneity

$$\tau_i = \tau \quad \text{for all } i$$

  - If Y_{1i} and Y_{0i} are constant across individual units, then cross-sectional comparisons will recover $\tau = \tau_i$
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- This may be sometimes plausible in physical sciences

- Unfortunately, rarely true in social sciences

Stable Unit Treatment Value Assumption (SUTVA)

- Recall: $Y_i = Y_{D_i i}$, or equivalently $Y_i = D_i Y_{1i} + (1 - D_i) Y_{0i}$
- This notation implicitly makes the following assumption:

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Assumption (SUTVA)

$$Y_{(D_1, D_2, \ldots, D_N)i} \;=\; Y_{(D'_1, D'_2, \ldots, D'_N)i} \quad \textit{if} \quad D_i = D'_i$$

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Violation examples: Vaccination, fertilizer on plot yield, communication

Causal Inference without SUTVA

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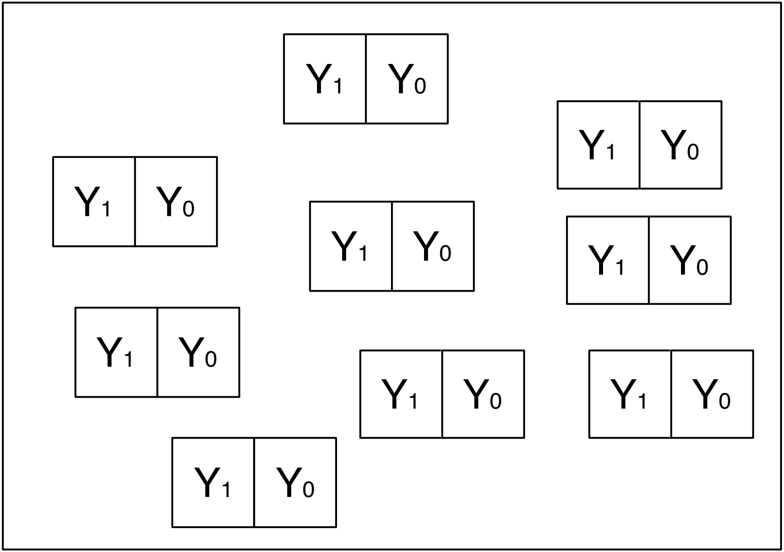
Without SUTVA, causal inference becomes exponentially more difficult as N increases.

Back to the Neyman Urn Model



Causal Quantities of Interest, or Estimands

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Definition (Average treatment effect, ATE)

$$\tau_{ATE} = \frac{1}{N}\sum_{i=1}^{N}\{Y_{1i} - Y_{0i}\}$$

or equivalently

$$\tau_{ATE} = \mathbb{E}[Y_{1i} - Y_{0i}]$$

- Note that τ_{ATE} is still unidentified
- In the rest of this course, we will consider various assumptions under which τ_{ATE} can be identified from observed information

Other Causal Estimands

Definition (Average treatment effect on the treated, ATT)

$$\tau_{ATT} = \frac{1}{N_1}\sum_{i=1}^{N} D_i \{Y_{1i} - Y_{0i}\} \quad \text{where} \quad N_1 = \sum_{i=1}^{N} D_i$$

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where X_i is a pre-treatment covariate for unit i

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- In words, $\tau_{CATE}(x)$ is a subgroup effect, treatment effect on units who have particular characteristics x.

Illustration: Average Treatment Effect

Suppose we observe a population of 4 units:

i	D_i	Y_i
1	1	3
2	1	1
3	0	0
4	0	1

What is $\tau_{ATE} = \mathbb{E}[Y_{1i} - Y_{0i}]$?

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i	D_i	Y_i
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3	0	0
4	0	1
$\mathbb{E}[Y_i \mid D_i = 1]$		2
$\mathbb{E}[Y_i \mid D_i = 0]$		0.5
$\mathbb{E}[Y_i \mid D_i = 1] - \mathbb{E}[Y_i \mid D_i = 0]$		1.5

What is $\tau_{ATE} = \mathbb{E}[Y_{1i} - Y_{0i}]$?

