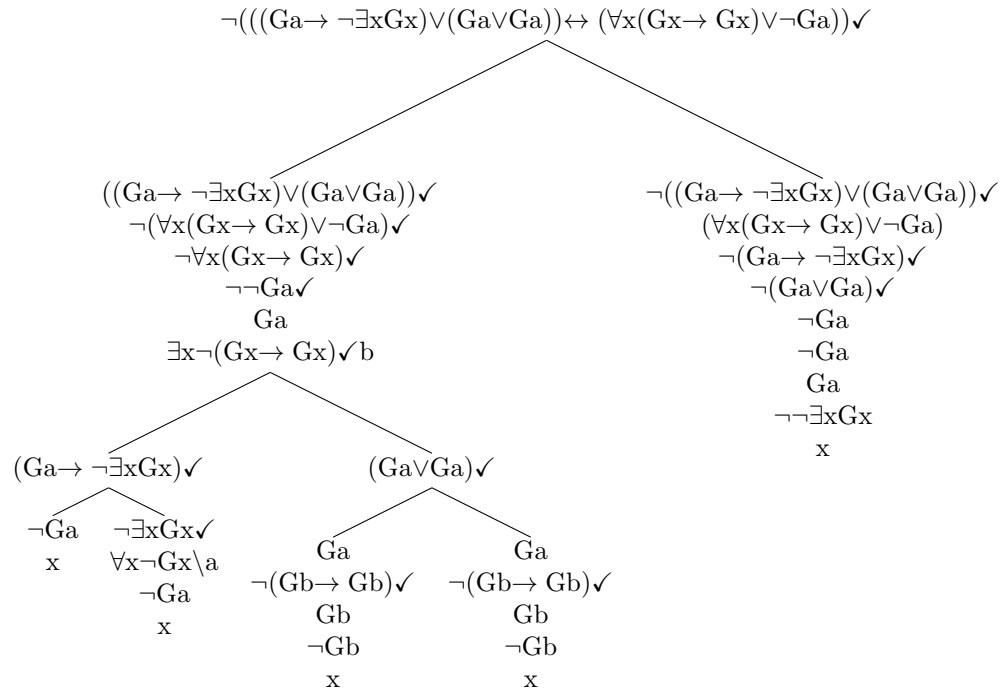# Problem Set 07: MPL Trees (Answers)

1. Use a tree to test whether the following are equivalent. If they are not, then read a countermodel off the tree.

   (a) ((Ga→ ¬∃xGx)∨(Ga∨Ga)), (∀x(Gx→ Gx)∨¬Ga)

   (b) Answer:

¬(((Ga→ ¬∃xGx)∨(Ga∨Ga))↔ (∀x(Gx→ Gx)∨¬Ga))✓

((Ga→ ¬∃xGx)∨(Ga∨Ga))✓
¬(∀x(Gx→ Gx)∨¬Ga)✓
¬∀x(Gx→ Gx)✓
¬¬Ga✓
Ga
∃x¬(Gx→ Gx)✓ b

(Ga→ ¬∃xGx)✓

¬Ga   ¬∃xGx✓
x   ∀x¬Gx\a
         ¬Ga
         x

(Ga∨Ga)✓

Ga   Ga
¬(Gb→ Gb)✓   ¬(Gb→ Gb)✓
Gb   Gb
¬Gb   ¬Gb
x   x

¬((Ga→ ¬∃xGx)∨(Ga∨Ga))✓
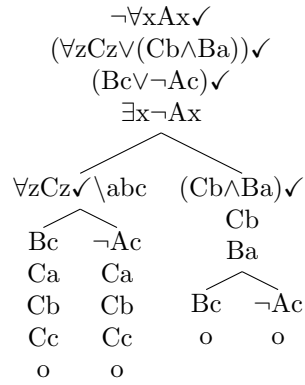(∀x(Gx→ Gx)∨¬Ga)
¬(Ga→ ¬∃xGx)✓
¬(Ga∨Ga)✓
¬Ga
¬Ga
Ga
¬¬∃xGx
x

2. Use a tree to test whether the following propositions are jointly satisfiable. If they are not, then read a counter-model off the tree.

   (a) ¬∀xAx, (∀zCz∨(Cb∧Ba)), (Bc∨¬Ac)

   (b) Answer:

$\neg\forall xAx$✓
$(\forall zCz\lor(Cb\land Ba))$✓
$(Bc\lor\neg Ac)$✓
$\exists x\neg Ax$

$\forall zCz$✓ \abc $(Cb\land Ba)$✓
                    Cb
Bc $\neg Ac$ Ba
Ca   Ca
Cb   Cb   Bc $\neg Ac$
Cc   Cc   o   o
 o   o

Domain: [1,2,3]
Referents: [(a,1),(b,2)]
Extensions: [(B,[3])(C,[1])(C,[2])(C,[3])]

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