lecture 07, validity and logical form

phil1012 introductory logic

overview

this lecture

- in the last few lectures we've been examining the semantics of PL and truth tables
- in this lecture we look in more detail at the idea of the **logical form** of a proposition
- recall that logical form was central to the definition of **validity**

learning outcomes

- after doing the relevant reading for this lecture, listening to the lecture, and attending the relevant tutorial, you will be able to:
    - identify the forms of propositions
    - identify instances of forms
    - identify the forms of arguments
    - identify instances of argument forms
    - explain how truth tables provide a test not only for necessary truth preservation but for validity

required reading

- all of chapter 5

validity and logical form

validity and logical form

- recall: an argument is **valid** if and only if, **in virtue of the form of the argument**, it is impossible for its conclusion to be false if its premises are all true
- our truth table test tells us whether it is possible for the conclusion to be false if all of the premises are true
- does it tell us whether this is in virtue of the form of the argument?
- it turns out that the answer is yes, since we can show that every argument with the same form is valid

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- practical upshot: once we know that a given argument is valid, we know that every other argument of the same form is valid, without having to do truth tables for them individually

the form of an argument

- what is the form of an argument?
- suppose we set out to test whether the following argument is

valid:

P1.	John is a philosopher
P2.	if John is a philosopher, then he drinks
C1.	John drinks

- we begin by writing up our glossary:
  - PP: John is a philosopher
  - DD: John drinks
- we translate accordingly:
  - PP, (P→D)(P \rightarrow D) ∴\therefore DD

- we use a truth table to test for validity:

PP	DD	PP	(P→D)(P \rightarrow D)	DD
T	T	T	T	T
T	F	T	F	F
F	T	F	T	T
F	F	F	T	F

- great, now suppose we set out to test whether the following argument is valid:

P1.	Jane is a philosopher
P2.	if Jane is a philosopher, then she smokes
C1.	Jane smokes

- we begin by writing up our glossary:
  - JJ: Jane is a philosopher
  - SS: Jane smokes
- we translate accordingly:
  - JJ, (J→S)(J \rightarrow S) ∴\therefore SS

- and we use a truth table to test for validity:

JJ	SS	JJ	(J→S)(J\rightarrow S)	SS
T	T	T	T	T
T	F	T	F	F
F	T	F	T	T
F	F	F	T	F

- after a while we might start to feel that the whole process is a bit laborious and redundant
- don’t these arguments have something in common which means that if we know that one is valid, we know the other is, and vice versa?
- yep, these arguments are instances of the same **form**
- how can we make the notion of form more precise?

abstracting from content: from propositions to forms

from propositions to forms

- consider the following proposition:
    - $(A \land (B \rightarrow \lnot C))$(A \land (B \rightarrow \lnot C))
- what is its form?
- there are many correct answers ranging from very course-grained to very fine-grained

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- at its most course-grained it is just a proposition:
    - $\underline{(A \land (B \rightarrow \lnot C))}$\underline{(A \land (B \rightarrow \lnot C))}
- so it has the following form: α\alpha

- at a more fine-grained level of description it is a conjunction:
    - $(\underline{A } \land \underline{(B \rightarrow \lnot C)})$(\underline{A } \land \underline{(B \rightarrow \lnot C)})
- so it has the following form: $(\alpha \land \beta)$(\alpha \land \beta)

- at an even more fine-grained level of description it is a conjunction whose second conjunct is a conditional:
    - $(\underline{A } \land \underline{(\underline{B} \rightarrow \underline{\lnot C})})$(\underline{A } \land \underline{(\underline{B} \rightarrow \underline{\lnot C})})
- so it has the following form:
    - $(\alpha \land (\beta \rightarrow \gamma))$(\alpha \land (\beta \rightarrow \gamma))

