

$$\begin{array}{c}
\frac{\Gamma, x:(A \times B) \vdash x:(A \times B)}{\Gamma, x:(A \times B) \vdash (\text{fst } x):A} \quad \frac{\Gamma, x:(A \times B) \vdash x:(A \times B)}{\Gamma, x:(A \times B) \vdash (\text{snd } x):B} \\
\hline
\Gamma, x:(A \times B) \vdash \{(\text{snd } x), (\text{fst } x)\}:(B \times A) \\
\hline
\emptyset \vdash \lambda x:(A \times B). \{(\text{snd } x), (\text{fst } x)\} : ((A \times B) \rightarrow (B \times A))
\end{array}$$

$$\begin{array}{c}
\frac{\Gamma, x:A, y:B \vdash x:A}{\Gamma, x:A \vdash \lambda y:B. x:(B \rightarrow A)} \\
\hline
\emptyset \vdash \lambda x:A. \lambda y:B. x:(A \rightarrow (B \rightarrow A))
\end{array}$$

$$\begin{array}{c}
\frac{\Gamma, x1:(A \rightarrow (B \rightarrow C)), x2:A, x3:B \vdash x2:A \quad \Gamma, x1:(A \rightarrow (B \rightarrow C)), x2:A, x3:B \vdash x1:(A \rightarrow (B \rightarrow C))}{\Gamma, x1:(A \rightarrow (B \rightarrow C)), x2:A, x3:B \vdash (x1 \ x2):(B \rightarrow C)} \\
\hline
\Gamma, x1:(A \rightarrow (B \rightarrow C)), x2:A, x3:B \vdash ((x1 \ x2) \ x3):C \\
\hline
\Gamma, x1:(A \rightarrow (B \rightarrow C)), x3:B \vdash \lambda x2:A. ((x1 \ x2) \ x3):(A \rightarrow C) \\
\hline
\Gamma, x1:(A \rightarrow (B \rightarrow C)) \vdash \lambda x3:B. \lambda x2:A. ((x1 \ x2) \ x3):(B \rightarrow (A \rightarrow C)) \\
\hline
\emptyset \vdash \lambda x1:(A \rightarrow (B \rightarrow C)). \lambda x3:B. \lambda x2:A. ((x1 \ x2) \ x3) : ((A \rightarrow (B \rightarrow C)) \rightarrow (B \rightarrow (A \rightarrow C)))
\end{array}$$

$$\begin{array}{c}
\frac{\Gamma, x1:(A \rightarrow (B \rightarrow C)), x4:(A \times B) \vdash x4:(A \times B)}{\Gamma, x1:(A \rightarrow (B \rightarrow C)), x4:(A \times B) \vdash (snd\ x4):B} \quad \frac{\Gamma, x1:(A \rightarrow (B \rightarrow C)), x4:(A \times B) \vdash x4:(A \times B)}{\Gamma, x1:(A \rightarrow (B \rightarrow C)), x4:(A \times B) \vdash (fst\ x4):A} \quad \Gamma, x1:(A \rightarrow (B \rightarrow C)), x4:(A \times B) \vdash x1:(A \rightarrow (B \rightarrow C)) \\
\hline
\Gamma, x1:(A \rightarrow (B \rightarrow C)), x4:(A \times B) \vdash (x1\ (fst\ x4)):(B \rightarrow C) \\
\hline
\Gamma, x1:(A \rightarrow (B \rightarrow C)), x4:(A \times B) \vdash ((x1\ (fst\ x4))\ (snd\ x4)):C \\
\hline
\Gamma, x1:(A \rightarrow (B \rightarrow C)) \vdash \lambda x4:(A \times B).((x1\ (fst\ x4))\ (snd\ x4)):((A \times B) \rightarrow C) \\
\hline
\emptyset \vdash \lambda x1:(A \rightarrow (B \rightarrow C)).\lambda x4:(A \times B).((x1\ (fst\ x4))\ (snd\ x4)):((A \rightarrow (B \rightarrow C)) \rightarrow ((A \times B) \rightarrow C))
\end{array}$$

