lecture 22, trees for GPLI

phil1012 introductory logic

overview

this lecture

- trees for GPLI
- two new tree rules
    - the closure rule
    - substitution of identicals
- finished trees and saturated paths
    - new definition of saturation
- reading models off open paths
    - provisional domains and assignments of referents
    - trimming domains and assignments of referents

learning outcomes

- after doing the relevant reading for this lecture, listening to the lecture, and attending the relevant tutorial, you will be able to:
    - explain what it is for a GPLI tree to be finished and a path to be saturated
    - construct GPLI trees
    - read models off open paths of GPLI trees

required reading

- section 13.7 of chapter 13

tree rules for gpli

tree rules for gpli

- as in the transition from MPL to GPL we will build on our pre-existing tree rules
- since == is part of the logical vocabulary, we need to say something about the tree rules for it

the closure rule

- the first new rule is called the **closure rule**

$$\frac{\underline{a \neq a}}{\otimes}$$

- this basically says that you can close a path if there is a proposition on it which asserts that something is not identical to itself!
- the motivation for this rule should be obvious

new notation explained

- recall that α(x_)\alpha(\underline{x}) stands for any arbitrary wff that has no free variables other than x_\underline{x} and that α(a_/x_)\alpha(\underline{a}/\underline{x}) stands for the wff that results from α(x_)\alpha(\underline{x}) by replacing *all* free occurrences of x_\underline{x} in α(x_)\alpha(\underline{x}) with the name a_\underline{a}
- we use α(a_)\alpha(\underline{a}) to stand for any arbitrary wff in which the name a_\underline{a} occurs (one or more times)
- we then use α(b_//a_)\alpha(\underline{b}//\underline{a}) to stand for any wff which results from α(a_)\alpha(\underline{a}) by replacing *some* occurrences of a_\underline{a} with the name b_\underline{b}

some examples using the new notation

- if α(a)\alpha(a) is GaGa, then α(b//a)\alpha(b//a) is GbGb and α(c//a)\alpha(c//a) is GcGc
- if α(a)\alpha(a) is RabRab, then α(b//a)\alpha(b//a) is RbbRbb and α(c//a)\alpha(c//a) is RcbRcb
- if α(a)\alpha(a) is (Ga**V**Rab)(Ga \lor Rab), then α(c//a)\alpha(c//a) is (Gc**V**Rab)(Gc \lor Rab) or (Ga**V**Rcb)(Ga \lor Rcb) or (Gc**V**Rcb)(Gc \lor Rcb)
- if α(a)\alpha(a) is (Ga→Rbb)(Ga \rightarrow Rbb), then α(a//b)\alpha(a//b) is (Ga→Rab)(Ga \rightarrow Rab) or (Ga→Rba)(Ga \rightarrow Rba) or (Ga→Raa)(Ga \rightarrow Raa)

substitution of identicals

- the second new rule is called the **substitution of identicals** rule

$$\begin{array}{c} \alpha(\underline{a}) \\ \underline{a} = \underline{b} \\ \alpha(\underline{b}/\!/\underline{a}) \end{array} \qquad (\text{or } \underline{b} = \underline{a})$$

- let's run through an example . . .

To prove: whether $\forall x Lxr \land \forall y(Lry \rightarrow I^2ym) \therefore I^2mr$ is valid.

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1. $\forall x Lxr \land \forall y(Lry \rightarrow I^2ym)$
2. $\neg I^2mr$

To prove: whether $\forall x Lxr \land \forall y(Lry \rightarrow I^2ym) \therefore I^2mr$ is valid.

1. $\forall x Lxr \land \forall y(Lry \rightarrow I^2ym)$
2. $\neg I^2mr$

To prove: whether $\forall x Lxr \wedge \forall y(Lry \rightarrow I^2ym) \therefore I^2mr$ is valid.

1. $\forall x Lxr \wedge \forall y(Lry \rightarrow I^2ym)$ ✓
2. $\neg I^2mr$
3. $\forall x Lxr$
4. $\forall y(Lry \rightarrow I^2ym)$

To prove: whether $\forall x Lxr \wedge \forall y(Lry \rightarrow I^2ym) \therefore I^2mr$ is valid.

1. $\forall x Lxr \wedge \forall y(Lry \rightarrow I^2ym)$ ✓
2. $\neg I^2mr$
3. $\forall x Lxr \setminus r$
4. $\forall y(Lry \rightarrow I^2ym) \setminus r$
5. Lrr
6. $Lrr \rightarrow I^2rm$

To prove: whether $\forall x Lxr \wedge \forall y(Lry \rightarrow I^2ym) \therefore I^2mr$ is valid.

