1 Maxwell Equation and Electromagnetic Induction

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Motivation: Ten most important equations that changed the world. Among those:

$$\begin{aligned} a^2 + b^2 = c^2 & \qquad \text{(Pythagoras)} \\ e^{i\pi} + 1 = 0 & \qquad \text{(Euler)} \\ \vec{F} = m\vec{a}; \vec{F} = G\tfrac{mM}{r^2} & \qquad \text{(Newton)} \\ \text{(...Einstein, Schrödinger)} & \end{aligned}$$

and Maxwell:

$$\begin{aligned} \nabla \cdot \vec{\mathbf{D}} &= \rho \qquad \text{(Gauss)} \\ \nabla \cdot \vec{\mathbf{B}} &= 0 \qquad \text{(Gauss)} \\ \nabla \times \vec{\mathbf{E}} &= -\frac{\partial \vec{\mathbf{B}}}{\partial t} \qquad \text{(Faraday)} \\ \nabla \times \vec{\mathbf{H}} &= \vec{\mathbf{J}} + \frac{\partial \vec{\mathbf{D}}}{\partial t} \qquad \text{(Ampére-Maxwell)} \\ \vec{\mathbf{D}} &= \varepsilon\varepsilon_0 \vec{\mathbf{E}} \qquad \text{(materials: electric field, dielectric field)} \\ \vec{\mathbf{H}} &= \mu\mu_0 \vec{\mathbf{B}} \qquad \text{(materials: magnetizing field, magnetic induction)} \\ \vec{j} &= \sigma \vec{\mathbf{E}} \qquad \text{(Ohm's law)} \end{aligned}$$

and Integral:

$$\begin{aligned} \int\int_{\partial\Omega} \vec{\mathbf{D}} \cdot d\vec{\mathbf{S}} &= \int\int\int_{\Omega} \rho \qquad \text{(Gauss)} \\ \int\int_{\partial\Omega} \vec{\mathbf{B}} \cdot d\vec{\mathbf{S}} &= 0 \qquad \text{(Gauss)} \\ \int_{\partial\Sigma} \vec{\mathbf{E}} \cdot d\vec{\mathbf{l}} &= -\frac{\partial}{\partial t} \int\int_{\Sigma} \vec{\mathbf{B}} \cdot d\vec{S} \qquad \text{(Faraday)} \\ \int_{\partial\Sigma} \vec{\mathbf{H}} \cdot d\vec{\mathbf{l}} &= \int\int_{\Sigma} \vec{\mathbf{J}} \cdot d\vec{\mathbf{S}} + \frac{\partial}{\partial t} \int\int_{\Sigma} \vec{\mathbf{D}} \cdot d\vec{\mathbf{S}} \qquad \text{(Ampére-Maxwell)} \\ \Omega &: \text{Volume} \\ \Sigma &: \text{Surface} \\ \partial\Omega &: \text{Surface of volume} \\ \partial\Sigma &: \text{Edge of surface} \end{aligned}$$

Have a picture in mind for each one of them: (1) Source of a static E-field. (2) Dipole b-Field. (3) Induction with Magnet. (4) Displacement Currents and Bio-Savart law

Integral vs. Differential form: EMF, loops, LENZ law

Principle of self-induction

Table resistance frequency dependency

$Z_L = j\omega L$ V lags I $Z_c = \frac{1}{i\omega C}$ I lags V $Z_R = R$ in phase

Classification of electrical methods

Principles of the Slingram Method

$I_{l1} = I_1 e^{i\omega t}$
$V_{l3} = L_{13}\frac{dI_{l1}}{dt} = i\omega L_{13} I_1 e^{i\omega t}$ $V_{l2} = L_{12}\frac{dI_{l1}}{dt} = i\omega L_{12} I_1 e^{i\omega t}$
What is I_2 with a R-L Subsurface model? Ohms law:
$Z_{l2} = R + i\omega L_{l2}$ $V_{l2} = (R + i\omega L_{l2}) I_2$
Induced voltage is balanced by inductance and resistance:
$i\omega L_{12} I_1 e^{i\omega t} + (R + i\omega L_{l2}) I_2 = 0$
$I_{l2} = I_2 e^{i\omega} = \frac{-i\omega L_{12}}{R_{l2} + i\omega L_{l2}} I_1 e^{i\omega t}$ $I_{l2} = I_2 e^{i\omega t} = \frac{-i\omega \frac{L_{l2}}{R_{l2}}}{1 + i\omega \frac{L_{l2}}{R_{l2}}} \frac{L_{12}}{L_{l2}} I_1 e^{i\omega t}$

Il2 will produce a secondary B. How does that appear in Loop 3:
$V_{l3} = L_{13}\frac{dI_l 1}{dt}$ primary $V_{l3} = L_{23}\frac{dI_l 2}{dt}$ secondary
$\frac{U_s}{U_p} = -\frac{L_{12}L_{23}}{L_{13}L_{l2}}\left(\frac{i\omega \frac{L_{l2}}{R_{l2}}}{1 + i\omega \frac{L_{l2}}{R_{l2}}}\right)$
induction number: $\alpha = \omega \frac{L_{l2}}{R_{l2}}$ helps to write the complex number in standard form: $\frac{U_s}{U_p} == -\frac{L_{12}L_{23}}{L_{13}L_{l2}}\left(\frac{1}{1+\alpha^2}(\alpha^2 + i\alpha)\right.$