Expectations for Exercises

Exercises are an important part of the Geophysics lecture. They will treat some aspects of the lecture in more detail, but also cover new ground. We expect that you work on the exercises at home and we will discuss questions and solutions interactively together. The joint meetings may start with randomly chosen students presenting their approach. It is ok if the solution is not available at this stage and there will be no interrogation. However, please don't show up unprepared, because this will inevitably be awkward.

1 Exercises for Gravity Method 1

1.1 Detection of a spherical object in the sub-surface

(a) Consider a spherical object with radius R located at depth z below the surface, and a gravimeter that is moved along the surface across the anomaly (Fig.1). Derive an analytical expression for the expected *vertical* anomaly g_z as a function of the lateral distance x and depth z. Calculate the maximum anomalies that you would expect for some realistic settings (e.g., a limestone cave.)

(b) Use a piece of paper or a software of your choice (e.g., Excel, SciDAVis, Matlab, Python) and visualize the expected vertical gravity anomaly for a specific setting as a function of lateral distance x. Show how this profile changes as you vary the depth of the object.

(c) Sometimes results in the gravity method are expressed in *mGal* with 1 Gal = 0.01 m s^{-2}. Transfer your results to those units and post a picture of your graph in the forum.

(d) We now investigate how we can estimate the depth of the object from an anomaly as derived in (a,b). For this you have to derive an expression that links the half-width of the anomaly (i.e. $g_z = \frac{1}{2} g_{z,max}$ for $x = x_{1/2}$) to the depth z of the object. Derive this relationship which has the form $z = cx_{1/2}$. Why is such a relationship be useful?

1.2 Gravitational acceleration inside the Earth

Knowing the gravitational acceleration *inside* the Earth is important, e.g., for understanding processes related to mantel convection.

(a) Use the consequences of the shell theorem to predict the gravitational acceleration $g(r)$ inside a spherical Earth. First assume that the Earth's density is constant, then that it decreases linearly with distance from the center. Assume that the density in the core is approximately $\rho_{core} = 13\frac{g}{cm^3}$ and in the crust approximately $\rho_{crust} = 2.7\frac{g}{cm^3}$.

(b) Draw the results on a x-y graph on paper or using a software of your choice.

(c) The PREM model provides an observationally constrained estimate of the density distribution inside the Earth. Knowing the shape of this profile, what is your guess of the gravitational acceleration inside the Earth? Why is it more difficult for you to calculate this quantitatively compared to the previous cases of constant and linearly varying density?

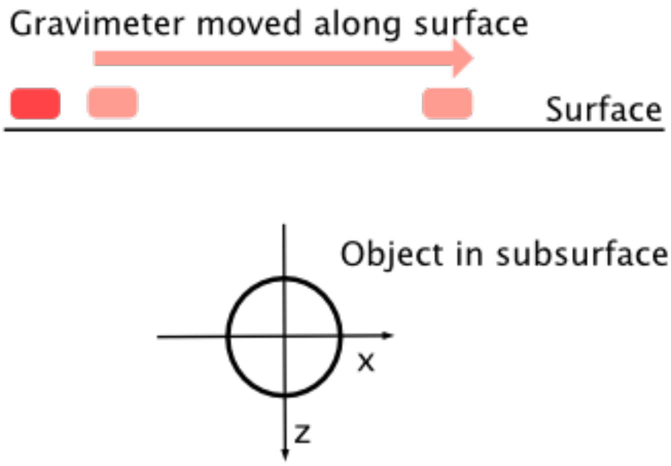
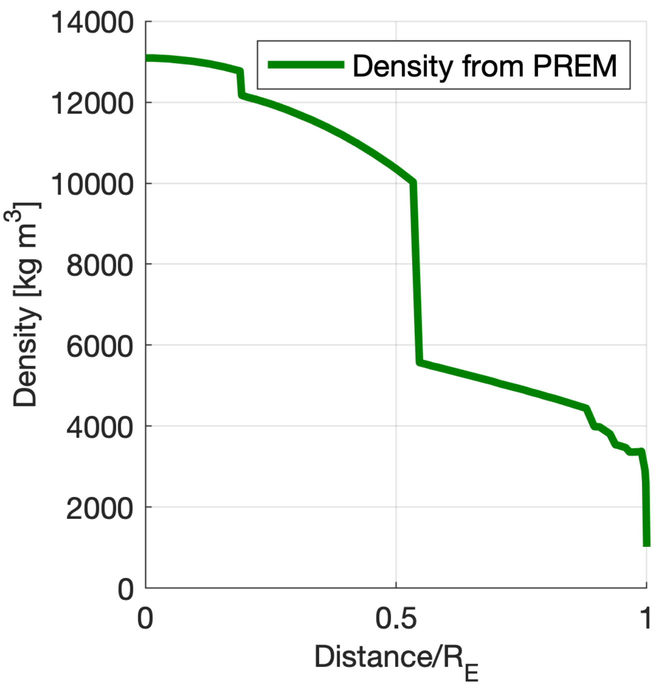


Figure 1: Sketch for problem 1.1. It is easiest to choose a coordinate system with the origin inside the subsurface object.

(d) Extra: Calculate the gravitational acceleration inside the Earth based on the PREM model using Matlab/Python/Excel. The data can be found on Ilias.



1.3 Determining the mean density of the Earth with your own gravimeter

Group Work

Self-organize in groups with 5-6 team members. Choose a group name and a team captain that communicates with the instructors. It is ideal if this group stays together throughout the term for the applied exercises. Make sure that your are inclusive during the group formation.

The vertical gravitational acceleration can be determined by measuring the traveltime of a freely falling object for a known distance. Assuming that the radius of the Earth and the gravitational constant are known ($G \approx 6.674 \cdot 10^{-11}$ m^{3}kg^{-1}s^{-2}, $R_E \approx 6370$) the mass of the Earth can be determined with the basic principles derived in class.

(a) Your task is to design a home-built free-fall gravimeter that does the job. You will quickly realize that the time measurement (i.e. the time from the start to the end of the fall) is one critical aspect in the system design. A helper tool that we suggest is the *acoustic stopwatch* that can be accessed via a smartphone and the *phyphox* app. Figure out an efficient way how the traveltime of a freely falling object can be determined with this type of acoustic trigger. (Tip: You may need a hammer, a metal bar and a bar clamp for your acoustic trigger. Be creative.)

(b) Collect a dataset of traveltimes and derive the mass of the earth. Provide an error estimate. Can you identify a measurement bias? How does your result compare to literature values?

(c) Given your result, what is the mean density of the Earth? How does that compare to rock densities found at the Earth's surface? What is a main conclusion that you can draw from this?

(d) Post your result, your dataset and a picture of your system setup in the ILIAS forum.

(e) Extra: Are you a tinkerer/Bastler? If so, feel free to improve the system design, e.g., by using light barriers in combination with a raspberry pi nano. We are more than happy to buy material for you, the only constraint is that it works reliably and that it remains cheapish (let's say <100 Euro). **If you succeed, you will win a gift certificate of that can be used in a Tübingen pub of your choice to celebrate your victory with your peers.** Moreover, your system will then be used for eternity for the following classes giving you much honor within the geo- and environmental student community.