lecture 07, validity and logical form

phil1012 introductory logic

overview

this lecture

- in the last few lectures we've been examining the semantics of PL and truth tables
- in this lecture we look in more detail at the idea of the **logical form** of a proposition
- recall that logical form was central to the definition of **validity**

learning outcomes

- after doing the relevant reading for this lecture, listening to the lecture, and attending the relevant tutorial, you will be able to:
    - identify the forms of propositions
    - identify instances of forms
    - identify the forms of arguments
    - identify instances of argument forms
    - explain how truth tables provide a test not only for necessary truth preservation but for validity

required reading

- all of chapter 5

validity and logical form

validity and logical form

- recall: an argument is **valid** if and only if, **in virtue of the form of the argument**, it is impossible for its conclusion to be false if its premises are all true
- our truth table test tells us whether it is possible for the conclusion to be false if all of the premises are true
- does it tell us whether this is in virtue of the form of the argument?
- it turns out that the answer is yes, since we can show that every argument with the same form is valid

---

- practical upshot: once we know that a given argument is valid, we know that every other argument of the same form is valid, without having to do truth tables for them individually

the form of an argument

- what is the form of an argument?

- suppose we set out to test whether the following argument is valid:

P1.	John is a philosopher
P2.	if John is a philosopher, then he drinks
C1.	John drinks

- we begin by writing up our glossary:
  - P: John is a philosopher
  - D: John drinks
- we translate accordingly:
  - P, $(P \to D)$ $\therefore$ D

- we use a truth table to test for validity:

P	D	P	$(P \to D)$	D
T	T	T	T	T
T	F	T	F	F
F	T	F	T	T
F	F	F	T	F

- great, now suppose we set out to test whether the following argument is valid:

P1.	Jane is a philosopher
P2.	if Jane is a philosopher, then she smokes
C1.	Jane smokes

- we begin by writing up our glossary:
  - J: Jane is a philosopher
  - S: Jane smokes
- we translate accordingly:
  - J, $(J \to S)$ $\therefore$ S

- and we use a truth table to test for validity:

J	S	J	$(J \to S)$	S
T	T	T	T	T
T	F	T	F	F
F	T	F	T	T
F	F	F	T	F

- after a while we might start to feel that the whole process is a bit laborious and redundant
- don’t these arguments have something in common which means that if we know that one is valid, we know the other is, and vice versa?
- yep, these arguments are instances of the same **form**
- how can we make the notion of form more precise?

abstracting from content: from propositions to forms

from propositions to forms

- consider the following proposition:
    - $(A \wedge (B \to \neg C))$
- what is its form?
- there are many correct answers ranging from very course-grained to very fine-grained

---

- at its most course-grained it is just a proposition:
    - $(A \wedge (B \to \neg C))$
- so it has the following form: α

- at a more fine-grained level of description it is a conjunction:
    - $(A \wedge (B \to \neg C))$
- so it has the following form: $(\alpha \wedge \beta)$

- at an even more fine-grained level of description it is a conjunction whose second conjunct is a conditional:
    - $(A \wedge (B \to \neg C))$
- so it has the following form:
    - $(\alpha \wedge (\beta \to \gamma))$

- at the most fine-grained level of description it is a conjunction whose second conjunct is a conditional, whose consequent is a negation:
    - $(A \wedge (B \to \neg C))$
- so it has the following form:
    - $(\alpha \wedge (\beta \to \neg\gamma))$

- propositions do not have a single form
- they have many forms ranging from the course-grained to the fine-grained

instances: from forms to propositions

instances: from forms to propositions

- consider the following logical form:
    - $(\alpha \leftrightarrow (\neg\beta \wedge \alpha))$
- a **logical form** is like a formula, except that in place of basic propositions it has variables in the place of formulas
- given a logical form, we can ask: what propositions have this form? We call such propositions **instances** of the form

---

- an instance of a logical form can be obtained by replacing the variables with propositions
- all occurrences of the same variable must be replaced by the same proposition

---

- for example . . .
    - $(\alpha \leftrightarrow (\neg\beta \wedge \alpha))$
        - $(A \leftrightarrow (\neg B \wedge A))$
        - $(B \leftrightarrow (\neg E \wedge B))$

- $(\ (\ A \to B)\ \leftrightarrow (\ \neg E \wedge (\ A \to B)\ )\ )$
- $(\ B \leftrightarrow (\ \neg B \wedge B)\ )$
- $(\ (\ A \to B)\ \leftrightarrow (\ \neg(\ A \to B)\ \wedge (\ A \to B)\ )\ )$

- but not . . .
  - $(\ \alpha \leftrightarrow (\ \neg\beta \wedge \alpha)\ )$
    - $(\ A \leftrightarrow (\ \neg B \wedge B)\ )$
    - $(\ B \leftrightarrow (\ \neg E \wedge D)\ )$
    - $(\ (\ A \to B)\ \leftrightarrow (\ \neg E \wedge B)\ )$

- two propositions share a form when they are both instances of it
- it doesn't make sense to say that they have the same form, they may both be instances of some form and not both instances of some other form

