Expectations for Exercises

Exercises are an important part of the Geophysics lecture. They will treat some aspects of the lecture in more detail, but also cover new ground. We expect that you work on the exercises at home and we will discuss questions and solutions interactively together. Questions that are marked with 'Extra' are not required but geared to stir your further interest. We will surely support you if you tackle those as well.

2 Exercises for Magnetics 1

Version: May 3, 2022
Context: Magnetics 01
Timing: All magnetics exercises should be completed by Thursday in week 5.

2.1 The magnetic dipole field

Derive the two-dimensional dipole field $\vec{B}$ and its magnitude $|\vec{B}|$ from the potential field using:

$$\begin{aligned}\vec{B} &= -\nabla A \\ &\approx -\nabla \frac{|\vec{m}|\cos(\theta)}{r^2}\end{aligned}$$

and show that

$$|B| = \frac{|\vec{m}|}{r^3}\sqrt{(3\cos^2(\theta)+1)}$$

. Also show that:

$$\nabla \cdot \vec{B} = 0.$$

Discuss what this means in terms of the magnetic dipole field. Be aware that the gradient ∇ and that the divergence $\nabla\cdot$ have a specific form in polar coordinates which you should look up. No need to memorize the specific form of these operators in different coordinate systems, but remember that they differ from coordinate system to coordinate system.

Solutions

We use the ∇ operator in spherical coordinates:

$$\nabla = \hat{r}\frac{\partial}{\partial r} + \hat{\theta}\frac{1}{r}\frac{\partial}{\partial \theta}$$

The derivation with respect to r is:

$$\hat{r}\frac{\partial}{\partial r}\left(\frac{|\vec{m}|\cos(\theta)}{r^2}\right) = -2\frac{|\vec{m}|\cos(\theta)}{r^3}\hat{r}$$

The derivation with respect to θ is:

$$\hat{\theta}\frac{1}{r}\frac{\partial}{\partial \theta}\left(\frac{|\vec{m}|\cos(\theta)}{r^2}\right) = \frac{|\vec{m}|\sin(\theta)}{r^3}\hat{\theta}$$

Taken together:

$$\vec{B} = \frac{|\vec{m}|}{r^3}\left(2\cos(\theta)\hat{r} + \sin(\theta)\hat{\theta}\right)$$

The norm of $|\vec{B}| = \sqrt{B_r^2 + B_\theta}$ as the unit vectors $\hat{r}$ and $\hat{\theta}$ are orthonormal. It is similar to a cartesian coordinate system but rotated.

