Introduction to Geophysics

R. Drews

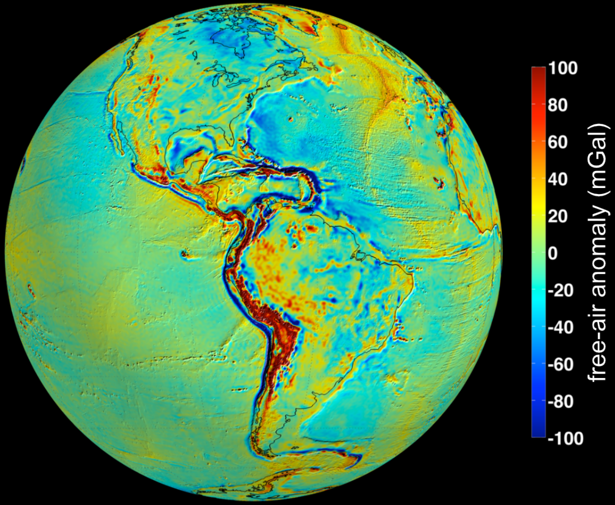
Gravimetry

Learning Goals

Learning goals today:

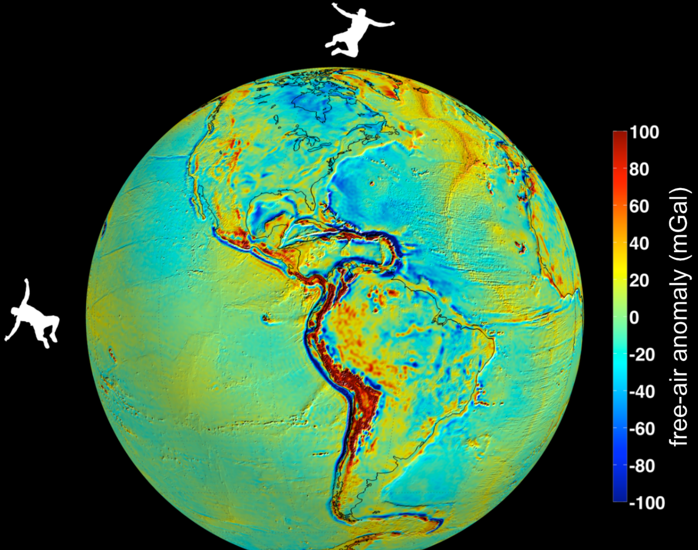
- The gravitational force, its potential field, and how to measure it.
- The reference gravitational field of the Earth.

Example: Global variability



Example: Global variability

Your mass is constant but your weight is not.



What is a force?

[Newton (1642-1726) / G. Johnson.]

$$\vec{F} = m\vec{g}$$

$\vec{F}$: Force (N; $\mathrm{kg\,m\,s^{-2}}$)

$\vec{g}$: Acceleration ($\mathrm{m\,s^{-2}}$)

m : Mass (kg)

The gravitational force

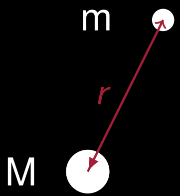
[Newton (1642-1726) / G. Johnson.]

$$\vec{F} = G\frac{mM}{r^2}\hat{r}$$

$G = 6.674 \cdot 10^{-11}\,(m^3 kg^{-1} s^{-2})$

$\hat{r}$: unit vector

r : distance between point masses



Example: The gravitational constant

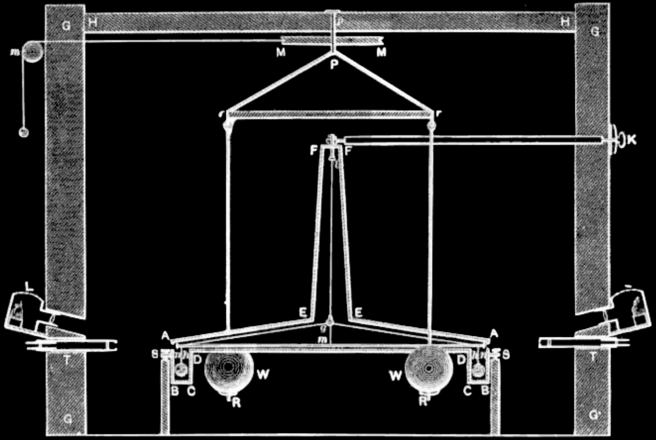
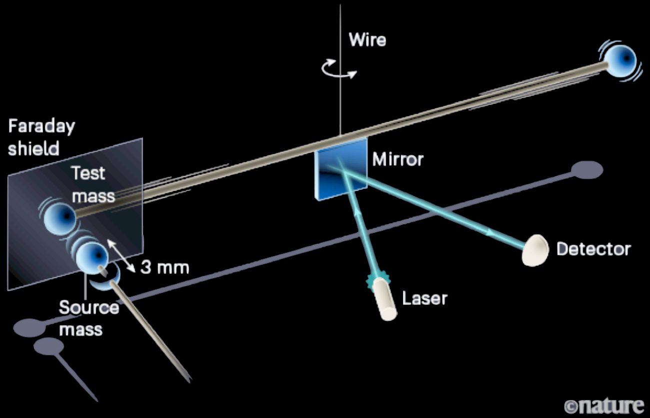


Fig. 1

Cavnedish, PNAS, 1798

Example: The gravitational constant



Westphal et al., Nature, 2021

G is the worst known constant in physics.
Why?

Example: Measuring acceleration

$$\vec{F} = m\vec{g}$$

$$\vec{F} = G\frac{mM}{r^2}\hat{r}$$

$$\rightarrow \vec{g} = G\frac{M}{r^2}\hat{r}$$

$$\rightarrow \frac{d^2\vec{x}}{dt^2} = G\frac{M}{r^2}\hat{r}$$

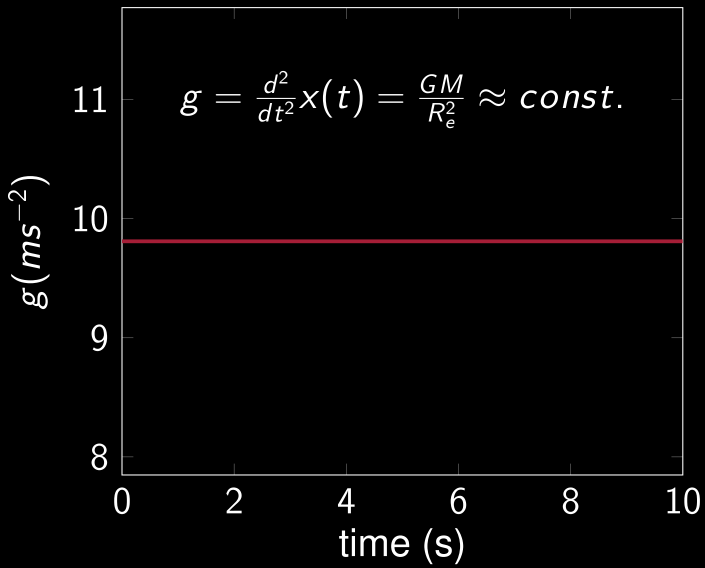
This is a differential equation.

