lecture 15, the semantics of MPL (part 1)

phil1012 introductory logic

overview

this lecture

- the first of two lectures on the semantics of MPL
- an introduction to the central notion of a model
- how the truth values of MPL formulas are determined on a given model
- just the semantics of MPL for uncomplicated propositions

learning outcomes

- after doing the relevant reading for this lecture, listening to the lecture, and attending the relevant tutorial, you will be able to:
    - explain what a model consists of
    - explain what a model of a fragment of MPL consists of
    - explain how the truth values of atomic propositions are determined in MPL
    - explain how the truth values of compound propositions are determined by the truth values of their components in MPL
    - explain how the truth values of simple quantified propositions are determine in MPL
    - determine whether a proposition is true or false on a model

required reading

- sections 9.1, 9.2, and 9.3 of chapter 9

the semantics of logical languages

the semantics of logical languages

- the guiding idea behind the semantics of any logical language:
- the values of the non-logical symbols are unconstrained: for any distribution at all of values to nonlogical symbols of the language, there is a possible scenario in which these symbols have those values.
- complex expressions—in particular, propositions—have their (truth) values determined by the values of their non-logical components, together with the laws of truth governing the logical symbols.

- in PL the nonlogical symbols are basic propositions.
- the appropriate kind of value for a basic proposition is a truth value.
- a scenario is a truth table row: an assignment of truth values to basic propositions.

- the limits of PL (again) …
- there is a possible assignment of values to the basic propositions in the following argument on which the premises are all true and the conclusion false: All philosophers are drinkers. John is a philosopher. Therefore, John is a drinker.
- in MPL, these propositions are complex expressions, and their values are constrained.
- they are constrained in such a way that there is no possible assignment of values on which the premises of the argument are all true and the conclusion false.

- two questions …
  - what are the values of the nonlogical symbols of MPL?
    - possible scenario (model) will then simply be any assignment of values to nonlogical symbols (the analogue of a truth-table row).
  - what are the laws of truth that determine the truth values of propositions of MPL on the basis of the values of their components?

the semantics of atomic propositions in MPL

the semantics of atomic propositions in MPL

- the value of a proposition is its **truth value**.
- the value of a name is its **referent**.
- the value of a predicate is its **extension**.

- the value of an atomic proposition—its truth value—is determined by the value of the name—the name’s referent—and the value of its predicate—the predicate’s extension.
- an atomic proposition is true if and only if the name’s referent is in the predicate’s extension.
- the value of a name, its referent, is an **object**.
- the value of a predicate, its extension, is a **set of objects**.

- an atomic proposition is true if and only if the name’s referent, an object, is in the predicate’s extension, a set of objects.
- FaFa is true if and only if the referent of aa is in the extension of FF.

the semantics of simple quantified propositions in MPL

- ∀x_F_x_\forall \underline{x}\underline{F}\underline{x} is true if and only if everything is in the extension of F_\underline{F}.
- but what do we mean by ‘everything’ here?
- we make the idea precise by introducing the notion of a **model**.

- a **model** consists of:
  - a domain (a set of objects)—this specifies what ‘everything’ means according to the model.
  - a specification of a referent (an object) for each name.

    - a specification of an extension (a set of objects) for each predicate.

- ∀x_F_x_\forall \underline{x}\underline{F}\underline{x} is true in a model if and only if everything in the domain of the model is in the extension of F_\underline{F} on that model.
- ∃x_F_x_\exists \underline{x}\underline{F}\underline{x} is true in a model if and only if something in the domain of the model is in the extension of F_\underline{F} on that model.

- some constraints on domains . . .
- the domain of a model must be a non-empty set.
- every name and predicate in the fragment must be assigned a referent or extension.
- the extension of each predicate must be a subset of the domain of the model.