Naïve estimator:

$$\begin{aligned} \tilde{\tau} &= \mathbb{E}[Y_i \mid D_i = 1] - \mathbb{E}[Y_i \mid D_i = 0] \quad \text{(observed difference in means)} \\ &= \frac{3+1}{2} - \frac{0+1}{2} = 1.5 \quad \text{Could this be wrong?} \end{aligned}$$

Illustration: Average Treatment Effect

Suppose we observe a population of 4 units:

i	D_i	Y_i	Y_{1i}	Y_{0i}	τ_i
1	1	3	3	?	?
2	1	1	1	?	?
3	0	0	?	0	?
4	0	1	?	1	?

What is $\tau_{ATE} = \mathbb{E}[Y_{1i} - Y_{0i}]$? We need potential outcomes that we do not observe!

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Suppose hypothetically: $Y_{01} = 0, Y_{02} = Y_{13} = Y_{14} = 1$.

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$$\tau_{ATE} = \mathbb{E}[Y_{1i} - Y_{0i}] = \mathbb{E}[\tau_i] = \frac{3+0+1+0}{4} = 1.$$

Why $\tau_{ATE} \neq \tilde{\tau}$? When would they be equal?

Illustration: Average Treatment Effect on the Treated

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Why does $\tau_{ATT} \neq \tau_{ATE}$?

Selection Bias

- Comparisons of observed outcomes for the treated and the untreated do not usually give the right answer:

$$
\begin{aligned}
\tilde{\tau} &= \mathbb{E}[Y_i | D_i = 1] - \mathbb{E}[Y_i | D_i = 0] \\
&= \mathbb{E}[Y_{1i} | D_i = 1] - \mathbb{E}[Y_{0i} | D_i = 0] \\
&= \underbrace{\mathbb{E}[Y_{1i} - Y_{0i} | D_i = 1]}_{\tau_{ATT}} + \underbrace{\mathbb{E}[Y_{0i} | D_i = 1] - \mathbb{E}[Y_{0i} | D_i = 0]}_{\text{Bias}}
\end{aligned}
$$

- Bias term $\neq 0$ if selection into treatment is associated with potential outcomes

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\end{aligned}$$

- Bias term $\neq 0$ if selection into treatment is associated with potential outcomes

Example: Church attendance and turnout

- churchgoers differ from individuals who do not attend church in many ways (e.g. civic duty)
- turnout for churchgoers would be higher than for non-churchgoers even if churchgoers never attended church ($E[Y_{0i}|D_i = 1] - E[Y_{0i}|D_i = 0] > 0$)

Selection Bias

- Comparisons of observed outcomes for the treated and the untreated do not usually give the right answer:

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Example: Job training program for the disadvantaged

- participants are self-selected from a subpopulation of individuals in difficult labor situations
- post-training period earnings for participants would be lower than those for nonparticipants in the absence of the program ($E[Y_{0i}|D_i = 1] - E[Y_{0i}|D_i = 0] < 0$)

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4. Treatment is self-selected and no plausible control is available ☹

An Alternative Causal Model: Causal Graphs

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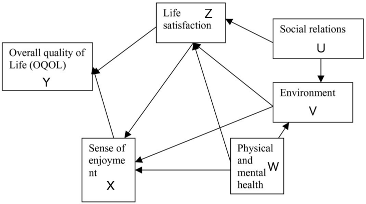
- The old paradigm: structural equation modeling (SEM) and path analysis
  1. Postulate a causal mechanism and draw a corresponding path diagram



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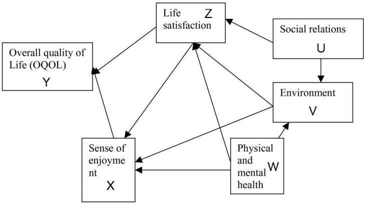

