lecture 06, uses of truth tables

phil1012 introductory logic

overview

this lecture

- the uses of truth tables
- constructing truth tables
- using truth tables to test for various logical properties

learning outcomes

- after doing the relevant reading for this lecture, listening to the lecture, and attending the relevant tutorial, you will be able to:
  - construct truth tables for arguments, single propositions, pairs of propositions and sets of propositions
  - use truth tables to determine whether an argument is valid, providing a counterexample if not
  - use truth tables to determine whether an proposition is a tautology, a contradiction, or neither
  - use truth tables to determine whether pairs of formulas are jointly satisfiable, equivalent, contradictory, or contraries.
  - use truth tables to determine whether sets of propositions are jointly satisfiable

required reading

- all of chapter 4

truth tables for complex propositions

truth tables for complex propositions

- sometimes we want to know the truth value of a complex proposition for every possible assignment of truth values to basic propositions.
- or this purpose we construct a **truth table** for the complex proposition.
- each row of a truth table corresponds to a possible assignment of truth values to basic propositions

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- we set up our table so that the rows in the columns on the left correspond to the possible assignments of truth values to basic propositions
- and, we “copy over” those values to the basic propositions on the right for each row
- we proceed as in the case of a single assignment, calculating the truth value of the more complex propositions in terms of the truth value of their constituents

- until we have the values of the complex proposition for each possible assignment of values to the basic propositions

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- let’s work through a simple example involving ¬(P∧¬Q)\lnot(P \land \lnot Q)

truth tables for multiple propositions

truth tables for multiple propositions

- we can also construct tables for multiple propositions
- we calculate the possible truth values for every possible assignment of values to basic propositions
- for any basic proposition which occurs in any formula

PP	QQ	(P→Q)(P \rightarrow Q)	¬(P∧¬P)\lnot(P \land \lnot P)
T	T	T	T
T	F	F	T
F	T	T	T
F	F	T	T

general points about drawing truth tables

general points about drawing truth tables

- one distinct wff variable
- column 1 has one T and one F

α\alpha	…
T	F
F	T

- two distinct wff variables
- the previous matrix occurs in the top right-hand corner of the matrix

α\alpha	β\beta	…
T	T	…
T	F	…
F	T	…
F	F	…

- three distinct wff variables
- the previous matrix occurs in the top right-hand corner of the matrix

α\alpha	β\beta	γ\gamma	…
T	T	T	…
T	T	F	…
T	F	T	…

T	F	F	…
F	T	T	…
F	T	F	…
F	F	T	…
F	F	F	…

- four distinct wff variables
- the previous matrix occurs in the top right-hand corner of the matrix

α\alpha	β\beta	γ\gamma	δ\delta	…
T	T	T	T	…
T	T	T	F	…
T	T	F	T	…
T	T	F	F	…
T	F	T	T	…
T	F	T	F	…
T	F	F	T	…
T	F	F	F	…
F	T	T	T	…
F	T	T	F	…
F	T	F	T	…
F	T	F	F	…
F	F	T	T	…
F	F	T	F	…
F	F	F	T	…
F	F	F	F	…

- in general …

number of basic components	**number of rows**
1	2
2	4
3	8
4	16
5	32
6	64
…	…
n	2^n

- the right-most column always alternates T, F, and so on
- then TT, FF, and so on
- then TTTT, FFFF, and so on
- then TTTTTTTT, FFFFFFFF, and so on
- always doubling the Ts and Fs
- basic propositions are sorted alphabetically

- for the problem sets and exam . . .
  - make sure you set up your tables correctly
  - keep your rows and columns neat
  - show the truth values of all sub-formulas

using truth tables to test for

validity

using truth tables to test for validity

- recall the definition of validity . . .
- an argument is **valid** if and only if, in virtue of the form of the argument, it is impossible for its conclusion to be false if its premises are all true
- in PL, a possibility is an assignment of truth values to the basic propositions
- a joint truth table for a group of propositions shows all of the possible ways of making the propositions or true or false

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- the question we put to a truth table in order to determine whether an argument is valid it this:
  - is there a row on which all of the premises are true and the conclusion is false?
- if so, the argument is invalid. if not, then it is valid.

- is this argument valid?

P1.	Either John is not a philosopher or he drinks
P2.	John is a philosopher
C1.	John drinks

- we begin by translating the argument into PL
- here’s our glossary:
  - PP: John is a philosopher
  - DD: John drinks
- here’s our translation:
  - (¬P∨D),P∴D(\lnot P \lor D), P \therefore D
- now consider the joint truth table for the premises and the conclusion.

