Expectations for Exercises

Exercises are an important part of the Geophysics lecture. They will treat some aspects of the lecture in more detail, but also cover new ground. We expect that you work on the exercises at home and we will discuss questions and solutions interactively together. Questions that are marked with 'Extra' are not required but geared to stir your further interest. We will surely support you if you tackle those as well.

6 Exercises for Resistivity Method and Induced Polarization

Version: June 23, 2022
Context: Resistivity Method 02, Induced Polarization 01 & Self-Potential 01
Timing: All resistivity exercises should be completed the latest by June 13th 2022

6.1 Forward Modelling of a vertical electrical sounding survey

(a) Use the attached file *RD_VES_ForwardModel_Ex6.ipynb* which contains a Jupyter Notebook with Py-Gimli as discussed in the latest video. Set-up a sub-surface model with three different resistivities for cases:

- $\rho_1 > \rho_2 > \rho_3$
- $\rho_1 < \rho_2 < \rho_3$
- $\rho_1 < \rho_2, \rho_2 > \rho_3$

Also explore possibilities of four layers cases. This type of forward modelling can be useful for your applied exercises in geoelectrics.
(b) Change the Jupyter notebook code so that you can visualize output from two forward runs (use a copy and past where you can). Illustrate to different sub-surface models which result in a similar vertical electrical sounding observations. Memorize that does ambiguities (same as, e.g., for gravity surveys) are very prevalent in geophysical applications.

6.2 Geoelectric Array Types

(a) Show explicitly that for a Wenner (α) array the geometry factor is $K = 2\pi a$ where a is the distance between between all electrodes.
(b) In preparation of the applied exercises, make a table with the different array types (Wenner α, Schlumberger, and half-Schlumberger (or pole-dipole), and dipole-dipole). Without going into too much detail, mention two importantpoints that require consideration when choosing a specific type.

6.3 Geoelectric Depth Of Investigation 2

(tricky) In a previous exercises we have shown that the horizontal component of the two current density in the center between the current electrodes is given by:

$$J_x(x = L/2) = \frac{I}{2\pi} \frac{L}{(z^2 + y^2 + L^2/4)^{3/2}}$$

We now want to calculate the cumulative signal contribution from a given depth interval between z_1 and z_2. This will give as an estimate for the depth of investigation as a function of the electrode spacing. The cumulative signal contribution can be calculated via integrating:

$$\frac{I_x}{I} = \frac{L}{2\pi}\int_{z_1}^{z_2} dz \int_{-\infty}^{\infty} dy \frac{1}{(z^2+y^2+L^2/4)^{3/2}}$$

Note that like this we calculate the ratio of current I_X (not current density) in that depth interval relative to the overall injected current I. Show that for $z_2 \to \infty$:

$$\frac{I_x}{I} = 1 - \frac{2}{\pi}\tan^{-1}(\frac{2z_1}{L})$$

What does this mean, e.g., for $z_1 = 0$ or $z_1 = L/2$?

Hint 1:

$$\int \frac{1}{(c+z^2)^{3/2}} = \frac{z}{c\sqrt{c+z^2}}$$

Hint 2:

$$\int \frac{1}{a^2+z^2} = \frac{1}{a}\tan^{-1}(\frac{z}{a})$$

6.4 Induced Polarization

(a) Explain explicitly why an oscillating input current can be described with

$$V(t) = V_0 e^{j(\omega t+\phi_0)}$$

where $j = \sqrt{-1}$ is the imaginary number. Assign the terms "Amplitude", "Phase Offset", and "Frequency" to the variables involved and memorize that those three parameters are always required to describe any type of waves and oscillations. Sketch the real part of V(t).
(b) Why does multiplication with "j" correspond to a phase shift of 90 degrees (or $\pi/2$)?
(c) Explain in a few words why the relationship between current I and potential difference V_c across a capacitor is given by:

$$I = C\frac{dV_c}{dt}$$

and calculate the ratio (or the impedance) of $\frac{U}{I}$ for an ac potential V(t).
(d) Explain in a few words as to why this can be understood as the resistance of a capacitor in an AC circuit. How does the impedance change with lower and higher frequencies?
Induced Polarization surveys can also be done in the frequency-domain. Instead of measuring the chargeability (defined in class) the frequency effect of the apparent resistitivy is obtained:

$$FE = \frac{\rho_{a,dc}}{\rho_{a,ac}} - 1$$

In practice this is done by measuring the dc-resistivity at very low frequencies and the ac-resistivity at intermediate frequencies.
(e) Why are in an induced polarization survey the induced current and measured voltage out of phase? Is this also the case if the sub-surface has no polarization characteristics?