

# lecture 18, uses of trees for MPL

phil1012 introductory logic

## overview

### this lecture

- uses of trees for MPL
- how to set up trees to test for various logical properties in MPL
- how to read off models from completed trees
- identifying infinite trees and reading models off the open paths of infinite trees

### learning outcomes

- after doing the relevant reading for this lecture, listening to the lecture, and attending the relevant tutorial, you will be able to:
  - use trees to test for various logical properties of MPL formulas
  - read off (counter)models from open paths of MPL trees
  - identify infinite trees and read models of infinite paths

### required reading

- sections 10.2 and 10.3 of chapter 10

## using trees

### using trees

- MPL trees, like PL trees, test for satisfiability, in the first instance.
- but we can use them to test for much more
- again, you need to know how to set up the tree, and how to interpret its results
- in the case of MPL trees, we read off models on which the initial propositions are jointly satisfiable
- these are called **countermodels** in the case of arguments.

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- we set up and use MPL trees to test for various properties in much the same way as we did for PL trees.
  - let's look at an example of testing some proposition to see whether it is a tautology
- 

To prove: whether  $((\exists xFx \wedge \forall x(Fx \rightarrow Gx)) \rightarrow \exists xGx)$  is a tautology.

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To prove: whether  $((\exists xFx \wedge \forall x(Fx \rightarrow Gx)) \rightarrow \exists xGx)$  is a tautology.

1.  $\neg(\exists xFx \wedge \forall x(Fx \rightarrow Gx)) \rightarrow \exists xGx$       Assumption
-

To prove: whether  $((\exists xFx \wedge \forall x(Fx \rightarrow Gx)) \rightarrow \exists xGx)$  is a tautology.

- |    |   |                     |
|----|---|---------------------|
| 1. | $\neg(\exists xFx \wedge \forall x(Fx \rightarrow Gx)) \rightarrow \exists xGx$ ✓ | Assumption          |
| 2. | $(\exists xFx \wedge \forall x(Fx \rightarrow Gx))$                               | Negated Conditional |
| 3. | $\neg\exists xGx$   | Negated Conditional |

To prove: whether  $((\exists xFx \wedge \forall x(Fx \rightarrow Gx)) \rightarrow \exists xGx)$  is a tautology.

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|----|---|---------------------|
| 1. | $\neg(\exists xFx \wedge \forall x(Fx \rightarrow Gx)) \rightarrow \exists xGx$ ✓ | Assumption          |
| 2. | $(\exists xFx \wedge \forall x(Fx \rightarrow Gx))$ ✓                             | Negated Conditional |
| 3. | $\neg\exists xGx$   | Negated Conditional |
| 4. | $\exists xFx$   | Conjunction         |
| 5. | $\forall x(Fx \rightarrow Gx)$  | Conjunction         |

To prove: whether  $((\exists xFx \wedge \forall x(Fx \rightarrow Gx)) \rightarrow \exists xGx)$  is a tautology.

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| 1. | $\neg(\exists xFx \wedge \forall x(Fx \rightarrow Gx)) \rightarrow \exists xGx$ ✓ | Assumption          |
| 2. | $(\exists xFx \wedge \forall x(Fx \rightarrow Gx))$ ✓                             | Negated Conditional |
| 3. | $\neg\exists xGx$ ✓   | Negated Conditional |
| 4. | $\exists xFx$   | Conjunction         |
| 5. | $\forall x(Fx \rightarrow Gx)$  | Conjunction         |
| 6. | $\forall x\neg Gx$  | Negated Existential |

To prove: whether  $((\exists xFx \wedge \forall x(Fx \rightarrow Gx)) \rightarrow \exists xGx)$  is a tautology.

