

lecture 13, the formal language MPL

phil1012 introductory logic

overview

this lecture

- an introduction to the formal language MPL
- the limitations of PL and the motivation for MPL
- an introduction to the notions of names, predicates, variables, and quantifiers
- translation of propositions into MPL

learning outcomes

- after doing the relevant reading for this lecture, listening to the lecture, and attending the relevant tutorial, you will be able to:
 - explain what MPL allows us to do that we cannot do in PL
 - translate propositions with a simple subject-predicate form into MPL
 - translate existential and universal propositions into MPL
 - translate propositions involving restricted quantification into MPL

required reading

- sections 8.1, 8.2, 8.3 of chapter 8

beyond propositional logic

beyond propositional logic

- propositional logic is great
- but there are some arguments which seem to be such that, in virtue of their form, their premises can't all be true and their conclusion false

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- consider the following argument

P1.		all philosophers are drinkers
P2.		John is a philosopher
<hr/>		
C1.		John is a drinker

- a valid argument

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- let's translate the argument into PL
 - we have the following glossary
 - PP: all philosophers are drinkers
 - JJ: John is a philosopher
 - DD: John is a drinker
 - and the following translation

- PP, JJ, ∴D\therefore D
- but this argument is invalid in PL!

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- we must go beyond PL!
 - for the remainder of the course we will go beyond PL three times over
 - MPL: names, one-place predicates, quantifiers, and variables.
 - GPL: names, many-place predicates, quantifiers, and variables.
 - GPLI: names, many-place predicates, quantifiers, variables, and the identity predicate.
 - for now: MPL

names and predicates

names and predicates

- introducing names and predicates
- consider: 'John is a philosopher'
 - this expresses a basic proposition. but let's now take a look at the constituents of this sentence
- there is a **name** 'John', which **refers** to an individual thing, namely John
- there is a **predicate**, 'is a philosopher' which is **satisfied** by certain individual things if and only if they have the property of being a philosopher

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- the symbol for a **name** in MPL is a lowercase letter, aa, bb, cc, dd, ee, . . . , rr, ss, tt
 - not including the last six letters in the alphabet, uu, vv, ww, xx, yy, zz (these will be used for something else)
 - the symbol for a **predicate** in MPL is an uppercase letter, AA, BB, CC, DD, . . . , XX, YY, ZZ (no reservations here.)
 - don't confuse sentences letters from PL with predicate letters from MPL!
 - and note that there are no sentence letters in PL!

glossaries in MPL (preliminary)

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- glossaries in MPL work like glossaries in PL
 - they give meanings for the non-logical symbols of MPL
 - unlike PL whose non-logical symbols were sentence letters, the non-logical symbols of MPL are names and predicates
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- we give the meanings of names in our MPL glossaries like this:
 - bb: Bruce
 - jj: Jane
 - mm: *Australian Materialism*
 - aa: The author of *Australian Materialism*
 - we give the meanings of predicates in our MPL glossaries like this:
 - PP: is a philosopher
 - BB: is a book
 - this is provisional. we'll see a better way of doing glossaries

from MPL below.

atomic propositions

atomic propositions

- an **atomic proposition** is a proposition made up from one name and one predicate.
- we translate the atomic proposition 'Bruce is a philosopher' like this:
 - $PbPb$
- we translate the atomic proposition 'The author of *Australian Materialism* is a philosopher' like this:
 - $PaPa$
- we translate the atomic proposition '*Australian Materialism* is a book' like this:
 - $BmBm$

connectives in MPL

- we retain all the connectives of PL in MPL.
- we translate a disjunction like 'Bruce is a philosopher or Jane is a philosopher' like this:
 - $(Pb \vee Pj) (Pb \text{ \texttt{\textbackslash}lor} Pj)$
- we translate a conjunction like 'Bruce is a philosopher and Jane is a philosopher' like this:
 - $(Pb \wedge Pj) (Pb \text{ \texttt{\textbackslash}land} Pj)$

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- we translate a conditional like 'If Bruce is a philosopher, then Jane is a philosopher' like this:
 - $Pb \rightarrow Pj Pb \text{ \texttt{\textbackslash}rightarrow} Pj$
 - we translate a biconditional like 'Bruce is a philosopher if and only if Jane is a philosopher' like this:
 - $(Pb \leftrightarrow Pj) (Pb \text{ \texttt{\textbackslash}leftrightarrow} Pj)$
 - we translate a negation like 'Bruce is not a philosopher' like this:
 - $\neg Pb \text{ \texttt{\textbackslash}not} Pb$

variables and quantifiers

- introducing **variables** and **quantifiers**
- how should we translate the following?
 - everyone is a philosopher
 - someone is a philosopher
 - so-one is a philosopher

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- not like this:
 - here's my glossary:
 - ee: everyone
 - ss: someone
 - nn: no-one
 - here's my translation:
 - ee is a philosopher
 - ss is a philosopher
 - nn is a philosopher
 - this is wrong!

