Introduction to Geophysics

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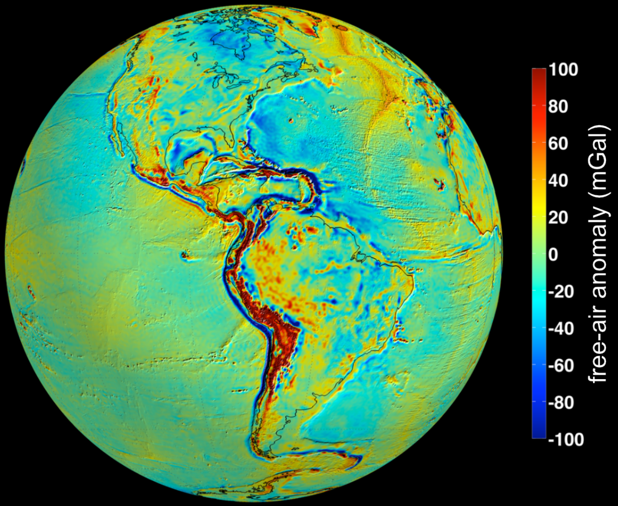
Gravimetry

Learning Goals

Learning goals today:

- The gravitational force, its potential field, and how to measure it.
- The gravitational field of the Earth.
- ....

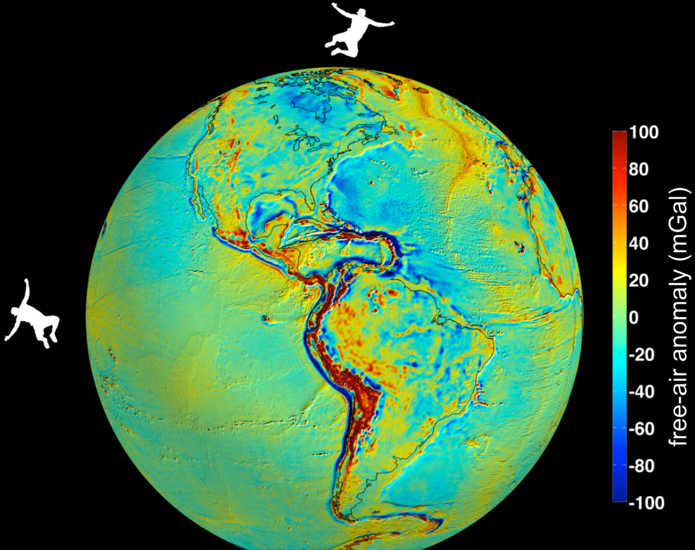
Example: Global variability



[JPL/Caltech, GRACE-FO (RT-10)]

Example: Global variability

Your mass is constant but your weight is not.



What is a force?

[Newton (1642-1726) / G. Johnson.]

$$\vec{F} = m\vec{g}$$

$\vec{F}$: Force (N; $\mathrm{kg\,m\,s^{-2}}$)

$\vec{a}$: Acceleration ($\mathrm{m\,s^{-2}}$)

m : Mass (kg)

The gravitational force

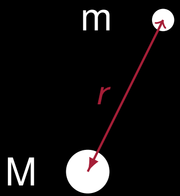
[Newton (1642-1726) / G. Johnson.]