Example: Measuring acceleration

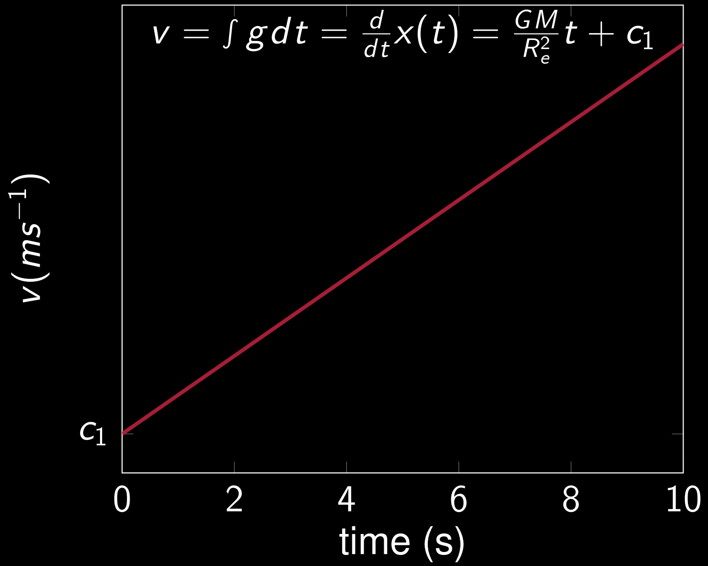
$$\frac{d^2\vec{x}}{dt^2} = G\frac{M}{R_E^2} \approx const.$$

At the Earth's surface (R_E) g is close to constant and only vertical. (Later we will see that none of this is not quite true).

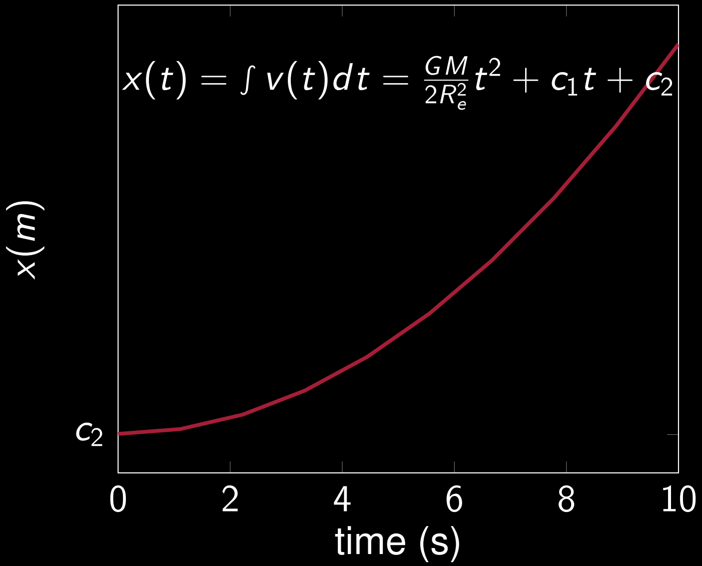
Example: Measuring acceleration



Example: Measuring acceleration



Example: Measuring acceleration



Example: Measuring acceleration

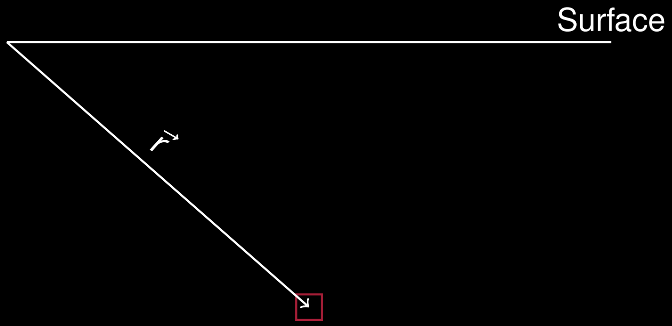
$$x(t) = \frac{GM}{2R_e^2}t^2 + c_1 t + c_2$$

- Setting, e.g., $c1 = 0$ (initial velocity) and $c_2 = 0$ (initial position) is quite convenient.
- This is the principal of a free-fall gravimeter.

Exercises: Group-Work Thursdays

- Thanks to the Greeks we know the radius R_E for the Earth. However, its mass was unknown for a while.
- Go ahead and determine the mass of the Earth M with your Smartphone!
- There is an important first-order finding in Earth Sciences that you can (re-) discover. Which one?

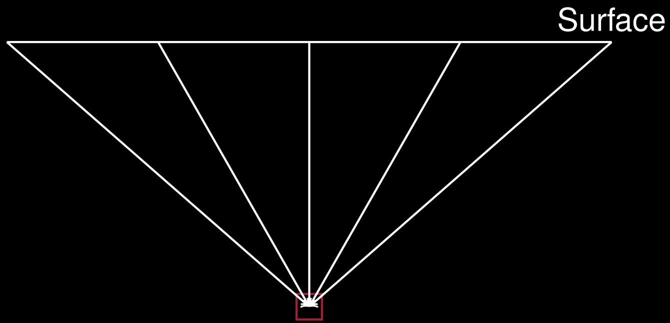
Beyond point masses



$$\vec{F} = G\frac{dM}{r^2}\hat{r}$$

For a small mass dM the point mass approximation holds.

