

# lecture 03, the formal language PL

phil1012 introductory logic

## overview

### this lecture

- an introduction to the formal language PL
- the vocabulary and syntax of PL
- how formulas of PL are constructed
- translating propositions from English into PL with the use of a glossary

### learning outcomes

- after doing the relevant reading for this lecture, listening to the lecture, and attending the relevant tutorial, you will be able to:
  - explain what the different symbols of PL mean
  - translate PL formulas into English
  - translate propositions from English into PL, providing a glossary for your translation
  - identify well-formed formulas of PL
  - construct construction tables for formulas of PL

### required reading

- all of chapter 2

## the language of propositional logic (PL)

### the language of propositional logic (PL)

- the formal symbolic language of propositional logic (PL)
- the language of PL consists of a **vocabulary** and a **syntax**
- the vocabulary concerns the basic symbols
- the syntax concerns how basic symbols can be put together to form complex symbols

## the vocabulary of PL

### the vocabulary of PL

- basic propositions are represented in PL by simple capital letters with or without numerical subscripts
  - they are called **sentence letters**
    - $A, A_2, A_3, \dots, B, B_2, B_3, \dots, C, C_2, C_3, \dots, Z, Z_2, Z_3, A, \underline{A}_2, \underline{A}_3, \dots, B, \underline{B}_2, \underline{B}_3, \dots, C, \underline{C}_2, \underline{C}_3, \dots, Z, \underline{Z}_2, \underline{Z}_3$
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- suppose you want to use 'A' to represent the proposition expressed by the sentence 'John is short'
- in order to do this you would write a **glossary** like this:
  - AA: John is short

- we use sentence letters only for basic propositions. so the following is not okay:
  - AA: John is short and Jane is tall
- remember that a negation is not a basic proposition. so the following is not okay:
  - AA: John is not short

- we have introduced symbols to represent propositions (sentence letters)
- now we introduce symbols to represent the **connectives**
  - $\neg, \wedge, \vee, \rightarrow, \leftrightarrow$  \not, \land, \lor, \rightarrow, \leftrightarrow

## conjunction

- the connective conjunction is represented in PL by the symbol:
  - $\wedge$  \land
- the name of the symbol is **caret**

- we can use caret to represent the conjunction of AA and BB as follows:
  - $(A \wedge B)$  (A \land B)
- we can use this to represent the proposition that John is short and Jane is tall

## disjunction

- the connective disjunction is represented in PL by the symbol:
  - $\vee$  \lor
- the name of the symbol is **vel**

- we can use vel to represent the disjunction of AA and BB as follows:
  - $(A \vee B)$  (A \lor B)
- we can use this to represent the proposition that John is short or Jane is tall

## conditional

- the connective conditional is represented in PL by the symbol:
  - $\rightarrow$  \rightarrow
- the name of the symbol is **arrow**

- we can use arrow to represent a conditional with AA as its antecedent and BB as its consequent as follows:
  - $(A \rightarrow B)$  (A \rightarrow B)
- we can use this to represent the proposition that if John is short, then Jane is tall

## biconditional

- the connective biconditional is represented in PL by the symbol:
  - $\leftrightarrow$
- the name of the symbol is **double arrow**

- we can use double arrow to represent a biconditional with AA as its left-hand side and BB as its right-hand side as follows:
  - $(A \leftrightarrow B)$
- we can use this to represent the proposition that John is short if and only if Jane is tall

## negation

- the connective negation is represented in PL by the symbol:
  - $\neg$
- the name of the symbol is **neg**

- we can use neg to represent a negation with AA as its negand as follows:
  - $\neg A$
- we can use this to represent the proposition that John is not tall

## the connectives of PL

- here are the symbols for the connectives in PL:

connective	name	symbol
negation	neg	$\neg$
conjunction	caret	$\wedge$
disjunction	vel	$\vee$
conditional	arrow	$\rightarrow$
biconditional	double arrow	$\leftrightarrow$

- here are some alternative symbols for the connectives

connective	symbol
negation	$\sim$
conjunction	$\&$
disjunction	$\vee$
conditional	$\rightarrow$
biconditional	$\leftrightarrow$

- note! although the choice of symbols for the connectives is entirely arbitrary, the choice we have made is final, and henceforth we will only use these symbols

- we now have the most important parts of the vocabulary of PL: sentence letters, connective symbols, and punctuation symbols '(' and ')'
- now we need to say how these can be put together

## the syntax of PL

## the syntax of PL

- we are now ready to precisely state the syntax of PL
  - see the handout on the syntax of PL
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- we use lowercase Greek letters like  $\alpha$  and  $\beta$  to express general claims about the formulas of PL
  - these are called **well-formed formula variables**
  - well-formed formula variables can stand in for single sentence letters like  $A$  or for complex formulas like  $(A \wedge B)$
  - they are not part of the language of PL
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- the symbols of PL are:
    - sentence letters (basic propositions):
      - $A, A_2, A_3, \dots, B, B_2, B_3, \dots, C, C_2, C_3, \dots, Z, Z_2, Z_3$
    - connectives:
      - $\neg, \wedge, \vee, \rightarrow, \leftrightarrow$
    - punctuation symbols (parentheses):
      - $(, )$
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- wffs of PL are defined as follows:
    - any basic proposition is a wff
    - if  $\alpha$  and  $\beta$  are wffs, then so are:
      - $\neg\alpha$
      - $(\alpha \wedge \beta)$
      - $(\alpha \vee \beta)$
      - $(\alpha \rightarrow \beta)$
      - $(\alpha \leftrightarrow \beta)$
    - nothing else is a wff
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- the syntax of PL is specified by a **recursive definition**
  - the **base clause** of the definition states certain things are wffs
  - the **recursive clause** states that certain other things are wffs in terms of things we already know to be wffs
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- all of the following can be generated using this definition:
    - $((R \rightarrow P) \leftrightarrow R)$
    - $(B \leftrightarrow ((C \wedge A) \vee C))$
    - $\neg(B \vee (B \leftrightarrow A))$
    - $((H \rightarrow G) \rightarrow (H \leftrightarrow (G \vee H)))$
    - $((Z \vee ((Y \vee Y) \wedge (Z \rightarrow Y))) \vee Y)$
    - $((C \vee ((G \leftrightarrow (D \wedge A)) \rightarrow A)) \wedge ((D \leftrightarrow J) \leftrightarrow J))$
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- the symbols of PL can be divided into the following categories:
  - **logical symbols** (the connective symbols)
  - **nonlogical symbols** (sentence letters)
  - **auxiliary symbols** (parentheses)
- the nonlogical symbols do not have a fixed meaning and need to be specified in a glossary

- this is why you must always provide a glossary for your translations

- given this syntax for PL we can show how any well-formed formula of PL is constructed
- suppose we want to construct  $(\neg P \wedge (Q \vee R))$
- we might construct it as follows using a **construction table**

step	wff constructed at this step	from steps/by clause
1	PP	/(2i)
2	QQ	/(2i)
3	RR	/(2i)

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5	$(Q \vee R)$	2,3 / (2ii $\vee$ )

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5	$(Q \vee R)$	2,3 / (2ii $\vee$ )
6	$(\neg P \wedge (Q \vee R))$	4, 5 / (2ii $\wedge$ )

- the other wffs in the construction are **subformulas** of the formula
- the connective added last in the construction is the **main connective**
- the main connectives gives its name to the formula it constructs

## wrapping up

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### next lectures

- lecture 4, issues in translation: assertability and implicature
- lecture 5, the semantics of PL