

# lecture 12, issues in translation: conditional

phil1012 introductory logic

## overview

### this lecture

- issues arising with respect to translating conditional statements from English into PL

### learning outcomes

- after doing the relevant reading for this lecture, listening to the lecture, and attending the relevant tutorial, you will be able to:
  - explain some of the problems arising from the assumption that ordinary conditionals are to be translated as material conditionals
  - distinguish between the unacceptability of certain conditions and their truth conditions

### required reading

- section 6.3 of chapter 6

## issues with conditional

### issues with conditional

- one of the biggest issues in philosophical logic and philosophy of language concerns how we should translate conditional statements into formal language like PL
- logicians and philosophers are fond of the idea that  $\rightarrow$  adequately captures the truth conditions of ordinary conditionals
- we call  $\rightarrow$  the **material conditional**.
- so the question is whether the material conditional provides an adequate translation for ordinary conditionals

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- people write entire books defending the claim against evidence to the contrary
  - the matter remains controversial
  - it is important that you get a sense of the controversy
  - but then forget about it for the rest of the course, and just assume that  $\rightarrow$  adequately captures ordinary language conditionals

### the truth table for conditional

- recall the truth table for conditional
- if the antecedent is true and the consequent is false then the conditional is false. otherwise it is true.

$\alpha$	$\beta$	$(\alpha \rightarrow \beta)$
T	T	T
T	F	F
F	T	T
F	F	T

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- so far, we have assumed that the following sentences translate into the following PL formula.
    - if the train breaks down, then I will be late.
    - I will be late if the train breaks down.
    - the train broke down, only if I am late.
  - which we translate with a glossary like this:
    - $T$ : The train breaks down
    - $L$ : I am late
  - using  $\rightarrow$  we translate like this:
    - $*(T \rightarrow L)$

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- let's see if we can do more to motivate translating the ordinary conditional as a material conditional

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- consider:
    - if the train breaks down, I will be late.
  - with the following glossary
    - $B$ : The train breaks down
    - $L$ : I am late
  - we can all agree that (1) is false if  $T$  is true and  $L$  is false.
  - now let's ask:
    - is (1) true if  $B$  is true and  $L$  is true?
    - is (1) true if  $B$  is false and  $L$  is true?
    - is (1) true if  $B$  is false and  $L$  is false?

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- now consider:
    - if the train breaks down, the train breaks down.
  - with the following glossary
    - $B$ : The train breaks down
    - $L$ : I am late
  - had better be true whether  $B$  is false or whether  $B$  is true.
  - so we can agree that (2) is true if  $B$  is true and  $L$  is true
  - and we can agree that (2) is true if  $B$  is false and  $L$  is false.
  - now let's ask:
    - is (1) or (2) true if  $B$  is false and  $L$  is true?

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- now let's ask:
    - is (2) true if  $B$  is false and  $L$  is true?

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- well, it had better not be
  - if (1) is false if  $B$  is false and  $L$  is true, then the conditional must have the same truth table as the biconditional.
  - this is absurd.
  - so we can agree that (1) is true if  $B$  is false and  $L$  is true
  - and now we have agreed that the ordinary conditional must have the truth table we have given it if it is a truth functional connective at all

## proving equivalence

- let's have another go. this time we set out to prove the equivalence of the ordinary conditional.
- we do so in two parts
- first part: if "If  $A$  then  $B$ " is true, then  $A \rightarrow B$  is true.
- second part: If  $A \rightarrow B$  is true, then "If  $A$  then  $B$ ".

## the first part

- if "If  $A$  then  $B$ " is true, then  $A \rightarrow B$  is true.
- proof:
  - suppose that  $A \rightarrow B$  is false.
  - if  $A \rightarrow B$  is false, then  $A$  is true and  $B$  is false.
  - so  $A$  is true and  $B$  is false.
  - but if  $A$  is true and  $B$  is false, then "If  $A$  then  $B$ " is certainly false.
  - so if "If  $A$  then  $B$ " is true,  $A \rightarrow B$  must be true.

## the second part (version 1)

- if  $A \rightarrow B$  is true, then "If  $A$  then  $B$ ".
- "proof" 1:
  - the proposition  $A \rightarrow B$  is equivalent to  $\neg(A \wedge \neg B)$ , so if  $A \rightarrow B$  is true,  $\neg(A \wedge \neg B)$  is true.
  - but from  $\neg(A \wedge \neg B)$ , "if  $A$ , then  $B$ " obviously follows. for given that it is not the case that both  $A$  and that  $\neg B$ , if  $A$  is the case, then  $\neg B$ , must not be the case—that is,  $B$  must be the case.

## the second part (version 2)

- if  $A \rightarrow B$  is true, then "If  $A$  then  $B$ ".
- "proof" 2:
  - the proposition  $A \rightarrow B$  is equivalent to  $(\neg A \vee B)$ , so if  $A \rightarrow B$  is true,  $(\neg A \vee B)$  is true. ...
  - but from  $(\neg A \vee B)$ , "if  $A$ , then  $B$ " obviously follows. For given that at least one of  $\neg A$  and  $B$  is the case, if  $A$  is the case—that is,  $\neg A$  is not the case—then  $B$  must be the case.

