

$$\begin{array}{c}
\frac{\Gamma, x:(A \times B) \vdash x:(A \times B)}{\Gamma, x:(A \times B) \vdash (\text{fst } x):A} \quad \frac{\Gamma, x:(A \times B) \vdash x:(A \times B)}{\Gamma, x:(A \times B) \vdash (\text{snd } x):B} \\
\hline
\Gamma, x:(A \times B) \vdash \{(\text{snd } x), (\text{fst } x)\}:(B \times A) \\
\hline
\emptyset \vdash \lambda x:(A \times B). \{(\text{snd } x), (\text{fst } x)\}:(A \times B) \rightarrow (B \times A) \\
\\
\frac{\Gamma, x:A \vdash x:A}{\Gamma, x:A \vdash (\text{abort } x):A} \\
\hline
\emptyset \vdash \lambda x:A. (\text{abort } x):(A \rightarrow A) \\
\\
\frac{\Gamma, x:A, y:B \vdash x:A}{\Gamma, x:A \vdash \lambda y.B.x:(B \rightarrow A)} \\
\hline
\emptyset \vdash \lambda x:A. \lambda y.B.x:(A \rightarrow (B \rightarrow A)) \\
\\
\frac{\Gamma, x1:(A \rightarrow (B \rightarrow C)), x2:A, x3:B \vdash x3:B \quad \frac{\Gamma, x1:(A \rightarrow (B \rightarrow C)), x2:A, x3:B \vdash x2:A \quad \Gamma, x1:(A \rightarrow (B \rightarrow C)), x2:A, x3:B \vdash x1:(A \rightarrow (B \rightarrow C))}{\Gamma, x1:(A \rightarrow (B \rightarrow C)), x2:A, x3:B \vdash (x1 \ x2):(B \rightarrow C)}}{\Gamma, x1:(A \rightarrow (B \rightarrow C)), x2:A, x3:B \vdash ((x1 \ x2) \ x3):C} \\
\hline
\frac{\Gamma, x1:(A \rightarrow (B \rightarrow C)), x3:B \vdash \lambda x2.A. ((x1 \ x2) \ x3):(A \rightarrow C)}{\Gamma, x1:(A \rightarrow (B \rightarrow C)) \vdash \lambda x3.B. \lambda x2.A. ((x1 \ x2) \ x3):(B \rightarrow (A \rightarrow C))} \\
\hline
\emptyset \vdash \lambda x1:(A \rightarrow (B \rightarrow C)). \lambda x3.B. \lambda x2.A. ((x1 \ x2) \ x3):((A \rightarrow (B \rightarrow C)) \rightarrow (B \rightarrow (A \rightarrow C))) \\
\\
\frac{\Gamma, x1:(A \rightarrow (B \rightarrow C)), x4:(A \times B) \vdash x4:(A \times B) \quad \frac{\Gamma, x1:(A \rightarrow (B \rightarrow C)), x4:(A \times B) \vdash (\text{fst } x4):A \quad \Gamma, x1:(A \rightarrow (B \rightarrow C)), x4:(A \times B) \vdash x1:(A \rightarrow (B \rightarrow C))}{\Gamma, x1:(A \rightarrow (B \rightarrow C)), x4:(A \times B) \vdash (x1 \ (\text{fst } x4)):(B \rightarrow C)}}{\Gamma, x1:(A \rightarrow (B \rightarrow C)), x4:(A \times B) \vdash ((x1 \ (\text{fst } x4)) \ (\text{snd } x4)):C} \\
\hline
\frac{\Gamma, x1:(A \rightarrow (B \rightarrow C)) \vdash \lambda x4.(A \times B). ((x1 \ (\text{fst } x4)) \ (\text{snd } x4)):((A \times B) \rightarrow C)}{\emptyset \vdash \lambda x1:(A \rightarrow (B \rightarrow C)). \lambda x4.(A \times B). ((x1 \ (\text{fst } x4)) \ (\text{snd } x4)):((A \rightarrow (B \rightarrow C)) \rightarrow ((A \times B) \rightarrow C))} \\
\\
\frac{\Gamma, x1:(A \rightarrow (B \rightarrow C)), x4:(B \times A) \vdash x4:(B \times A) \quad \frac{\Gamma, x1:(A \rightarrow (B \rightarrow C)), x4:(B \times A) \vdash (\text{snd } x4):A \quad \Gamma, x1:(A \rightarrow (B \rightarrow C)), x4:(B \times A) \vdash x1:(A \rightarrow (B \rightarrow C))}{\Gamma, x1:(A \rightarrow (B \rightarrow C)), x4:(B \times A) \vdash (x1 \ (\text{snd } x4)):(B \rightarrow C)}}{\Gamma, x1:(A \rightarrow (B \rightarrow C)), x4:(B \times A) \vdash ((x1 \ (\text{snd } x4)) \ (\text{fst } x4)):C} \\
