

lecture 21, the formal language GPLI

phil1012 introductory logic

overview

this lecture

- an introduction to the formal language GPLI
- some of the limitations of GPL which motivate the shift to GPLI
- the identity relation
- translations into GPLI
- the semantics of identity

learning outcomes

- after doing the relevant reading for this lecture, listening to the lecture, and attending the relevant tutorial, you will be able to:
 - explain the limitations of GPL and the motivations for GPLI
 - explain what the identity relation is
 - translate propositions from English into GPLI
 - explain the motivation behind the semantics of identity
 - determine whether a GPLI proposition is true or false on a model

required reading

- sections 13.1, 13.2, and 13.3 of chapter 13

expressive limitations of GPL

expressive limitations of GPL

- consider the following argument

P1.	John loves something
P2.	John does not love himself
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C1.	John loves something other than himself

- we have the following glossary:
 - jj : John
 - $LxyLxy$: xx loves yy
- and we translate the premises as follows:
 - $\exists xLjx$ \exists exists x Ljx
 - $\neg Ljj$ \not Ljj
- but we have no way of translating the conclusion
- we have no way of saying that John loves someone other than himself

introducing GPLI

introducing GPLI

- we introduce a new two-place predicate into our logical language: the **identity** predicate
 - $I2I^2$
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- in GPL $I2I^2$ is not used for a two-place predicate. Although $I1I^1$, $I3I^3$, $I4I^4$, and so on are
 - we are extending GPL to GPLI which includes the symbol $I2I^2$ in addition to all the symbols of GPL
 - syntactically, $I2I^2$ functions just like a two-place predicate in GPL
 - semantically, $I2I^2$ is treated differently
 - its meaning does not vary from model to model
 - it is not given a glossary
 - in every model it expresses the **identity relation**
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- since the two-place predicate $I2I^2$ is so special in GPLI we usually abbreviate it as follows:
 - $==$
- and we abbreviate negations involving $I2I^2$ using:
 - \neq
- it is important to remember, however, that $==$ is just an abbreviation of a two-place predicate

the identity relation

the identity relation

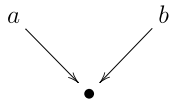
- $==$ expresses the identity relation
 - but what is the relation of identity?
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- B1 and B2 are identical
 - B1 is identical to B2
 - B1 and B2 are *exactly the same in all respects*
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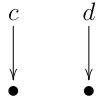
- B1 and B2 are identical
 - B1 is identical to B2
 - B1 and B2 are *one and the same thing*
-

- we have two relations:
 - **identity** (being one and the same thing)
 - **exact similarity** (being the same in all respects)
 - how are they related?
 - Leibniz's Law/Indiscernability of Identicals: if xx is identical to yy then xx is exactly similar to yy . (a logical truth)
 - Identity of Indiscernibles: if xx is exactly similar to yy then xx is identical to yy . (controversial)
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- aa and bb are identical (the names ' aa ' and ' bb ' pick out the same thing):



- cc and dd are non-identical (the names 'cc' and 'dd' pick out different things):



the predicate ==

the predicate ==

- consider the following sentences:
- B1 is a banana
- B1 is B2
- the word 'is' has a different meaning in each. in the first case, it is the 'is' of predication. in the second, it is the 'is' of identity.

- we have the following glossary:
 - aa: B1
 - bb: B2
 - ByBy: xx is a banana

- we have the following translations:
 - BaBa
 - a=ba=b

- recall that $I2I^2$ (or ==) is part of the logical vocabulary
- we do not put an entry for it in the glossary

translations into GPLI

translations into GPLI

- GPLI allows us to express more than just that a and b are identical
- let's take a look at the expressive power we have gained

- return to our example:

P1.	John loves something
P2.	John does not love himself
C1.	John loves something other than himself

