

lecture 06, uses of truth tables

phil1012 introductory logic

overview

this lecture

- the uses of truth tables
- constructing truth tables
- using truth tables to test for various logical properties

learning outcomes

- after doing the relevant reading for this lecture, listening to the lecture, and attending the relevant tutorial, you will be able to:
 - construct truth tables for arguments, single propositions, pairs of propositions and sets of propositions
 - use truth tables to determine whether an argument is valid, providing a counterexample if not
 - use truth tables to determine whether a proposition is a tautology, a contradiction, or neither
 - use truth tables to determine whether pairs of formulas are jointly satisfiable, equivalent, contradictory, or contraries.
 - use truth tables to determine whether sets of propositions are jointly satisfiable

required reading

- all of chapter 4

truth tables for complex propositions

truth tables for complex propositions

- sometimes we want to know the truth value of a complex proposition for every possible assignment of truth values to basic propositions.
- for this purpose we construct a **truth table** for the complex proposition.
- each row of a truth table corresponds to a possible assignment of truth values to basic propositions

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- we set up our table so that the rows in the columns on the left correspond to the possible assignments of truth values to basic propositions
 - and, we “copy over” those values to the basic propositions on the right for each row
 - we proceed as in the case of a single assignment, calculating the truth value of the more complex propositions in terms of the truth value of their constituents

- until we have the values of the complex proposition for each possible assignment of values to the basic propositions

- let's work through a simple example involving $\neg(P \wedge \neg Q) \wedge \neg(P \wedge \neg Q)$

truth tables for multiple propositions

truth tables for multiple propositions

- we can also construct tables for multiple propositions
- we calculate the possible truth values for every possible assignment of values to basic propositions
- for any basic proposition which occurs in any formula

PP	QQ	$(P \rightarrow Q)$	$(P \rightarrow Q)$	$\neg(P \wedge \neg P) \wedge \neg(P \wedge \neg P)$
T	T	T	T	T
T	F	F	F	T
F	T	T	T	T
F	F	T	T	T

general points about drawing truth tables

general points about drawing truth tables

- one distinct wff variable
- column 1 has one T and one F

α	...
T	F
F	T

- two distinct wff variables
- the previous matrix occurs in the top right-hand corner of the matrix

α	β	...
T	T	...
T	F	...
F	T	...
F	F	...

- three distinct wff variables
- the previous matrix occurs in the top right-hand corner of the matrix

α	β	γ	...
T	T	T	...
T	T	F	...
T	F	T	...

T	F	F	...
F	T	T	...
F	T	F	...
F	F	T	...
F	F	F	...

- four distinct wff variables
- the previous matrix occurs in the top right-hand corner of the matrix

α \alpha	β \beta	γ \gamma	δ \delta	...
T	T	T	T	...
T	T	T	F	...
T	T	F	T	...
T	T	F	F	...
T	F	T	T	...
T	F	T	F	...
T	F	F	T	...
T	F	F	F	...
F	T	T	T	...
F	T	T	F	...
F	T	F	T	...
F	T	F	F	...
F	F	T	T	...
F	F	T	F	...
F	F	F	T	...
F	F	F	F	...

- in general ...

number of basic components	number of rows
1	2
2	4
3	8
4	16
5	32
6	64
...	...
n	2^n

- the right-most column always alternates T, F, and so on
- then TT, FF, and so on
- then TTTT, FFFF, and so on
- then TTTTTTTT, FFFFFFFF, and so on
- always doubling the Ts and Fs
- basic propositions are sorted alphabetically

- for the problem sets and exam . . .
 - make sure you set up your tables correctly
 - keep your rows and columns neat
 - show the truth values of all sub-formulas

using truth tables to test for

validity

using truth tables to test for validity

- recall the definition of validity . . .
- an argument is **valid** if and only if, in virtue of the form of the argument, it is impossible for its conclusion to be false if its premises are all true
- in PL, a possibility is an assignment of truth values to the basic propositions
- a joint truth table for a group of propositions shows all of the possible ways of making the propositions or true or false

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- the question we put to a truth table in order to determine whether an argument is valid is this:
 - is there a row on which all of the premises are true and the conclusion is false?
 - if so, the argument is invalid. if not, then it is valid.
-

- is this argument valid?

P1.	Either John is not a philosopher or he drinks
P2.	John is a philosopher
C1.	John drinks

- we begin by translating the argument into PL
- here's our glossary:
 - PP: John is a philosopher
 - DD: John drinks
- here's our translation:
 - $(\neg P \vee D), P \therefore D$
- now consider the joint truth table for the premises and the conclusion.