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2. $\neg I^2mr$
3. $\forall x Lxr \setminus r$
4. $\forall y(Lry \rightarrow I^2ym) \setminus r$
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6. $Lrr \rightarrow I^2rm$ ✓
7. $\neg Lrr$ I^2rm

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$\otimes$

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3. $\forall x Lxr \setminus r$
4. $\forall y(Lry \rightarrow I^2ym) \setminus r$
5. Lrr
6. $Lrr \rightarrow I^2rm$ ✓
7. $\neg Lrr$ I^2rm

$\otimes$ $\otimes$

- wrong!
- why? neither the closure rule nor the ordinary rule for closing a path applies here

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1.	$\forall xLxr \wedge \forall y(Lry \rightarrow I^2ym)$ ✓		
2.	$\neg I^2mr$		
3.	$\forall xLxr \backslash r$		
4.	$\forall y(Lry \rightarrow I^2ym) \backslash r$		
5.	Lrr		
6.	$Lrr \rightarrow I^2rm$ ✓		
7.	$\neg Lrr$	I^2rm	
8.		$\neg I^2mm$	2,7 SI
		$\otimes$	

- right!

finished trees, closed and saturated paths

finished trees, closed and saturated paths

- a tree is finished when all paths are closed or saturated
- a tree is saturated when no more rules can be applied to it
- recall the qualification for the rule for the universal quantifier
- we add the following qualification for saturation

saturation

- **saturation:** a path is not saturated unless every application of SI that could be made on that path and that would result in the addition to the path of a formula that does not already appear on the path has been made

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- let's consider another example . . .

1.	Rab	
2.	I^2ab	

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2.	I^2ab	
3.	Raa	1,2, SI 1=$\alpha(b)$

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2.	I^2ab	
3.	Raa	1,2, SI 1=$\alpha(b)$
4.	Rbb	1,2, SI 1=$\alpha(a)$

1.	Rab	
2.	I^2ab	
3.	Raa	1,2, SI 1=$\alpha(b)$
4.	Rbb	1,2, SI 1=$\alpha(a)$
5.	Rba	4,2, SI 4=$\alpha(b)$

- this tree is finished, as we cannot apply the SI rule to generate any new basic propositions (or negations of basic propositions)

reading models off open paths

reading models off open paths

- given a saturated open path, we can read off from it a model on which the proposition(s) at the top of the tree are true
- the process involves three stages
  - stage one: construct a provisional domain and assignment of referents to names
  - stage two: trim the domain and assignments of referents to names to obtain a final domain and assignment of referents to names
  - stage three: assign extensions to predicates on the trimmed domain and assignments of referents

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- let's consider an example . . .

To prove: whether $Rab, I^2ab, \therefore Rbc$ is valid.

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4. Raa
5. Rba
6. Rbb
7. $\neg Rac$

To prove: whether $Rab, I^2ab, \therefore Rbc$ is valid.

1. Rab
2. I^2ab
3. $\neg Rbc$
4. Raa
5. Rba
6. Rbb
7. $\neg Rac$

- domain:
- referents:
- extensions:

To prove: whether $Rab, I^2ab, \therefore Rbc$ is valid.

1. Rab
2. I^2ab
3. $\neg Rbc$
4. Raa
5. Rba
6. Rbb
7. $\neg Rac$

- domain: {1, 2, 3} (provisional)
- referents: aa:1, bb:2, cc: 3 (provisional)
- extensions:

To prove: whether $Rab, I^2ab, \therefore Rbc$ is valid.

1. Rab
2. $\boxed{I^2ab}$
3. $\neg Rbc$
4. Raa
5. Rba
6. Rbb
7. $\neg Rac$

- domain: {1, 2, 3} (provisional)
- referents: aa:1, bb:2, cc: 3 (provisional)
- extensions:

To prove: whether $Rab, I^2ab, \therefore Rbc$ is valid.

1. Rab
2. $\boxed{I^2ab}$
3. $\neg Rbc$
4. Raa
5. Rba
6. Rbb
7. $\neg Rac$

- domain: {1, 2, 3} (provisional)
- referents: aa:1, bb:1, cc: 3 (revised)
- extensions:

To prove: whether $Rab, I^2ab, \therefore Rbc$ is valid.

1. Rab
2. $\boxed{I^2ab}$
3. $\neg Rbc$
4. Raa
5. Rba
6. Rbb
7. $\neg Rac$

- domain: {1, 2} (revised)
- referents: aa:1, bb:1, cc: 2 (revised)
- extensions:

To prove: whether $Rab, I^2ab, \therefore Rbc$ is valid.

