lecture 21, the formal language GPLI

phil1012 introductory logic

overview

this lecture

- an introduction to the formal language GPLI
- some of the limitations of GPL which motivate the shift to GPLI
- the identity relation
- translations into GPLI
- the semantics of identity

learning outcomes

- after doing the relevant reading for this lecture, listening to the lecture, and attending the relevant tutorial, you will be able to:
  - explain the limitations of GPL and the motivations for GPLI
  - explain what the identity relation is
  - translate propositions from English into GPLI
  - explain the motivation behind the semantics of identity
  - determine whether a GPLI proposition is true or false on a model

required reading

- sections 13.1, 13.2, and 13.3 of chapter 13

expressive limitations of GPL

expressive limitations of GPL

- consider the following argument

P1.	John loves something
P2.	John does not love himself
C1.	John loves something other than himself

- we have the following glossary:
  - j: John
  - Lxy: x loves y
- and we translate the premises as follows:
  - $\exists xLjx$
  - $\neg Ljj$
- but we have no way of translating the conclusion
- we have no way of saying that John loves someone other than himself

introducing GPLI

introducing GPLI

- we introduce a new two-place predicate into our logical language: the **identity** predicate
    - I^2

- in GPL I^2 is not used for a two-place predicate. Although I^1, I^3, I^4, and so on are
- we are extending GPL to GPLI which includes the symbol I^2 in addition to all the symbols of GPL
- syntactically, I^2 functions just like a two-place predicate in GPL
- semantically, I^2 is treated differently
    - its meaning does not vary from model to model
    - it is not given a glossary
    - in every model it expresses the **identity relation**

- since the two-place predicate I^2 is so special in GPLI we usually abbreviate it as follows:
    - $=$
- and we abbreviate negations involving I^2 using:
    - $\neq$
- it is important to remember, however, that $=$ is just an abbreviation of a two-place preciate

the identity relation

the identity relation

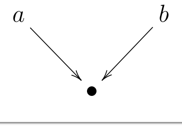
- $=$ expresses the identity relation
- but what is the relation of identity?

- B1 and B2 are identical
- B1 is identical to B2
- B1 and B2 are *exactly the same in all respects*

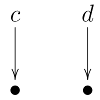
- B1 and B2 are identical
- B1 is identical to B2
- B1 and B2 are *one and the same thing*

- we have two relations:
    - **identity** (being one and the same thing)
    - **exact similarity** (being the same in all respects)
- how are they related?
    - Leibniz's Law/Indiscernability of Identicals: if x is identical to y then x is exactly similar to y. (a logical truth)
    - Identity of Indiscernibles: if x is exactly similar to y then x is identical to y. (controversial)

- a and b are identical (the names 'a' and 'b' pick out the same thing):



- c and d are non-identical (the names 'c' and 'd' pick out different things):



the predicate $=$

the predicate $=$

- consider the following sentences:
- B1 is a banana
- B1 is B2
- the word 'is' has a different meaning in each. in the first case, it is the 'is' of predication. in the second, it is the 'is' of identity.

- we have the following glossary:
  - a: B1
  - b: B2
  - By: x is a banana

- we have the following translations:
  - Ba
  - $a = b$

- recall that I^2 (or $=$) is part of the logical vocabulary
- we do not put an entry for it in the glossary

translations into GPLI

translations into GPLI

- GPLI allows us to express more than just that a and b are identical
- let's take a look at the expressive power we have gained

- return to our example:

P1. John loves something
P2. John does not love himself
C1. John loves something other than himself

- here's a glossary:
  - j: John
  - Lxy: x loves y

- how should we translate the conclusion of the argument?
  - $\exists x\left(Ljx \wedge \neg I^2jx\right)$
- there is something that John loves and that thing is not John
- John loves something other than himself!

- let's look at some other examples . . .

- here's a glossary:

  j: John, m: Mary, s: Sam
  Lxy: x loves y, Sx: x is a baker, Px: x is a person

- translate the following into GPLI:

  John isn't Mary

  $\neg I^2 jm$
  $j \neq m$

- here's a glossary:

  j: John, m: Mary, s: Sam
  Lxy: x loves y, Sx: x is a baker, Px: x is a person

- translate the following into GPLI:

  John loves someone other than himself

  $\exists x\big(\; Px \wedge Ljx \wedge \neg I^2 xj\big)$
  $\exists x(\; Px \wedge Ljx \wedge x \neq j)$

- here's a glossary:

  j: John, m: Mary, s: Sam
  Lxy: x loves y, Sx: x is a baker, Px: x is a person

- translate the following into GPLI:

  John loves everyone except Sam

  $\forall x\big(\; \big(\; Px \wedge \neg I^2 xs\big) \;\to Ljx\big)$
  $\forall x(\; (\; Px \wedge x \neq s) \;\to Ljx)$

- here's a glossary:

  j: John, m: Mary, s: Sam
  Lxy: x loves y, Sx: x is a baker, Px: x is a person

- translate the following into GPLI:

  Some baker other than John loves Mary

  $\exists x\big(\; Bx \wedge \neg I^2 xj \wedge Lxm\big)$
  $\exists x(\; Bx \wedge x \neq j \wedge Lxm)$

- here's a glossary:

  j: John, m: Mary, s: Sam
  Lxy: x loves y, Sx: x is a baker, Px: x is a person

- translate the following into GPLI:

  Everyone loves themselves

$\forall x(\ Px \to Lxx)$

- here's a glossary:

  j: John, m: Mary, s: Sam
  Lxy: x loves y, Sx: x is a baker, Px: x is a person

- translate the following into GPLI:

  John loves everyone other than himself

  $\forall x\Big(\ \big(\ Px \land \neg I^2xj\big)\ \to Ljx\Big)$
  $\forall x(\ (\ Px \land x \neq j)\ \to Ljx)$

- here's a glossary:

  j: John, m: Mary, s: Sam
  Lxy: x loves y, Sx: x is a baker, Px: x is a person

- translate the following into GPLI:

  John loves everyone other than himself

  $\forall x\Big(\ \big(\ Px \land \neg I^2xj\big)\ \to Ljx\Big)\ \land \neg Ljj$
  $\forall x(\ (\ Px \land x \neq j)\ \to Ljx)\ \land \neg Ljj$

- here's a glossary:

  j: John, m: Mary, s: Sam
  Lxy: x loves y, Sx: x is a baker, Px: x is a person

- translate the following into GPLI:

  John loves everything but does not love himself

  $\forall xLjx \land \neg Ljj$

- (this is a contradiction. it is false in every model.)

the semantics of $=$

the semantics of $=$

- since I^2 is a two-place predicate, its extension is a set of ordered pairs of members of the domain
- once we have an extension for I^2, the semantics for GPLI is just like the semantics of GPL
- so all we need is an extension for I^2

- in a model in which the domain is {1, 2, 3}, the extension of I^2 will be the following set of ordered pairs:
  - $\{\langle 1,1\rangle, \langle 2,2\rangle, \langle 3,3\rangle\}$

- in a model in which the domain is $\{Bill, Ben\}$, the extension of I^2 will be the following set of ordered pairs:
  - $\{\langle Bill, Bill\rangle, \langle Ben, Ben\rangle\}$

- whatever the domain is, the extension of I^2 will contain exactly one ordered pair for each object in the domain: the pair containing that object in both the first and second place
- the identity relation is a relation which holds between each object and itself and between no other objects

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- note! The extension of the identity predicate is not exactly the same on all models
- given a domain, however, the extension of the identity predicate is fixed
- this is not true of other predicates
- this is why it is part of the logical vocabulary

propositions of the form I^2ab

- I^2ab is true in a model if and only if the pair consisting of the referent of a and the referent of b is in the extension of I^2
  - Domain: $\{Bill, Ben\}$
  - Referents: a: $Bill$, b: $Bill$
  - Extension of I^2
    - $\{\langle Bill, Bill\rangle, \langle Ben, Ben\rangle\}$
- I^2ab is true in a model if and only if a and b have the same referent on that model

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- let's run through a couple of examples

- here's a model:
  - Domain: $\{Sydney, Canberra, Melbourne\}$
  - Referents: a: $Melbourne$, b: $Canberra$, c: $Sydney$
  - Extensions: N: $\{\langle Sydney, Canberra\rangle, \langle Canberra, Sydney\rangle, \langle Melbourne, Canberra\rangle, \langle Canberra, Melbourne\rangle\}$
- is the following true or false in the model?
  - $\forall x \forall y\Big(Nxy \to \Big(I^2xb \vee I^2yb\Big) \Big)$

- here's a model:
  - Domain: $\{Sydney, Canberra, Melbourne\}$
  - Referents: a: $Melbourne$, b: $Canberra$, c: $Sydney$
  - Extensions: N: $\{\langle Sydney, Canberra\rangle, \langle Canberra, Sydney\rangle, \langle Melbourne, Canberra\rangle, \langle Canberra, Melbourne\rangle\}$
- is the following true or false in the model?
  - $\Big(I^2ab \to \forall x \forall y Nxy\Big)$

wrapping up

this lecture

- some of the limitations of GPL which motivate the shift to GPLI
- introducing GPLI
- the identity relation
- translations into GPLI
- the semantics of identity

next lecture

- lecture 22, trees for GPLI