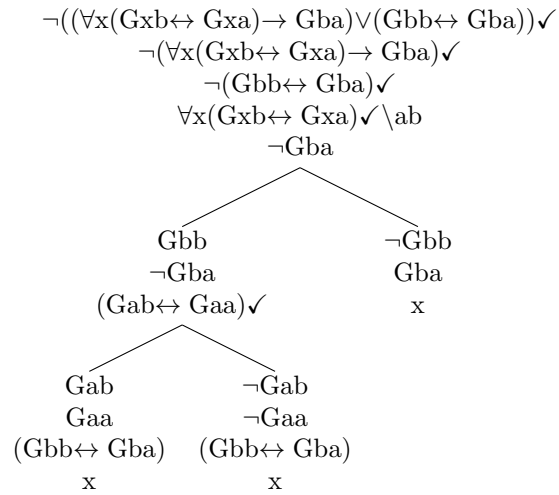# Problem Set 09: GPL Trees (Answers)

1. Use a tree to test whether the following is a tautology. If it is not, then read a countermodel off the tree.

   (a) $((\forall x(Gxb \leftrightarrow Gxa) \rightarrow Gba) \vee (Gbb \leftrightarrow Gba))$
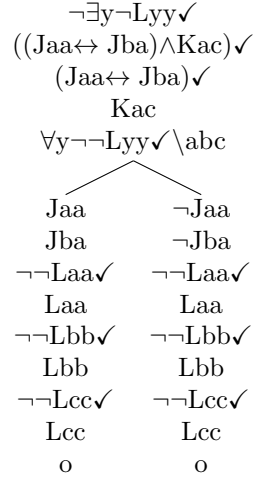
   (b) Answer:

$$\neg((\forall x(Gxb \leftrightarrow Gxa) \rightarrow Gba) \vee (Gbb \leftrightarrow Gba)) \checkmark$$
$$\neg(\forall x(Gxb \leftrightarrow Gxa) \rightarrow Gba) \checkmark$$
$$\neg(Gbb \leftrightarrow Gba) \checkmark$$
$$\forall x(Gxb \leftrightarrow Gxa) \checkmark \backslash ab$$
$$\neg Gba$$

Gbb	¬Gbb
¬Gba	Gba
$(Gab \leftrightarrow Gaa)\checkmark$	x

Gab	¬Gab
Gaa	¬Gaa
$(Gbb \leftrightarrow Gba)$	$(Gbb \leftrightarrow Gba)$
x	x

2. Use a tree to test whether the following propositions are jointly satisfiable. If they are not, then read a countermodel off the tree.

   (a) $\neg \exists y \neg Lyy$, $((Jaa \leftrightarrow Jba) \wedge Kac)$

   (b) Answer:

$$\neg\exists y\neg Lyy\checkmark$$
$$((Jaa\leftrightarrow Jba)\wedge Kac)\checkmark$$
$$(Jaa\leftrightarrow Jba)\checkmark$$
$$Kac$$
$$\forall y\neg\neg Lyy\checkmark \backslash abc$$

Jaa	¬Jaa
Jba	¬Jba
¬¬Laa✓	¬¬Laa✓
Laa	Laa
¬¬Lbb✓	¬¬Lbb✓
Lbb	Lbb
¬¬Lcc✓	¬¬Lcc✓
Lcc	Lcc
o	o

Domain: [1,2,3]
Referents: [('a',1),('b',2),('c',3)]
Extensions: [('K',[(1,3)])('J',[(1,1)])('J',[(2,1)])('L',[(1,1)])('L',[(2,2)])('L',[(3,3)])]

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