1. $\boxed{Rab}$
2. I^2ab
3. $\neg Rbc$
4. $\boxed{Raa}$
5. $\boxed{Rba}$
6. $\boxed{Rbb}$
7. $\neg Rac$

- `domain: {1, 2} (revised)`
- `referents: aa:1, bb:1, cc: 2 (revised)`
- `extensions: RR: {⟨1,1⟩}\{\langle 1, 1 \rangle \}`

- `let’s consider another example . . .`

To prove: whether $\forall x(Fx \to Gx), \exists x Fx, I^2ab, \therefore Ga$ is valid.

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$$\forall x(Fx \to Gx)$$
$$\exists x Fx$$
$$I^2ab$$
$$\neg Ga$$

To prove: whether $\forall x(Fx \to Gx), \exists x Fx, I^2ab, \therefore Ga$ is valid.

$\forall x(Fx \to Gx) \ \backslash a, b, c$
$\exists x Fx \ \checkmark c$
I^2ab
$\neg Ga$
Fa
$Fa \to Ga \ \checkmark$

- $\neg Fa$
  $Fb \to Gb \ \checkmark$
  - $\neg Fb$
    $Fc \to Gc \ \checkmark$
    - $\neg Fc$
      $\otimes$
    - Gc
      $\neg Gb$
  - Gb
    $\neg Gb$
    $\otimes$
- Ga
  $\otimes$

- `domain:`
- `referents:`
- `extensions:`

To prove: whether $\forall x(Fx \rightarrow Gx), \exists xFx, I^2ab, \therefore Ga$ is valid.

$$\forall x(Fx \rightarrow Gx) \setminus a, b, c$$
$$\exists xFx \checkmark c$$
$$I^2ab$$
$$\neg Ga$$
$$Fa$$
$$Fa \rightarrow Ga \checkmark$$
$$\neg Fa \qquad Ga$$
$$Fb \rightarrow Gb \checkmark \qquad \otimes$$
$$\neg Fb \qquad Gb$$
$$Fc \rightarrow Gc \checkmark \qquad \neg Gb$$
$$\neg Fc \quad Gc \qquad \otimes$$
$$\otimes \quad \neg Gb$$

- domain: {1, 2, 3}
- referents: aa: 1, bb:2, cc:3
- extensions:

To prove: whether $\forall x(Fx \rightarrow Gx), \exists xFx, I^2ab, \therefore Ga$ is valid.

$$\forall x(Fx \rightarrow Gx) \setminus a, b, c$$
$$\exists xFx \checkmark c$$
$$\boxed{I^2ab}$$
$$\neg Ga$$
$$Fa$$
$$Fa \rightarrow Ga \checkmark$$
$$\neg Fa \qquad Ga$$
$$Fb \rightarrow Gb \checkmark \qquad \otimes$$
$$\neg Fb \qquad Gb$$
$$Fc \rightarrow Gc \checkmark \qquad \neg Gb$$
$$\neg Fc \quad Gc \qquad \otimes$$
$$\otimes \quad \neg Gb$$

- domain: {1, 2}
- referents: aa: 1, bb:1, cc:2
- extensions:

To prove: whether $\forall x(Fx \to Gx), \exists xFx, I^2ab, \therefore Ga$ is valid.

$\forall x(Fx \to Gx) \setminus a, b, c$

$\exists xFx$ ✓ c

I^2ab

$\neg Ga$

$\boxed{Fc}$

$Fa \to Ga$ ✓

$\neg Fa$ $\quad\quad$ Ga

$Fb \to Gb$ ✓ $\quad$ $\otimes$

$\neg Fb$ $\quad\quad$ Gb

$Fc \to Gc$ ✓ $\quad$ $\neg Gb$

$\neg Fc$ $\quad$ $\boxed{Gc}$ $\quad\quad$ $\otimes$

$\otimes$ $\quad$ $\neg Gb$

- domain: {1, 2}
- referents: aa: 1, bb:1, cc:2
- extensions: F:{2},G:{2}F: \{2\}, G: \{2\}

wrapping up

this lecture

- trees for GPLI
- two new tree rules
  - the closure rule
  - substitution of identicals
- finished trees and saturated paths
  - new definition of saturation
- reading models off open paths
  - provisional domains and assignments of referents
  - trimming domains and assignments of referents

live lecture and tutorials

- you will practice constructing GPLI trees on your own

next lecture

- lecture 23, numerical quantifiers and definite descriptions in GPLI