Expectations for Exercises

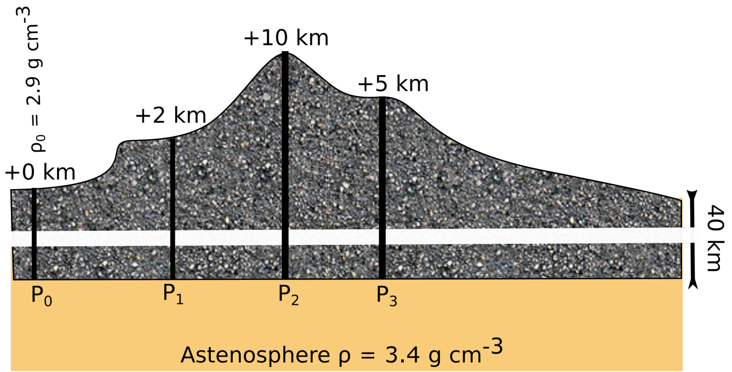
Exercises are an important part of the Geophysics lecture. They will treat some aspects of the lecture in more detail, but also cover new ground. We expect that you work on the exercises at home and we will discuss questions and solutions interactively together. Questions that are marked with 'Extra' are not required but geared to stir your further interest. We will surely support you if you tackle those as well.

2 Exercises for Gravity Method 2

Version: April 26, 2022
Context: Videos Introduction & Gravity 01, Gravity 02
Timing: All gravity exercises should be completed by Thursday in week 3.

2.1 Airy and Pratt hypothesis for mountain ranges

(a)



The figure above illustrates a crust with with a inhomogenous density and a mountain change floating on the asthenosphere. Consider this as an idealised case in which every vertical slice is locally balanced (i.e. everything is in hydrostatic equilibrium which reduces to an effective 1D problem). Calculate the required densities in the vertical slices at $P_1 - -P_3$. (Tip: Below the crust the pressure is equal everywhere $P_1 - -P_3$)

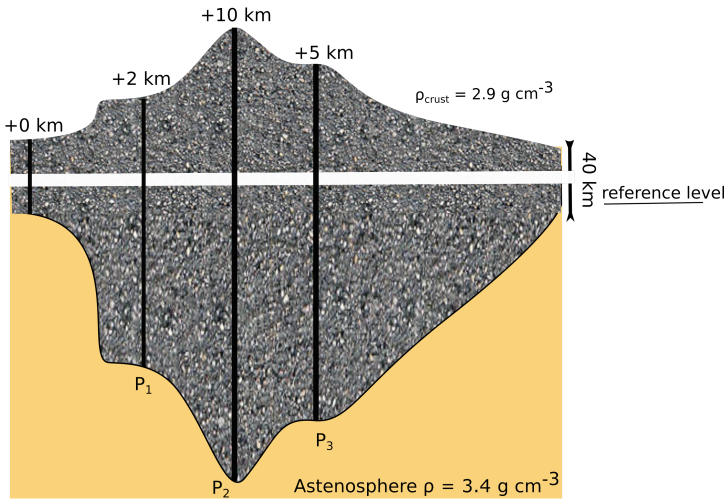
Solutions

The hydrostatic pressure p at P_1 is $p\ \rho g H_1$ with $H_1 = 40$ km. This is the same for the other locations. Hence:

$$\rho_0 H_1 = \rho_1 (H_1 + 2)$$
$$\rightarrow \rho_1 = \rho_0 \frac{H_1}{H_1 + 2} \approx 2.76\,\mathrm{g\ cm^{-3}}$$

Equivalent for the other locations.

(b)



The figure above illustrates a crust with a homogenous density and a mountain change floating on the asthenosphere. Consider this as an idealised case in which every vertical slice is locally balanced (i.e. everything is in hydrostatic equilibrium which reduces to an effective 1D problem). Calculate the thickness differences between $P_1 - -P_3$.

