

# lecture 07, validity and logical form

phil1012 introductory logic

## overview

### this lecture

- in the last few lectures we've been examining the semantics of PL and truth tables
- in this lecture we look in more detail at the idea of the **logical form** of a proposition
- recall that logical form was central to the definition of **validity**

### learning outcomes

- after doing the relevant reading for this lecture, listening to the lecture, and attending the relevant tutorial, you will be able to:
  - identify the forms of propositions
  - identify instances of forms
  - identify the forms of arguments
  - identify instances of argument forms
  - explain how truth tables provide a test not only for necessary truth preservation but for validity

### required reading

- all of chapter 5

## validity and logical form

### validity and logical form

- recall: an argument is **valid** if and only if, **in virtue of the form of the argument**, it is impossible for its conclusion to be false if its premises are all true
- our truth table test tells us whether it is possible for the conclusion to be false if all of the premises are true
- does it tell us whether this is in virtue of the form of the argument?
- it turns out that the answer is yes, since we can show that every argument with the same form is valid

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- practical upshot: once we know that a given argument is valid, we know that every other argument of the same form is valid, without having to do truth tables for them individually

### the form of an argument

- what is the form of an argument?
- suppose we set out to test whether the following argument is

valid:

**P1.** John is a philosopher  
**P2.** if John is a philosopher, then he drinks  
**C1.** John drinks

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- we begin by writing up our glossary:
    - PP: John is a philosopher
    - DD: John drinks
  - we translate accordingly:
    - PP,  $(P \rightarrow D) (P \rightarrow D) \therefore D$
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- we use a truth table to test for validity:

| PP | DD | PP | $(P \rightarrow D)$ | $(P \rightarrow D)$ | DD |
|----|----|----|---------------------|---------------------|----|
| T  | T  | T  |                     |                     | T  |
| T  | F  | T  |                     |                     | F  |
| F  | T  | F  |                     |                     | T  |
| F  | F  | F  |                     |                     | F  |

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- great, now suppose we set out to test whether the following argument is valid:

**P1.** Jane is a philosopher  
**P2.** if Jane is a philosopher, then she smokes  
**C1.** Jane smokes

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- we begin by writing up our glossary:
    - JJ: Jane is a philosopher
    - SS: Jane smokes
  - we translate accordingly:
    - JJ,  $(J \rightarrow S) (J \rightarrow S) \therefore S$
- 

- and we use a truth table to test for validity:

| JJ | SS | JJ | $(J \rightarrow S)$ | $(J \rightarrow S)$ | SS |
|----|----|----|---------------------|---------------------|----|
| T  | T  | T  |                     |                     | T  |
| T  | F  | T  |                     |                     | F  |
| F  | T  | F  |                     |                     | T  |
| F  | F  | F  |                     |                     | F  |

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- after a while we might start to feel that the whole process is a bit laborious and redundant
- don't these arguments have something in common which means that if we know that one is valid, we know the other is, and vice versa?
- yep, these arguments are instances of the same **form**
- how can we make the notion of form more precise?

## abstracting from content: from propositions to forms

### from propositions to forms

- consider the following proposition:
  - $(A \wedge (B \rightarrow \neg C)) (A \wedge (B \rightarrow \neg C))$
- what is its form?
- there are many correct answers ranging from very course-grained to very fine-grained

- at its most course-grained it is just a proposition:
  - $(A \wedge (B \rightarrow \neg C))$
- so it has the following form:  $\alpha$

- at a more fine-grained level of description it is a conjunction:
  - $(A \wedge (B \rightarrow \neg C))$
- so it has the following form:  $(\alpha \wedge \beta)$

- at an even more fine-grained level of description it is a conjunction whose second conjunct is a conditional:
  - $(A \wedge (B \rightarrow \neg C))$
- so it has the following form:
  - $(\alpha \wedge (\beta \rightarrow \gamma))$

- at the most fine-grained level of description it is a conjunction whose second conjunct is a conditional, whose consequent is a negation:
  - $(A \wedge (B \rightarrow \neg C))$
- so it has the following form:
  - $(\alpha \wedge (\beta \rightarrow \neg \gamma))$

- propositions do not have a single form
- they have many forms ranging from the course-grained to the fine-grained

## instances: from forms to propositions

### instances: from forms to propositions

- consider the following logical form:
  - $(\alpha \leftrightarrow (\neg \beta \wedge \alpha))$
- a **logical form** is like a formula, except that in place of basic propositions it has variables in the place of formulas
- given a logical form, we can ask: what propositions have this form? We call such propositions **instances** of the form

- an instance of a logical form can be obtained by replacing the variables with propositions
- all occurrences of the same variable must be replaced by the same proposition

- for example . . .