$$\vec{F} = G\frac{mM}{r^2}\hat{r}$$

$G = 6.674 \cdot 10^{-11}\,(m^3 kg^{-1} s^{-2})$

$\hat{r}$: unit vector

r : distance between point masses



Example: The gravitational constant

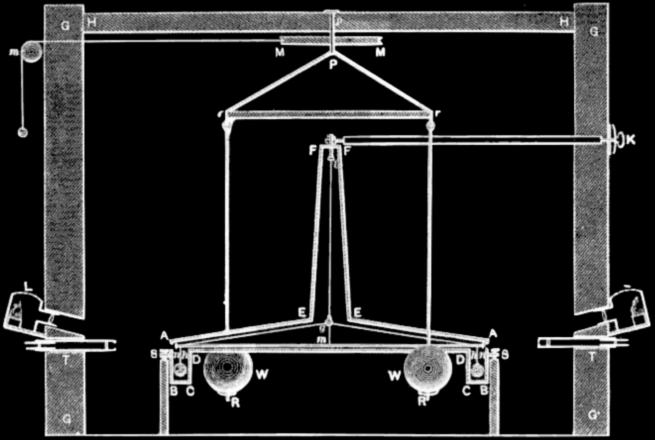
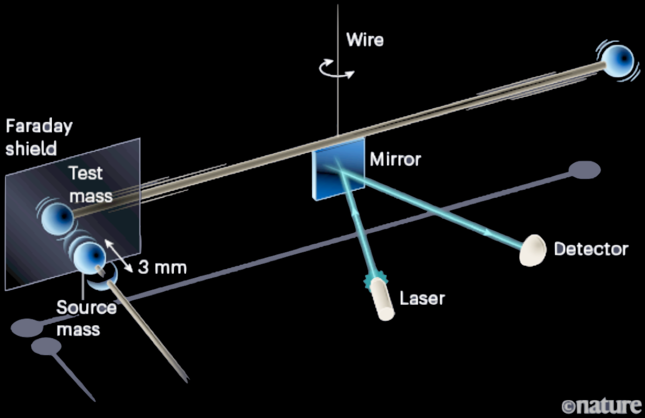


Fig. 1

Cavnedish, PNAS, 1798

Example: The gravitational constant



Westphal et al., Nature, 2021

G is the worst known constant in physics.
Why?

Example: Measuring acceleration

$$\vec{F} = m\vec{g}$$

$$\vec{F} = G\frac{mM}{r^2}\hat{r}$$

$$\rightarrow \vec{g} = G\frac{M}{r^2}\hat{r}$$

$$\rightarrow \frac{d^2\vec{x}}{dt^2} = G\frac{M}{r^2}\hat{r}$$

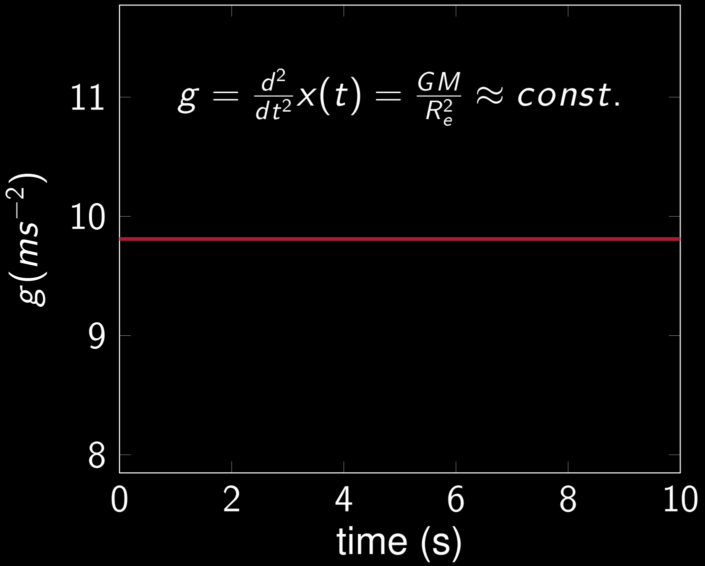
This is a differential equation.