- here's a glossary:
 - jj: John
 - LxyLxy: xx loves yy

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- how should we translate the conclusion of the argument?
 - $\exists x(Ljx \wedge \neg I^2jx) \setminus \text{exists } x \text{ (Ljx } \setminus \text{land } \setminus \text{not } I^2jx)$
 - there is something that John loves and that thing is not John
 - John loves something other than himself!
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- let's look at some other examples . . .
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- here's a glossary:

jj: John, mm: Mary, ss: Sam
 LxyLxy: xx loves yy, SxSx: xx is a baker, PxPx: xx is a person

- translate the following into GPLI:

John isn't Mary
 $\neg I^2jm \setminus \text{not } I^2jm$
 $j \neq m \setminus \text{neq } m$

- here's a glossary:

jj: John, mm: Mary, ss: Sam
 LxyLxy: xx loves yy, SxSx: xx is a baker, PxPx: xx is a person

- translate the following into GPLI:

John loves someone other than himself
 $\exists x(Px \wedge Ljx \wedge \neg I^2xj) \setminus \text{exists } x \text{ (Px } \setminus \text{land } Ljx \setminus \text{land } \setminus \text{not } I^2xj)$
 $\exists x(Px \wedge Ljx \wedge x \neq j) \setminus \text{exists } x \text{ (Px } \setminus \text{land } Ljx \setminus \text{land } x \setminus \text{neq } j)$

- here's a glossary:

jj: John, mm: Mary, ss: Sam
 LxyLxy: xx loves yy, SxSx: xx is a baker, PxPx: xx is a person

- translate the following into GPLI:

John loves everyone except Sam
 $\forall x((Px \wedge \neg I^2xs) \rightarrow Ljx) \setminus \text{forall } x \text{ ((Px } \setminus \text{land } \setminus \text{not } I^2xs) \rightarrow Ljx)$
 $\forall x((Px \wedge x \neq s) \rightarrow Ljx) \setminus \text{forall } x \text{ ((Px } \setminus \text{land } x \setminus \text{neq } s) \rightarrow Ljx)$

- here's a glossary:

jj: John, mm: Mary, ss: Sam
 LxyLxy: xx loves yy, SxSx: xx is a baker, PxPx: xx is a person

- translate the following into GPLI:

Some baker other than John loves Mary

$\exists x (Bx \wedge \neg I^2 x j \wedge L x m) \setminus \text{exists } x (Bx \setminus \text{and } \setminus \text{not } I^2 x j \setminus \text{and } L x m)$
 $\exists x (Bx \wedge x \neq j \wedge L x m) \setminus \text{exists } x (Bx \setminus \text{and } x \setminus \text{neq } j \setminus \text{and } L x m)$

- here's a glossary:

jj: John, mm: Mary, ss: Sam
 LxyLxy: xx loves yy, SxSx: xx is a baker, PxPx: xx is a person

- translate the following into GPLI:

Everyone loves themselves

$\forall x (Px \rightarrow Lxx) \setminus \text{forall } x (Px \setminus \text{rightarrow } Lxx)$

- here's a glossary:

jj: John, mm: Mary, ss: Sam
 LxyLxy: xx loves yy, SxSx: xx is a baker, PxPx: xx is a person

- translate the following into GPLI:

John loves everyone other than himself

$\forall x ((Px \wedge \neg I^2 x j) \rightarrow Ljx) \setminus \text{forall } x ((Px \setminus \text{and } \setminus \text{not } I^2 x j) \setminus \text{rightarrow } Ljx)$
 $\forall x ((Px \wedge x \neq j) \rightarrow Ljx) \setminus \text{forall } x ((Px \setminus \text{and } x \setminus \text{neq } j) \setminus \text{rightarrow } Ljx)$

- here's a glossary:

jj: John, mm: Mary, ss: Sam
 LxyLxy: xx loves yy, SxSx: xx is a baker, PxPx: xx is a person

- translate the following into GPLI:

John loves everyone other than himself

$\forall x ((Px \wedge \neg I^2 x j) \rightarrow Ljx) \wedge \neg Ljj \setminus \text{forall } x ((Px \setminus \text{and } \setminus \text{not } I^2 x j) \setminus \text{rightarrow } Ljx) \setminus \text{and } \setminus \text{not } Ljj$
 $\forall x ((Px \wedge x \neq j) \rightarrow Ljx) \wedge \neg Ljj \setminus \text{forall } x ((Px \setminus \text{and } x \setminus \text{neq } j) \setminus \text{rightarrow } Ljx) \setminus \text{and } \setminus \text{not } Ljj$