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|----|---|---------------------|
| 1. | $\neg(\exists xFx \wedge \forall x(Fx \rightarrow Gx)) \rightarrow \exists xGx$ ✓ | Assumption          |
| 2. | $(\exists xFx \wedge \forall x(Fx \rightarrow Gx))$ ✓                             | Negated Conditional |
| 3. | $\neg\exists xGx$ ✓   | Negated Conditional |
| 4. | $\exists xFx$ ✓ $a$   | Conjunction         |
| 5. | $\forall x(Fx \rightarrow Gx)$  | Conjunction         |
| 6. | $\forall x\neg Gx$  | Negated Existential |
| 7. | $Fa$  | Existential         |

To prove: whether  $((\exists xFx \wedge \forall x(Fx \rightarrow Gx)) \rightarrow \exists xGx)$  is a tautology.

- |    |   |                     |
|----|---|---------------------|
| 1. | $\neg(\exists xFx \wedge \forall x(Fx \rightarrow Gx)) \rightarrow \exists xGx$ ✓ | Assumption          |
| 2. | $(\exists xFx \wedge \forall x(Fx \rightarrow Gx))$ ✓                             | Negated Conditional |
| 3. | $\neg\exists xGx$ ✓   | Negated Conditional |
| 4. | $\exists xFx$ ✓ $a$   | Conjunction         |
| 5. | $\forall x(Fx \rightarrow Gx) \setminus a$  | Conjunction         |
| 6. | $\forall x\neg Gx$  | Negated Existential |
| 7. | $Fa$  | Existential         |
| 8. | $Fa \rightarrow Ga$   | Universal           |

To prove: whether  $((\exists xFx \wedge \forall x(Fx \rightarrow Gx)) \rightarrow \exists xGx)$  is a tautology.

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|----|---|---------------------|
| 1. | $\neg(\exists xFx \wedge \forall x(Fx \rightarrow Gx)) \rightarrow \exists xGx$ ✓     | Assumption          |
| 2. | $(\exists xFx \wedge \forall x(Fx \rightarrow Gx))$ ✓                                 | Negated Conditional |
| 3. | $\neg\exists xGx$ ✓   | Negated Conditional |
| 4. | $\exists xFx$ ✓ $a$   | Conjunction         |
| 5. | $\forall x(Fx \rightarrow Gx) \setminus a$  | Conjunction         |
| 6. | $\forall x\neg Gx$  | Negated Existential |
| 7. | $Fa$  | Existential         |
| 8. | $Fa \rightarrow Ga$ ✓   | Universal           |
| 9. | $\begin{array}{c} \neg Fa \quad Ga \\ \diagup \quad \diagdown \\ \otimes \end{array}$ | Conditional         |

To prove: whether  $((\exists xFx \wedge \forall x(Fx \rightarrow Gx)) \rightarrow \exists xGx)$  is a tautology.

1.	$\neg(\exists xFx \wedge \forall x(Fx \rightarrow Gx)) \rightarrow \exists xGx$	✓	Assumption
2.	$(\exists xFx \wedge \forall x(Fx \rightarrow Gx))$	✓	Negated Conditional
3.	$\neg\exists xGx$	✓	Negated Conditional
4.	$\exists xFx$	✓	Conjunction
5.	$\forall x(Fx \rightarrow Gx)$	$\backslash a$	Conjunction
6.	$\forall x\neg Gx$	$\backslash a$	Negated Existential
7.	$Fa$		Existential
8.	$Fa \rightarrow Ga$	✓	Universal
$\begin{array}{cc} \swarrow & \searrow \\ \neg Fa & Ga \end{array}$			
9.	$\otimes$		Conditional
10.	$\neg Ga$		Universal
	$\otimes$		

- we can conclude that  $((\exists xFx \wedge \forall x(Fx \rightarrow Gx)) \rightarrow \exists xGx)$  ( $(\exists x Fx \wedge \forall x (Fx \rightarrow Gx)) \rightarrow \exists x Gx$ ) is a tautology, since its negation is not satisfiable.

- to take another example, suppose we want to test whether some argument is a valid argument

To prove: whether  $\exists xFx, \exists xGx \therefore \exists x(Fx \wedge Gx)$  is a valid argument.