  2. Translate it into a (typically linear) system of equations:

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Y &= \alpha_0 + \alpha_1 X + \alpha_2 Z + \varepsilon_\alpha \\
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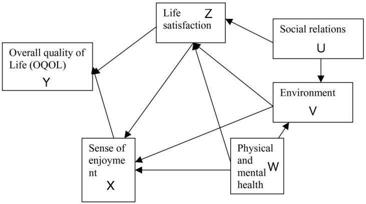

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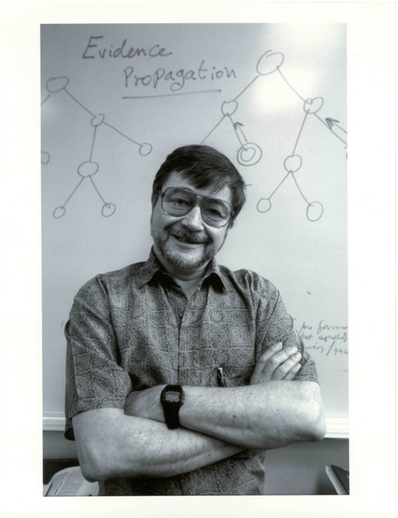
- Went out of fashion:
  - Strong distributional/functional form assumptions
  - No language to distinguish causation from association

Pearl's Attack



Judea Pearl (1936–) proposed a new causal inference framework based on nonparametric structural equation modeling (NPSEM)

- Originally a computer scientist
- Previous important work on artificial intelligence
- *Causality* (2000, Cambridge UP)
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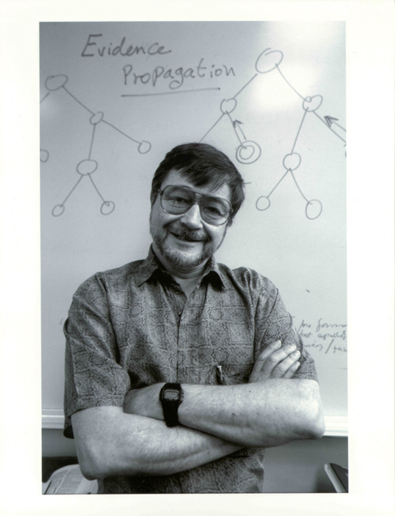

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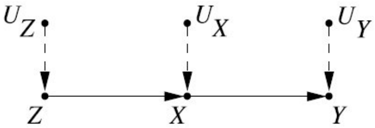
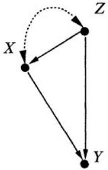
Pearl's framework builds on SEMs and revives it as a formal language of causality.

Causal Diagram





- A causal diagram is a directed acyclic graph (DAG) composed of:
  - Nodes (representing variables in the causal model)
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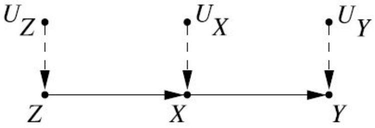

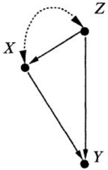

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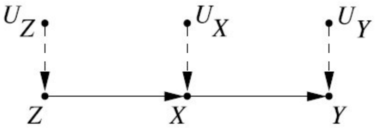

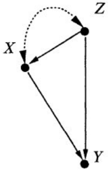

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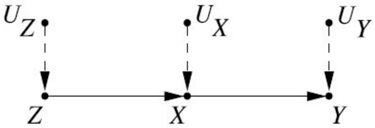
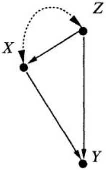
- Note that it is *missing* edges that encode causal assumptions
  - Missing arrows encode exclusion restrictions
  - Missing dashed arcs encode independencies between error terms

NPSEM and Treatments





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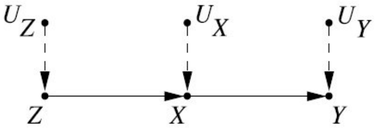

