

Expectations for Exercises

Exercises are an important part of the Geophysics lecture. They will treat some aspects of the lecture in more detail, but also cover new ground. We expect that you work on the exercises at home and we will discuss questions and solutions interactively together. Questions that are marked with 'Extra' are not required but geared to stir your further interest. We will surely support you if you tackle those as well.

6 Exercises for Resistivity Method and Induced Polarization

Version: June 2, 2022

Context: Resistivity Method 02, Induced Polarization 01 & Self-Potential 01

Timing: All resistivity exercises should be completed the latest by June 13th 2022

6.1 Forward Modelling of a vertical electrical sounding survey

(a) Use the attached file *RD_VES_ForwardModel_Ex6.ipynb* which contains a Jupyter Notebook with PyGimli as discussed in the latest video. Set-up a sub-surface model with three different resistivities for cases:

- $\rho_1 > \rho_2 > \rho_3$
- $\rho_1 < \rho_2 < \rho_3$
- $\rho_1 < \rho_2, \rho_2 > \rho_3$

Also explore possibilities of four layers cases. This type of forward modelling can be useful for your applied exercises in geoelectrics.

(b) Change the Jupyter notebook code so that you can visualize output from two forward runs (use a copy and past where you can). Illustrate to different sub-surface models which result in a similar vertical electrical sounding observations. Memorize that does ambiguities (same as, e.g., for gravity surveys) are very prevalent in geophysical applications.

Solutions

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../Src/Resistivity/RD_VES_ForwardModel_Ex6Solution.ipynb
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6.2 Geoelectric Array Types

(a) Show explicitly that for a Wenner (α) array the geometry factor is $K = 2\pi a$ where a is the distance between between all electrodes.

(b) In preparation of the applied exercises, make a table with the different array types (Wenner α , Schlumberger, and half-Schlumberger (or pole-dipole), and dipole-dipole). Without going into too much detail, mention two important points that require consideration when choosing a specific type.