To prove: whether  $\exists xFx, \exists xGx \therefore \exists x(Fx \wedge Gx)$  is a valid argument.

1.	$\exists xFx$	Assumption
2.	$\exists xGx$	Assumption
3.	$\neg\exists x(Fx \wedge Gx)$	Negated Conclusion

To prove: whether  $\exists xFx, \exists xGx \therefore \exists x(Fx \wedge Gx)$  is a valid argument.

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To prove: whether  $\exists xFx, \exists xGx \therefore \exists x(Fx \wedge Gx)$  is a valid argument.

1.	$\exists xFx$	Assumption
2.	$\exists xGx$	Assumption
3.	$\neg\exists x(Fx \wedge Gx)$	✓ Negated Conclusion
4.	$\forall x\neg(Fx \wedge Gx)$	Negated Existential

To prove: whether  $\exists xFx, \exists xGx \therefore \exists x(Fx \wedge Gx)$  is a valid argument.

1.	$\exists xFx$	✓	Assumption
2.	$\exists xGx$		Assumption
3.	$\neg\exists x(Fx \wedge Gx)$	✓	Negated Conclusion
4.	$\forall x\neg(Fx \wedge Gx)$		Negated Existential
5.	$Fa$		Existential

To prove: whether  $\exists xFx, \exists xGx \therefore \exists x(Fx \wedge Gx)$  is a valid argument.

- |    |   |                     |
|----|---|---------------------|
| 1. | $\exists xFx \checkmark a$                | Assumption          |
| 2. | $\exists xGx \checkmark b$                | Assumption          |
| 3. | $\neg \exists x(Fx \wedge Gx) \checkmark$ | Negated Conclusion  |
| 4. | $\forall x \neg(Fx \wedge Gx)$            | Negated Existential |
| 5. | $Fa$                                      | Existential         |
| 6. | $Gb$                                      | Existential         |
- 

To prove: whether  $\exists xFx, \exists xGx \therefore \exists x(Fx \wedge Gx)$  is a valid argument.

- |    |  |                     |
|----|--|---------------------|
| 1. | $\exists xFx \checkmark a$                 | Assumption          |
| 2. | $\exists xGx \checkmark b$                 | Assumption          |
| 3. | $\neg \exists x(Fx \wedge Gx) \checkmark$  | Negated Conclusion  |
| 4. | $\forall x \neg(Fx \wedge Gx) \setminus a$ | Negated Existential |
| 5. | $Fa$                                       | Existential         |
| 6. | $Gb$                                       | Existential         |
| 7. | $\neg(Fa \wedge Ga)$                       | Universal           |
- 

To prove: whether  $\exists xFx, \exists xGx \therefore \exists x(Fx \wedge Gx)$  is a valid argument.

- |    |  |                     |
|----|--|---------------------|
| 1. | $\exists xFx \checkmark a$   | Assumption          |
| 2. | $\exists xGx \checkmark b$   | Assumption          |
| 3. | $\neg \exists x(Fx \wedge Gx) \checkmark$  | Negated Conclusion  |
| 4. | $\forall x \neg(Fx \wedge Gx) \setminus a$   | Negated Existential |
| 5. | $Fa$   | Existential         |
| 6. | $Gb$   | Existential         |
| 7. | $\neg(Fa \wedge Ga) \checkmark$  | Universal           |
| 8. | $\begin{array}{c} \swarrow \quad \searrow \\ \neg Fa \quad \neg Ga \\ \otimes \end{array}$ | Negated Conjunction |
- 

To prove: whether  $\exists xFx, \exists xGx \therefore \exists x(Fx \wedge Gx)$  is a valid argument.