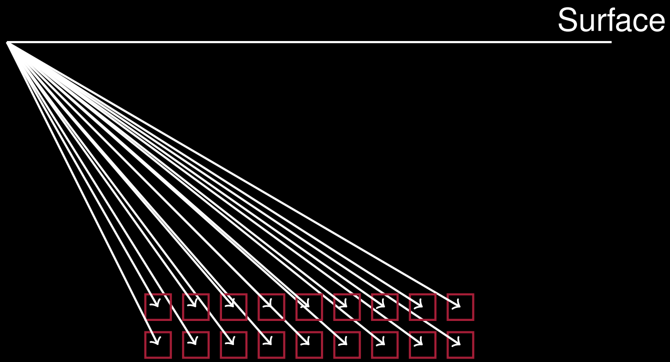
Beyond point masses



$$\vec{F} = G\frac{dM}{r^2}\hat{r}$$

Profiling across a sub-surface target results in a gravity anomaly ($\rightarrow$ Exercises).

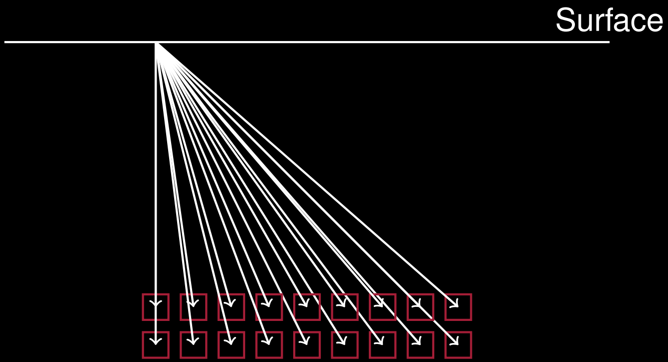
Beyond point masses



$$\vec{F}(\vec{r}) = \sum_i G \frac{dM_i}{r_i^2} \hat{r}_i$$

For i point masses the effect adds up.

Beyond point masses



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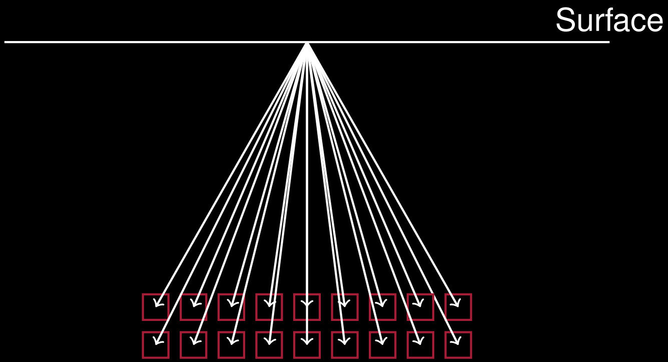
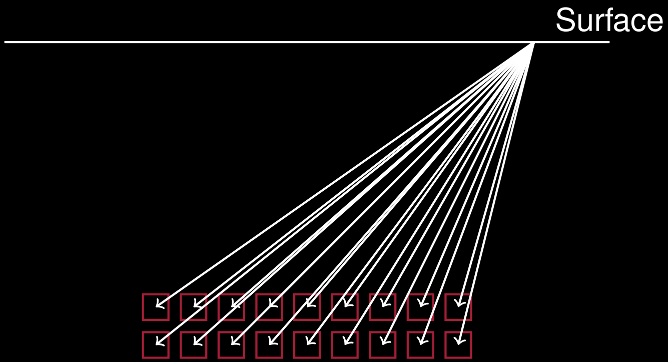
Beyond point masses



$$\vec{F}(\vec{r}) = \sum G \frac{dM_i}{r_i^2} \hat{r}_i$$

Beyond point masses

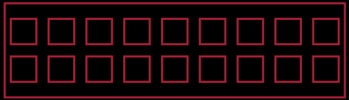


$$\vec{F}(\vec{r}) = \sum G \frac{dM_i}{r_i^2} \hat{r}_i$$

Beyond point masses

Surface

$$\vec{F}(\vec{r}) = G \int \rho \frac{1}{r^2} \hat{r} \, dV$$

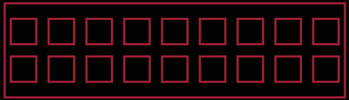


The summation can be replaced by an integration over a volume enclosing a continuous density.

Beyond point masses

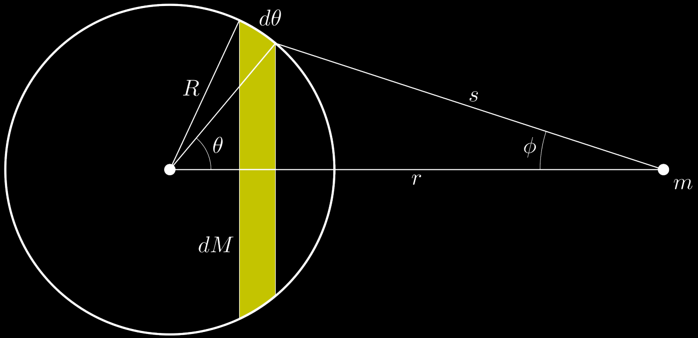
Surface

$$\vec{F}(\vec{r}) = G \int \rho \frac{1}{r^2} \hat{r} dV$$



The integration is a triple integral. Integration limits and coordinates depend on the viewpoint. Example is a Bouger plate, in general not easy to solve ($\rightarrow$ Exercises).

Example: Shell



[Xaononl CC BY-SA 4.0]

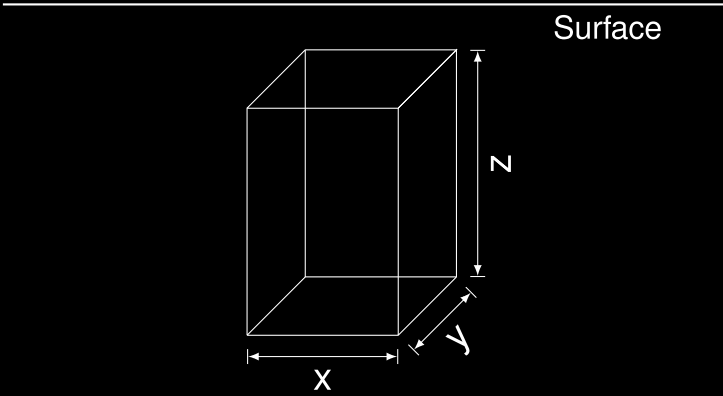
Newton's shell theorem solves the volume integral inside and outside spherical objects ($\rightarrow$ Ex.-Discussion)

