Expectations for Exercises

Exercises are an important part of the Geophysics lecture. They will treat some aspects of the lecture in more detail, but also cover new ground. We expect that you work on the exercises at home and we will discuss questions and solutions interactively together. Questions that are marked with 'Extra' are not required but geared to stir your further interest. We will surely support you if you tackle those as well.

5 Exercises for Resistivity Method 1

Version: May 18, 2022
Context: Resistivity Method 01
Timing: All resistivity exercises should be completed the latest by June 4th 2022

5.1 Resistivity Survey in a Sandbox

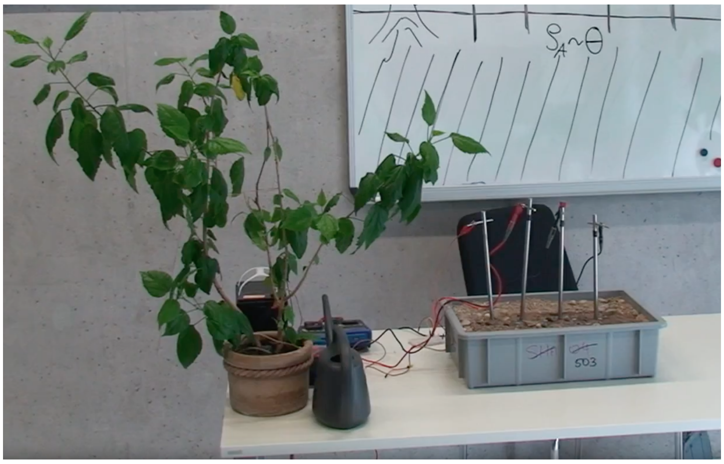


Figure 1: Toy setup for understanding the principles of the resistivity method.

Group Work

Self-organize with your groups to do this during our contact time on Thursdays. Ideally two to three groups should do the experiment in a total of 2h. We will assign two sessions for that. After you are done, disconnect all cables and keep everything as you found it for the next group. **When using the battery with open-end cables, be careful that you do not short them.**

(*practical*) Use a battery, four electrodes and two multimeters to determine the apparent resistivity of sand in a sandbox. You can align the electrodes in a co-linear Wenner array and also check how/if the resistivity changes with variable array types and moisture content. Post your results in the forum and compare them with the literature values. Don't overestimate the accuracy here, it is more about internalizing the measurement principle and to strengthen your cabling skills. All material (Fig. 1) will be provided on-site.

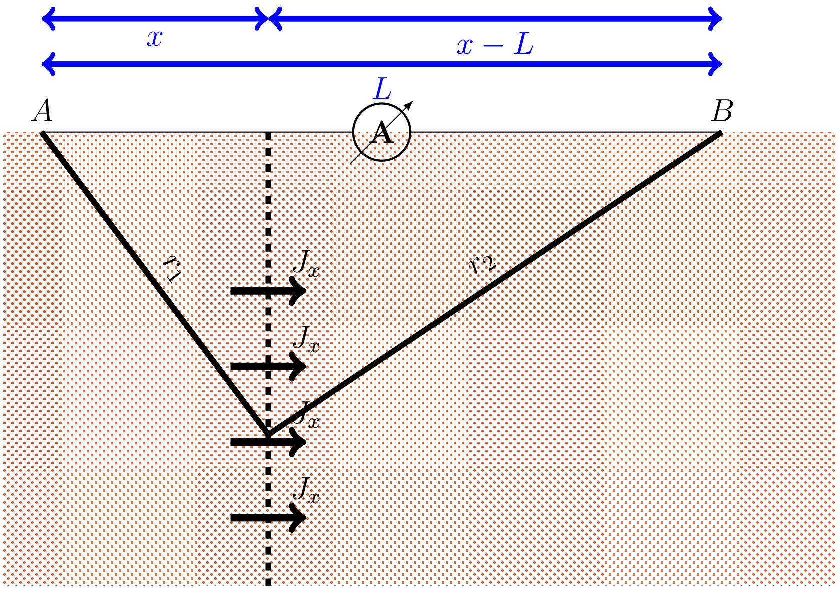
5.2 The LaPlace Equation

(*easy*) In the lecture we derived that the potential field of a single electrode (second electrode is located at infinity) satisfies the LaPlace equation:

$$\vec{\nabla}^2 V = \Delta V = 0.$$

Show that our *Ansatz* $V(r) = \frac{A}{r}$ satisfies this equation in spherical coordinates. Note that the LaPlace in spherical coordinates is $\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial}{\partial r}\right)$.

5.3 Depth of Investigation



(*tricky*) In this exercise we would like to approximate which depth interval the resistivity method is sensitive to. We already understand intuitively that a larger electrode spacing between A and B increases the depth penetration. Here we aim to do it quantitatively. For this we consider a vertical plane located at distance x away from electrode A, and calculate how much of the current flow density crosses that plane. This means we would like to calculate $J_x(z)$. From the lecture we know that:

$$\vec{j} = \sigma \vec{E} = \sigma \nabla V \rightarrow J_x = -\frac{I}{2\pi}\frac{\partial}{\partial x}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

Adopting the coordinate system from the Figure above we choose $r_1 = \sqrt{x^2 + y^2 + z^2}$ with z point along depth and y into the plane. First calculate J_x for any point P, then show that

$$J_x(x = L/2) = \frac{I}{2\pi}\frac{L}{(z^2 + \frac{L^2}{4})^{3/2}}$$

at the center point. Sketch this function roughly on paper for a fixed L. Now consider a fixed layer at constant depth z. How does J_x change as L is varied? Show that $J_x(x = L/2, z)$ has a maximum at $L = \sqrt{2}z$. This shows that depth penetration increases as we increase the distance between the current electrodes A and B. Still, most of the current flow will be near the surface. In order to define a depth of investigation we will need to apply some thresholding and include placement of the M,N electrodes. This varies from array to array.