PP	DD	(¬P∨D)(\lnot P \lor D)	PP	DD
T	T	T	T	T
T	F	F	T	F
F	T	T	F	T
F	F	T	F	F

- is there a row on which the premises are true and the conclusion false?
- if so, the argument is invalid. if not, then it is valid.

- is this argument valid?

P1.	Either John is a philosopher or he drinks
P2.	John drinks
C1.	John is a philosopher

- we begin by translating the argument into PL
- here’s our glossary:
  - PP: John is a philosopher
  - DD: John drinks
- here’s our translation:

    - (P∨D),D∴P(P \lor D), D \therefore P
- now consider the joint truth table for the premises and the conclusion

PP	DD	(P∨D)(P \lor D)	DD	PP
T	T	T	T	T
T	F	T	F	T
F	T	T	T	F
F	F	F	F	F

- is there a row on which the premises are true and the conclusion false?
- if so, the argument is invalid. if not, then it is valid.

- if an argument is invalid, then it is possible for all the premises to be true and for the conclusion to be false.
- a **counterexample** is an assignment of truth values to basic propositions on which the premises are true and the conclusion is false.
- look back at the matrix. PP is true and DD is false on the relevant row.

PP	DD	(¬P∨D)(\lnot P \lor D)	DD	PP
T	T	T	T	T
T	F	T	F	T
F	**T**	**T**	**T**	**F**
F	F	F	F	F

- we write a counterexample as follows:
    - PP: F, DD: T

- what if there is a row where both premises are true and the conclusion is also true?

PP	DD	(¬P∨D)(\lnot P \lor D)	DD	PP
T	**T**	**T**	**T**	**T**
T	F	T	F	T
F	T	T	T	F
F	F	F	F	F

- this does not show that the argument is valid. (recall the definition of validity)

using truth tables to test single propositions

using truth tables to test single propositions

- we can use truth tables to determine whether a single propositions is:
    - a tautology
    - a contradiction
    - a non-contradiction
    - satisfiable

- intuitively, a proposition is a tautology if it *must* be true
    - e.g. the ball is round or the ball is not round
- a proposition is a **tautology** if and only if it is true on every row of its truth table

PP	(P∨¬P)(P \lor \lnot P)
T	T
F	T

- we can use a truth table to determine whether a proposition is a tautology: we check whether it is true on every row of its truth table

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- intuitively, a proposition is a contradiction if it *cannot* be true
    - e.g. the ball is round and the ball is not round
- a proposition is a **contradiction** if and only if it is false on every row of its truth table

PP	(P∧¬P)(P \land \lnot P)
T	F
F	F

- we can use a truth table to determine whether a proposition is a contradiction: we check whether it is false on every row of its truth table

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- intuitively, a proposition is satisfiable if it *can* be true
- a proposition is **satisfiable** if and only if it is true in some or all rows of its truth table

PP	(P→¬P)(P \rightarrow \lnot P)
T	F
F	T

- we can use a truth table to determine whether a proposition is satisfiable: we check whether it is true on some row of its truth table

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- contradictions and satisfiable propositions are opposites
    - if a proposition is not a contradiction, then it is satisfiable
    - if a proposition is not satisfiable, then it is a contradiction
- contradictions and tautologies are not opposites
    - a proposition may be neither a contradiction nor a tautology
- satisfiable propositions and tautologies are not opposites
    - a tautology is a satisfifiable proposition

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values in truth table	**type of proposition**
T in every row	**tautology**
F in every row	**contradiction**
T in some or all rows	**satisfiable proposition**
F in some or all rows	**nontautology**

using truth tables to test pairs of

propositions

using truth tables to test pairs of propositions

- we can use truth tables to determine whether a pair of propositions are:
    - equivalent or not equivalent
    - jointly satisfiable or not jointly satisfiable
    - contrary or contradictory

- intuitively, a pair of propositions are equivalent if it is not possible for one proposition to be true without the other being true and vice versa
- a pair of propositions are **equivalent** if and only if they have the same value in every row of the truth table

PP	QQ	(P→Q)(P \rightarrow Q)	(¬P∨Q)(\lnot P \lor Q)
T	T	T	T
T	F	F	F
F	T	T	T
F	F	T	T

- we can use a truth table to determine whether a pair of propositions are equivalent: we check whether they have the same value on every row of the truth table

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- intuitively, a pair of propositions are jointly satisfiable if they can both be true together
- a pair of propositions are **jointly satisfiable** if and only if there is some row of the truth table in which they are both true

PP	QQ	¬(P→Q)\lnot(P \rightarrow Q)	(P∨Q)(P \lor Q)
T	T	F	T
T	F	T	T
F	T	F	T
F	F	F	F