Solutions

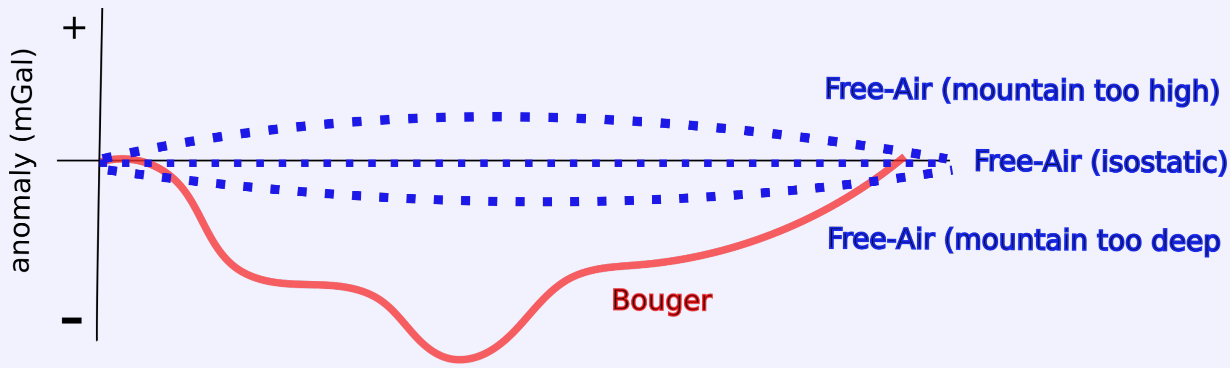
At the depth of P_1 we have:

$$\rho_c H_1 + \rho_A H_A = \rho_c (H_1 + 2)$$
$$\rightarrow H_A = 2 \frac{\rho_c}{\rho_A - \rho_c} \approx 11.6\,\text{km}$$

This means thickness at P_1 is 40 + 2 + 11.6 = 53.6 km. Equivalent for the other locations.

(c) Draw an approximate profile for the free-air and the Bouger anomalies. How would the free-air anomaly profile change if the mountain change is not in hydrostatic equilibrium? Which conclusions regarding the temporal evolution of the mountain chain would you draw from that? In which areas along this profile do you think is the assumption of local hydrostatic equilibrium most unlikely and how would this be reflected in the free-air anomaly?

Solutions



Text directly copied from: Link2PDF We can use gravity measurements to determine whether an area is in isostatic equilibrium. If a region is in isostatic equilibrium, there should be no gravity anomaly and hence no excess or lack of mass above the compensation depth. However, in practice, interpreting gravity measurements is a convoluted process. As an example, take the mountains shown above which are in 100% isostatic compensation of the Airy type. The Bouguer anomaly across these mountains is negative, since below sea level there is a mass deficit under the mountains, ie, the low density root is holding the overlying mountains up. The Bouguer anomaly reflects the fact that the overlying mountains have been removed from the correction, which leaves only the mass deficit at depth unaccounted for, which causes the negative Bouguer anomaly. The free air anomaly, on the other hand, will be slightly positive, since this anomaly only takes into account the fact that we're above sea level in our measurements and doesn't take into account the distribution of mass below us. The slight positive reading comes from the fact that the overlying mountain is closer to us and and our point of measurement than is the compensating low density material at depth, and since gravitational acceleration drops off as $1/r^2$, the closer, mountain attraction io stronger than the more distant lack of attraction due to the mass deficit in the root, which results in a slight positive free air anomaly. The simplest way to determine whether a large-scale structure such as a mountain chain is in isostatic equilibrium is to use the free air anomaly. If a structure is totally compensated, away from the edges of the structure the free air anomaly will be very small. Near the edges is difficult to discern. If the structure is only partially compensated, the or not at all, then the free air anomaly will be strongly positive, up to several hundred millgals, while the Bouguer anomaly will be about zero. Free air anomalies are always almost isostatic anomalies. They do not tell you what type of compensation is ocurring (ie, Pratt versus Airy), but if compensation of any mechanism is complete, then the free air anomaly will be nearly zero.

2.2 Forward modelling and non-uniqueness in potential field methods

Matlab (or Python)

Basic programming (Matlab/Python/R) will likely be part of your study experience when you move to MSc level courses. It is a useful skill to have, but here we do not cover any introduction. What we do is that we start with codes that need little user interaction to give you a feel for what programming can be about. In order to run this exercise you should have a working Matlab version on your Computer, please follow the installation instructions provided by the ZDV. Alternatively, we can also give you a laptop for the joint meeting.

In order to predict how any kind of object will appear in a gravity survey, we need to solve the volume integral:

$$\vec{g}(r) = G \int \frac{1}{r^3} \rho(r) \vec{r} dV$$

which simplifies slightly to:

$$g_z(r) = G \int \frac{1}{r^2} \cos(\phi) \rho(r) dV = G \int \frac{z}{r^3} \rho(r) dV$$

because often only the vertical component is of interest (same as in Ex 1.1). However, the problem remains complicated as the integration bounds depend on the object's geometry and the integral needs to be solved for every r along the gravimetry profile. Some solutions for special shapes you already know (e.g.sphere, bouger plate). Here we use the solution for a rectangular prism which fortunately others have already

calculated for us (*Naggy 1966, Geophysics VOL. XXX, SO. 2*). Using this solution, we can build up more complicated shapes out of individual prisms.
In the specific model applied individual prisms are defined with their widths in the horizontal (wx, wy) and the vertical (wz), together with the positions in the subsurface. The key is that the position coordinates (dx1, dx2, dy1, dy2, dz1, dz2) need to be prescribed relative to the measurement position which changes along the profile. The expected anomaly is then calculated based on the analytical solution.