- $(\alpha \leftrightarrow (\neg \beta \wedge \alpha)) (\alpha \leftrightarrow (\neg \beta \wedge \alpha)) (\alpha \leftrightarrow (\neg \beta \wedge \alpha))$ 
  - $(A \leftrightarrow (\neg B \wedge A)) (A \leftrightarrow (\neg B \wedge A)) (A \leftrightarrow (\neg B \wedge A))$
  - $(B \leftrightarrow (\neg E \wedge B)) (B \leftrightarrow (\neg E \wedge B)) (B \leftrightarrow (\neg E \wedge B))$
  - $((A \rightarrow B) \leftrightarrow (\neg E \wedge (A \rightarrow B))) ((A \rightarrow B) \leftrightarrow (\neg E \wedge (A \rightarrow B))) ((A \rightarrow B) \leftrightarrow (\neg E \wedge (A \rightarrow B)))$
  - $(B \leftrightarrow (\neg B \wedge B)) (B \leftrightarrow (\neg B \wedge B)) (B \leftrightarrow (\neg B \wedge B))$
  - $((A \rightarrow B) \leftrightarrow (\neg (A \rightarrow B) \wedge (A \rightarrow B))) ((A \rightarrow B) \leftrightarrow (\neg (A \rightarrow B) \wedge (A \rightarrow B))) ((A \rightarrow B) \leftrightarrow (\neg (A \rightarrow B) \wedge (A \rightarrow B)))$

- but not . . .
  - $(\alpha \leftrightarrow (\neg \beta \wedge \alpha)) (\alpha \leftrightarrow (\neg \beta \wedge \alpha)) (\alpha \leftrightarrow (\neg \beta \wedge \alpha))$ 
    - $(A \leftrightarrow (\neg B \wedge B)) (A \leftrightarrow (\neg B \wedge B)) (A \leftrightarrow (\neg B \wedge B))$
    - $(B \leftrightarrow (\neg E \wedge D)) (B \leftrightarrow (\neg E \wedge D)) (B \leftrightarrow (\neg E \wedge D))$
    - $((A \rightarrow B) \leftrightarrow (\neg E \wedge B)) ((A \rightarrow B) \leftrightarrow (\neg E \wedge B)) ((A \rightarrow B) \leftrightarrow (\neg E \wedge B))$

- two propositions share a form when they are both instances of it
- it doesn't make sense to say that they have the same form, they may both be instances of some form and not both instances of some other form

- okay, now we have a better idea of the forms of a proposition
- and we have a better idea of the propositions which are instances of forms
- we can now generalise this idea to the forms of an argument

## the form(s) of an argument

### the form(s) of an argument

- consider the following argument form:
  - $\alpha \wedge \alpha, (\alpha \rightarrow (\beta \rightarrow \alpha)) (\alpha \rightarrow (\beta \rightarrow \alpha)) (\alpha \rightarrow (\beta \rightarrow \alpha)) \therefore \beta \rightarrow \alpha$
- an **argument form** is just like an argument except that, in place of basic propositions, it has variables in the place of propositions
- given an argument form, we can ask: what arguments have this form?
- we call such arguments **instances** of the form

- for example . . .
  - $\alpha \wedge \alpha, \alpha \rightarrow (\beta \rightarrow \alpha) \therefore \beta \rightarrow \alpha$ 
    - $PP, P \rightarrow (Q \rightarrow P) \therefore Q \rightarrow P$
    - $(RV \rightarrow Q) (R \vee \neg Q), (RV \rightarrow Q) \rightarrow (P \rightarrow (RV \rightarrow Q)) (R \vee \neg Q) \therefore P \rightarrow (RV \rightarrow Q)$

- consider the following argument:
  - $PP, (P \rightarrow (Q \rightarrow P)) (P \rightarrow (Q \rightarrow P)) \therefore Q \rightarrow P$
- an argument can be an instance of more than one form

- for example . . .
  - $PP, (P \rightarrow (Q \rightarrow P)) (P \rightarrow (Q \rightarrow P)) \therefore Q \rightarrow P$

$Q \rightarrow P$   
 •  $\alpha \rightarrow (\beta \rightarrow \alpha) \rightarrow (\alpha \rightarrow \beta) \rightarrow \alpha$   
 •  $\alpha \rightarrow (\beta \rightarrow \alpha) \rightarrow (\beta \rightarrow \alpha) \rightarrow \alpha$   
 •  $\alpha \rightarrow (\beta \rightarrow \alpha) \rightarrow (\beta \rightarrow \alpha) \rightarrow \alpha$   
 •  $\alpha \rightarrow (\beta \rightarrow \alpha) \rightarrow (\beta \rightarrow \alpha) \rightarrow \alpha$   
 •  $\alpha \rightarrow (\beta \rightarrow \alpha) \rightarrow (\beta \rightarrow \alpha) \rightarrow \alpha$

