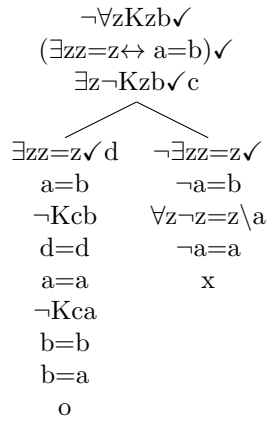# Problem Set 10: GPLI Trees (Answers)

1. Use a tree to test whether the following propositions are jointly satisfiable. If they are, then read a model off the tree.

   (a) ¬∀zKzb, (∃zz=z↔ a=b)

   (b) Answer:

$$
\begin{array}{c}
\neg\forall zKzb\checkmark \\
(\exists zz=z\leftrightarrow a=b)\checkmark \\
\exists z\neg Kzb\checkmark c
\end{array}
$$

∃zz=z✓d	¬∃zz=z✓
a=b	¬a=b
¬Kcb	∀z¬z=z\a
d=d	¬a=a
a=a	x
¬Kca	
b=b	
b=a	
o	

2. Use a tree to test whether the following argument is valid. If it is not, then read a countermodel off the tree.

   (a) (Gbb↔ (b=c∧Gcb)), ¬(¬(¬¬Gbb∧c=b)∨c=a) ∴ ((Gcc↔ ∃yy=a)∧(c=b↔ ∃x(x=b∨(x=x∨Gcx))))

   (b) Answer:

$(Gbb \leftrightarrow (b=c \land Gcb))$✓
$\neg(\neg(\neg\neg Gbb \land c=b) \lor c=a)$✓
$\neg((Gcc \leftrightarrow \exists y y=a) \land (c=b \leftrightarrow \exists x(x=b \lor (x=x \lor Gcx))))$✓
$\neg\neg(\neg\neg Gbb \land c=b)$✓
$\neg c=a$
$(\neg\neg Gbb \land c=b)$✓
$\neg\neg Gbb$✓
$c=b$
Gbb

$\neg(Gcc \leftrightarrow \exists y y=a)$✓

Gbb
$(b=c \land Gcb)$✓
$b=c$
Gcb

Gcc
$\neg\exists y y=a$✓
$\forall y \neg y=a \backslash a$
$\neg a=a$
x

$\neg Gcc$
$\exists y y=a$✓ d
$d=a$
$c=c$
Gbc
Gcc
x

$\neg Gbb$
$\neg(b=c \land Gcb)$
x

$\neg(c=b \leftrightarrow \exists x(x=b \lor (x=x \lor Gcx)))$✓

Gbb
$(b=c \land Gcb)$✓
$b=c$
Gcb

$c=b$
$\neg\exists x(x=b \lor (x=x \lor Gcx))$✓
$\forall x \neg(x=b \lor (x=x \lor Gcx)) \backslash a$
$\neg(a=b \lor (a=a \lor Gca))$✓
$\neg a=b$
$\neg(a=a \lor Gca)$✓
$\neg a=a$
$\neg Gca$
x

$\neg c=b$
$\exists x(x=b \lor (x=x \lor Gcx))$
x

$\neg Gbb$
$\neg(b=c \land Gcb)$
x

2