(a) This exercises uses Matlab. However, only minimal Matlab skills are required to follow along. Download the files *Gravimetry02_ForwardModelling.m* and *gravprism.m* into the same folder on your computer. Check out case 1 which simulates a rectangular object in the subsurface. Change it's location and size so that you know what is going on.

(b) Switch to case 2. This one treats the combined effect of two prisms. See what's different compared to case 1. Play around with positions to see what is going on.

(c) Switch to case 3. This one treats the individual effects of two prisms meaning that it doesn't sum them up. This one will not run until you fill out the parts marked with XXX. Use this case to illustrate that multiple situations in the sub-surface (e.g. a shallow prism with low density contrast vs. a lower prism with larger density contrast) can result in similar anomalies. This is an important finding. Forward models are often not unique, and therefore your interpretation won't be either. This situation occurs in many geophysical situations. Remember that.

Solutions

```
clear all;
close all;

%% This code quantifies the gravitational potential of a rectangular
%% Prism located in the sub-surface using an analytical solution of
%% Nagy 1966 (Geophysics)



%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Case Numbers
% RectangleCentered: 1 ;
% Two Rectangles: 2;
% Ambiguities: 3
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
CaseNumber = 1;

switch CaseNumber
    case 1
        display('Calculating centered rectangle.')
        %This is the density contrast
        drho = 400;

        % Location and geometry of Prisms.
        % ------------------------------------------------------------
        % Width height and depth of the prism.
        wx = 10;wy = 100;wz = 0.5;
        % Offset in depth z and lateral direction x.
```

```
        % The y-dimension is (but doesn not have to)
        % is symetric to the profile direction

        offsetz=1;offsetx=0;

        % Sample Points along profile in x-direction.
        % Coordinates are symetric with origin in center.
        % ----------------------------------------------------------------
        dx=0.5;xp = -20:dx:20;yp = xp*0;zp=xp*0;

        % Coordinates of the prism relative to sample points
        % ----------------------------------------------------------------
        dx1=fliplr(min(xp)-wx/2:dx:max(xp)-wx/2);dx2=dx1+wx;
        dy1=dx1*0-wy/2;dy2=dy1+wy;
        dz1=dx1*0+offsetz;dz2=dz1+wz;

        % This applies the analysitcal solution of Nagy 1966.
        % ----------------------------------------------------------------
        dg = gravprism(drho,dx1,dx2,dy1,dy2,dz1,dz2);

        % Here we visualize the results.
        fig = figure()
        subplot(3,1,1)
        plot(xp,dg)
        ylabel('Gravity Anomaly (mGal)');box off;set(gcf, 'color', 'none');set(gca, 'color', 'none');
        subplot(3,1,2)%[x y w h]
        rectangle('Position',[-wx/2,offsetz,wx,wz]);
        set(gca,'XAxisLocation','top','YAxisLocation','left','ydir','reverse');
        xlim([min(xp),max(xp)]);xlabel('Horizontal Distance x (km)');ylabel('Depth (m)');ylim([0,2]);set(gcf, 'color', 'none');set(gca, 'color', 'none');
        subplot(3,1,3)%[x y w h]
        rectangle('Position',[-wx/2,-wy/2,wx,wy]);
        xlim([min(xp),max(xp)]);xlabel('Horizontal Distance x (km)');ylabel('Horizontal Distance y (km)');
        %Export to a png. (This can be done much better.)
        set(gcf, 'color', 'none');set(gca, 'color', 'none');
        set(gcf,'PaperUnits','centimeters','PaperPosition',[0 0 10 20])
        set(findall(fig, '-property', 'FontSize'), 'FontSize', 12)
        print('-dpng','-r300','../../../LatexSlidesLectures/Figures/Gravity/Exported/ForwardModelPrism.png')
    case 2
        display('Calculating the effect of multiple rectangles.')
        % This is the density contrast
        % ----------------------------------------------------------------
        drho = [400];
        % Width, height and depth of the two prisms.
        % ----------------------------------------------------------------
        wx = [10 8];
        wy = [100 100];
        wz = [0.5 0.25];

        % Offset in depth z and lateral direction x.
        % The y-dimension is (but doesn not have to)
```

```
        % is symetric to the profile direction
        % ------------------------------------------------------------
        offsetz=[1 0.25];offsetx=[-10 12]

        % Number of prisms in this example
        % ------------------------------------------------------------
