lecture 02, connectives and logical form

phil1012 introductory logic

overview

this lecture

- in the previous lecture, we introduced the idea of logical form, the logical form of a proposition
- in this lecture we take a closer look at the logical forms of propositions
- we look at the logical form of propositions constructed using the truth functional connectives negation, conjunction, disjunction, conditional, and biconditional

learning outcomes

- after doing the relevant reading for this lecture, listening to the lecture, and attending the relevant tutorial, you will be able to:
 - identify different sorts of compound propositions and identify their components
 - o negations, and the negand of a negation
 - conjunctions, and the conjuncts of a conjunction
 - disjunctions, and the disjuncts of a disjunction
 - conditionals, and the antecedent and consequent of a conditional
 - biconditionals, and the right-hand-side and the left-handside of a conditional
 - o explain the difference between basic and compound propositions
 - explain what makes a connective a truth-functional connective

required reading

• section 1.6 of chapter 1

truth functional connectives and compound propositions

basic and compound propositions

- \bullet some propositions are made up of other propositions
 - Jane lives in Australia and Jack lives in China
 - if Jane lives in Australia, then Jack lives in China
- propositions made up of other propositions are called compound propositions
- if a proposition is not made up of other propositions, it is a **basic proposition**
 - Jane lives in Australia

- the observation that some propositions are made up of other propositions is the basic insight which motivates propositional logic
- it is a basic insight about the form of propositions

connectives and compound propositions

- compound propositions are made up of other propositions and bits that connect these other propositions together to make a compound proposition
- these 'bits' are called connectives
- \bullet a connective 'connects' propositions to make propositions
 - Jane lives in Australia and Jack lives in China
 - Jane lives in Australia or Jack lives in China
 - if Jane lives in Australia then Jack lives in China
 - Jane does not live in Australia
- in each of these examples we can see how a larger proposition is built out of other propositions by way of the connectives 'and', 'or', 'if, then', and 'not'
- 'not' may seem odd. 'not' doesn't 'connect' propositions. still, it takes one proposition and makes another. so we call it a connective.

truth-functional connectives

- some connectives are special in the following sense: when they are used to make a compound proposition from other propositions, we can be sure that the **truth** of the compound expression is determined by the truth or falsity of the propositions it is made out of
- a connective is **truth-functional connective** if and only if the truth or falsity of a compound proposition formed from the connective and some other propositions is completely determined by the truth and falsity of those component propositions

truth-functional connectives and propositional logic

- \bullet the truth-functional connectives are at the heart of ${\bf propositional}$ ${\bf logic}$
- we might say that propositional logic is the systematic study of the laws of truth with respect to the form propositions have as a consequence of their being built using truth-functional connectives
- since the propositions studied in other areas of logic are built using truth-functional connectives, propositional logic forms the basis of the logics studied in these other areas
- ullet the formal language **PL** is designed to assist us in the systematic study of these laws of truth

logical connectives in English

five central logical connectives

- the rest of this lecture introduces the **five** connectives which will be central to our study of propositional logic
- these are:

- conjunction
- disjunction
- conditional
- biconditional
- negation

conjunction

- conjunction
 - e.g. John is short and Jane is tall
- the compound proposition expressed by this sentence is made up of the proposition that John is short and the proposition that Jane is tall and the connective **conjunction**
- the compound proposition expressed by 'John is short and Jane is tall' is a **conjunction**
- the propositions expressed by 'John is short' and 'Jane is tall' are conjuncts of the conjunction
- conjunction is a two-place connective, because it connects two propositions
- conjunction is a truth-functional connective
- if both the conjuncts are true, the conjunction is true. if either is false, then the conjunction is false
 - o consider: John is short and Jane is tall

disjunction

- disjunction
 - e.g. John is short or Jane is tall
- the compound proposition expressed by this sentence is made up of the proposition that John is short and the proposition that Jane is tall and the connective **disjunction**
- the compound proposition expressed by 'John is short or Jane is tall' is a disjunction.
- the propositions expressed by 'John is short' and 'Jane is tall' are disjuncts of the disjunction.
- disjunction is a two-place connective, because it connects two propositions.
- disjunction is a truth-functional connective
- if either disjunct is true or both disjuncts are true, the disjunction is true. If both disjuncts are false, the disjunction is false
 - consider: John is short or Jane is tall

conditional

- conditional
 - e.g. if John is short, then Jane is tall
- the compound proposition expressed by this sentence is made up of the proposition that John is short and the proposition that Jane is tall and the connective **conditional**.

- the compound proposition expressed by 'If John is short, then Jane is tall' is a conditional
- the proposition expressed by 'John is short' is the **antecedent** of the conditional and the proposition expressed by 'Jane is tall' is the **consequent** of the conditional
- conditional is a two-place connective, because it connects two propositions
- the connective conditional is a truth-functional connective
- if the antecedent is true and the consequent is false then the conditional is false. otherwise it is true.
 - o consider: if John is short, then Jane is tall

biconditional

- biconditional
 - e.g. John is short if and only if Jane is tall
- the compound proposition expressed by this sentence is made up of the proposition that John is short and the proposition that Jane is tall and the connective **biconditional**
- the compound proposition expressed by 'John is short if and only if Jane is tall' is a **biconditional**
- the propositions expressed by 'John is short' is the **left-hand side** of the biconditional and the proposition expressed by 'Jane is tall' is the **right-hand side** of the biconditional
- biconditional is a two-place connective, because it connects two propositions
- biconditional is a truth-functional connective
- if the right-hand side and the left-hand side are either both true or both false, then the conditional is true. it is false otherwise.
 - consider: John is short if and only if Jane is tall

negation

- negation
 - John is not short
- the compound proposition expressed by this sentence is made up of the proposition that John is short and the connective **negation**
- the compound proposition expressed by 'John is not short' is the negation of the proposition expressed by 'John is short'
- the proposition expressed by 'John is short' is the **negand** of the proposition expressed by 'John is not short'
- the proposition expressed by 'It is not the case that John is not short' is the **double negation** of the proposition expressed by 'John is short'
- negation is a one-place connective, because it connects one proposition
- the connective negation is a truth-functional connective
- if the negand is true, the negation is false, and if the negand is false, the negation is true

wrapping up

this lecture

- basic and compound propositions
- truth functional connectives in English
 - conjunction
 - disjunction
 - conditional
 - biconditional
 - negation

next lecture

ullet lecture 03, the formal language PL