- an argument can be an instance of more than one form
- it is important to keep this in mind when we discuss validity and form

## validity and form

### validity and form

- now that we have a precise understanding of the forms of an argument, to what use can we put the idea?
- well, we can use it to make our notion of validity more precise
- and we can appeal to the form of an argument in arguing that it is valid
- we begin by introducing notions analogous to validity and invalidity, which are properties of arguments, for argument forms

- we define the properties **valid\*** and **invalid\*** of **argument forms**
- these are distinct from the properties **valid** and **invalid** of **arguments**

- an argument form is **valid\*** if and only if there is no row in which the premises are true and the conclusion false
- an argument form is **valid\*** if and only if there is no instance of the form in which the premises are true and the conclusion false
- an argument form is **invalid\*** if and only if there is a row in which the premises are true and the conclusion false

- we test for **validity\*** of an argument form using truth tables as before

| $\alpha$ | $\beta$ | $\alpha \rightarrow \beta$ | $(\alpha \rightarrow \beta) \rightarrow \alpha$ | $\beta$ |
|----------|---------|----------------------------|---|---------|
| T        | T       | T                          | T   | T       |
| T        | F       | F                          | F   | F       |
| F        | T       | T                          | T   | T       |
| F        | F       | T                          | T   | F       |

- now, a very nice feature of **valid\*** argument forms is that every instance of a **valid\*** argument form is valid argument

- consider the following argument:
  - $((A \rightarrow (B \vee D)) \rightarrow E) \rightarrow (A \rightarrow (B \vee D))$   
 $\rightarrow E$ ,  $(A \rightarrow (B \vee D)) \rightarrow E$ ,  $\therefore (A \rightarrow (B \vee D))$
- we need a truth table with 32 rows since we have five distinct basic propositions!

- but wait! this argument is an instance of the form:

- $(\alpha \rightarrow \beta) (\alpha \rightarrow \beta), \alpha \therefore \beta$
- and this form is **valid\*** (we just did the truth table for it)
- so we can immediately conclude that the argument is valid

- hint: unless stated otherwise, it is *always* permissible to save yourself time on a problem set or an exam by appealing to the form of an argument in proving its validity
- we will look at examples in the tutorials

## invalidity and form

### invalidity and form

- careful: it is not the case that every instance of an **invalid\*** argument form is an invalid argument
- the following is an **invalid\*** argument form:
  - $\beta \rightarrow \alpha, \alpha \therefore \beta$

- but the following is an instance of the form:
  - $(A \rightarrow A) (A \rightarrow A), A \therefore A$
- and this argument is valid
- we can see this by doing the truth table, or by noting that this is *also* an instance of a valid form

- indeed, we can distinguish between **A-properties** and **S-properties** of arguments and argument forms
- every instance of a form with an A-property has that A-property. not so with the S-properties.

| A-property       | S-property        |
|------------------|-------------------|
| validity         | invalidity        |
| logical truth    | non-logical truth |
| equivalence      | inequivalence     |
| unsatisfiability | satisfiability    |

- in other words, there are shortcuts available for establishing A-properties, but no shortcuts for S-properties

## notable argument forms

### notable argument forms

- now that we've introduced the idea of a **valid\*** argument form, and we've seen how to appeal to valid argument forms in establishing the validity of an argument, you should be on the lookout for shortcuts
- here are some **valid\*** argument forms to be on the lookout for

- modus ponens
  - $(\alpha \rightarrow \beta) \alpha \therefore \beta$
- modus tollens
  - $(\alpha \rightarrow \beta) \neg \beta \therefore \neg \alpha$

- hypothetical syllogism
  - $(\alpha \rightarrow \beta), (\beta \rightarrow \gamma) \therefore \alpha \rightarrow \gamma$
- constructive dilemma
  - $(\alpha \rightarrow \beta), (\gamma \rightarrow \delta), (\alpha \vee \gamma) \therefore (\beta \vee \delta)$
- disjunctive syllogism
  - $(\alpha \vee \beta), \neg \alpha \therefore \beta$

## wrapping up

### this lecture

- validity and logical form
- the forms of propositions
- the instances of forms
- valid\* argument forms

### next lecture

- lecture 08, functional completeness