        np = length(offsetz);

        % Sample Points along profile in x-direction.
        % Coordinates are symetric with origin in center.
        % ------------------------------------------------------------
        dx=0.5;xp = -20:dx:20;yp = xp*0;zp=xp*0;

        % Coordinates of the prisms relative to sample points
        % ------------------------------------------------------------
        for kk=1:np
            dx1(kk,:)=fliplr(min(xp)-wx(kk)/2:dx:max(xp)-wx(kk)/2)+offsetx(kk);dx2(kk,:)=dx1(kk,:)+wx(kk);
            dy1(kk,:)=dx1(kk,:)*0-wy(kk)/2;dy2(kk,:)=dy1(kk,:)+wy(kk);
            dz1(kk,:)=dx1(kk,:)*0+offsetz(kk);dz2(kk,:)=dz1(kk,:)+wz(kk);
        end
        dg = gravprism(drho,dx1,dx2,dy1,dy2,dz1,dz2);

        % Visualization of the combined effect
        % ------------------------------------------------------------
        figure()
        subplot(3,1,1)
        % Here we show the combined effect by summing the effects if individual prisms
        plot(xp,sum(dg,1))
        xlabel('Horizontal Distance x (km)');ylabel('Gravity Anomaly (mGal)');
        subplot(3,1,2)
        hold on;
        for kk=1:np
            rectangle('Position',[-wx(kk)/2+offsetx(kk),offsetz(kk),wx(kk),wz(kk)]);
        end
        set(gca,'XAxisLocation','top','YAxisLocation','left','ydir','reverse');
        xlim([min(xp),max(xp)]);xlabel('Horizontal Distance x (km)');ylabel('Depth (m)');
        subplot(3,1,3)%[x y w h]
        for kk=1:np
            rectangle('Position',[-wx(kk)/2+offsetx(kk),-wy(kk)/2,wx(kk),wy(kk)]);
        end
        xlim([min(xp),max(xp)]);xlabel('Horizontal Distance x (km)');ylabel('Horizontal Distance y (km)');
    case 3
        display('Showcasing ambiguity.')
        % Those are density contrasts. We now choose two.
        % ------------------------------------------------------------
        drho1 = 400;drho2=950;
        % Width, height and depth of the two prisms.
        % ------------------------------------------------------------
        wx = [10 8.7];
        wy = [100 100];
```

```
        wz = [0.5  0.25];
        %Offset in depth z and lateral direction x.
        %The y-dimension is (but doesn not have to)
        %is symetric to the profile direction
        offsetz=[1  2];offsetx=[0  0]

        % Number of prisms
        % -------------------------------------------------------
        np = length(offsetz);

        % Sample Points along profile in x-direction.
        % Coordinates are symetric with origin in center.
        % -------------------------------------------------------
        dx=0.5;xp = -20:dx:20;yp = xp*0;zp=xp*0;

        % Coordinates of prisms relative to sample points
        % -------------------------------------------------------
        for kk=1:np
            dx1(kk,:)=fliplr(min(xp)-wx(kk)/2:dx:max(xp)-wx(kk)/2)+offsetx(kk);dx2(
    kk,:)=dx1(kk,:)+wx(kk);
            dy1(kk,:)=dx1(kk,:)*0-wy(kk)/2;dy2(kk,:)=dy1(kk,:)+wy(kk);
            dz1(kk,:)=dx1(kk,:)*0+offsetz(kk);dz2(kk,:)=dz1(kk,:)+wz(kk);
        end

        % Now calculate the effects with variable densities.
        % For this we calculate all prisms for all densities.
        % It is a bit silly, but it works.
        % -------------------------------------------------------
        dg1 = gravprism(drho1,dx1,dx2,dy1,dy2,dz1,dz2);
        dg2 = gravprism(drho2,dx1,dx2,dy1,dy2,dz1,dz2);

        % Visualization
        % -------------------------------------------------------
        figure()
        subplot(3,1,1)
        hold on;
        plot(xp,dg1(1,:),'r') %This is prism 1 with density 1
        plot(xp,dg2(2,:),'m') %This is prism 2 with density 2
        xlabel('Horizontal Distance x (km)');ylabel('Gravity Anomaly (mGal)');
        subplot(3,1,2)
        hold on;
        plot(xp,0*xp,'b-x')
            kk=1
            rectangle('Position',[-wx(kk)/2+offsetx(kk),offsetz(kk),wx(kk),wz(kk)],'
    FaceColor','r');
            kk=2
            rectangle('Position',[-wx(kk)/2+offsetx(kk),offsetz(kk),wx(kk),wz(kk)],'
    FaceColor','m');


        set(gca,'XAxisLocation','top','YAxisLocation','left','ydir','reverse');
        xlim([min(xp),max(xp)]);xlabel('Horizontal Distance x (km)');ylabel('Depth (
    m)');
        subplot(3,1,3)%[x y w h]
```

```
            kk=1;
               rectangle('Position',[-wx(kk)/2+offsetx(kk),-wy(kk)/2,wx(kk),wy(kk)],'FaceColor','r');
               kk=2;
               rectangle('Position',[-wx(kk)/2+offsetx(kk),-wy(kk)/2,wx(kk),wy(kk)],'FaceColor','m');
          xlim([min(xp),max(xp)]);xlabel('Horizontal Distance x (km)');ylabel('Horizontal Distance y (km)');

end
```