- |    |   |                     |
|----|---|---------------------|
| 1. | $\exists xFx \checkmark a$  | Assumption          |
| 2. | $\exists xGx \checkmark b$  | Assumption          |
| 3. | $\neg \exists x(Fx \wedge Gx) \checkmark$                                       | Negated Conclusion  |
| 4. | $\forall x \neg(Fx \wedge Gx) \setminus a, b$                                   | Negated Existential |
| 5. | $Fa$  | Existential         |
| 6. | $Gb$  | Existential         |
| 7. | $\neg(Fa \wedge Ga) \checkmark$   | Universal           |
| 8. | $\begin{array}{c} \swarrow \quad \searrow \\ \neg Fa \quad \neg Ga \end{array}$ | Negated Conjunction |
| 9. | $\otimes \quad \neg(Fb \wedge Gb)$  | Universal           |
-

To prove: whether  $\exists xFx, \exists xGx \therefore \exists x(Fx \wedge Gx)$  is a valid argument.

1.	$\exists xFx \checkmark a$	Assumption
2.	$\exists xGx \checkmark b$	Assumption
3.	$\neg \exists x(Fx \wedge Gx) \checkmark$	Negated Conclusion
4.	$\forall x \neg (Fx \wedge Gx) \setminus a, b$	Negated Existential
5.	$Fa$	Existential
6.	$Gb$	Existential
7.	$\neg (Fa \wedge Ga) \checkmark$	Universal
$\swarrow \quad \searrow$		
8.	$\neg Fa \quad \neg Ga$	Negated Conjunction
9.	$\otimes \quad \neg (Fb \wedge Gb) \checkmark$	Universal
$\swarrow \quad \searrow$		
10.	$\neg Fb \quad \neg Gb$	Negated Conjunction
	$\otimes$	

- 
- we can conclude that  $\exists xFx, \exists xGx \therefore \exists x(Fx \wedge Gx)$  \exists exists xFx, \exists exists xGx \therefore \exists exists x(Fx \wedge Gx) is not a valid argument, since the premises,  $\exists xFx, \exists xGx$ , \exists exists xFx, \exists exists xGx, and the negated conclusion,  $\neg \exists x(Fx \wedge Gx)$  \not \exists exists x(Fx \wedge Gx) are jointly satisfiable.
  - we want to be able to read off a countermodel from our tree.
  - how?

## how to read off models from open paths

### how to read off models from open paths

- a model consists of:
    - a domain
    - a referent for each name which appears on the path
    - an extension for each predicate which appears on the path
- 
- where there are  $n$  names in the path, we write our domain as follows:
    - domain:  $\{1, \dots, n\}$
  - so if there are 3 names in the path, we write our domain as follows:
    - domain:  $\{1, 2, 3\}$
  - we then assign each name in the path to an object in the domain as follows:
    - referents: aa:1, bb:2, cc:3, ...
- 
- we then assign an extension to each predicate which makes atomic formulas involving the predicate true.
  - if FaFa, GaGa, and GbGb, are all on an open path, then we assign the following extensions to the predicates:
    - extensions: FF:  $\{1\}$ , GG:  $\{1, 2\}$ .
  - if the predicate HH is on the open path but does not occur in an atomic formula, then we assign the following extension to the predicate:
    - extensions: HH:  $\emptyset$  (Not HH:  $\{\emptyset\}$ ).
-

- let's consider an example ...

- suppose we want to read a model off of this tree:

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1.	$\exists x Fx \checkmark a$	Assumption
2.	$\exists x Gx \checkmark b$	Assumption
3.	$\neg \exists x (Fx \wedge Gx) \checkmark$	Negated Conclusion
4.	$\forall x \neg (Fx \wedge Gx) \setminus a, b$	Negated Existential
5.	$Fa$	Existential
6.	$Gb$	Existential
7.	$\neg (Fa \wedge Ga) \checkmark$	Universal
	$\swarrow \quad \searrow$	
8.	$\neg Fa \quad \neg Ga$	Negated Conjunction
9.	$\otimes \quad \neg (Fb \wedge Gb) \checkmark$	Universal
	$\swarrow \quad \searrow$	
10.	$\neg Fb \quad \neg Gb$	Negated Conjunction
	$\quad \quad \otimes$	