PP	DD	$(\neg P \vee D)$	$(\neg P \vee D)$	PP	DD
T	T		T	T	T
T	F		F	T	F
F	T		T	F	T
F	F		T	F	F

-
- is there a row on which the premises are true and the conclusion false?
 - if so, the argument is invalid. if not, then it is valid.
-

- is this argument valid?

P1.	Either John is a philosopher or he drinks
P2.	John drinks
C1.	John is a philosopher

-
- we begin by translating the argument into PL
 - here's our glossary:
 - PP: John is a philosopher
 - DD: John drinks
 - here's our translation:

- $(P \vee D), D \vdash P$ ($P \vee D$), $D \therefore P$
- now consider the joint truth table for the premises and the conclusion

PP	DD	$(P \vee D)$	$(P \vee D)$	DD	PP
T	T		T	T	T
T	F		T	F	T
F	T		T	T	F
F	F		F	F	F

- is there a row on which the premises are true and the conclusion false?
- if so, the argument is invalid. if not, then it is valid.

- if an argument is invalid, then it is possible for all the premises to be true and for the conclusion to be false.
- a **counterexample** is an assignment of truth values to basic propositions on which the premises are true and the conclusion is false.
- look back at the matrix. PP is true and DD is false on the relevant row.

PP	DD	$(\neg P \vee D)$	$(\neg P \vee D)$	DD	PP
T	T		T	T	T
T	F		T	F	T
F	T		T	T	F
F	F		F	F	F

- we write a counterexample as follows:
 - PP: F, DD: T

- what if there is a row where both premises are true and the conclusion is also true?

PP	DD	$(\neg P \vee D)$	$(\neg P \vee D)$	DD	PP
T	T		T	T	T
T	F		T	F	T
F	T		T	T	F
F	F		F	F	F

- this does not show that the argument is valid. (recall the definition of validity)

using truth tables to test single propositions

using truth tables to test single propositions

- we can use truth tables to determine whether a single proposition is:
 - a tautology
 - a contradiction
 - a non-contradiction
 - satisfiable

- intuitively, a proposition is a tautology if it *must* be true
 - e.g. the ball is round or the ball is not round
- a proposition is a **tautology** if and only if it is true on every row of its truth table

PP	$(P \vee \neg P)$	$(P \wedge \neg P)$
T	T	F
F	T	F

- we can use a truth table to determine whether a proposition is a tautology: we check whether it is true on every row of its truth table

- intuitively, a proposition is a contradiction if it *cannot* be true
 - e.g. the ball is round and the ball is not round
- a proposition is a **contradiction** if and only if it is false on every row of its truth table

PP	$(P \wedge \neg P)$	$(P \vee \neg P)$
T	F	T
F	F	T

- we can use a truth table to determine whether a proposition is a contradiction: we check whether it is false on every row of its truth table

- intuitively, a proposition is satisfiable if it *can* be true
- a proposition is **satisfiable** if and only if it is true in some or all rows of its truth table

PP	$(P \rightarrow \neg P)$	$(P \leftrightarrow \neg P)$
T	F	F
F	T	T

- we can use a truth table to determine whether a proposition is satisfiable: we check whether it is true on some row of its truth table

- contradictions and satisfiable propositions are opposites
 - if a proposition is not a contradiction, then it is satisfiable
 - if a proposition is not satisfiable, then it is a contradiction
- contradictions and tautologies are not opposites
 - a proposition may be neither a contradiction nor a tautology
- satisfiable propositions and tautologies are not opposites
 - a tautology is a satisfiable proposition

values in truth table	type of proposition
T in every row	tautology
F in every row	contradiction
T in some or all rows	satisfiable proposition
F in some or all rows	nontautology

using truth tables to test pairs of

propositions

using truth tables to test pairs of propositions

- we can use truth tables to determine whether a pair of propositions are:
 - equivalent or not equivalent
 - jointly satisfiable or not jointly satisfiable
 - contrary or contradictory

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- intuitively, a pair of propositions are equivalent if it is not possible for one proposition to be true without the other being true and vice versa
 - a pair of propositions are **equivalent** if and only if they have the same value in every row of the truth table

PP	QQ	$(P \rightarrow Q)$	$(\neg P \vee Q)$
T	T	T	T
T	F	F	F
F	T	T	T
F	F	T	T

- we can use a truth table to determine whether a pair of propositions are equivalent: we check whether they have the same value on every row of the truth table

-
- intuitively, a pair of propositions are jointly satisfiable if they can both be true together
 - a pair of propositions are **jointly satisfiable** if and only if there is some row of the truth table in which they are both true