../../Src/Gravimetry/PrismForwardModel/GravForwardModelPrismRD.m

2.3 Some general questions to reflect on

1. Why does the mean sea level follow the shape of the Geoid?

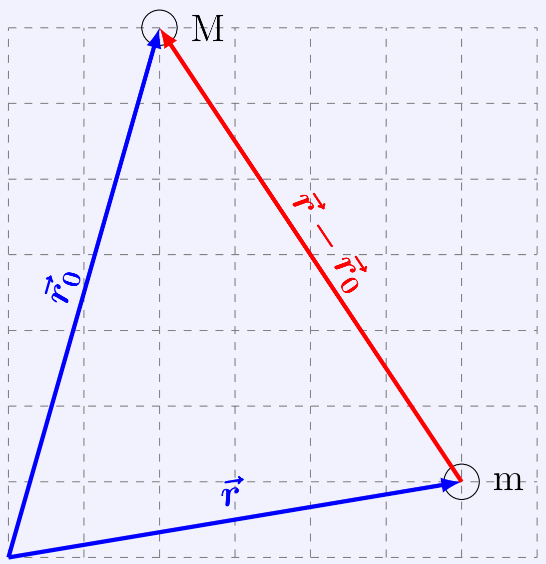
Solutions

Fluids cannot sustain shear stresses. If shear stresses exist the fluid surface will adjust so that the net force is normal to the surface. Without flow the gravitational field will therefore be perpendicular to the ocean surface, which is the definition of an equipotential line. The gravitational force along that line may vary.

2. How can you describe the gravitational attraction between two point masses if none of them is located in the origin of the coordinate system applied?

Solutions

$$\vec{F}(\vec{r}) = -GmM\frac{1}{||\vec{r}-\vec{r_0}||^2}\frac{\vec{r}-\vec{r_0}}{||\vec{r}-\vec{r_0}||} = -GmM\frac{1}{||\vec{r}-\vec{r_0}||^3}\vec{r}-\vec{r_0}$$



3. How does and equipotential line change by crossing an area of (a) mass deficit, and (b) mass excess?

Solutions

For a mass deficit the gravitational vectors will point away from the anomaly. Therefore the corresponding equipotential lines is curved downwards. The opposite holds for a mass excess. I didn't figure out how to draw that yet in tikz.

4. Why do we have Earth & Ocean tides? To understand the principle focus on the Moon's effect only.

Solutions

It comes down to two important effects: (1) The gravitational attraction of the moon towards the Earth is strong on the near-side than on the far side due to the r^{-2} dependence. This alone leads to an ellipsoidal deformation. (2) The centrifugal acceleration due to the rotation around the center of mass between Earth and Moon counterbalances this to a certain extent. The formation of tidal bulges on either side of the Earth relative to the moon can be derived by decomposing the centrifugal acceleration into a radial component and a component that is perpendicular to the rotation axis. This leads to the lunar differential gravitational field that explains tidal bulges at both sides leading to semi-diurnal tides. The sun complicates this further but does not introduce different geophysical concepts. A detailed explanation can be found in Matsuda et al. 2015 (provided on Ilias).

5. Discuss whether the sun or the moon is more important for tides.

Solutions

The sun is much larger and has a larger potential for tidal forces, however, it is also further away. The moon on the other hand has less weight but is closer. Doing the math shows that the sun accounts for about 40% of the tidal forces on Earth.

6. How does deglaciation support the idea of a ductile asthenosphere on top of which the continental plates 'float'?

Solutions

Area with strong former ice cover (e.g., Scandinavia) experience uplift today. Those uplift rates are a response to the disappearance of ice sheets, which have pushed the continents down. Now that the weight is gone, they experience isostatic uplift. This uplift does not occur instantaneously but takes many thousands of years.