Example: Measuring acceleration

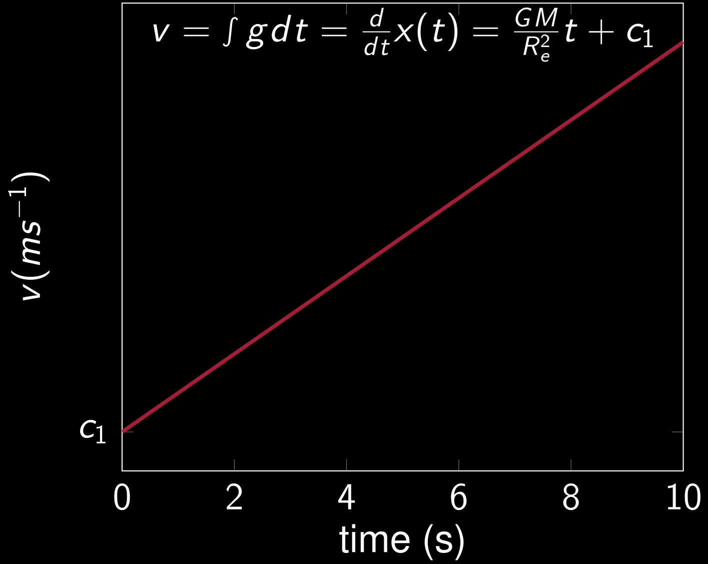
$$\frac{d^2\vec{x}}{dt^2} = G\frac{M}{R_E^2} \approx const.$$

At the Earth's surface (R_E) g is close to constant and only vertical. (Later we will see that none of this is not quite true).

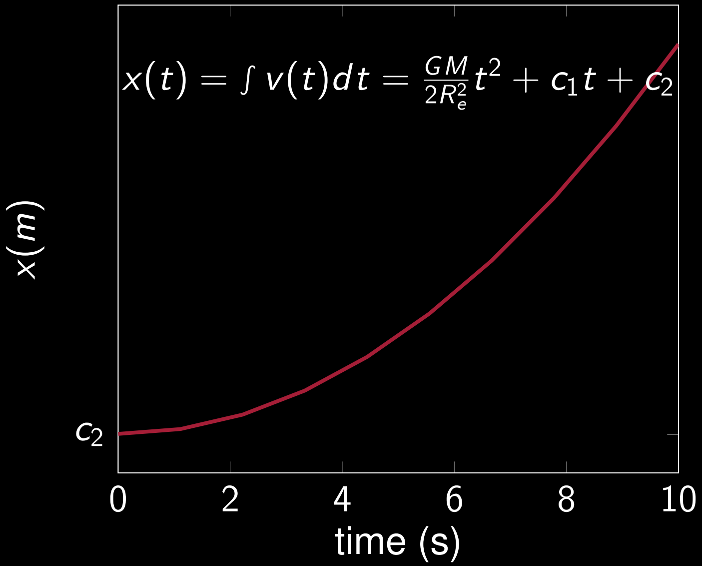
Example: Measuring acceleration



Example: Measuring acceleration



Example: Measuring acceleration



Example: Measuring acceleration

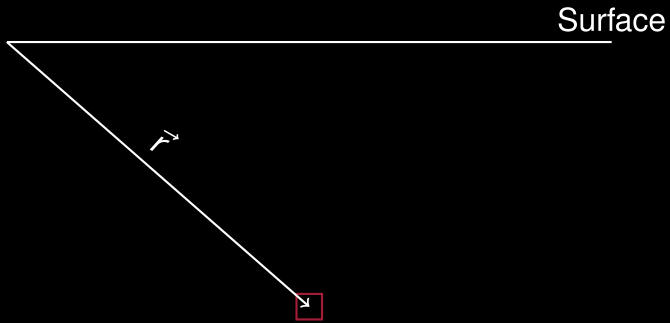
$$x(t) = \frac{GM}{2R_e^2}t^2 + c_1 t + c_2$$

- Setting, e.g., $c1 = 0$ (initial velocity) and $c_2 = 0$ (initial position) is quite convenient.
- This is the principal of a free-fall gravimeter.

Exercises: Group-Work Thursdays

- Thanks to the Greeks we know the radius R_E for the Earth. However, its mass was unknown for a while.
- Go ahead and determine the mass of the Earth M with your Smartphone!
- There is an important first-order finding in Earth Sciences that you can (re-) discover. Which one?