Newton's Shell Theorem

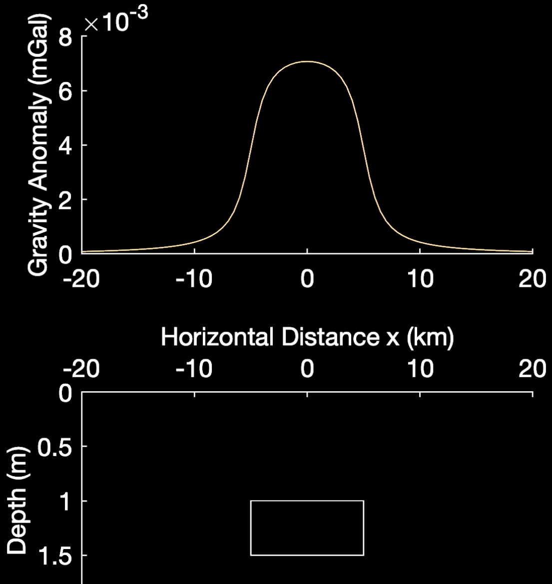
- The field outside a shell is the same as the one from an equivalent point mass
- The field inside a shell is zero. Everywhere.

Other shapes

- There are analytical solutions for other shapes (e.g., Nagy 1966 for Prism).

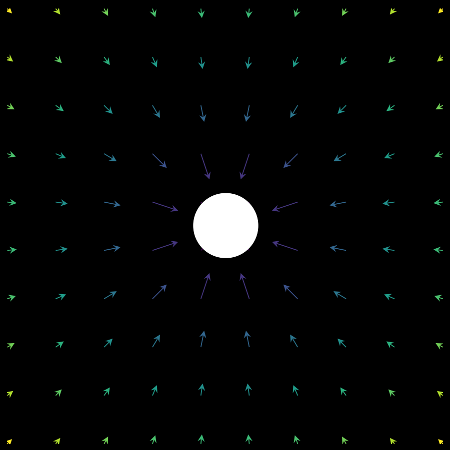


Numerical forward modelling ($\rightarrow$ Ex)



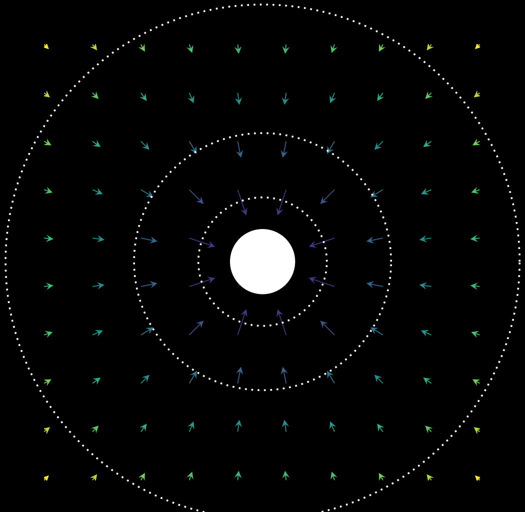
Vector fields

$$\vec{g} = G\frac{M}{r^2}\hat{r}$$



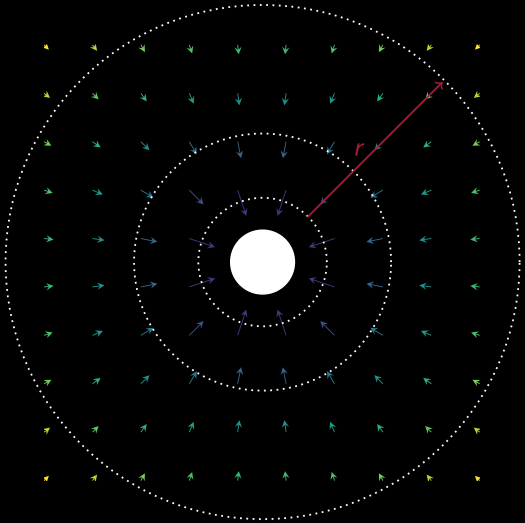
Potential Field

$$\vec{g} = G\frac{M}{r^2}\hat{r}$$



Potential Field

What is the amount of work required?



Potential Fields

$$
\begin{aligned}
U(r) &= -\int_{\infty}^{r} \vec{g}\, d\vec{r} \\
&= -\int_{\infty}^{r} g\, dr \\
&= -GM \int_{\infty}^{r} \frac{1}{r^2} dr \\
&= -GM \left[-\frac{1}{r} \right]_{\infty}^{r} \\
&= GM \frac{1}{r}
\end{aligned}
$$

Potential for a point mass.

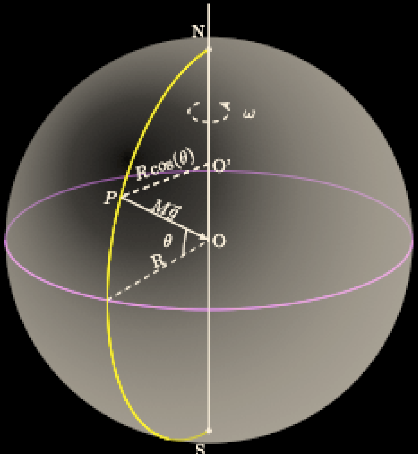
Potential Fields

$$\vec{g}(r) = -\nabla U(r)$$

- It is sometimes easier to calculate the potential of an anomaly and to infer the acceleration via the gradient.
- Equipotential lines are perpendicular to the field direction.
- Equipotential lines are in general NOT lines of equal field strength (cf. with down-hill slope force in landscape)

Gravitational field of a spherical Earth

The Earth's rotation minimizes gravitational acceleration at the equator. At the poles it does nothing.