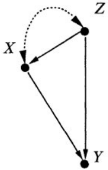

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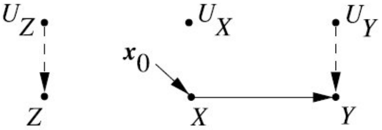

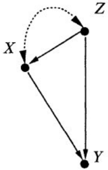

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- Treatments (interventions) are represented by the *do*() operator
- For example, $do(x_0)$ holds X at x_0 exogenously and thus replaces the structural equation for X with this value:

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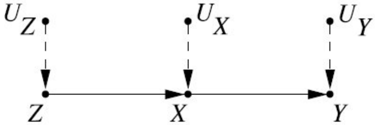
Causal Effects and Identification



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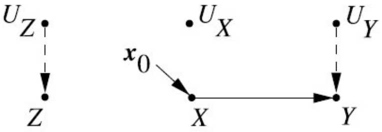

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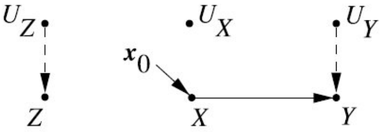

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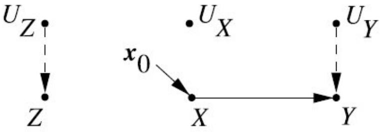

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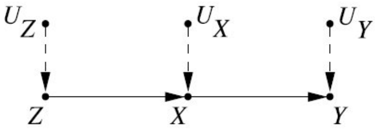

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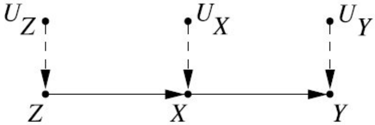

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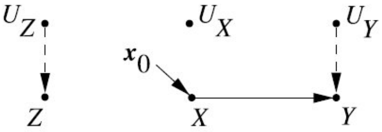

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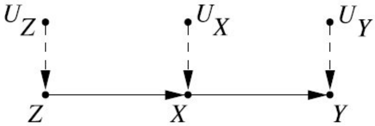

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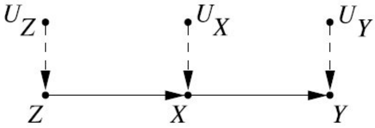

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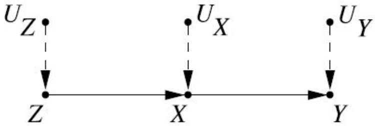

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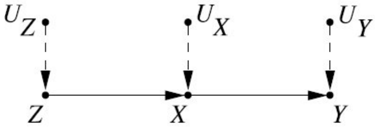

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- Because of this fundamental equivalence, we will mostly work with potential outcomes, currently the standard framework in social sciences.
- Graphs are useful for expressing and visualizing a causal model in empirical research.

Aside: Modern Schools of Statistical Causal Inference

1. Potential Outcomes (Rubin, Rosenbaum, Imbens)
    - Most widely used in applied sciences, especially economics and political science
    - Causal inference as missing data problem
2. NPSEM (Pearl)
    - Uses mathematical theory of graphs
    - Borrows concepts from Bayes net and neural networks
3. Sufficient Component Causes (Rothman, VanderWeele)
    - Originates in epidemiology; "causal pies"
    - Resembles the qualitative comparative analysis (QCA)
4. Decision-theoretic causality (Dawid)
    - Does not assume existence of counterfactuals

- Causal inference as an independent field of science: *Journal of Causal Inference*

Summing Up

- Potential outcomes framework (Neyman-Rubin model) as a dominant framework for causal inference
- Causal quantities are defined by potential outcomes (counterfactuals), not by realized (observed) outcomes
- No assumption of unit homogeneity; causal effects allowed to vary unit by unit
- Observed association is neither necessary nor sufficient for causality
- Estimation of causal effects often starts with studying the treatment assignment mechanism