

Here is a derivation of: $\lambda x:(A \times B). \{(\text{snd } x), (\text{fst } x)\} : ((A \times B) \rightarrow (B \times A))$

$$\frac{\frac{\frac{\Gamma, x:(A \times B) \vdash x:(A \times B)}{\Gamma, x:(A \times B) \vdash (\text{fst } x):A} \times \text{Elim1} \quad \frac{\Gamma, x:(A \times B) \vdash x:(A \times B)}{\Gamma, x:(A \times B) \vdash (\text{snd } x):B} \times \text{Elim2}}{\Gamma, x:(A \times B) \vdash \{(\text{snd } x), (\text{fst } x)\} : (B \times A)} \times \text{Intro} \rightarrow \text{Intro}$$

Here is the cooresponding natural deduction proof:

$$\frac{\frac{\frac{(A \wedge B)}{A} \wedge \text{Elim1} \quad \frac{(A \wedge B)}{B} \wedge \text{Elim2}}{(B \wedge A)} \wedge \text{Intro}}{((A \wedge B) \rightarrow (B \wedge A))} \rightarrow \text{Intro}$$

Here is a derivation of: $\lambda x_1:(A+B). (\text{case } x_1 \text{ of } \lambda x_2:A. (\text{inr } x_2 \text{ as } (B+A)) \mid \lambda x_3:B. (\text{inl } x_3 \text{ as } (B+A))) : ((A+B) \rightarrow (B+A))$

$$\frac{\frac{\frac{\Gamma, x_1:(A+B), x_3:B \vdash x_3:B}{\Gamma, x_1:(A+B), x_3:B \vdash (\text{inl } x_3 \text{ as } (B+A)):(B+A)} + \text{Intro2} \quad \frac{\frac{\Gamma, x_1:(A+B), x_2:A \vdash x_2:A}{\Gamma, x_1:(A+B), x_2:A \vdash (\text{inr } x_2 \text{ as } (B+A)):(B+A)} + \text{Intro1}}{\Gamma, x_1:(A+B) \vdash \lambda x_3:B. (\text{inl } x_3 \text{ as } (B+A)):(B \rightarrow (B+A))} \rightarrow \text{Intro} \quad \frac{\Gamma, x_1:(A+B) \vdash \lambda x_2:A. (\text{inr } x_2 \text{ as } (B+A)):(A \rightarrow (B+A))}{\Gamma, x_1:(A+B) \vdash \lambda x_3:B. (\text{inl } x_3 \text{ as } (B+A)):(A \rightarrow (B+A))} \rightarrow \text{Intro} \quad \Gamma, x_1:(A+B) \vdash x_1:(A+B)}{\Gamma, x_1:(A+B) \vdash (\text{case } x_1 \text{ of } \lambda x_2:A. (\text{inr } x_2 \text{ as } (B+A)) \mid \lambda x_3:B. (\text{inl } x_3 \text{ as } (B+A))):(B+A)} + \text{Elim} \rightarrow \text{Intro}$$

Here is the cooresponding natural deduction proof:

$$\frac{\frac{\frac{B}{(B \vee A)} \vee \text{Intro2} \quad \frac{A}{(B \vee A)} \vee \text{Intro1}}{(B \rightarrow (B \vee A))} \rightarrow \text{Intro} \quad \frac{(A \vee B)}{(B \vee A)} \vee \text{Elim}}{((A \vee B) \rightarrow (B \vee A))} \rightarrow \text{Intro}$$