---

- first step: find the number of names on the open path

---

1.	$\exists x Fx \checkmark a$	Assumption
2.	$\exists x Gx \checkmark b$	Assumption
3.	$\neg \exists x (Fx \wedge Gx) \checkmark$	Negated Conclusion
4.	$\forall x \neg (Fx \wedge Gx) \setminus a, b$	Negated Existential
5.	$Fa$	Existential
6.	$Gb$	Existential
7.	$\neg (Fa \wedge Ga) \checkmark$	Universal
	$\swarrow \quad \searrow$	
8.	$\neg Fa \quad \neg Ga$	Negated Conjunction
9.	$\otimes \quad \neg (Fb \wedge Gb) \checkmark$	Universal
	$\swarrow \quad \searrow$	
10.	$\neg Fb \quad \neg Gb$	Negated Conjunction
	$\quad \quad \otimes$	

---

- there are two, so we have the following domain:
  - domain:  $\{1, 2\}$

- second step: assign each name in the path to an object in the domain
-

1.	$\exists x Fx \checkmark a$	Assumption
2.	$\exists x Gx \checkmark b$	Assumption
3.	$\neg \exists x (Fx \wedge Gx) \checkmark$	Negated Conclusion
4.	$\forall x \neg (Fx \wedge Gx) \setminus a, b$	Negated Existential
5.	$Fa$	Existential
6.	$Gb$	Existential
7.	$\neg (Fa \wedge Ga) \checkmark$	Universal
	$\swarrow \quad \searrow$	
8.	$\neg Fa \quad \neg Ga$	Negated Conjunction
9.	$\otimes \quad \neg (Fb \wedge Gb) \checkmark$	Universal
	$\swarrow \quad \searrow$	
10.	$\neg Fb \quad \neg Gb$	Negated Conjunction
	$\otimes$	

- we assign referents in the most natural manner:
  - referents: aa: 1, bb: 2

- third step: assign an extension to each predicate which makes atomic formulas involving the predicate true

1.	$\exists x Fx \checkmark a$	Assumption
2.	$\exists x Gx \checkmark b$	Assumption
3.	$\neg \exists x (Fx \wedge Gx) \checkmark$	Negated Conclusion
4.	$\forall x \neg (Fx \wedge Gx) \setminus a, b$	Negated Existential
5.	$\boxed{Fa}$	Existential
6.	$Gb$	Existential
7.	$\neg (Fa \wedge Ga) \checkmark$	Universal
	$\swarrow \quad \searrow$	
8.	$\neg Fa \quad \neg Ga$	Negated Conjunction
9.	$\otimes \quad \neg (Fb \wedge Gb) \checkmark$	Universal
	$\swarrow \quad \searrow$	
10.	$\neg Fb \quad \neg Gb$	Negated Conjunction
	$\otimes$	

- we assign an extension to FF which makes FaFa true:
  - extensions: FF: {1}

1.	$\exists x Fx \checkmark a$	Assumption
2.	$\exists x Gx \checkmark b$	Assumption
3.	$\neg \exists x (Fx \wedge Gx) \checkmark$	Negated Conclusion
4.	$\forall x \neg (Fx \wedge Gx) \setminus a, b$	Negated Existential
5.	$\boxed{Fa}$	Existential
6.	$\boxed{Gb}$	Existential
7.	$\neg (Fa \wedge Ga) \checkmark$	Universal
	$\swarrow \quad \searrow$	
8.	$\neg Fa \quad \neg Ga$	Negated Conjunction
9.	$\otimes \quad \neg (Fb \wedge Gb) \checkmark$	Universal
	$\swarrow \quad \searrow$	
10.	$\neg Fb \quad \neg Gb$	Negated Conjunction
	$\otimes$	

- we assign an extension to GG which makes GbGb true:
  - extensions: FF: {1}, GG: {2}

- 
- here is our completed model:
    - domain: {1, 2}
    - deferents: aa: 1, bb: 2
    - extensions: FF: {1}, GG: {2}

## oh no! infinite trees

### oh no! infinite trees

- unlike PL trees, MPL trees have an interesting feature: they can be infinitely long
- see pp. 372–373 of the textbook for how to avoid infinite trees in MPL
- let's consider an example of an infinite tree

---

To prove: whether  $\forall x(Fx \wedge \exists yGy)$  is satisfiable.