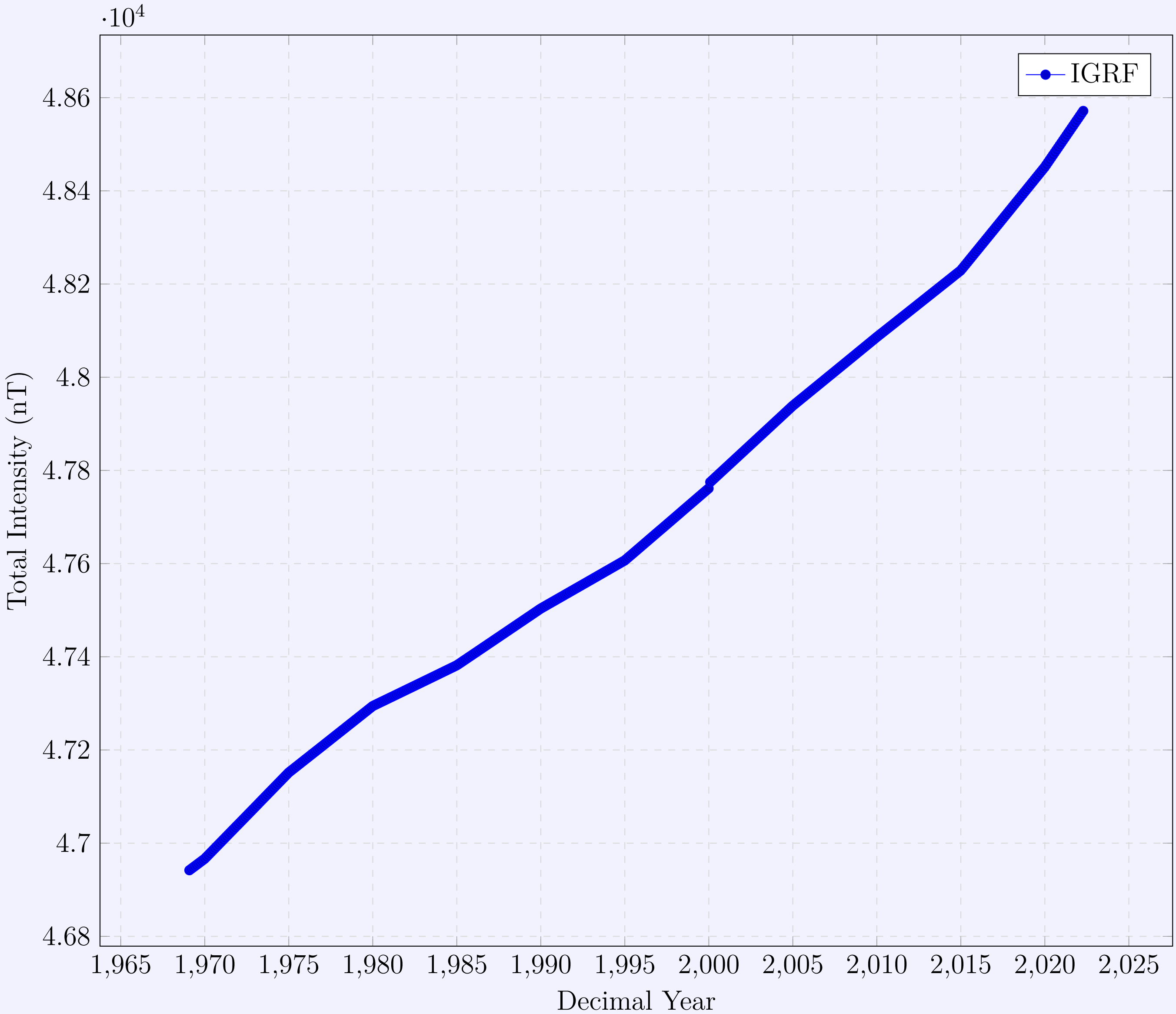
$$\begin{aligned}
B_r^2 &= \left(\frac{|\vec{m}|}{r^3}2\cos(\theta)\right)^2 \\
B_\theta^2 &= \left(\frac{|\vec{m}|}{r^3}\sin(\theta)\right)^2 \\
B_r^2 + B_\theta^2 &= \frac{|\vec{m}|^2}{r^6}\left(4\cos^2(\theta) + \sin^2(\theta)\right) \\
B_r^2 + B_\theta^2 &= \frac{|\vec{m}|^2}{r^6}\left(3\cos^2(\theta) + 1\right) \\
\sqrt{B_r^2 + B_\theta^2} &= \frac{|\vec{m}|}{r^3}\sqrt{(3\cos^2(\theta) + 1)}
\end{aligned}$$

For the divergence we use $\nabla \cdot \vec{B} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(\right)$

2.2 The Earth's magnetic field

Use online resources and characterize the Earth's magnetic field in Tübingen in terms of (declination, inclination, horizontal component, total field strength etc.). Can you find information about temporal variability? How large is it and over which time frame? Visualize the time series. Post values and graphics in the forum.

Solutions



Declination Inclination Horizontal Intensity NorthComp East Comp Vertical Comp TotalField
3.2940 64.4662 20,937.4 nT 20,902.8 nT 1,203.1 nT 43,829.7 nT 48,573.8 nT

2.3 Preparation for report writing

Read the document Geophysics_Report_Suggestions provided under general documents on Ilias. Provide feedback from other report-writing experiences if you have any.