lecture 13, the formal language MPL

phil1012 introductory logic

overview

this lecture

- an introduction to the formal language MPL
- the limitations of PL and the motivation for MPL
- an introduction to the notions of names, predicates, variables, and quantifiers
- translation of propositions into MPL

learning outcomes

- after doing the relevant reading for this lecture, listening to the lecture, and attending the relevant tutorial, you will be able to:
  - explain what MPL allows us to do that we cannot do in PL
  - translate propositions with a simple subject-predicate form into MPL
  - translate existential and universal propositions into MPL
  - translate propositions involving restricted quantification into MPL

required reading

- sections 8.1, 8.2, 8.3 of chapter 8

beyond propositional logic

beyond propositional logic

- propositional logic is great
- but there are some arguments which seem to be such that, in virtue of their form, their premises can't all be true and their conclusion false

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- consider the following argument

P1.	all philosophers are drinkers
P2.	John is a philosopher
C1.	John is a drinker

- a valid argument

- let's translate the argument into PL
- we have the following glossary
  - P: all philosophers are drinkers
  - J: John is a philosopher
  - D: John is a drinker
- and the following translation

- P, J, $\therefore D$
- but this argument is invalid in PL!

- we must go beyond PL!
- for the remainder of the course we will go beyond PL three times over
  - MPL: names, one-place predicates, quantifiers, and variables.
  - GPL: names, many-place predicates, quantifiers, and variables.
  - GPLI: names, many-place predicates, quantifiers, variables, and the identity predicate.
- for now: MPL

names and predicates

names and predicates

- introducing names and predicates
- consider: 'John is a philosopher'
  - this expresses a basic proposition. but let's now take a look at the constituents of this sentence
- there is a **name** 'John', which **refers** to an individual thing, namely John
- there is a **predicate**, 'is a philosopher' which is **satisfied** by certain individual things if and only if they have the property of being a philosopher

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- the symbol for a **name** in MPL is a lowercase letter, a, b, c, d, e, . . ., r, s, t
  - not including the last six letters in the alphabet, u, v, w, x, y, z (these will be used for something else)
- the symbol for a **predicate** in MPL is an uppercase letter, A, B, C, D, . . ., X, Y, Z (no reservations here.)
- don't confuse sentences letters from PL with predicate letters from MPL!
- and note that there are no sentence letters in PL!

glossaries in MPL (preliminary)

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- glossaries in MPL work like glossaries in PL
- the give meanings for the non-logical symbols of MPL
- unlike PL whose non-logical symbols were sentence letters, the non-logical symbols of MPL are names and predicates

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- we give the meanings of names in our MPL glossaries like this:
  - b: Bruce
  - j: Jane
  - m: *Australian Materialism*
  - a: The author of *Australian Materialism*
- we give the meanings of predicates in our MPL glossaries like this:
  - P: is a philosopher
  - B: is a book
- this is provisional. we'll see a better way of doing glossaries from MPL below.

atomic propositions

atomic propositions

- an **atomic proposition** is a proposition made up from one name and one predicate.
- we translate the atomic proposition 'Bruce is a philosopher' like this:
  - Pb
- we translate the atomic proposition 'The author of *Australian Materialism* is a philosopher' like this:
  - Pa
- we translate the atomic proposition '*Australian Materialism* is a book' like this:
  - Bm

connectives in MPL

- we retain all the connectives of PL in MPL.
- we translate a disjunction like 'Bruce is a philosopher or Jane is a philosopher' like this:
  - $(Pb \vee Pj)$
- we translate a conjunction like 'Bruce is a philosopher and Jane is a philosopher' like this:
  - $(Pb \wedge Pj)$

- we translate a conditional like 'If Bruce is a philosopher, then Jane is a philosopher' like this:
  - $Pb \rightarrow Pj$
- we translate a biconditional like 'Bruce is a philosopher if and only if Jane is a philosopher' like this:
  - $(Pb \leftrightarrow Pj)$
- we translate a negation like 'Bruce is not a philosopher' like this:
  - $\neg Pb$

variables and quantifiers

- introducing **variables** and **quantifiers**
- how should we translate the following?
  - everyone is a philosopher
  - someone is a philosopher
  - so-one is a philosopher

- not like this:
- here's my glossary:
  - e: everyone
  - s: someone
  - n: no-one
- here's my translation:
  - e is a philosopher
  - s is a philosopher
  - n is a philosopher
- this is wrong!
- expressions like 'everyone', 'someone', and 'no-one' are not names.