Gravitational field of a spherical Earth

Centripedal acceleration at P perpendicular to rotation axis parallel to O'-P:

$$g_{r.} = \omega^2 R \cos(\theta)$$

Centripedal acceleration at P perpendicular to rotation axis parallel to O'-P:

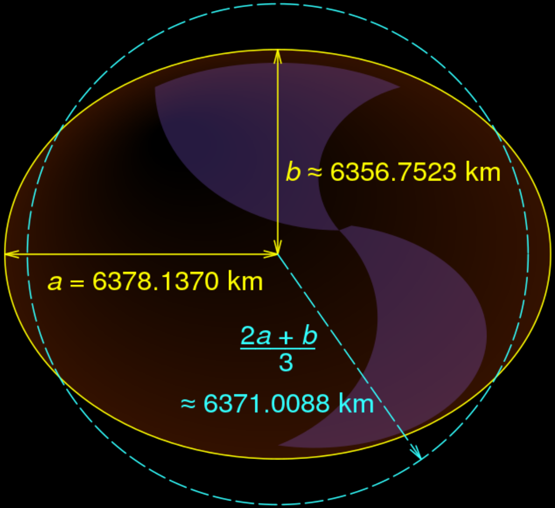
$$g_{r.,proj.} = \omega^2 R \cos^2(\theta)$$

Angular Frequency: ω

Angular Velocity: $\vec{v}_r = \vec{\omega} \times \vec{R} \cos(\theta)$

Angular Acceleration: $\vec{g}_r = \dot{\vec{v}}_r = \vec{\omega} \times \vec{\omega} \times \vec{R} \cos(\theta)$

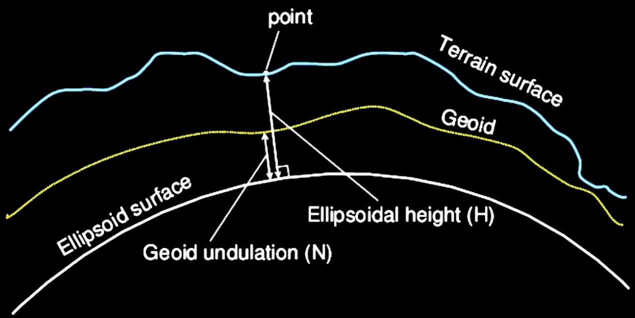
An ellipsoidal Earth



Potential Fields

- Rotation induces ellipsoidal shape approximate with reference ellipsoid.
- Latitudinal correction of gravitation is adjusted accordingly.

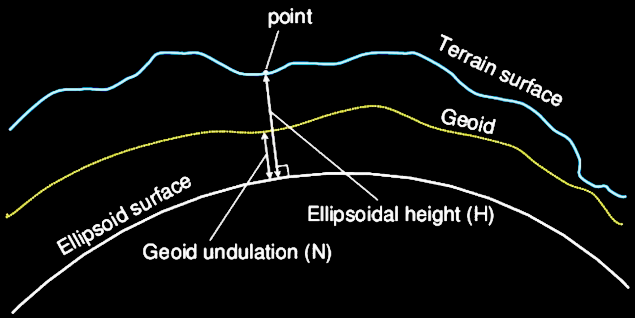
An ellipsoidal Earth



[Ziebart et al., 2004]

- Geoid is a real-world equipotential line approximating sea level.
- It is referenced to the geometric ellipsoid.

An ellipsoidal Earth



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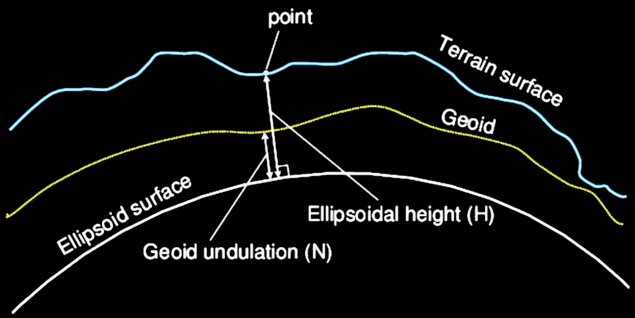
An ellipsoidal Earth



[Ziebart et al., 2004]

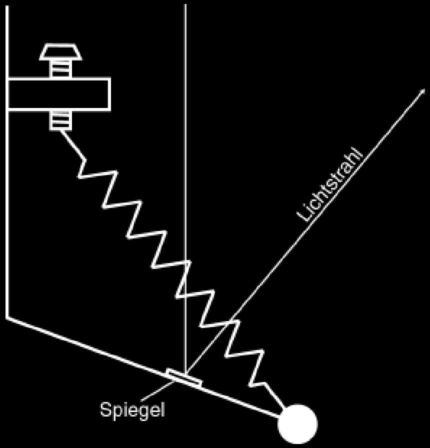
- Upwarping of geoid indicates mass excess.
- Downwarping of geoid indicates mass deficit.

Learning Goals

Learning goals today:

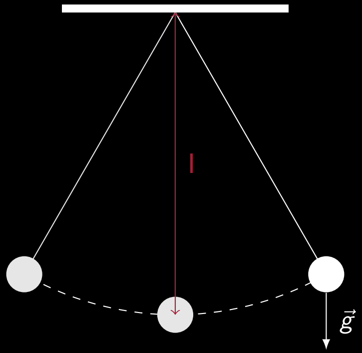
- Principles of gravimeters.
- Gravity survey types (absolute & relative) and general considerations of survey layouts and structure types.
- Reduction of gravity data.

Spring-based gravimeters



Springconstant & Extension.

Pendulum-based gravimeters

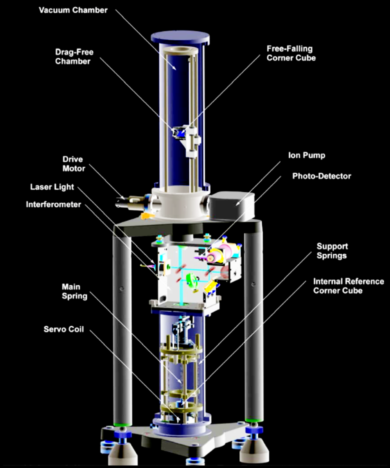


Eigenfrequency & Length.

$$\omega = \sqrt{\frac{g}{l}}$$

Free-fall gravimeters ($\rightarrow$ Ex.)