---

- okay, now we have a better idea of the forms of a proposition
- and we have a better idea of the propositions which are instances of forms
- we can now generalise this idea to the forms of an argument

the form(s) of an argument

the form(s) of an argument

- consider the following argument form:
  - α, $(\ \alpha \to (\ \beta \to \alpha)\ )\quad \therefore \beta \to \alpha$
- an **argument form** is just like an argument except that, in place of basic propositions, it has variables in the place of propositions
- given an argument form, we can ask: what arguments have this form?
- we call such arguments **instances** of the form

- for example . . .
  - α, $\alpha \to (\ \beta \to \alpha)\quad \therefore \beta \to \alpha$
    - P, $P \to (\ Q \to P)$, $\therefore Q \to P$
    - $(\ R \vee \neg Q)$, $(\ R \vee \neg Q)\ \to \big(\ P \to (\ R \vee \neg Q)\ \big)$, $\therefore P \to (\ R \vee \neg Q)$

- consider the following argument:
  - P, $\big(\ P \to (\ Q \to P)\ \big)$, $\therefore Q \to P$
- an argument can be an instance of more than one form

- for example . . .
  - P, $\big(\ P \to (\ Q \to P)\ \big)\quad \therefore Q \to P$
    - α, $(\ \alpha \to (\ \beta \to \alpha)\ )$, $\therefore (\ \beta \to \alpha)$
    - α, $(\alpha \to \gamma)$, $\therefore (\ \beta \to \alpha)$

- an argument can be an instance of more than one form
- it is important to keep this in mind when we discuss validity and form

validity and form

validity and form

- now that we have a precise understanding of the forms of an argument, to what use can we put the idea?
- well, we can use it to make our notion of validity more precise
- and we can appeal to the form of an argument in arguing that it is valid
- we begin by introducing notions analogous to validity and invalidity, which are properties of arguments, for argument forms

---

- we define the properties **valid*** and **invalid*** of **argument forms**
- these are distinct from the properties **valid** and **invalid** of **arguments**

- an argument form is **valid*** if and only if there is no row in which the premises are true and the conclusion false
- an argument form is **valid*** if and only if there is no instance of the form in which the premises are true and the conclusion false
- an argument form is **invalid*** if and only if there is a row in which the premises are true and the conclusion false

---

- we test for **validity*** of an argument form using truth tables as before

α	β	α	$(\alpha \to \beta)$	β
T	T	T	T	T
T	F	T	F	F
F	T	F	T	T
F	F	F	T	F

- now, a very nice feature of **valid*** argument forms is that every instance of a **valid*** argument form is valid argument

---

- consider the following argument:
  - $((A \wedge C) \to ((B \vee D) \to E))$, $(A \wedge C)$, $\therefore ((B \vee D) \to E)$
- we need a truth table with 32 rows since we have five distinct basic propositions!

- but wait! this argument is an instance of the form:
  - $(\alpha \to \beta)$, α $\therefore \beta$
- and this form is **valid*** (we just did the truth table for it)
- so we can immediately conclude that the argument is valid

- hint: unless stated otherwise, it is *always* permissible to save yourself time on a problem set or an exam by appealing to the form of an argument in proving its validity
- we will look at examples in the tutorials

invalidity and form

invalidity and form

- careful: it is not the case that every instance of an **invalid*** argument form is an invalid argument

- the following is an **invalid*** argument form:
  - $\beta \rightarrow \alpha$, $\alpha \therefore \beta$

- but the following is an instance of the form:
  - $(A \rightarrow A)$, $A \therefore A$
- and this argument is valid
- we can see this by doing the truth table, or by noting that this is *also* an instance of a valid form

---

- indeed, we can distinguish between **A-properties** and **S-properties** of arguments and argument forms
- every instance of a form with an A-property has that A-property. not so with the S-properties.

A-property	S-property
validity	invalidity
logical truth	non-logical truth
equivalence	inequivalence
unsatisfiability	satisfiability

- in other words, there are shortcuts available for establishing A-properties, but no shortcuts for S-properties

notable argument forms

notable argument forms

- now that we've introduced the idea of a valid* argument form, and we've seen how to appeal to valid argument forms in establishing the validity of an argument, you should be on the lookout for shortcuts
- here are some valid* argument forms to be on the lookout for

---

- modus ponens
  - $(\alpha \rightarrow \beta)$, $\alpha \therefore \beta$
- modus tollens
  - $(\alpha \rightarrow \beta)$, $\neg\beta \therefore \neg\alpha$
- hypothetical syllogism
  - $(\alpha \rightarrow \beta)$, $(\beta \rightarrow \gamma) \therefore (\alpha \rightarrow \gamma)$
- constructive dilemma
  - $(\alpha \rightarrow \beta)$, $(\gamma \rightarrow \delta)$, $(\alpha \vee \gamma) \therefore (\beta \rightarrow \delta)$
- disjunctive syllogism
  - $(\alpha \vee \beta)$, $\neg\alpha \therefore \beta$

wrapping up

this lecture

- validity and logical form
- the forms of propositions
- the instances of forms
- valid* argument forms

next lecture

- lecture 08, functional completeness