$$\begin{array}{c}
\frac{\Gamma, x1:(A \rightarrow (B \rightarrow C)), x4:(B \times A) \vdash x4:(B \times A)}{\Gamma, x1:(A \rightarrow (B \rightarrow C)), x4:(B \times A) \vdash (fst\ x4):B} \quad \frac{\Gamma, x1:(A \rightarrow (B \rightarrow C)), x4:(B \times A) \vdash x4:(B \times A)}{\Gamma, x1:(A \rightarrow (B \rightarrow C)), x4:(B \times A) \vdash (snd\ x4):A} \quad \Gamma, x1:(A \rightarrow (B \rightarrow C)), x4:(B \times A) \vdash x1:(A \rightarrow (B \rightarrow C)) \\
\hline
\Gamma, x1:(A \rightarrow (B \rightarrow C)), x4:(B \times A) \vdash (x1\ (snd\ x4)):(B \rightarrow C) \\
\hline
\Gamma, x1:(A \rightarrow (B \rightarrow C)), x4:(B \times A) \vdash ((x1\ (snd\ x4))\ (fst\ x4)):C \\
\hline
\Gamma, x1:(A \rightarrow (B \rightarrow C)) \vdash \lambda x4:(B \times A).((x1\ (snd\ x4))\ (fst\ x4)):((B \times A) \rightarrow C) \\
\hline
\emptyset \vdash \lambda x1:(A \rightarrow (B \rightarrow C)).\lambda x4:(B \times A).((x1\ (snd\ x4))\ (fst\ x4)):((A \rightarrow (B \rightarrow C)) \rightarrow ((B \times A) \rightarrow C))
\end{array}$$

$$\begin{array}{c}
\Gamma, x:A, y:(A \rightarrow \perp) \vdash x:A \quad \Gamma, x:A, y:(A \rightarrow \perp) \vdash y:(A \rightarrow \perp) \\
\hline
\Gamma, x:A, y:(A \rightarrow \perp) \vdash (y\ x):\perp \\
\hline
\Gamma, x:A \vdash \lambda y:(A \rightarrow \perp).(y\ x):((A \rightarrow \perp) \rightarrow \perp) \\
\hline
\emptyset \vdash \lambda x:A.\lambda y:(A \rightarrow \perp).(y\ x):(A \rightarrow ((A \rightarrow \perp) \rightarrow \perp))
\end{array}$$

$$\begin{array}{c}
\frac{\Gamma, x1:(A \times (B \times C)) \vdash x1:(A \times (B \times C))}{\Gamma, x1:(A \times (B \times C)) \vdash (\text{snd } x1):(B \times C)} \quad \frac{\Gamma, x1:(A \times (B \times C)) \vdash x1:(A \times (B \times C))}{\Gamma, x1:(A \times (B \times C)) \vdash (\text{snd } x1):(B \times C)} \quad \frac{\Gamma, x1:(A \times (B \times C)) \vdash x1:(A \times (B \times C))}{\Gamma, x1:(A \times (B \times C)) \vdash (\text{fst } (\text{snd } x1)):B} \quad \frac{\Gamma, x1:(A \times (B \times C)) \vdash x1:(A \times (B \times C))}{\Gamma, x1:(A \times (B \times C)) \vdash (\text{fst } x1):A} \\
\frac{\Gamma, x1:(A \times (B \times C)) \vdash (\text{snd } (\text{snd } x1)):C}{\Gamma, x1:(A \times (B \times C)) \vdash \{(\text{fst } x1), (\text{fst } (\text{snd } x1))\}:(A \times B)} \quad \frac{\Gamma, x1:(A \times (B \times C)) \vdash \{(\text{fst } x1), (\text{fst } (\text{snd } x1))\}:(A \times B)}{\Gamma, x1:(A \times (B \times C)) \vdash \{\{(\text{fst } x1), (\text{fst } (\text{snd } x1))\}, (\text{snd } (\text{snd } x1))\}:(A \times B) \times C)} \\
\frac{\Gamma, x1:(A \times (B \times C)) \vdash \{\{(\text{fst } x1), (\text{fst } (\text{snd } x1))\}, (\text{snd } (\text{snd } x1))\}:(A \times B) \times C}{\emptyset \vdash \lambda x1:(A \times (B \times C)).\{\{(\text{fst } x1), (\text{fst } (\text{snd } x1))\}, (\text{snd } (\text{snd } x1))\}:(A \times (B \times C)) \rightarrow ((A \times B) \times C)}
\end{array}$$