



Introduction to Geophysics  
R. Drews

# Gravity Method





## Introduction to Geophysics

- ▶ Who?
- ▶ What?
- ▶ How?



- ▶ R. Drews (Prof. Geophysics at UT)
- ▶ P. Dietrich (Prof. Env./Eng.- Geophysics at UFZ, Leipzig)
- ▶ R. Ershadi (PhD Geophysics)
- ▶ A. Vinson (TA, MSc Geosciences)
- ▶ L. Naumann (TA, BSc Geosciences)



## Introduction to Geophysics - R. Drews

Prof. for Geophysics since 2022



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# What are we teaching?



**Geophysics** is a branch of earth science dealing with the physical processes and phenomena occurring especially in the earth and in its vicinity.

[Merriam-Webster]

**Applied Geophysics** is a branch of Geophysics dealing with different observational methods imaging the sub-surface.

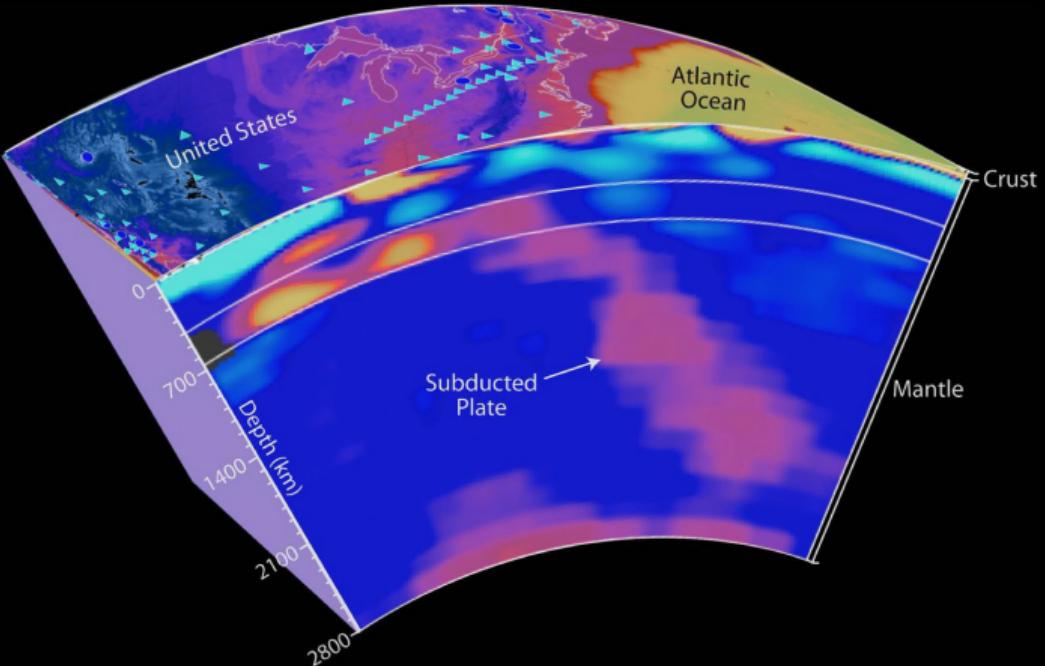
[The lecture focus will be here.]



**Applied Geophysics** contains, e.g, gravity, magnetics, electrics, electrical induction, electromagnetics (radar), seismics.

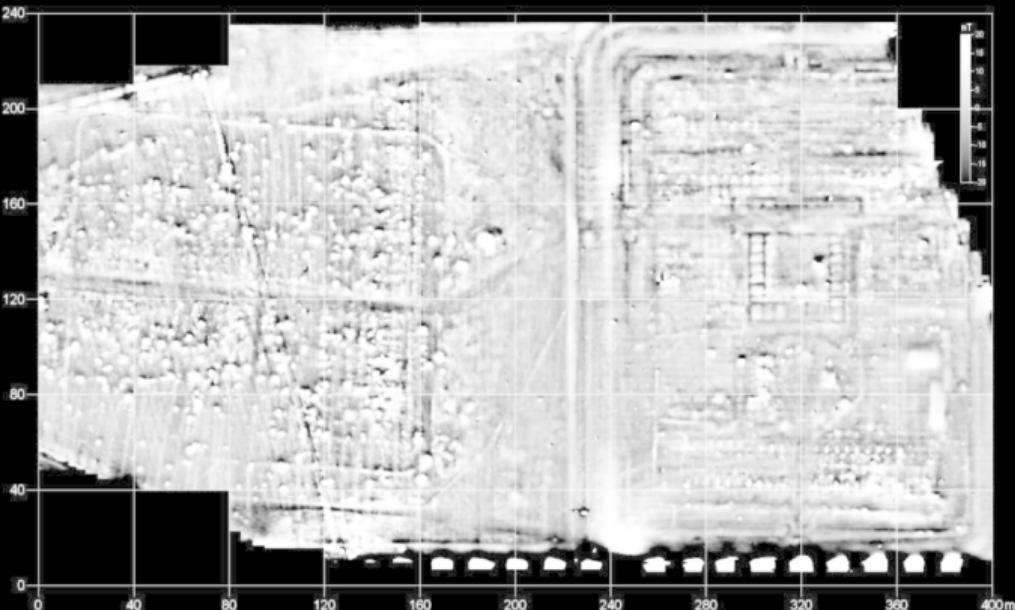
[Focus on physical principals rather than aquistition specifics.]