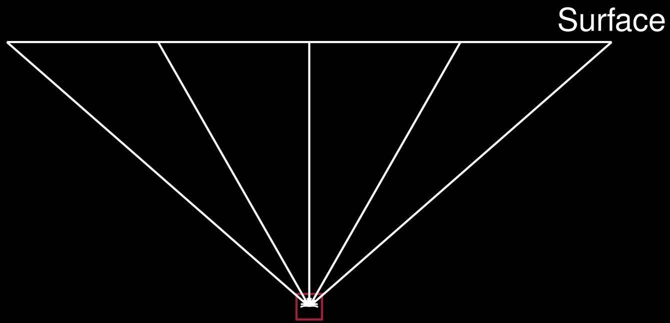
Beyond point masses



$$\vec{F} = G \frac{dM}{r^2} \hat{r}$$

For a small mass dM the point mass approximation holds.

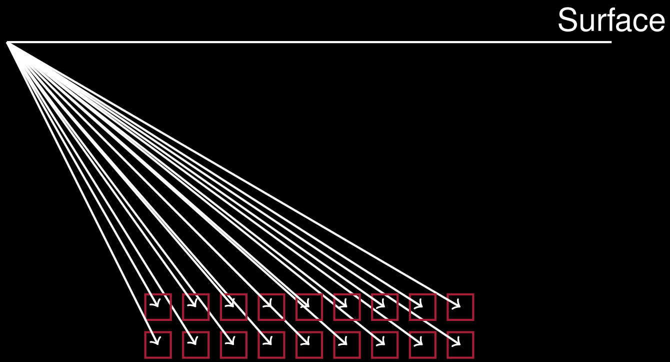
Beyond point masses



$$\vec{F} = G\frac{dM}{r^2}\hat{r}$$

Profiling across a sub-surface target results in a gravity anomaly ($\rightarrow$ Exercises).

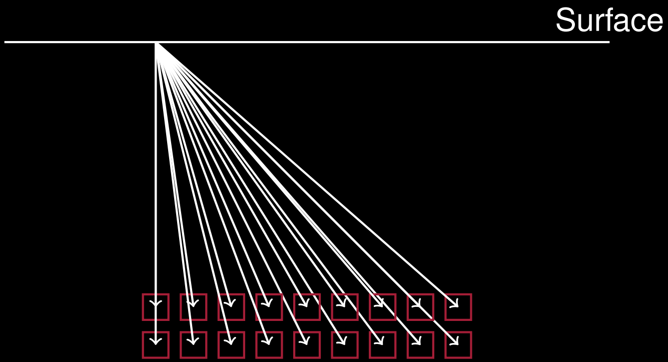
Beyond point masses



$$\vec{F}(\vec{r}) = \sum_i G \frac{dM_i}{r_i^2} \hat{r}_i$$

For i point masses the effect adds up.

Beyond point masses



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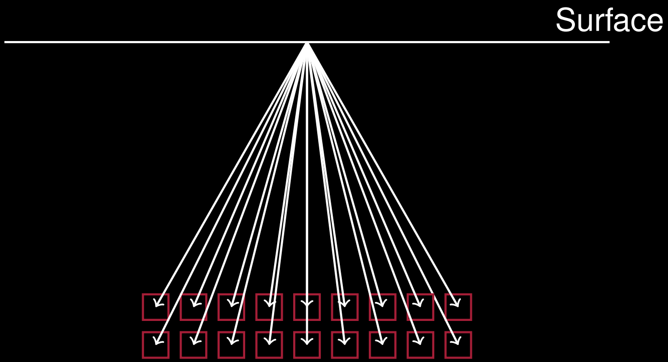
Beyond point masses