- at the most fine-grained level of description it is a conjunction whose second conjunct is a conditional, whose consequent is a negation:
    - $(\underline{A } \land \underline{(\underline{B} \rightarrow \lnot \underline{C})})$(\underline{A } \land \underline{(\underline{B} \rightarrow \lnot \underline{C})})
- so it has the following form:
    - $(\alpha \land (\beta \rightarrow \lnot \gamma))$(\alpha \land (\beta \rightarrow \lnot \gamma))

- propositions do not have a single form
- they have many forms ranging from the course-grained to the fine-grained

instances: from forms to propositions

instances: from forms to propositions

- consider the following logical form:
    - $(\alpha \leftrightarrow (\lnot \beta \land \alpha))$(\alpha \leftrightarrow (\lnot \beta \land \alpha))
- a **logical form** is like a formula, except that in place of basic propositions it has variables in the place of formulas
- given a logical form, we can ask: what propositions have this form? We call such propositions **instances** of the form

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- an instance of a logical form can be obtained by replacing the variables with propositions
- all occurrences of the same variable must be replaced by the same proposition

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- for example . . .

    - (α↔(¬β∧α))(\alpha \leftrightarrow (\lnot \beta \land \alpha))
        - (A↔(¬B∧A))(A \leftrightarrow (\lnot B \land A))
        - (B↔(¬E∧B))(B \leftrightarrow (\lnot E \land B))
        - ((A→B)↔(¬E∧(A→B)))((A \rightarrow B) \leftrightarrow (\lnot E \land (A \rightarrow B)))
        - (B↔(¬B∧B))(B \leftrightarrow (\lnot B \land B))
        - ((A→B)↔(¬(A→B)∧(A→B)))((A \rightarrow B) \leftrightarrow (\lnot (A \rightarrow B) \land (A \rightarrow B)))

- but not . . .
    - (α↔(¬β∧α))(\alpha \leftrightarrow (\lnot \beta \land \alpha))
        - (A↔(¬B∧B))(A \leftrightarrow (\lnot B \land B))
        - (B↔(¬E∧D))(B \leftrightarrow (\lnot E \land D))
        - ((A→B)↔(¬E∧B))((A \rightarrow B) \leftrightarrow (\lnot E \land B))

- two propositions share a form when they are both instances of it
- it doesn’t make sense to say that they have the same form, they may both be instances of some form and not both instances of some other form

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- okay, now we have a better idea of the forms of a proposition
- and we have a better idea of the propositions which are instances of forms
- we can now generalise this idea to the forms of an argument

the form(s) of an argument

the form(s) of an argument

- consider the following argument form:
    - α\alpha, (α→(β→α))(\alpha \rightarrow (\beta \rightarrow \alpha)) ∴β→α\therefore \beta \rightarrow \alpha
- an **argument form** is just like an argument except that, in place of basic propositions, it has variables in the place of propositions
- given an argument form, we can ask: what arguments have this form?
- we call such arguments **instances** of the form

- for example . . .
    - α\alpha, α→(β→α)\alpha \rightarrow (\beta \rightarrow \alpha) ∴β→α\therefore \beta \rightarrow \alpha
        - PP, P→(Q→P)P \rightarrow (Q \rightarrow P), ∴Q→P\therefore Q \rightarrow P
        - (R∨¬Q)(R \lor \lnot Q), (R∨¬Q)→(P→(R∨¬Q))(R \lor \lnot Q) \rightarrow (P \rightarrow (R \lor \lnot Q)), ∴P→(R∨¬Q)\therefore P \rightarrow (R \lor \lnot Q)

- consider the following argument:
    - PP, (P→(Q→P))(P \rightarrow (Q \rightarrow P)), ∴Q→P\therefore Q \rightarrow P
- an argument can be an instance of more than one form

- for example . . .
    - PP, (P→(Q→P))(P \rightarrow (Q \rightarrow P)) ∴Q→P\therefore

$Q \rightarrow P$
- α, $(\alpha \rightarrow (\beta \rightarrow \alpha))$, $\therefore (\beta \rightarrow \alpha)$
- α, $(\alpha \rightarrow \gamma)$, $\therefore (\beta \rightarrow \alpha)$

