1 Exercises for Gravimetry

1.1 The shell theorem

1.2 Gravitational potential inside and outside a sphere with constant density

1.3 Detection of a spherical object in the sub-surface

(a) In the lecture we have discussed the general shape of a gravity anomaly for a sphere with radius R located at depth z below the surface. Formulate the problem in a relative sense using density contrasts ($\Delta\rho$). Derive an analytical expression for the vertical anomaly as a function of the lateral distance x and depth z. Calculate the maximum anomalies that you would expect for some realistic settings (e.g., a limestone cave.)

Solution: Because we have a spherical object, there is not much difference to the point mass scenario discussed in class. The gravitational force exhibited by the sphere is:

$$\vec{g} = G\frac{M}{r^2}\hat{r}$$

The mass anomaly M (formulated in a relative sense) of the sphere is given by:

$$M = \frac{4}{3}\pi R^3 \Delta\rho$$

At distance x the distance to the sphere is $r = \sqrt{x^2 + z^2}$. The gravimeter only measures the vertical component of $\vec{g}$:

$$g_z = |\vec{g}| \sin\alpha = |\vec{g}|\frac{z}{r}$$

which brings us to:

$$\begin{aligned} g_z &= \frac{4}{3}\pi R^3 \Delta\rho \frac{G}{r^2}\frac{z}{r} \\ &= \frac{4}{3}\pi R^3 \Delta\rho \frac{G}{r^3} z \\ &= \frac{4}{3}\pi R^3 \Delta\rho G \frac{z}{(x^2 + z^2)^{\frac{3}{2}}} \end{aligned}$$

We will encounter the maximum value of the anomaly at $x = 0$ (directly above the target):

$$g_{z,max} = \frac{4}{3}\pi R^3 \Delta\rho G \frac{1}{z^2}$$

which for the limestone cave example is approximately -0.35 mGal (see below).

(b) Use a piece of software of your choice (e.g., Excel, SciDAVis, Matlab, Python) and plot the expected vertical gravity anomaly for a specific setting as a function of lateral distance x. Visualize how this profile changes as you vary, e.g., the depth of the object. Label your axis and post a picture in the forum alongside with a comment which software you used.

```
clear all
close all

%This is vector x for the lateral distance
x = -150:0.01:150;
%Depth of the sphere
z = 50;
%Radius of the sphere
R = 25;
%Constants for the limestone cave scenario
%(SI stands for System International units)
DensityLimestone_InGramsPerCentimeterCubed = 2.0;
DensityLimestone_SI = DensityLimestone_InGramsPerCentimeterCubed *1/1000 * 100^3;
DenistyAir_SI = 1.2;
%Density Contrast in kg/m^3
DeltaRho_SI = DenistyAir_SI-DensityLimestone_SI;
%Gravitational constants
G_SI = 6.674e-11;


%This is the formula we want to plot
gz_SI = 4/3*pi*R^3*DeltaRho_SI*G_SI*z./((z.^2+x.^2).^(3/2));
gz_SI_max = 4/3*pi*R^3*DeltaRho_SI*G_SI*1/(z^2);
gz_mGal = gz_SI*100*1000;

fig=figure(1)
plot(x,gz_mGal-gz_mGal(1),'r-','LineWidth',3);hold on

%And for a different depths
z=75;
gz_SI = 4/3*pi*R^3*DeltaRho_SI*G_SI*z./((z.^2+x.^2).^(3/2));
gz_mGal = gz_SI*100*1000;
plot(x,gz_mGal-gz_mGal(1),'b-','LineWidth',3);hold on

z=100;
gz_SI = 4/3*pi*R^3*DeltaRho_SI*G_SI*z./((z.^2+x.^2).^(3/2));
gz_mGal = gz_SI*100*1000;
plot(x,gz_mGal-gz_mGal(1),'g-','LineWidth',3);hold on

legend('Cave at 50 m', 'Cave at 75 m','Cave at 100 m','Location','SouthEast')
xlabel('Distance (m)')
ylabel('g_z anomaly (mGal)')
%Export to a png. (This can be done much better.)
set(findall(fig, '-property', 'FontSize'), 'FontSize', 15, 'fontWeight', 'bold')
print('-dpng','-r300','GravityAnomaly.png')


-2*G_SI*6.0e24/((6370*1000)^3)*100*1000
```

../Src/Exercises/Gravimetry/Gravimetry01.m