[FG Gravimeter from MicroGLaCoste]

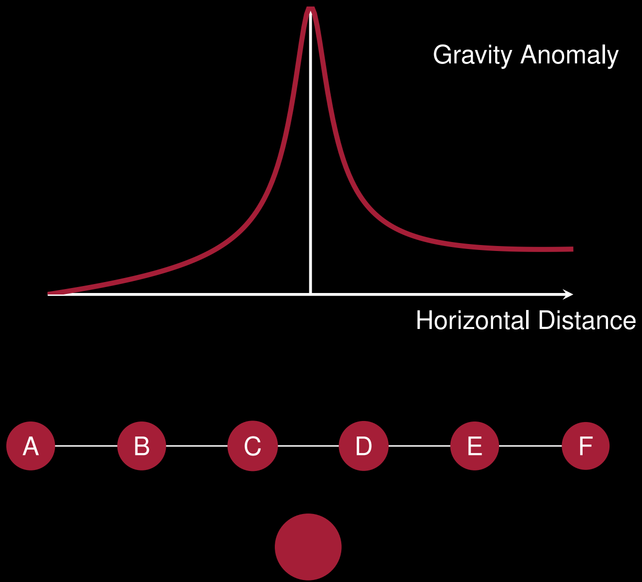


Traveltime & Distance.

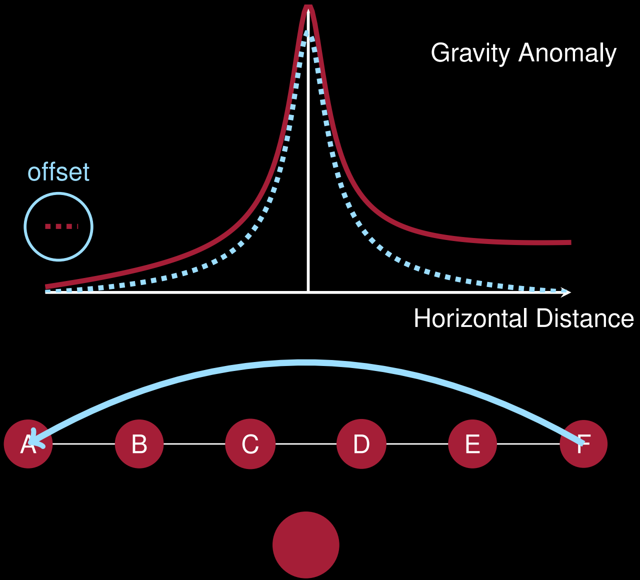
Gravimeters

- Unit used is *Gal* (=0.01 ms^{-2})
- Top *absolute* gravimeters $\sim 1\mu$Gal $(10^{-9}g)$
- Top *relative* gravimeters $\sim 10\mu$Gal $(10^{-9}g)$
- Typically only g_z is measured.

Instrument drift / temporal variability



Instrument drift / temporal variability



Absolute vs. relative

Absolute gravimeters are needed

- if loop closure if impossible (e.g. intercontinental surveys),
- for long-term changes such as isostatic uplift,
- as basestations for relative surveys.

Relative surveys are always easier to conduct and loop closure can cancel many error sources (e.g., instrument drift).

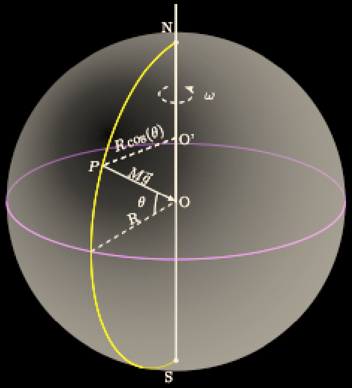
Reduction of gravity data

Every gravity survey measures:

- latitudinal variability,
- dependency on elevation,
- the surrounding terrain,
- excess mass above anomaly,
- earth & ocean tides,
- (instr. drift, motion compons.).
- density variability in the subsurface.

Latitudinal variability ($\rightarrow$ Ex.)

Extension to ellipsoid contains the same physics.



Max 5 Gal (this is large!).

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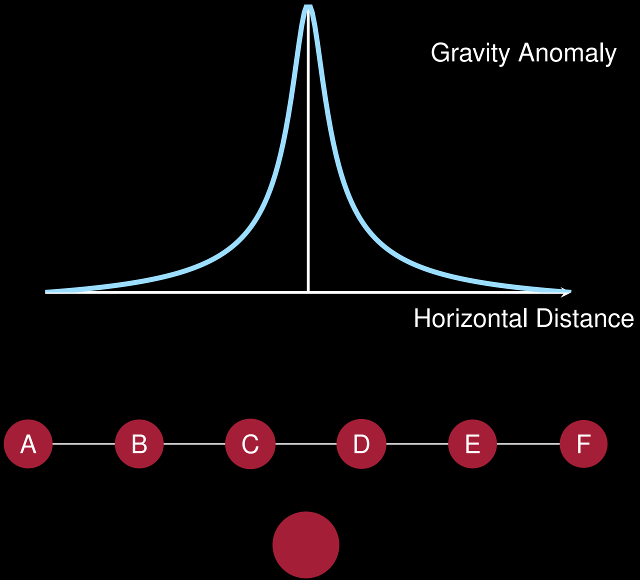
Elevation correction (i.e. free-air)



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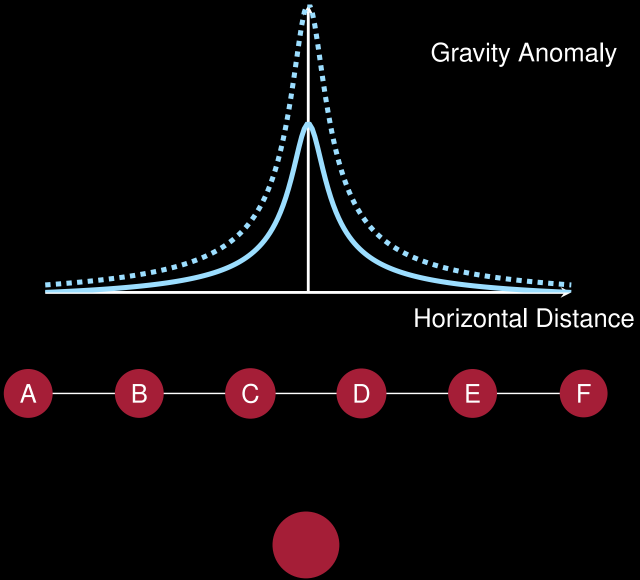

Elevation correction

The elevation correction references the gravity anomaly to the same datum (e.g., the geoid).

How does the gravitational acceleration change with elevation near the Earth’s surface?

