Expectations for Exercises

Exercises are an important part of the Geophysics lecture. They will treat some aspects of the lecture in more detail, but also cover new ground. We expect that you work on the exercises at home and we will discuss questions and solutions interactively together. Questions that are marked with 'Extra' are not required but geared to stir your further interest. We will surely support you if you tackle those as well.

7 Exercises for Electromagnetic Induction

Version: June 15, 2022
Context: ElectromagneticInduction_01 + three short EMI videos
Timing: All EMI exercises should be completed the latest by July 1st 2022

7.1 Details of the Slingram Method

Parts (a)-(c) are fairly technical and geared to train your math skills (e.g. by dealing with complex numbers). This is an important skill to have because it enables you to follow more advanced textbooks and papers. They also fill some gaps that we left open in our lecture. However, it is not needed to memorize any of the specific expressions derived. Only the techniques and underlying principles matter (e.g. it is useful to memorize how the phase angle of a complex number is defined.). If you get stuck at one part, move to the next one. Explicit solutions will be provided in class.

(a) Show that the ratio of secondary over the primary voltage in the Slingram method can be expressed as:

$$\frac{V_{l3,s}}{V_{l3,p}} = -\frac{L_{12}L_{23}}{L_{13}L_{l2}}\left(\frac{i\omega\frac{L_{l2}}{R_{l2}}}{1+i\omega\frac{L_{l2}}{R_{l2}}}\right)$$

by using Faraday's law of induction and expressions for for the currents in the loops derived in class.
(b) Show that:

$$\left(\frac{i\omega\frac{L_{l2}}{R_{l2}}}{1+i\omega\frac{L_{l2}}{R_{l2}}}\right) = \left(\frac{1}{1+\alpha^2}(\alpha^2+i\alpha)\right)$$

using the induction paramter $\alpha = \omega\frac{L_{l2}}{R_{l2}}$.
(c) Show that:

$$\phi_p - \phi_s = \frac{\pi}{2} + atan(\frac{\omega L_{l2}}{R_{l2}})$$

whereas ϕ_p and ϕ_p are the phase differences relative to the primary loop. **(d)** (*Already sketch out in class. This is a repetition for yourself.*) Suppose you have an idealised loop in the sub-surface (as sketched out in lecture) located in the center of an arbitrary profile. Sketch out the shape of the slingram anomaly with distance on the x-axis and the ratio $\frac{V_s}{V_p}$on the y-axis.

7.2 Details of the Skin Effect

Same as in 7.1 (a) is fairly technical. It is helfpul for (b).

(a) Show that

$$m^2 = i\alpha \rightarrow m = -(1+i)\sqrt{\frac{\alpha}{2}}$$

(b) Show that

$$Re(E) = E_0 e^{-kz} \cos(\omega t - kz)$$

is the real part of a solution satisfying

$$\frac{\partial^2}{\partial z^2} E = i\omega\mu\sigma E$$

with the Ansatz: $E = E_0 e^{i\omega t + mz}$. What type of differential equation is this? What process does it describe?