1.  $\forall x(Fx \wedge \exists yGy)$

---

To prove: whether  $\forall x(Fx \wedge \exists yGy)$  is satisfiable.

1.  $\forall x(Fx \wedge \exists yGy) \setminus a$
2.  $(Fa \wedge \exists yGy)$

---

To prove: whether  $\forall x(Fx \wedge \exists yGy)$  is satisfiable.

1.  $\forall x(Fx \wedge \exists yGy) \setminus a$
2.  $(Fa \wedge \exists yGy) \checkmark$
3.  $\exists yGy$
4.  $Fa$

---

To prove: whether  $\forall x(Fx \wedge \exists yGy)$  is satisfiable.

1.  $\forall x(Fx \wedge \exists yGy) \setminus a$
2.  $(Fa \wedge \exists yGy) \checkmark$
3.  $\exists yGy \checkmark b$
4.  $Fa$
5.  $Gb$

---

To prove: whether  $\forall x(Fx \wedge \exists yGy)$  is satisfiable.

1.  $\forall x(Fx \wedge \exists yGy) \setminus a, b$
  2.  $(Fa \wedge \exists yGy) \checkmark$
  3.  $\exists yGy \checkmark b$
  4.  $Fa$
  5.  $Gb$
  6.  $(Fb \wedge \exists yGy) \checkmark$
  7.  $\exists yGy \checkmark c$
  8.  $Fb$
  9.  $Gc$
-



To prove: whether  $\forall x(Fx \wedge \exists yGy)$  is satisfiable.

1.  $\forall x(Fx \wedge \exists yGy) \setminus a, b, c$
2.  $(Fa \wedge \exists yGy) \checkmark$
3.  $\exists yGy \checkmark b$
4.  $Fa$
5.  $Gb$
6.  $(Fb \wedge \exists yGy) \checkmark$
7.  $\exists yGy \checkmark c$
8.  $Fb$
9.  $Gc$
10.  $(Fc \wedge \exists yGy) \checkmark$
11.  $\exists yGy \checkmark d$
12.  $Fc$
13.  $Gd$

- 
- this tree is going to go on like this forever
  - it will never be complete
  - it has an infinite path
- 

- is the proposition satisfiable or not?
  - is the path saturated or not?
  - it is saturated
  - so it is satisfiable
  - we can read off a model
- 

- will the proposition always be satisfiable if we have an infinite tree?
  - will infinite paths always be saturated?
  - no and no.
  - here is an example
- 

To prove: whether  $\forall x(Fx \wedge \exists yGy) \wedge (Ga \wedge \neg Ga)$  is satisfiable.

1.  $\forall x(Fx \wedge \exists yGy) \wedge (Ga \wedge \neg Ga)$

To prove: whether  $\forall x(Fx \wedge \exists yGy) \wedge (Ga \wedge \neg Ga)$  is satisfiable.

1.  $\forall x(Fx \wedge \exists yGy) \wedge (Ga \wedge \neg Ga) \checkmark$
2.  $\forall x(Fx \wedge \exists yGy)$
3.  $(Ga \wedge \neg Ga)$

To prove: whether  $\forall x(Fx \wedge \exists yGy) \wedge (Ga \wedge \neg Ga)$  is satisfiable.

1.  $\forall x(Fx \wedge \exists yGy) \wedge (Ga \wedge \neg Ga) \checkmark$
2.  $\forall x(Fx \wedge \exists yGy) \setminus a$
3.  $(Ga \wedge \neg Ga)$
4.  $Fa \wedge \exists yGy$

To prove: whether  $\forall x(Fx \wedge \exists yGy) \wedge (Ga \wedge \neg Ga)$  is satisfiable.