- we can use a truth table to determine whether a pair of propositions are jointly satisfiable: we check whether there is some row of the truth table on which they are both true

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- intuitively, a pair of propositions are jointly unsatisfiable if they cannot both be true together
- a pair of propositions are **jointly unsatisfiable** if and only if there is some row of the truth table in which they are both true

PP	QQ	¬(P∨Q)\lnot(P \lor Q)	(P∨Q)(P \lor Q)
T	T	F	T
T	F	F	T
F	T	F	T
F	F	T	F

- we can use a truth table to determine whether a pair of propositions are jointly unsatisfiable: we check whether they have

the same value on every row of the truth table
- note: as the example above shows, two satisfiable propositions may be jointly unsatisfiable

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- when a pair or propositions are jointly unsatisfiable they can be classified either as contradictories or contraries
- let’s consider the two cases now

- intuitively, a pair of propositions are contradictory when if one is true the other is false and vice versa
- a pair of propositions are **contradictory** if and only if they are jointly unsatisfiable and there is no row of the truth table in which they are both false

PP	QQ	(P∧Q)(P \land Q)	¬(P∧Q)\lnot(P \land Q)
T	T	T	F
T	F	F	T
F	T	F	T
F	F	F	T

- we can use a truth table to determine whether a pair of propositions are contradictory: we check whether their truth values are opposed on every row of the truth table

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- if a pair of propositions are contradictory, we can always infer from the falsity of one proposition to the truth of the other
- but this will not always be the case when a pair of propositions are jointly unsatisfiable
- why?
- they might both be false

- intuitively, a pair of propositions are (merely) contraries if they are jointly unsatisfiable, but aren’t contradictory: they can’t be true together, but they can be false together
- a pair of propositions are **contraries** if and only if they are jointly unsatisfiable and there is some row in which both are false

PP	QQ	(P∧¬P)(P \land \lnot P)	¬(P∧Q)\lnot(P \land Q)
T	T	F	F
T	F	F	T
F	T	F	T
F	F	F	T

- we can use a truth table to determine whether a pair of propositions are contraries: we check whether there is no row of the truth table on which they are both true and some row on which they are both false

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values in truth table	**type of propositions**
same value in every row	**equivalent**
some row in which both T	**jointly satisfiable**
no row in which both T	**jointly unsatisfiable**
j-unsat and no row in which both F	**contradictory**
j-unsat and some row in which both F	**contraries**

using truth tables to test sets of propositions

using truth tables to test sets of propositions

- equivalence and joint satisfiability can be generalised from pairs of propositions to sets of propositions in obvious ways
- I will leave equivalence as an exercise for you
- but let us look at the satisfiability and unsatisfiability of sets of propositions

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- intuitively, a set of propositions is satisfiable if they can all be true together
- a set of propositions is **satisfiable** if and only if there is some row in the truth table on which all the members of the set are true
- intuitively, a set of propositions is unsatisfiable if they cannot all be true together
- set of propositions is **unsatisfiable** if and only if there is no row in the truth table on which all the members of the set are true

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- we can use truth tables to test for the satisfiability of sets of propositions
- suppose we want to know whether {(P→Q)(P \rightarrow Q), (P∧Q)(P \land Q), (P∨Q)(P \lor Q)} is satisfiable
- we draw up a joint truth table as follows:

PP	QQ	(P→Q)(P \rightarrow Q)	(P∧Q)(P \land Q)	(P∨Q)(P \lor Q)
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	F

- now, the notion of the satisfiability of a set of propositions will play a central role in the course going forward
- notice that the truth table not only tells us that the set of propositions are satisfiable, it also provides assignments on which the set of propositions are satisfiable
- as noted earlier, truth tables get unwieldy very quickly: what if we have 5 basic propositions?
- might there be a more efficient method for testing the satisfiability of sets of propositions?
- might there be a more efficient search procedure for assignments on which the propositions are satisfiable?
- yes: truth trees

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values in truth table	**set of propositions**
some row on which all T	**satisfiable**
no row on which all T	**unsatisfiable**

wrapping up

this lecture

- truth tables for complex propositions and multiple propositions
- using truth tables to test for various logical properties
- single propositions
    - tautologies
    - contradictions
    - satisfiable propositions
    - noncontradictions
- pairs of propositions
    - equivalence
    - joint satisfiability
    - joint unsatisfiability
    - contradictories
    - contraries
- sets of propositions
    - joint satisfiability
    - joint unsatisfiability

next lecture

- lecture 07, validity and logical form