Solutions

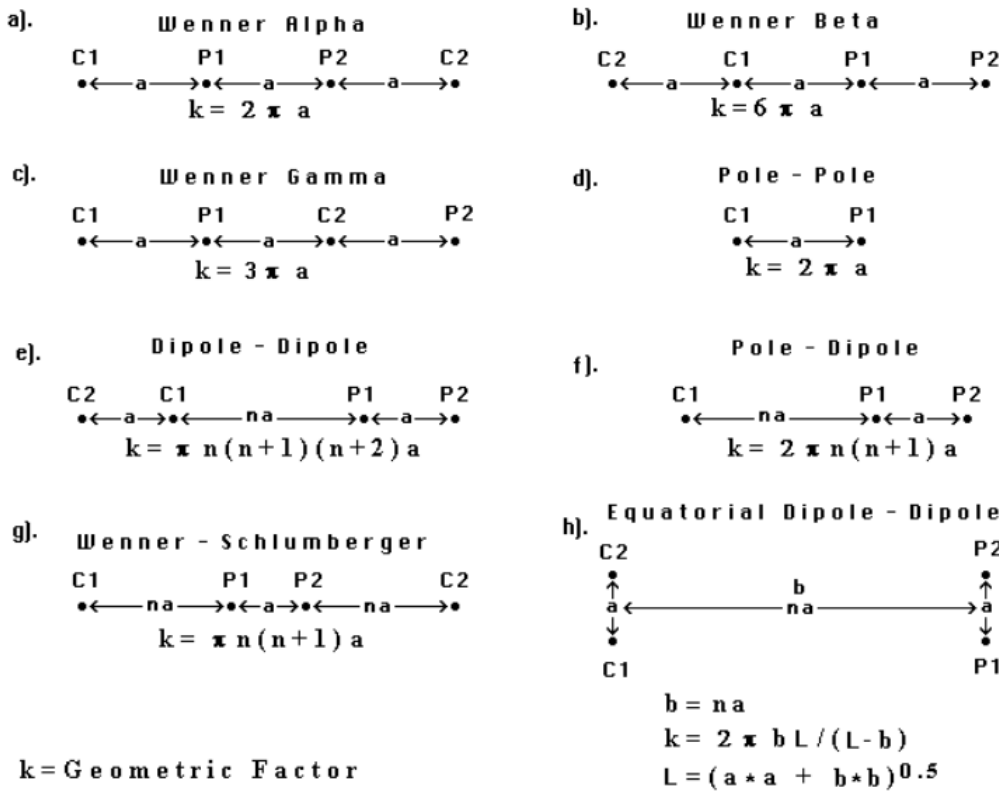


Table 7.3 Comparison of dipole-dipole, Schlumberger, square and Wenner electrode arrays.

Criteria	Wenner	Schlumberger	Dipole-dipole	Square
Vertical resolution	✓✓✓	✓✓	✓	✓✓
Depth penetration	✓	✓✓	✓✓✓	✓✓
Suitability to VES	✓✓	✓✓✓	✓	×
Suitability to CST	✓✓✓	×	✓✓✓	✓✓✓
Sensitivity to orientation	Yes	Yes	Moderate	No
Sensitivity to lateral inhomogeneities	High	Moderate	Moderate	Low
Labour intensive	Yes (no*)	Moderate (no*)	Moderate (no*)	Low
Availability of interpretational aids	✓✓✓	✓✓✓	✓✓	✓

✓ = poor; ✓✓ = moderate; ✓✓✓ = good; × = unsuitable.
 * When using a multi-core cable and automated electrode array.

6.3 Geoelectric Depth Of Investigation 2

(tricky) In a previous exercises we have shown that the horizontal component of the two current density in the center between the current electrodes is given by:

$$J_x(x = L/2) = \frac{I}{2\pi} \frac{L}{(z^2 + y^2 + L^2/4)^{3/2}}$$

We now want to calculate the cumulative signal contribution from a given depth interval between z_1 and z_2 . This will give as an estimate for the depth of investigation as a function of the electrode spacing. The

cumulative signal contribution can be calculated via integrating:

$$\frac{I_x}{I} = \frac{L}{2\pi} \int_{z_1}^{z_2} dz \int_{-\infty}^{\infty} dy \frac{1}{(z^2 + y^2 + L^2/4)^{3/2}}$$

Note that like this we calculate the ratio of current I_X (not current density) in that depth interval relative to the overall injected current I . Show that for $z_2 \rightarrow \infty$:

$$\frac{I_x}{I} = 1 - \frac{2}{\pi} \tan^{-1}\left(\frac{2z_1}{L}\right)$$

What does this mean, e.g., for $z_1 = 0$ or $z_1 = L/2$?

Hint 1:

$$\int \frac{1}{(c + z^2)^{3/2}} = \frac{z}{c\sqrt{c + z^2}}$$

Hint 2:

$$\int \frac{1}{a^2 + z} = \frac{1}{a} \tan^{-1}\left(\frac{z}{a}\right)$$

Solutions

First the y-integration we set $c = z^2 + L^2/4$ and use hint 1 then

$$\begin{aligned} \int_{-\infty}^{\infty} dy \frac{1}{(z^2 + y^2 + L^2/4)^{3/2}} &= \\ \left[\frac{y}{(z^2 + L^2/4)\sqrt{z^2 + L^2/4 + y^2}} \right]_{-\infty}^{\infty} &= \\ \frac{1}{(z^2 + L^2/4)} \left[\frac{y}{\sqrt{z^2 + L^2/4 + y^2}} \right]_{-\infty}^{\infty} &= \\ \frac{1}{(z^2 + L^2/4)} \left[\frac{y}{|y|\sqrt{z^2/y^2 + L^2/4/y^2 + 1}} \right]_{-\infty}^{\infty} &= \frac{2}{(z^2 + L^2/4)} \end{aligned}$$

Next the z-integration and using $a = L/2$:

$$\begin{aligned} \int_{z_1}^{z_2} dz \frac{2}{(z^2 + L^2/4)} &= \\ \frac{4}{L} \left[\tan^{-1}(2z/L) \right]_{z_1}^{z_2} \end{aligned}$$