- expressions like 'everyone', 'someone', and 'no-one' are not names.

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- expressions like 'someone' and 'everyone' are **quantifiers**
 - 'someone' is an **existential quantifier**
 - 'everyone' is a **universal quantifier**
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- how should we think about quantified propositions then? what do they say?
 - well, 'Everything is a philosopher' says, roughly, that every *thing* is such that *it* is a philosopher
 - and 'Something is a philosopher' says, roughly, that some *thing* is such that *it* is a philosopher
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- to put it slightly more formally (but still informally):
 - well, 'Everything is a philosopher' says, roughly, that every x is such that x is a philosopher
 - and 'Something is a philosopher' says, roughly, that some x is such that x is a philosopher
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- we use the variable x in this informal presentation.
 - in MPL the symbols for *individual variables* are uu, vv, ww, xx, yy, zz
 - using variables, we can get closer to what we are after.
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- given our glossary for 'is a philosopher' we have:
 - every xx is such that $PxPx$
 - some xx is such that $PxPx$
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- we introduce two new symbols in MPL which mean 'every xx is such that' and 'some xx is such that'
 - $\forall xPx$ (universal quantifier)
 - $\exists xPx$ (existential quantifier)
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- using quantifiers and variables we can translate 'Everything is a philosopher' and 'Something is a philosopher' as follows:
 - everything is a philosopher
 - $\forall xPx$
 - something is a philosopher
 - $\exists xPx$
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- okay, we are getting somewhere
- but we set out to translate the following
 - everyone is a philosopher
 - someone is a philosopher
 - no-one is a philosopher
- 'everything is a philosopher' doesn't mean the same thing as 'everyone is a philosopher'
- things get just a little bit more complicated
- but before turning to this complication . . .

glossaries in MPL (official)

- with the introduction of variables, here is a new convention for writing glossaries for predicates
 - PPxx: xx is a philosopher
 - BBxx: xx is a book
- notice how one variable occurs before the colon and immediately after the predicate letter, and another variable occurs after it in the place in the sentence where a name might go

restricted quantification

restricted quantification

- back to the complication . . .
- how should we translate 'every philosopher is a drinker' and 'some philosopher is a drinker'?
- let's think it through carefully

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- think about 'every philosopher is a drinker'
 - you can think of it as saying that every x is such that if x is a philosopher, then x is a drinker.
 - so we translate it as follows:
 - $\forall x(Px \rightarrow Dx)$ \forall x (Px \rightarrow Dx)
 - where:
 - Dx: xx is a person
 - Px: xx is a philosopher
 - you can think of the predicate in the antecedent of the conditional as 'restricting' the things over which we are quantifying
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- now think about 'Some philosopher is a drinker'
 - you can think of it as saying that some x is such that x is a philosopher and x is a drinker
 - so we translate it as follows:
 - $\exists x(Px \wedge Dx)$ \exists x (Px \wedge Dx)
 - where:
 - Dx: xx is a person
 - Px: xx is a philosopher
 - you can think of the predicate in the first conjunct of the conjunction as 'restricting' the things over which we are quantifying
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- notice that for restricted universal quantification we use a conditional, and the antecedent of the conditional does the restricting, and that for restricted existential quantification we use a conjunction, and one conjunct does the restricting.
- don't confuse the two
- this says that everything is a philosopher and a drinker:
 - $\forall x(Px \wedge Dx)$ \forall x (Px \wedge Dx)
- and this says that there is something which is either not a philosopher or is a drinker.
 - $\exists x(Px \rightarrow Dx)$ \exists x (Px \rightarrow Dx)
- it is true if anything is not a philosopher or if anything is a drinker!

wrapping up

this lecture

- an introduction to the formal language MPL
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next lecture

- lecture 14, the syntax of MPL