lecture 02, connectives and logical form

phil1012 introductory logic

overview

this lecture

- in the previous lecture, we introduced the idea of logical form, the logical form of a proposition
- in this lecture we take a closer look at the logical forms of propositions
- we look at the logical form of propositions constructed using the **truth functional connectives** negation, conjunction, disjunction, conditional, and biconditional

learning outcomes

- after doing the relevant reading for this lecture, listening to the lecture, and attending the relevant tutorial, you will be able to:
  - identify different sorts of compound propositions and identify their components
    - negations, and the negand of a negation
    - conjunctions, and the conjuncts of a conjunction
    - disjunctions, and the disjuncts of a disjunction
    - conditionals, and the antecedent and consequent of a conditional
    - biconditionals, and the right-hand-side and the left-hand-side of a conditional
  - explain the difference between basic and compound propositions
  - explain what makes a connective a truth-functional connective

required reading

- section 1.6 of chapter 1

truth functional connectives and compound propositions

basic and compound propositions

- some propositions are made up of other propositions
  - Jane lives in Australia and Jack lives in China
  - if Jane lives in Australia, then Jack lives in China
- propositions made up of other propositions are called **compound propositions**
- if a proposition is not made up of other propositions, it is a **basic proposition**
  - Jane lives in Australia

- the observation that some propositions are made up of other propositions is the basic insight which motivates propositional logic
- it is a basic insight about the form of propositions

connectives and compound propositions

- compound propositions are made up of other propositions and bits that connect these other propositions together to make a compound proposition
- these 'bits' are called connectives
- a **connective** 'connects' propositions to make propositions
    - Jane lives in Australia **and** Jack lives in China
    - Jane lives in Australia **or** Jack lives in China
    - **if** Jane lives in Australia **then** Jack lives in China
    - Jane does **not** live in Australia

- in each of these examples we can see how a larger proposition is built out of other propositions by way of the connectives 'and', 'or', 'if, then', and 'not'
- 'not' may seem odd. 'not' doesn't 'connect' propositions. still, it takes one proposition and makes another. so we call it a connective.

truth-functional connectives

- some connectives are special in the following sense: when they are used to make a compound proposition from other propositions, we can be sure that the **truth** of the compound expression is determined by the truth or falsity of the propositions it is made out of

- a connective is **truth-functional connective** if and only if the truth or falsity of a compound proposition formed from the connective and some other propositions is completely determined by the truth and falsity of those component propositions

truth-functional connectives and propositional logic

- the truth-functional connectives are at the heart of **propositional logic**
- we might say that propositional logic is the systematic study of the laws of truth with respect to the form propositions have as a consequence of their being built using truth-functional connectives
- since the propositions studied in other areas of logic are built using truth-functional connectives, propositional logic forms the basis of the logics studied in these other areas
- the formal language **PL** is designed to assist us in the systematic study of these laws of truth

logical connectives in English

five central logical connectives

- the rest of this lecture introduces the **five** connectives which will be central to our study of propositional logic
- these are:
  - conjunction
  - disjunction
  - conditional
  - biconditional
  - negation

conjunction

- **conjunction**
  - e.g. John is short and Jane is tall
- the compound proposition expressed by this sentence is made up of the proposition that John is short and the proposition that Jane is tall and the connective **conjunction**

- the compound proposition expressed by 'John is short and Jane is tall' is a **conjunction**
- the propositions expressed by 'John is short' and 'Jane is tall' are **conjuncts** of the conjunction
- conjunction is a **two-place connective**, because it connects two propositions

---

- conjunction is a **truth-functional connective**
- if both the conjuncts are true, the conjunction is true. if either is false, then the conjunction is false
  - consider: John is short and Jane is tall

disjunction

- **disjunction**
  - e.g. John is short or Jane is tall
- the compound proposition expressed by this sentence is made up of the proposition that John is short and the proposition that Jane is tall and the connective **disjunction**

- the compound proposition expressed by 'John is short or Jane is tall' is a **disjunction**.
- the propositions expressed by 'John is short' and 'Jane is tall' are **disjuncts** of the disjunction.
- disjunction is a **two-place connective**, because it connects two propositions.

- disjunction is a **truth-functional connective**
- if either disjunct is true or both disjuncts are true, the disjunction is true. If both disjuncts are false, the disjunction is false
  - consider: John is short or Jane is tall

conditional

- **conditional**
  - e.g. if John is short, then Jane is tall
- the compound proposition expressed by this sentence is made up of the proposition that John is short and the proposition that Jane

is tall and the connective **conditional**.

- the compound proposition expressed by 'If John is short, then Jane is tall' is a **conditional**
- the proposition expressed by 'John is short' is the **antecedent** of the conditional and the proposition expressed by 'Jane is tall' is the **consequent** of the conditional
- conditional is a **two-place connective**, because it connects two propositions

---

- the connective conditional is a **truth-functional connective**
- if the antecedent is true and the consequent is false then the conditional is false. otherwise it is true.
  - consider: if John is short, then Jane is tall

---

biconditional

- **biconditional**
  - e.g. John is short if and only if Jane is tall
- the compound proposition expressed by this sentence is made up of the proposition that John is short and the proposition that Jane is tall and the connective **biconditional**

- the compound proposition expressed by 'John is short if and only if Jane is tall' is a **biconditional**
- the propositions expressed by 'John is short' is the **left-hand side** of the biconditional and the proposition expressed by 'Jane is tall' is the **right-hand side** of the biconditional
- biconditional is a **two-place connective**, because it connects two propositions

---

- biconditional is a **truth-functional connective**
- if the right-hand side and the left-hand side are either both true or both false, then the conditional is true. it is false otherwise.
  - consider: John is short if and only if Jane is tall

negation

- **negation**
  - John is not short
- the compound proposition expressed by this sentence is made up of the proposition that John is short and the connective **negation**

- the compound proposition expressed by 'John is not short' is the **negation** of the proposition expressed by 'John is short'
- the proposition expressed by 'John is short' is the **negand** of the proposition expressed by 'John is not short'
- the proposition expressed by 'It is not the case that John is not short' is the **double negation** of the proposition expressed by 'John is short'
- negation is a **one-place connective**, because it connects one proposition

---

- the connective negation is a **truth-functional connective**
- if the negand is true, the negation is false, and if the negand is false, the negation is true
    - consider: John is not short

wrapping up

this lecture

- basic and compound propositions
- truth functional connectives in English
    - conjunction
    - disjunction
    - conditional
    - biconditional
    - negation

next lecture

- lecture 03, the formal language PL