- an argument can be an instance of more than one form
- it is important to keep this in mind when we discuss validity and form

validity and form

validity and form

- now that we have a precise understanding of the forms of an argument, to what use can we put the idea?
- well, we can use it to make our notion of validity more precise
- and we can appeal to the form of an argument in arguing that it is valid
- we begin by introducing notions analogous to validity and invalidity, which are properties of arguments, for argument forms

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- we define the properties **valid*** and **invalid*** of **argument forms**
- these are distinct from the properties **valid** and **invalid** of **arguments**

- an argument form is **valid*** if and only if there is no row in which the premises are true and the conclusion false
- an argument form is **valid*** if and only if there is no instance of the form in which the premises are true and the conclusion false
- an argument form is **invalid*** if and only if there is a row in which the premises are true and the conclusion false

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- we test for **validity*** of an argument form using truth tables as before

α	β	α	$(\alpha \rightarrow \beta)$	β
T	T	T	T	T
T	F	T	F	F
F	T	F	T	T
F	F	F	T	F

- now, a very nice feature of **valid*** argument forms is that every instance of a **valid*** argument form is valid argument

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- consider the following argument:
  - $((A \land C) \rightarrow ((B \lor D) \rightarrow E))$, $(A \land C)$, $\therefore ((B \lor D) \rightarrow E)$
- we need a truth table with 32 rows since we have five distinct basic propositions!

- but wait! this argument is an instance of the form:

- $(\alpha \rightarrow \beta)$, α $\therefore \beta$
- and this form is **valid*** (we just did the truth table for it)
- so we can immediately conclude that the argument is valid

- hint: unless stated otherwise, it is *always* permissible to save yourself time on a problem set or an exam by appealing to the form of an argument in proving its validity
- we will look at examples in the tutorials

invalidity and form

invalidity and form

- careful: it is not the case that every instance of an **invalid*** argument form is an invalid argument
- the following is an **invalid*** argument form:
  - β $\rightarrow$ α, α $\therefore \beta$

- but the following is an instance of the form:
  - $(A \rightarrow A)$, A $\therefore$ A
- and this argument is valid
- we can see this by doing the truth table, or by noting that this is *also* an instance of a valid form

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- indeed, we can distinguish between **A-properties** and **S-properties** of arguments and argument forms
- every instance of a form with an A-property has that A-property. not so with the S-properties.

A-property	**S-property**
validity	invalidity
logical truth	non-logical truth
equivalence	inequivalence
unsatisfiability	satisfiability

- in other words, there are shortcuts available for establishing A-properties, but no shortcuts for S-properties

notable argument forms

notable argument forms

- now that we've introduced the idea of a valid* argument form, and we've seen how to appeal to valid argument forms in establishing the validity of an argument, you should be on the lookout for shortcuts
- here are some valid* argument forms to be on the lookout for

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- modus ponens
  - $(\alpha \rightarrow \beta)$, α $\therefore$ β
- modus tollens
  - $(\alpha \rightarrow \beta)$, $\lnot\beta$ $\therefore$ $\lnot\alpha$

- hypothetical syllogism
  - (α(\alpha →\rightarrow β)\beta), (β(\beta →\rightarrow γ)\gamma) ∴\therefore (α(\alpha →\rightarrow γ)\gamma)
- constructive dilemma
  - (α(\alpha →\rightarrow β)\beta), (γ(\gamma →\rightarrow δ)\delta), (α(\alpha ∨\lor γ)\gamma) ∴\therefore (β(\beta ∨\lor δ)\delta)
- disjunctive syllogism
  - (α(\alpha ∨\lor β)\beta), ¬\lnotα\alpha ∴\therefore β\beta

wrapping up

this lecture

- validity and logical form
- the forms of propositions
- the instances of forms
- valid* argument forms

next lecture

- lecture 08, functional completeness