\hline
\frac{\Gamma, x1:(A \rightarrow (B \rightarrow C)) \vdash \lambda x4.(B \times A). ((x1 \ (\text{snd } x4)) \ (\text{fst } x4)):((B \times A) \rightarrow C)}{\emptyset \vdash \lambda x1:(A \rightarrow (B \rightarrow C)). \lambda x4.(B \times A). ((x1 \ (\text{snd } x4)) \ (\text{fst } x4)):((A \rightarrow (B \rightarrow C)) \rightarrow ((B \times A) \rightarrow C))}
\end{array}$$

$$\begin{array}{c}
\frac{\Gamma, x1:(A \rightarrow (B \rightarrow C)), x4:(A \times B) \vdash x4:(A \times B)}{\Gamma, x1:(A \rightarrow (B \rightarrow C)), x4:(A \times B) \vdash (snd\ x4):B} \quad \frac{\Gamma, x1:(A \rightarrow (B \rightarrow C)), x4:(A \times B) \vdash x4:(A \times B)}{\Gamma, x1:(A \rightarrow (B \rightarrow C)), x4:(A \times B) \vdash (fst\ x4):A} \quad \Gamma, x1:(A \rightarrow (B \rightarrow C)), x4:(A \times B) \vdash x1:(A \rightarrow (B \rightarrow C)) \\
\hline
\Gamma, x1:(A \rightarrow (B \rightarrow C)), x4:(A \times B) \vdash (x1\ (fst\ x4)):(B \rightarrow C) \\
\hline
\Gamma, x1:(A \rightarrow (B \rightarrow C)), x4:(A \times B) \vdash ((x1\ (fst\ x4))\ (snd\ x4)):C \\
\hline
\Gamma, x1:(A \rightarrow (B \rightarrow C)) \vdash \lambda x4:(A \times B).((x1\ (fst\ x4))\ (snd\ x4)):((A \times B) \rightarrow C) \\
\hline
\emptyset \vdash \lambda x1:(A \rightarrow (B \rightarrow C)).\lambda x4:(A \times B).((x1\ (fst\ x4))\ (snd\ x4)):((A \rightarrow (B \rightarrow C)) \rightarrow ((A \times B) \rightarrow C)) \\
\\
\frac{\Gamma, x:A, y:(A \rightarrow \perp) \vdash x:A \quad \Gamma, x:A, y:(A \rightarrow \perp) \vdash y:(A \rightarrow \perp)}{\Gamma, x:A, y:(A \rightarrow \perp) \vdash (y\ x):\perp} \\
\hline
\Gamma, x:A \vdash \lambda y:(A \rightarrow \perp).(y\ x):((A \rightarrow \perp) \rightarrow \perp) \\
\hline
\emptyset \vdash \lambda x:A.\lambda y:(A \rightarrow \perp).(y\ x):(A \rightarrow ((A \rightarrow \perp) \rightarrow \perp)) \\
\\
\frac{\Gamma, x1:(A \times (B \times C)) \vdash x1:(A \times (B \times C))}{\Gamma, x1:(A \times (B \times C)) \vdash (snd\ x1):(B \times C)} \quad \frac{\Gamma, x1:(A \times (B \times C)) \vdash x1:(A \times (B \times C))}{\Gamma, x1:(A \times (B \times C)) \vdash (fst\ (snd\ x1)):B} \quad \frac{\Gamma, x1:(A \times (B \times C)) \vdash x1:(A \times (B \times C))}{\Gamma, x1:(A \times (B \times C)) \vdash (fst\ x1):A} \\
\hline
\Gamma, x1:(A \times (B \times C)) \vdash (snd\ (snd\ x1)):C \quad \Gamma, x1:(A \times (B \times C)) \vdash \{(fst\ x1), (fst\ (snd\ x1))\}:(A \times B) \\
\hline
\Gamma, x1:(A \times (B \times C)) \vdash \{\{(fst\ x1), (fst\ (snd\ x1))\}, (snd\ (snd\ x1))\}:((A \times B) \times C) \\
\hline
\emptyset \vdash \lambda x1:(A \times (B \times C)).\{\{(fst\ x1), (fst\ (snd\ x1))\}, (snd\ (snd\ x1))\}:((A \times (B \times C)) \rightarrow ((A \times B) \times C))
\end{array}$$