PP	QQ	$\neg(P \rightarrow Q)$	$(P \vee Q)$
T	T	F	T
T	F	T	T
F	T	F	T
F	F	F	F

- we can use a truth table to determine whether a pair of propositions are jointly satisfiable: we check whether there is some row of the truth table on which they are both true

-
- intuitively, a pair of propositions are jointly unsatisfiable if they cannot both be true together
 - a pair of propositions are **jointly unsatisfiable** if and only if there is some row of the truth table in which they are both true

PP	QQ	$\neg(P \vee Q)$	$(P \wedge Q)$
T	T	F	T
T	F	F	F
F	T	F	F
F	F	T	F

- we can use a truth table to determine whether a pair of propositions are jointly unsatisfiable: we check whether they have

the same value on every row of the truth table

- note: as the example above shows, two satisfiable propositions may be jointly unsatisfiable

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- when a pair of propositions are jointly unsatisfiable they can be classified either as contradictory or contraries
 - let's consider the two cases now
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- intuitively, a pair of propositions are contradictory when if one is true the other is false and vice versa
- a pair of propositions are **contradictory** if and only if they are jointly unsatisfiable and there is no row of the truth table in which they are both false

PP	QQ	$(P \wedge Q)$	$(P \wedge \neg Q)$	$\neg(P \wedge Q)$
T	T	T	F	F
T	F	F	T	T
F	T	F	T	T
F	F	F	T	T

- we can use a truth table to determine whether a pair of propositions are contradictory: we check whether their truth values are opposed on every row of the truth table

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- if a pair of propositions are contradictory, we can always infer from the falsity of one proposition to the truth of the other
 - but this will not always be the case when a pair of propositions are jointly unsatisfiable
 - why?
 - they might both be false
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- intuitively, a pair of propositions are (merely) contraries if they are jointly unsatisfiable, but aren't contradictory: they can't be true together, but they can be false together
- a pair of propositions are **contraries** if and only if they are jointly unsatisfiable and there is some row in which both are false

PP	QQ	$(P \wedge \neg P)$	$(P \wedge \neg \neg P)$	$\neg(P \wedge \neg P)$
T	T	F	F	F
T	F	F	T	T
F	T	F	T	T
F	F	F	T	T

- we can use a truth table to determine whether a pair of propositions are contraries: we check whether there is no row of the truth table on which they are both true and some row on which they are both false

values in truth table	type of propositions
same value in every row	equivalent
some row in which both T	jointly satisfiable
no row in which both T	jointly unsatisfiable
j-unsat and no row in which both F	contradictory
j-unsat and some row in which both F	contraries

using truth tables to test sets of propositions

using truth tables to test sets of propositions

- equivalence and joint satisfiability can be generalised from pairs of propositions to sets of propositions in obvious ways
- I will leave equivalence as an exercise for you
- but let us look at the satisfiability and unsatisfiability of sets of propositions

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- intuitively, a set of propositions is satisfiable if they can all be true together
 - a set of propositions is **satisfiable** if and only if there is some row in the truth table on which all the members of the set are true
 - intuitively, a set of propositions is unsatisfiable if they cannot all be true together
 - set of propositions is **unsatisfiable** if and only if there is no row in the truth table on which all the members of the set are true

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- we can use truth tables to test for the satisfiability of sets of propositions
 - suppose we want to know whether $\{(P \rightarrow Q), (P \wedge Q) (P \wedge Q), (P \vee Q) (P \vee Q)\}$ is satisfiable
 - we draw up a joint truth table as follows:

PP	QQ	$(P \rightarrow Q)$	$(P \wedge Q)$	$(P \vee Q)$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	F

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- now, the notion of the satisfiability of a set of propositions will play a central role in the course going forward
 - notice that the truth table not only tells us that the set of propositions are satisfiable, it also provides assignments on which the set of propositions are satisfiable
 - as noted earlier, truth tables get unwieldy very quickly: what if we have 5 basic propositions?
 - might there be a more efficient method for testing the satisfiability of sets of propositions?
 - might there be a more efficient search procedure for assignments on which the propositions are satisfiable?
 - yes: truth trees

values in truth table	set of propositions
some row on which all T	satisfiable
no row on which all T	unsatisfiable

wrapping up

this lecture

- truth tables for complex propositions and multiple propositions
- using truth tables to test for various logical properties
- single propositions
 - tautologies
 - contradictions
 - satisfiable propositions
 - noncontradictions
- pairs of propositions
 - equivalence
 - joint satisfiability
 - joint unsatisfiability
 - contradictories
 - contraries
- sets of propositions
 - joint satisfiability
 - joint unsatisfiability

next lecture

- lecture 07, validity and logical form