Taken together this results in:

$$\frac{I_x}{I} = \frac{2}{\pi} \tan^{-1}\left(\frac{2z_2}{L}\right) - \frac{2}{\pi} \tan^{-1}\left(\frac{2z_1}{L}\right)$$

For $z_2 \rightarrow \infty$:

$$\frac{I_x}{I} = 1 - \frac{2}{\pi} \tan^{-1}\left(\frac{2z_1}{L}\right)$$

For $z_1 = 0$ $\frac{I_x}{I} = 1$ which is logical as it means all of the current flows through the interface at some stage as we integrate over the entire depth. For $z_1 = L/2$ half the currents flows above z_1 and half the current flows below z_1 as $\frac{I_x}{I} = 0.5$. This gives as good estimate for the depth of investigation as a function of the electrode spacing. However, keep in mind that the larger L the smaller the current and potential differences involved, and at some stage you will not be able to detect this anymore.

6.4 Induced Polarization

(a) Explain explicitly why an oscillating input current can be described with

$$V(t) = U_0 e^{j(\omega t + \phi_0)}$$

where $j = \sqrt{-1}$ is the imaginary number. Assign the terms "Amplitude", "Phase Offset", and "Frequency" to the variables involved and memorize that those three parameters are always required to describe any type of waves and oscillations. Sketch the real part of $V(t)$.

(b) Why does multiplication with "j" correspond to a phase shift of 90 degrees (or $\pi/2$)?

(c) Explain why the $I - V_c$ relationship for a capacitor is given by:

$$I = C \frac{dV_c}{dt}$$

and calculate the ratio (or the impedance) of $\frac{I}{V}$ for an ac potential $V(t)$.

(d) Explain in a few words as to why this can be understood as the resistance of a capacitor in an AC circuit. How does the impedance change with lower and higher frequencies?

Induced Polarization surveys can also be done in the frequency-domain. Instead of measuring the chargeability (defined in class) the frequency effect of the apparent resistivity is obtained:

$$FE = \frac{\rho_{a,dc}}{\rho_{a,ac}} - 1$$

In practice this is done by measuring the dc-resistivity at very low frequencies and the ac-resistivity at intermediate frequencies.

(e) Why are in an induced polarization survey the induced current and measured voltage out of phase? Is this also the case if the sub-surface has no polarization characteristics?

Solutions

(a)

$$V(t) = U_0 e^{j(\omega t + \phi_0)} = U_0 (\cos(\omega t + \phi_0) + i \sin(\omega t + \phi_0))$$

The real part of this equation corresponds to a *normal* cos with amplitude U_0 , angular frequency ω and phase offset ϕ_0 . (Drawing)

(b)

$$V(t) = U_0 e^{j(\omega t + \phi_0 + \pi/2)} \quad (1)$$

$$= U_0 e^{j(\omega t + \phi_0)} e^{j\pi/2} \quad (2)$$

$$= U_0 e^{j(\omega t + \phi_0)} (\cos(\pi/2) + i \sin(\pi/2)) \quad (3)$$

$$= i U_0 e^{j(\omega t + \phi_0)} \quad (4)$$

(c) From lecture we know that $V_c = \frac{q}{C}$:

$$\frac{dV_c}{dt} = \frac{1}{C} \frac{ds}{dt} = \frac{1}{C} I$$

Hence:

$$\frac{dV_c}{dt} = \frac{d}{dt} V_0 e^{j\omega t} = j\omega V_c(t)$$

and:

$$\frac{V_c}{I} = \frac{1}{j\omega C} = -\frac{j}{\omega C}$$

(d) This is an extension to Ohm's law in the ac case. At low frequencies the impedance is large, and at high frequencies the impedance is high.

(e) The multiplication with "-j" indicates that current leads the voltage by 90 degrees if the subsurface is polarized. This phase shift can also be analyzed and will be important for other geophysical methods as well. If there is no polarization in the sub-surface that we "only" have resistive properties in which (according to Ohm's law) the potential difference and currents are always in-phase with each other.