- some constraints on domains (continued) . . .
- the referent of each name must be an object in the domain of the model.
- the extension of a predicate may be the empty set.
- the extension of a predicate may be the entire domain.
- different names may be assigned the same object as a referent.
- different predicates may be assigned the same set of objects as extensions.

- open wffs do not get truth values on models. PxPx is not true or false, even once we specify an extension for PP
- this is because the variable xx does not get a referent (in a model)
- only names get referents

fragment and signature

fragment and signature

- recall the syntax of MPL.
- there were an infinite number of names and predicates.
- if we start with a nonempty set of predicates and a (possibly empty) set of names we get a **fragment** of the full language of MPL.
- the wffs of this fragment are generated from the starting set of names/predicates using the exact same rules as in the syntax of the full language MPL.
- the starting set of nonlogical symbols (names/predicates) is called the **signature** of the fragment.
- note: at least one predicate must be in the fragment so that we can generate wffs.

- a **model** of a fragment of MPL consists of:
    - a domain (a set of objects)
    - a specification of a referent (an object) for each name in the fragment
    - a specification of an extension (a set of objects) for each predicate in the fragment

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- here’s a potential model of a fragment of MPL:
  - ℳ1\mathcal{M}_1:
    - domain: {Alice, Ben, Carol}
    - referents: aa: Alice
    - extensions: PP: {Alice, Ben}
- this model meets the conditions on a model for a fragment of MPL.

- here’s a potential model of a fragment of MPL:
  - ℳ2\mathcal{M}_2:
    - domain: {Alice, Ben, Carol}
    - referents: aa: Alice, bb:
    - extensions: PP: {Alice}
- this model does not meet the conditions on a model for a fragment of MPL. why not?

- here’s a potential model of a fragment of MPL:
  - ℳ3\mathcal{M}_3:
    - domain: {Alice, Ben, Carol}
    - referents: aa: Alice
    - extensions: PP: {Alice}, QQ:
- this model does not meet the conditions on a model for a fragment of MPL. why not?

- here’s a potential model of a fragment of MPL:
  - ℳ4\mathcal{M}_4:
    - domain: {Alice, Ben, Carol}
    - referents: aa: Danny
    - extensions: PP: {Alice}
- this model does not meet the conditions on a model for a fragment of MPL. why not?

- here’s a potential model of a fragment of MPL:
  - ℳ5\mathcal{M}_5:
    - domain: {Alice, Ben, Carol}
    - referents: aa: Alice
    - extensions: PP: {Danny}
- this model does not meet the conditions on a model for a fragment of MPL. why not?

- here’s a potential model of a fragment of MPL:
  - ℳ6\mathcal{M}_6:
    - domain: {Alice, Ben, Carol}
    - referents: aa: Alice
    - extensions: PP: ∅\emptyset
- does this model meet the conditions on a model for a fragment of MPL?

- here’s a potential model of a fragment of MPL:
  - ℳ7\mathcal{M}_7:
    - domain: {Alice, Ben, Carol}
    - referents: aa: Alice
    - extensions: PP: {Alice, Ben, Carol}
- does this model meet the conditions on a model for a fragment of MPL?

- here’s a potential model of a fragment of MPL:
  - 𝓜8\mathcal{M}_8:
    - domain: {Alice, Ben, Carol}
    - referents: aa: Alice, bb: Alice
    - extensions: PP: {Alice}
- does this model meet the conditions on a model for a fragment of MPL?