- here's a glossary:

jj: John, mm: Mary, ss: Sam
 LxyLxy: xx loves yy, SxSx: xx is a baker, PxPx: xx is a person

- translate the following into GPLI:

John loves everything but does not love himself

$\forall x Ljx \wedge \neg Ljj \setminus \text{forall } x Ljx \setminus \text{and } \setminus \text{not } Ljj$

- (this is a contradiction. it is false in every model.)

the semantics of ==

the semantics of ==

- since $I2I^2$ is a two-place predicate, its extension is a set of ordered pairs of members of the domain
- once we have an extension for $I2I^2$, the semantics for $GPLI$ is just like the semantics of GPL
- so all we need is an extension for $I2I^2$

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- in a model in which the domain is $\{1, 2, 3\}$, the extension of $I2I^2$ will be the following set of ordered pairs:
 - $\{\langle 1, 1 \rangle, \langle 2, 2 \rangle, \langle 3, 3 \rangle\}$

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- in a model in which the domain is $\{Bill, Ben\}$, the extension of $I2I^2$ will be the following set of ordered pairs:
 - $\{\langle Bill, Bill \rangle, \langle Ben, Ben \rangle\}$

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- whatever the domain is, the extension of $I2I^2$ will contain exactly one ordered pair for each object in the domain: the pair containing that object in both the first and second place
 - the identity relation is a relation which holds between each object and itself and between no other objects

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- note! The extension of the identity predicate is not exactly the same on all models
 - given a domain, however, the extension of the identity predicate is fixed
 - this is not true of other predicates
 - this is why it is part of the logical vocabulary

propositions of the form $I2abI^2ab$

- $I2abI^2ab$ is true in a model if and only if the pair consisting of the referent of aa and the referent of bb is in the extension of $I2I^2$
 - Domain: $\{Bill, Ben\}$
 - Referents: $aa: Bill$, $bb: Bill$
 - Extension of $I2I^2$
 - $\{\langle Bill, Bill \rangle, \langle Ben, Ben \rangle\}$
- $I2abI^2ab$ is true in a model if and only if aa and bb have the same referent on that model

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- let's run through a couple of examples

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- here's a model:
 - Domain: $\{Sydney, Canberra, Melbourne\}$
 - Referents: $aa: Melbourne$, $bb: Canberra$, $cc: Sydney$
 - Extensions: $NN: \{\langle Sydney, Canberra \rangle\}$

- Canberra \rangle, \langle Canberra, Sydney \rangle, \langle Melbourne, \rangle
 - Canberra, Sydney \rangle, \rangle
 - Melbourne, Canberra \rangle, \rangle
 - Melbourne \rangle, \rangle
 - is the following true or false in the model?
 - $\forall x \forall y (Nxy \rightarrow (I^2xb \vee I^2yb)) \rightarrow \forall x \forall y (Nxy \rightarrow (I^2xb \vee I^2yb))$
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- here's a model:
 - Domain: {Sydney, Canberra, Melbourne} \setminus \{Sydney, Canberra, Melbourne\}
 - Referents: aa: Melbourne, bb: Canberra, cc: Sydney
 - Extensions: NN: { \langle Sydney, Canberra \rangle, \langle Canberra, Sydney \rangle, \langle Melbourne, Canberra \rangle, \langle Canberra, Melbourne \rangle }
 - is the following true or false in the model?
 - $(I^2ab \rightarrow \forall x \forall y (Nxy \rightarrow (I^2xb \vee I^2yb))) \rightarrow \forall x \forall y (Nxy \rightarrow (I^2xb \vee I^2yb))$

wrapping up

this lecture

- some of the limitations of GPL which motivate the shift to GPLI
- introducing GPLI
- the identity relation
- translations into GPLI
- the semantics of identity

next lecture

- lecture 22, trees for GPLI