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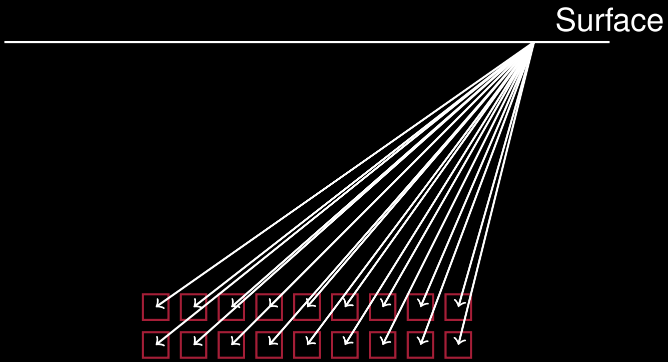
Beyond point masses

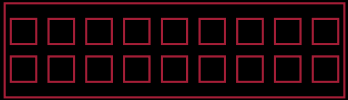


$$\vec{F}(\vec{r}) = \sum G \frac{dM_i}{r_i^2} \hat{r}_i$$

Beyond point masses

Surface

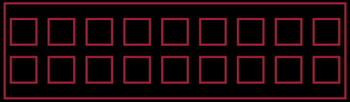
$$\vec{F}(\vec{r}) = G \int \rho \frac{1}{r^2} \hat{r} dV$$



The summation can be replaced by an integration over a volume enclosing a continuous density.

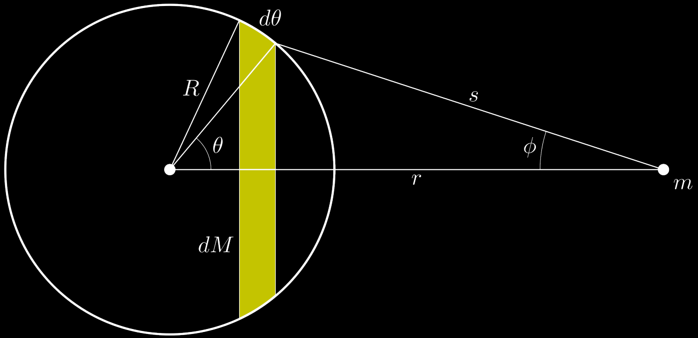
Beyond point masses

$$\vec{F}(\vec{r}) = G \int \rho \frac{1}{r^2} \hat{r} dV$$



The integration is a triple integral. Integration limits and coordinates depend on the viewpoint. Example is a Bouger plate, in general not easy to solve ($\rightarrow$ Exercises).

Example: Shell

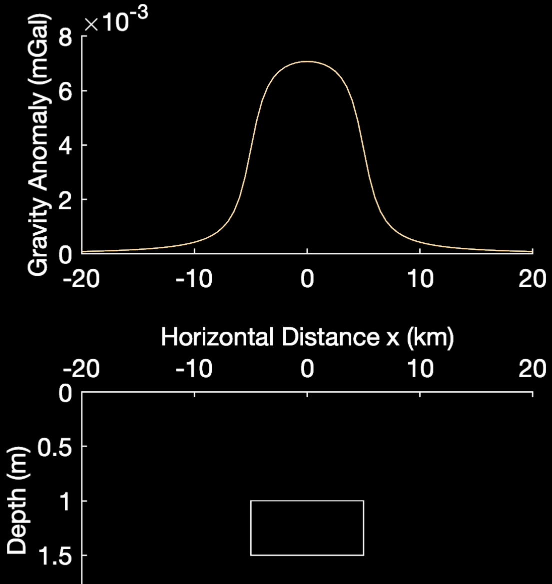


Newton's shell theorem solves the volume integral inside and outside spherical objects (→ Ex.-Discussion)

Newton’s Shell Theorem

- The field outside a shell is the same as the one from an equivalent point mass
- The field inside a shell is zero. Everywhere.

Numerical forward modelling ($\rightarrow$ Ex)



Vector fields