1.  $\forall x(Fx \wedge \exists yGy) \wedge (Ga \wedge \neg Ga) \checkmark$
2.  $\forall x(Fx \wedge \exists yGy) \setminus a$
3.  $(Ga \wedge \neg Ga)$
4.  $Fa \wedge \exists yGy \checkmark$
5.  $Fa$
6.  $\exists yGy$

---

To prove: whether  $\forall x(Fx \wedge \exists yGy) \wedge (Ga \wedge \neg Ga)$  is satisfiable.

1.  $\forall x(Fx \wedge \exists yGy) \wedge (Ga \wedge \neg Ga) \checkmark$
2.  $\forall x(Fx \wedge \exists yGy) \setminus a, b$
3.  $(Ga \wedge \neg Ga)$
4.  $Fa \wedge \exists yGy \checkmark$
5.  $Fa$
6.  $\exists yGy \checkmark b$
7.  $Gb$
8.  $Fb \wedge \exists yGy$
9.  $Fb$
10.  $\exists yGy \checkmark c$
11.  $Gc$

- 
- here is another infinite path (if we ignore  $Ga \wedge \neg Ga$  and  $\neg Ga$ ).
  - but the proposition is not satisfiable.
  - and the path is not saturated.
  - and if we were to saturate it, by applying the rule for  $\wedge$  and to  $Ga \wedge \neg Ga$  and  $\neg Ga$ , it would close straight away.

## reading models off infinite trees

### reading models off infinite trees

- I said we could read a model off of our infinite tree above.
- let's look at how.
- here is our tree again:

---

To prove: whether  $\forall x(Fx \wedge \exists yGy)$  is satisfiable.

1.  $\forall x(Fx \wedge \exists yGy) \setminus a, b, c$
2.  $(Fa \wedge \exists yGy) \checkmark$
3.  $\exists yGy \checkmark b$
4.  $Fa$
5.  $Gb$
6.  $(Fb \wedge \exists yGy) \checkmark$
7.  $\exists yGy \checkmark c$
8.  $Fb$
9.  $Gc$
10.  $(Fc \wedge \exists yGy) \checkmark$
11.  $\exists yGy \checkmark d$
12.  $Fc$
13.  $Gd$

- 
- what is the domain?
  - how many names occur on the open path?
  - an infinite number of names occur
  - so here is our domain:
    - domain:  $\{1, 2, 3, \dots\}$

- 
- how shall we assign referents to each of the infinite number of names?
  - like this of course:
    - referents: aa: 1, bb: 2, cc: 3 . . .
-

- and what about extensions for the predicates?
- well, there are only two predicates FF and GG.
- but we need an assignment which makes them true whenever they appear in atomic propositions on the path.
- let's look at our tree and think about it . . .

---

To prove: whether  $\forall x(Fx \wedge \exists yGy)$  is satisfiable.

1.  $\forall x(Fx \wedge \exists yGy) \setminus a, b, c$
  2.  $(Fa \wedge \exists yGy) \checkmark$
  3.  $\exists yGy \checkmark b$
  4.  $Fa$
  5.  $Gb$
  6.  $(Fb \wedge \exists yGy) \checkmark$
  7.  $\exists yGy \checkmark c$
  8.  $Fb$
  9.  $Gc$
  10.  $(Fc \wedge \exists yGy) \checkmark$
  11.  $\exists yGy \checkmark d$
  12.  $Fc$
  13.  $Gd$
- 

- the pattern for GG is: GbGb, GcGc, GdGd, ...
  - the pattern for FF is: FaFa, FbFb, FcFc, ...
  - our complete model, then:
    - domain: {1, 2, 3, . . . }
    - referents: aa: 1, bb: 2, cc: 3 . . .
    - extensions: FF: {11, 22, 33, . . .}, GG: {22, 33, . . .}.
- 

- will you have to read off a model from an infinite tree in the problem sets or the exam?
- possibly. but if so, it won't be a difficult pattern to identify.

## wrapping up

### this lecture

### next lecture

- lecture 19, the formal language GPL