Elevation correction via Taylor expansion

Taylor expansion near $r = R_E$:

$$g(r) = g(R_E) + \frac{dg}{dr}\Big|_{R_E}$$

Elevation correction via Taylor expansion

Taylor expansion near $r = R_E$:

$$g(r) \approx G\frac{M}{R_E^2} - 2G\frac{M}{R_E^3}(r - R_E) + \ldots$$

Elevation correction via Taylor expansion

Taylor expansion near $r = R_E$:

$$g(r) \approx \underbrace{G\frac{M}{R_E^2}}_{\text{g at Earth's surface}} - 2G\frac{M}{R_E^3}(r - R_E) + \ldots$$

Elevation correction via Taylor expansion

Taylor expansion near $r = R_E$:

$$g(r) \approx G\frac{M}{R_E^2} - \underbrace{2G\frac{M}{R_E^3}(r - R_E)}_{\text{change with elevation}} + \ldots$$

Elevation correction via Taylor expansion

Taylor expansion near $r = R_E$:

$$g(r) \approx G\frac{M}{R_E^2} - \underbrace{2G\frac{M}{R_E^3}(r - R_E)}_{\text{change with elevation}} + \ldots$$

Evaluation at let's say $r = R_E + 1$ (m) returns a change of $\delta g(r) \approx -0.3$ mGal per m.

Elevation correction via Taylor expansion

Taylor expansion near $r = R_E$:

$$g(r) \approx G\frac{M}{R_E^2} - \underbrace{2G\frac{M}{R_E^3}(r - R_E)}_{\text{change with elevation}} + \ldots$$

$\delta g(r) \approx -0.3$ mGal per m is large compared to the sensitivity of gravimeters, therefore the gravimeter elevation needs to be determined within centimeters using GNSS.

Reduction of gravity data

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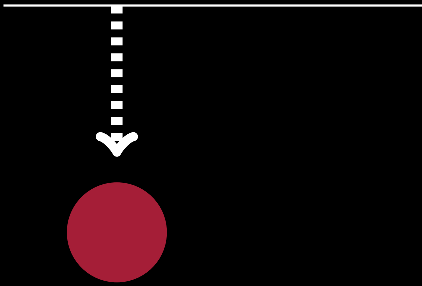
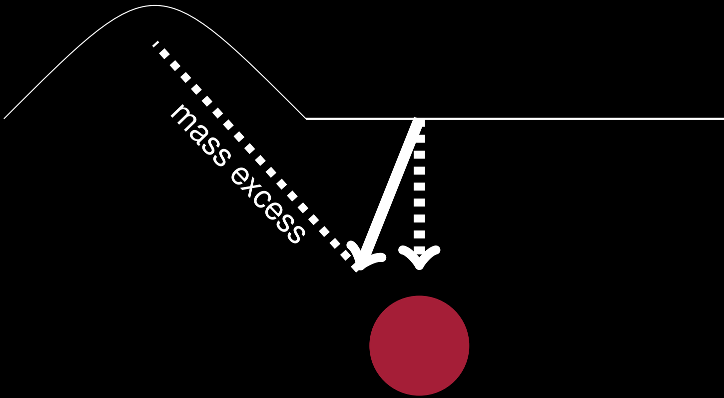
Reduction of gravity data: Terrain correction

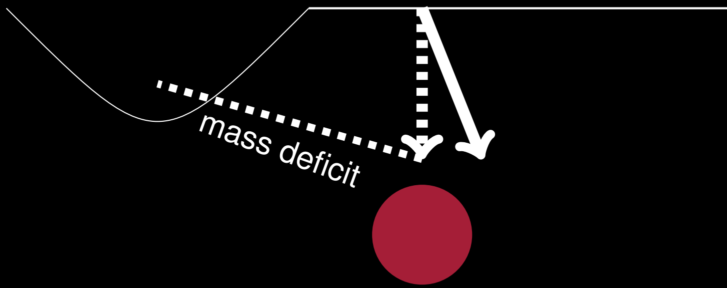


Reduction of gravity data: Terrain correction



A neighboring mountain will reduce the measured g_z independent of target properties.

Reduction of gravity data: Terrain correction



A neighboring valley will reduce the measured g_z independent of target properties.

Reduction of gravity data: Terrain correction

- The terrain correction requires an elevation model and assumptions about the broad-scale sub-surface density.
- The terrain correction is positive both for surrounding valleys and mountains.

Reduction of gravity data

Every gravity survey measures:

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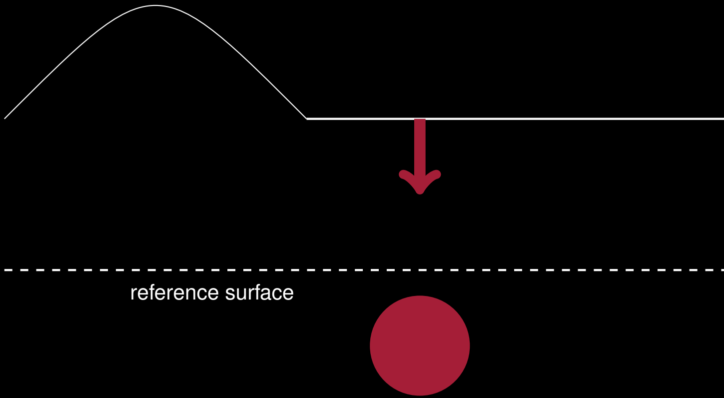
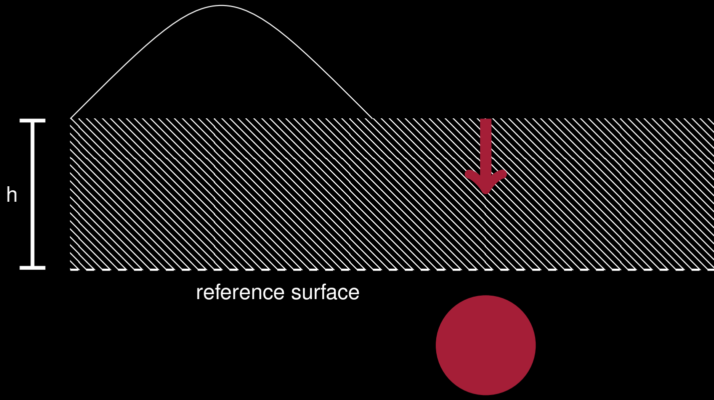
- density variability in the subsurface.