## a quick proof

- from Frank Jackson:

Instead of saying "if it rains, then the match will be cancelled," one could have said, to much the same effect, "either it will not rain, or it will and the match will be cancelled:" and the latter is true if and only if either "it will rain" is false or "the match will be cancelled" is true; that is if and only if [the material conditional] is true. (Jackson 1991 p. 2)

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- it is very plausible that if conditionals are truth-functional connectives, then they are the connective we have chosen.
  - but there are problems with the antecedent of this conditional!
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- let's consider some of the absurd consequences of treating the ordinary language conditional as  $\rightarrow$ :
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- absurdity of assuming that if  $C$  is true, then  $A \rightarrow C$  is true.
    - if Sydney is in New Zealand, then  $2+2=4$ !
    - if Sydney is in Australia, then  $2+2=4$ !
- 

- absurdity of assuming that if  $A$  is false, then  $A \rightarrow C$  is true.
    - if  $2+2=5$ , then Sydney is in Australia!
    - if  $2+2=5$ , then Sydney is in New Zealand!
- 

- one might argue that all of these conditionals are intuitively false
  - but according to the truth table for conditional, they should be true.
  - what's missing is some kind of connection between the antecedent and the consequent.
  - but truth functions do not seem to be able to provide the relevant connection.
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- perhaps our intuitions concern the appropriateness of uttering these conditionals, not their truth values.
- perhaps they are all true, although it would be inappropriate to utter them.
- perhaps it is appropriate to utter a conditional only if there is the relevant kind of connection between the antecedent and the consequent?

## a gricean explanation

- for it to be appropriate to assert "If  $A$ , then  $C$ ", one must believe it to be true. (Maxim of Quality).
- so one must either believe (i) that  $A$  is false, or (ii) that  $C$  is true, or

iii. that both  $A$  is true and  $C$  is true.

- if (i), then asserting  $\neg A$  would be more informative than asserting "If  $A$ , then  $C$ ". So if you were to assert former rather than the latter you would violate the Maxim of Quality.
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- if (ii), then asserting  $C$  would be more informative than asserting "If  $A$ , then  $C$ ". So if you were to assert former rather than the latter you would violate the Maxim of Quality.
  - if (iii) then, since you believe that  $A$  is true, and believe "If  $A$ , then  $C$ ", you should believe that  $C$ . Then asserting  $A$  or  $C$  or  $A \vee C$  would be more informative than asserting 'If  $A$ , then  $C$ '.
  - finally, if you believe that  $B$  is false, then since you believe "If  $A$ , then  $C$ ", you should believe that  $\neg A$ . Then asserting  $\neg A$  would be more informative than asserting "If  $A$ , then  $C$ ".
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- for it to be appropriate to utter 'If  $A$ , then  $C$ ' one must not be confident that  $A$  is true or that  $B$  is false.
- yet, one must be confident that it is not the case that  $A$  is true and  $C$  is false.
- but this is plausible only if is confident that there is some kind of connection between  $A$  and  $C$ .
- so, an utterance of "If  $A$ , then  $C$ " will implicate a connection between  $A$  and  $C$ .
- this is what goes wrong in the examples on a Gricean explanation.

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- but there are problems which can't be explained away in Gricean terms
  - here's a logical entailment involving  $\rightarrow$ .
    - $(A \wedge B) \rightarrow C \models (A \rightarrow C) \vee (B \rightarrow C)$
  - and here's an ordinary language equivalent:

If you close switch A and switch B the light will go on. Hence, it is the case that either if you close switch A the light will go on, or that if you close switch B the light will go on.  
(Priest 2008 p. 14)

- this example is a problem for the Gricean explanation since we are not confident in either the antecedent or the consequent
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- here's another logical entailment involving  $\rightarrow$ .
  - $(A \rightarrow B) \wedge (C \rightarrow D) \models (A \rightarrow D) \vee (C \rightarrow B)$
- here's an ordinary language equivalent:

If John is in Paris he is in France, and if John is in London he is in England. Hence, it is the case either that if John is in Paris he is in England, or that if he is in London he is in France. (Priest 2008 p. 15)

- trouble!
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- here's is one more logical entailment involving  $\rightarrow$ .
  - $\neg(A \rightarrow B) \models A$
- here's an ordinary language equivalent:

It is not the case that if there is a good god the prayers of evil people will be answered. Hence, there is a god. (Priest 2008 p. 15)

- trouble!
- 

- this should be enough to give you a sense of the issues with translating ordinary conditionals using the material conditional
- but as I said earlier, while it is important for you to know the controversy, controversial matters won't make much of a difference to what you are doing in this course

## issues with 'only if'

### issues with 'only if'

- many students have trouble with 'only if'
  - they get the antecedent and consequent confused
  - they are tempted to translate 'A only if B' as  $B \rightarrow A$
  - let's see if we can set your thinking straight on this one
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- why translate 'A only if B' as  $A \rightarrow B$ ?
- well, we want to make sure that the following argument is valid:

P1.	John is home only if the lights are on.
P2.	John is home.
C1.	The lights are on.

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- to do so we must translate 'John is home only if the lights are on' as follows.
  - glossary:
    - $J$  : John is home.
    - $L$  : The lights are on.
  - translation:
    - $(J \rightarrow L), J \therefore L$
- 

- we are pretty certain that the argument is valid.
- so we must at least translate 'A only if B' as  $A \rightarrow B$ .
- the question is whether it *also* goes 'the other way'.
- but if it did, there would be no difference between 'John is home if and only if the lights are on' and 'John is home only if the lights are on'.
- so it can only go one way.
- and if it doesn't go the way we standardly assume, then it doesn't make the argument at issue valid.

## wrapping up

### this lecture

- issues arising with respect to translating conditional statements from English into PL

### next lecture

- lecture 13, the formal language MPL