- here’s a potential model of a fragment of MPL:
  - 𝓜9\mathcal{M}_9:
    - domain: {Alice, Ben, Carol}
    - referents: aa: Alice
    - extensions: PP: {Alice}, QQ: {Alice}
- does this model meet the conditions on a model for a fragment of MPL?

the semantics of connectives in MPL

the semantics of connectives in MPL

- the treatment of the semantics of connectives carries over from PL

- rule for negation
  - ¬α\lnot\alpha is true in 𝓜\mathcal{M} if and only if α\alpha is false in 𝓜\mathcal{M}

- rule for conjunction
  - (α∧β)(\alpha \land \beta) is true in 𝓜\mathcal{M} if and only if α\alpha and β\beta are true in 𝓜\mathcal{M}

- rule for disjunction
  - (α∨β)(\alpha \lor \beta) is true in 𝓜\mathcal{M} if and only if either α\alpha or β\beta or both are true in 𝓜\mathcal{M}

- rule for conditional
  - (α→β)(\alpha \rightarrow \beta) is true in 𝓜\mathcal{M} if and only if either α\alpha is false or β\beta is true or both in 𝓜\mathcal{M}

- rule for biconditional
  - (α↔β)(\alpha \leftrightarrow \beta) is true in 𝓜\mathcal{M} if and only if either both α\alpha and β\beta are true or both α\alpha and β\beta are false in 𝓜\mathcal{M}

- we have looked at simple cases of universally and existentially quantified propositions . . .

- rule for simple universally quantified propositions
  - ∀x_F_x_\forall \underline{x}\underline{F}\underline{x} is true in 𝓜\mathcal{M} if and only if everything in the domain of 𝓜\mathcal{M} is in the extension of F_\underline{F} on 𝓜\mathcal{M}

- rule for simple existentially quantified propositions
  - $\exists \underline{x}\underline{F}\underline{x}$ is true in in $\mathcal{M}$ if and only if something in the domain of $\mathcal{M}$ is in the extension of $\underline{F}$ on $\mathcal{M}$

the semantics of MPL

the semantics of MPL, stated formally

- see handout "the semantics of MPL"
- note: the following includes the general versions of the semantics for the quantifiers. We will look at this in the next lecture.

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1. $\underline{P}\underline{a}$ is true in $\mathcal{M}$ iff the referent of $\underline{a}$ in $\mathcal{M}$ is in the extension of $\underline{P}$ in $\mathcal{M}$.

2. $\lnot \alpha$ is true in $\mathcal{M}$ iff α is false in $\mathcal{M}$.

3. $(\alpha \land \beta)$ is true in $\mathcal{M}$ iff α and β are both true in $\mathcal{M}$.

4. $(\alpha \lor \beta)$ is true in $\mathcal{M}$ iff one or both of α and β is true in $\mathcal{M}$.

5. $(\alpha \rightarrow \beta)$ is true in $\mathcal{M}$ iff α is false in $\mathcal{M}$ or β is true in $\mathcal{M}$ or both.

6. $(\alpha \leftrightarrow \beta)$ is true in $\mathcal{M}$ iff α and β are both true in $\mathcal{M}$ or both false in $\mathcal{M}$.

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7. $\forall \underline{x} \alpha (\underline{x})$ is true in $\mathcal{M}$ iff for every object o in the domain of $\mathcal{M}$, $\alpha(\underline{a}/\underline{x})$ is true in $\mathcal{M}_{o}^{\underline{a}}$, where $\underline{a}$ is some name that is not assigned a referent in $\mathcal{M}$, and $\mathcal{M}_{o}^{\underline{a}}$ is a model just like $\mathcal{M}$ except that in it the name $\underline{a}$ is assigned the referent o.

8. $\exists\underline{x} \alpha (\underline{x})$ is true in $\mathcal{M}$ iff there is at least one object o in the domain of $\mathcal{M}$ such that $\alpha(\underline{a}/\underline{x})$ is true in $\mathcal{M}_{o}^{\underline{a}}$, where $\underline{a}$ is some name that is not assigned a referent in $\mathcal{M}$, and $\mathcal{M}_{o}^{\underline{a}}$ is a model just like $\mathcal{M}$ except that in it the name $\underline{a}$ is assigned the referent o.

wrapping up

this lecture

- in this lecture we looked at the semantics for uncomplicated propositions in MPL
- we introduced models in order to provide the semantics for MPL

next lecture

- lecture 16, the semantics of MPL, part 2