- expressions like ‘someone’ and ‘everyone’ are **quantifiers**
- ‘someone’ is an **existential quantifier**
- ‘everyone’ is a **universal quantifier**

- how should we think about quantified propositions then? what do they say?
- well, ‘Everything is a philosopher’ says, roughly, that every *thing* is such that *it* is a philosopher
- and ‘Something is a philosopher’ says, roughly, that some *thing* is such that *it* is a philosopher

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- to put it slightly more formally (but still informally):
- well, ‘Everything is a philosopher’ says, roughly, that every *x* is such that *x* is a philosopher
- and ‘Something is a philosopher’ says, roughly, that some *x* is such that *x* is a philosopher

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- we use the variable *x* in this informal presentation.
- in MPL the symbols for *individual variables* are u, v, w, x, y, z
- using variables, we can get closer to what we are after.

- given our glossary for ‘is a philosopher’ we have:
  - every x is such that Px
  - some x is such that Px

- we introduce two new symbols in MPL which mean ‘every x is such that’ and ‘some x is such that’
  - $\forall x$ (universal quantifier)
  - $\exists x$ (existenial quantifier)

- using quantifiers and variables we can translate ‘Everything is a philosopher’ and ‘Something is a philosopher’ as follows:
  - everything is a philosopher
    - $\forall x Px$
  - something is a philosopher
    - $\exists x Px$

- okay, we are getting somewhere
- but we set out to translate the following
  - everyone is a philosopher
  - someone is a philosopher
  - no-one is a philosopher
- ‘everything is a philosopher’ doesn’t mean the same thing as ‘everyone is a philosopher’
- things get just a little bit more complicated
- but before turning to this complication . . .

glossaries in MPL (official)

- with the introduction of variables, here is a new convention for writing glossaries for predicates
  - Px: x is a philosopher
  - Bx: x is a book
- notice how one variable occurs before the colon and immediately after

the predicate letter, and another variable occurs after it in the place in the sentence where a name might go

restricted quantification

restricted quantification

- back to the complication . . .
- how should we translate ‘every philosopher is a drinker’ and ‘some philosopher is a drinker’?
- let’s think it through carefully

- think about ‘every philosopher is a drinker’
- you can think of it as saying that every x is such that <u>if</u> x is a philosopher, <u>then</u> x is a drinker.
- so we translate it as follows:
  - $\forall x(\ Px \rightarrow Dx)$
- where:
  - Dx: x is a person
  - Px: x is a philosopher
- you can think of the predicate in the antecent of the conditional as ‘restricting’ the things over which we are quantifying

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- now think about ‘Some philosopher is a drinker’
- you can think of it as saying that some x is such that x is a philosopher <u>and</u> x is a drinker
- so we translate it as follows:
  - $\exists x(\ Px \wedge Dx)$
- where:
  - Dx: x is a person
  - Px: x is a philosopher
- you can think of the predicate in the first conjunct of the conjunction as ‘restricting’ the things over which we are quantifying

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- notice that for restricted universal quantification we use a conditional, and the antecedent of the conditional does the restricting, and that for restricted existential quantification we use a conjunction, and one conjunct does the restricting.
- don’t confuse the two
- this says that everything is a philosopher and a drinker:
  - $\forall x(\ Px \wedge Dx)$
- and this says that there is something which is either not a philosopher or is a drinker.
  - $\exists x(\ Px \rightarrow Dx)$
- it is true if anything is not a philosopher or if anything is a drinker!

wrapping up

this lecture

- an introduction to the formal language MPL
- the limitations of PL and the motivation for MPL
- an introduction to the notions of names, predicates, variables, and

quantifiers
- translation of propositions into MPL

next lecture

- lecture 14, the syntax of MPL