lecture 05, the semantics of PL

phil1012 introductory logic

overview

this lecture

- the semantics of PL
- truth tables for the connectives
- truth tables and truth functions
- how the truth value of a compound proposition is determined by the truth values of its components given an assignment of truth values to basic propositions

learning outcomes

- after doing the relevant reading for this lecture, listening to the lecture, and attending the relevant tutorial, you will be able to:
  - produce the truth tables for the connectives negation, conjunction, disjunction, conditional, and biconditional from memory
  - determine the truth values of PL formulas given an assignment of truth values for their basic components
  - explain the general notion of a truth function and relate it to the truth tables for the connectives

required reading

- all of chapter 3

the semantics of PL

the semantics of PL

- we state the semantics in terms of truth tables for the connectives
- recall the idea behind **truth functional connectives**: when they are used to make a compound proposition from other propositions, we can be sure that the truth of the compound expression is determined by the truth or falsity of the propositions it is made out of
- the semantics of PL is going to capture this idea

a guiding assumption

- at the outset, we assume that every proposition is either true of false and not both true and false
- this assumption is known as **bivalence**

- given that every proposition is either true or false and not both true and false, we only need to consider two ‘cases’ for every proposition: the case in which it is true, and the case in which it is false

- the semantics for truth-functional connectives should tell us what the truth value of a compound proposition is for each possible combination of truth values for the propositions the connectives connect
- we represent these possibilities using **truth tables**

negation

- the truth table for **negation**
- if the negand is true, the negation is false, and if the negand is false, the negation is true
- if PP is true, ¬P\lnot P is false, and if PP is false, ¬P\lnot P is true
- more generally, using well-formed formula variables:

α\alpha	¬\lnotα\alpha
T	F
F	T

- we only need to consider two cases for negation
- graphically, we can think of the two cases like this:

- when the ‘input’ is true:

- the ‘output’ is false

- when the ‘input’ is false:

- the ‘output’ is false

- okay, so much for negation. negation is pretty straightforward

conjunction

- the truth table for **conjunction**
- if both the conjuncts are true, the conjunction is true. if either or both are false, then the conjunction is false
- if AA and BB are true, (A∧B(A\land B) is true. If either AA or BB is false, (A∧B)(A\land B) is false.
- more generally, using well-formed formula variables:

α\alpha	β\beta	(α∧β)(\alpha\land\beta)
T	T	T
T	F	F
F	T	F
F	F	F

- we have to consider 4 cases for conjunction
- graphically, we can think of the four cases like this:

- when both ‘inputs’ are true:

- the ‘output’ is true

- when the first ‘input’ is true, and the second is false:

- the ‘output’ is false

- when the first ‘input’ is false, and the second is true:

- the ‘output’ is false

- when both ‘inputs’ are false:

- the ‘output’ is false

- okay, so the relevant case to keep in mind for conjunction is the case where both conjuncts are true
- only then is the conjunction itself true

disjunction

- the truth table for **disjunction**
- if either disjunct is true or both disjuncts are true, the disjunction is true. if both disjuncts are false, the disjunction is false.
- if either AA or BB is true, (A∨B)(A \lor B) is true. if both AA and BB are false, (A∨B)(A \lor B) is false.
- more generally, using well-formed formula variables:

α\alpha	β\beta	(α∨β)(\alpha \lor \beta)
T	T	T
T	F	T
F	T	T
F	F	F

- we have to consider 4 cases for disjunction
- graphically, we can think of the four cases like this:

- when both ‘inputs’ are true:

- the ‘output’ is true

- when the first ‘input’ is true, and the second is false:

- the 'output' is true

- when the first 'input' is false, and the second is true:

- the 'output' is true

- when both 'inputs' are false:

- the 'output' is false

- okay, so the relevant case to keep in mind for disjunction is the case where both disjuncts are false
- only then is a disjunction false

conditional

- the truth table for **conditional**
- if the antecedent is true and the consequent is false then the conditional is false. otherwise it is true.
- if AA is true and BB is false then (A→B)(A \rightarrow B) is false. otherwise (A→B)(A \rightarrow B) is true.
- more generally, using well-formed formula variables:

α\alpha	β\beta	(α→β)(\alpha \rightarrow \beta)
T	T	T
T	F	F
F	T	T
F	F	T

- we have to consider 4 cases for conditional
- graphically, we can think of the four cases like this:

- when both 'inputs' are true:

- the 'output' is true

- when the first 'input' is true, and the second is false:

- the 'output' is false

- when the first 'input' is false, and the second is true:

- the 'output' is true

- when both 'inputs' are false:

- the ‘output’ is true

- okay, so the relevant case to keep in mind for condition is the case where the antecedent is true and the consequent is false, the case where the first input is true, and the second input is false
- only then is a conditional false

biconditional

- the truth table for **biconditional**
- if the right hand side and the left hand side are either both true or both false, then the biconditional is true. it is false otherwise.
- if AA and BB are either both true or both false, then (A↔B)(A \leftrightarrow B) is true. otherwise (A↔B)(A \leftrightarrow B) is false.
- more generally, using well-formed formula variables:

α\alpha	β\beta	(α↔β)(\alpha \leftrightarrow \beta)
T	T	T
T	F	F
F	T	F
F	F	T

- we have to consider 4 cases for biconditional
- graphically, we can think of the four cases like this:

- when both ‘inputs’ are true:

- the ‘output’ is true

- when the first ‘input’ is true, and the second is false:

- the ‘output’ is false

- when the first ‘input’ is false, and the second is true:

- the ‘output’ is false

- when both ‘inputs’ are false:

- the ‘output’ is true

- okay, so the relevant cases to keep in mind for biconditional are the cases where both inputs are true and both inputs are false

- only in these cases is a biconditional true

- we will discuss the plausibility of these assumptions about the meanings of the connectives, and their relation to ordinary English connectives, later

connectives and truth functions

connectives and truth functions

- okay, so we’ve given the semantics of the connectives of PL using truth tables
- but what exactly have we achieved?
- to see this, let’s look at the idea of a truth-function

- the general idea of a truth function is this . . .
- a **truth-function** is a function on {T, F}, that is, a function whose inputs and outputs are truth values

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- there are ever so many truth functions
- for this reason it is helpful to have a naming convention
- we name functions as follows: f_1^1, f_2^1, . . ., f_1^2, f_2^2 . . . .
  - superscripts represent the number of places of the function (one-place, two-place, and so on)
  - subscripts represent index numbers to distinguish between different functions with the same number of places

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- some one-place truth functions

input	**output of f_1^1**	**output of f_2^1**
T	T	F
F	T	T

- the matrix for f_1^2 should ring a bell. Consider the truth table for $\lnot$ again:

α	$\lnot\alpha$
T	F
F	T

- truth tables effectively do two things at once: (i) they define a truth function and (ii) state that the truth function is the meaning of the connective
- we can separate the two steps and (i) define a truth function in the way above and (ii) state that the truth function is the meaning of some connective

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- some two-place truth functions

input	**output of f_1^2**	**output of f_2^2**	**output of f_3^2**
(T,T)	T	F	T

input	output of f12f_1^2	output of f22f_2^2	output of f32f_3^2
(T,F)	F	F	T
(F,T)	F	T	T
(F,F)	T	F	F

- the matrix for f32f_3^2 should ring a bell. consider the truth table for ∨\lor again:

α\alpha	β\beta	(α(\alpha∨\lorβ)\beta)
T	T	T
T	F	T
F	T	T
F	F	F

- so, instead of doing two things at once with truth tables for connectives, we could instead define a truth function and then declare that the truth function is the meaning of the connective
- finally, we are able to say more precisely what a truth functional connective is: it is any connective whose meaning can be specified as a truth function

truth values of complex propositions

truth values of complex propositions

- consider the following complex proposition
  - (¬P∧(Q∨R))(\lnot P \land (Q \lor R))
- is this true if PP is true, QQ is false, and RR is false?
- we call this an **assignment** of truth values to basic propositions
- let’s work through the example together

- each step corresponds to a step in the construction table for the formula
- we apply the semantic rule for the relevant connective

wrapping up

this lecture

- we looked at the semantics of PL
- we looked at the general idea of a truth function

next lecture

- lecture 6, uses of truth tables