Reduction of gravity data: Terrain correction



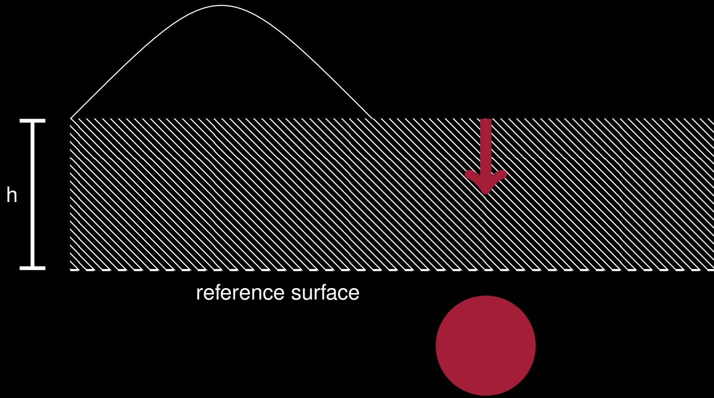
Elevation and terrain correction to not account for the mass between the measurement surface and the reference surface.

Reduction of gravity data: Terrain correction



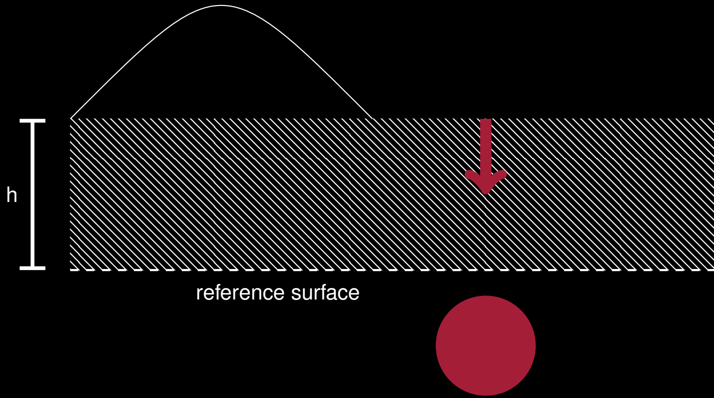
What is the effect δg_z of a horizontal plate with constant density?

Reduction of gravity data: Terrain correction



$$g_z = G\rho \int \int \int \frac{1}{r^2} \cos(\phi) dV = ?$$

Reduction of gravity data: Terrain correction



$$g_z = G\rho \int\int\int \frac{1}{r^2}\cos(\phi)dV = 2\pi G\rho h$$

Reduction of gravity data

Every gravity survey measures:

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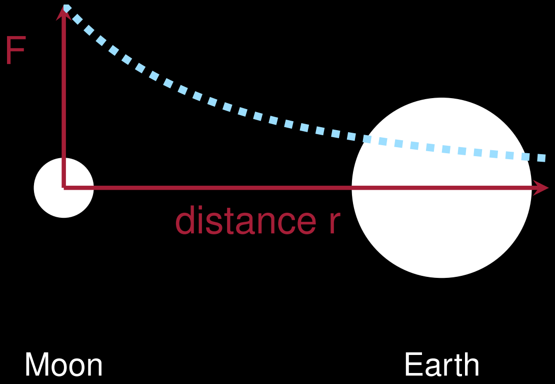
- excess mass above anomaly,
- earth & ocean tides,
- (instr. drift, motion compensation),
- density variability in the subsurface.

The origin of tides

- Tides are caused by gravity celestial bodies (i.e. Sun & Moon).
- Tidal forces vary across a spatially extended body.
- Tidal forces are balanced by centrifugal forces of two (three) body rotations.

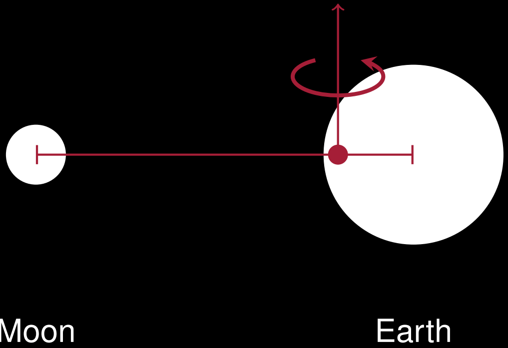
The origin of tides

The origin of tides

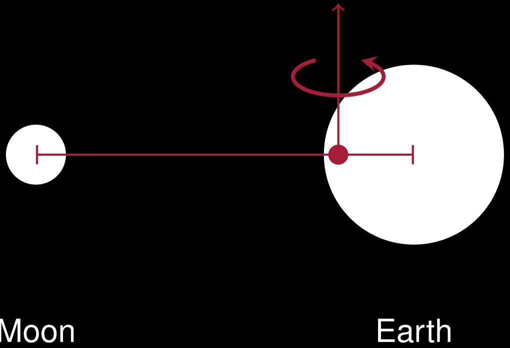


Gravitational attraction is stronger on the nearside than the farside.

The origin of tides

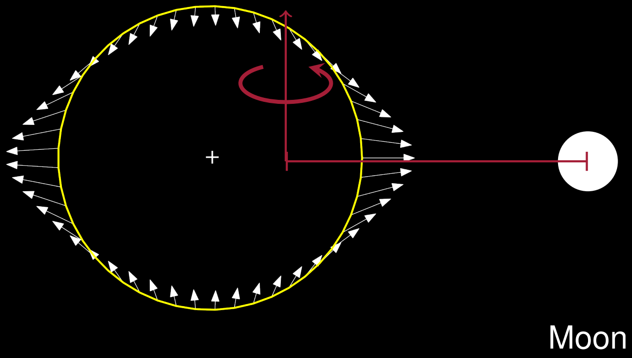


The origin of tides



Centrifugal force can be projected into radial (i.e. parallel to Earth's gravitation) and parallel component. This leads to the force balance.

The origin of tides



The lunar gravity differential field is responsible for two tidal bulges (i.e. tides twice a day).

The origin of tides

- Tidal forces vary across a spatially extended body.
- Tidal forces are balanced by centrifugal forces of two (three) body rotations.
- There is lots of confusion regarding the origin of tides (cf. Matsuda et al. 2015 $\rightarrow$ Ilias).
- Tide models can be used for correction

Reduction of gravity data

Every gravity survey measures:

- latitudinal variability,
- dependency on elevation,
- the surrounding terrain,
- excess mass above anomaly,
- earth & ocean tides,
- (instr. drift, motion compensation),
- density variability in the subsurface.

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