# Example: Seismics



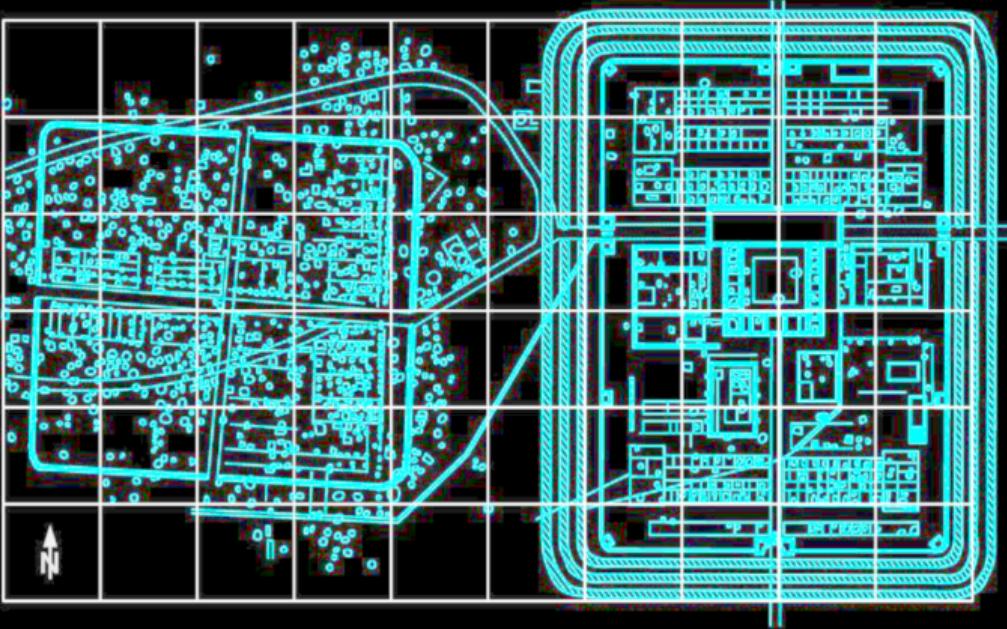
[S. van der Lee, Northwestern University, Evanston, IL]

# Example: Magnetics



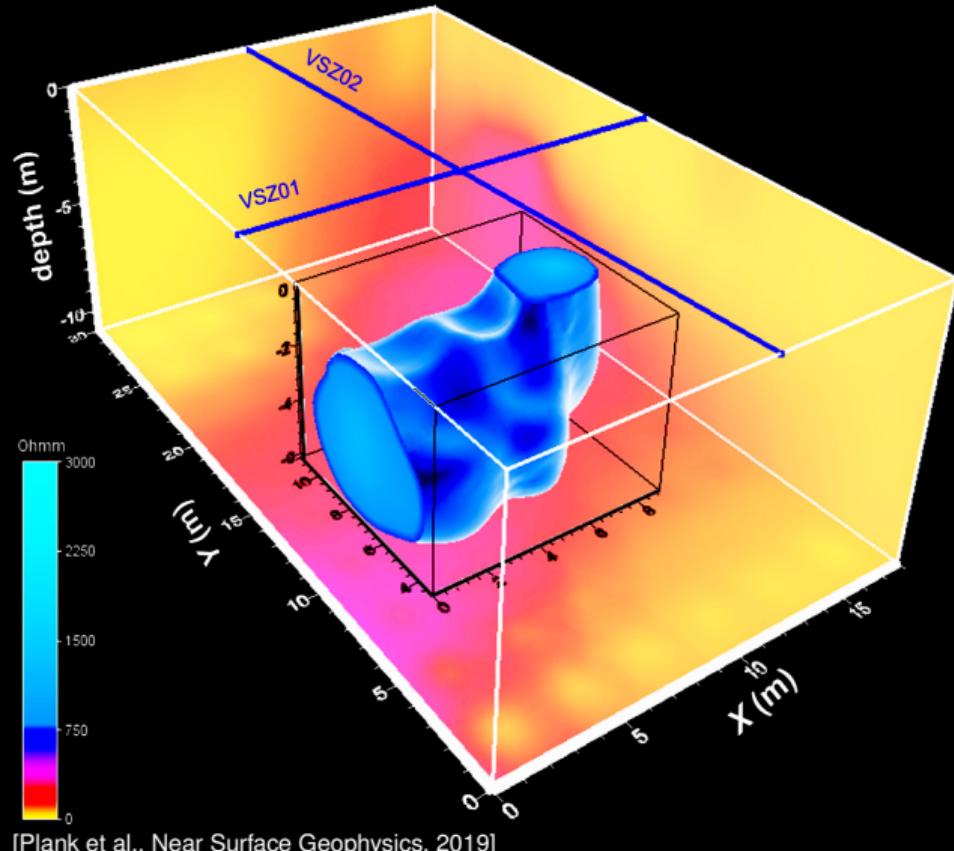
[Fassbinder, Bavarian Academy of Sciences]

# Example: Magnetics

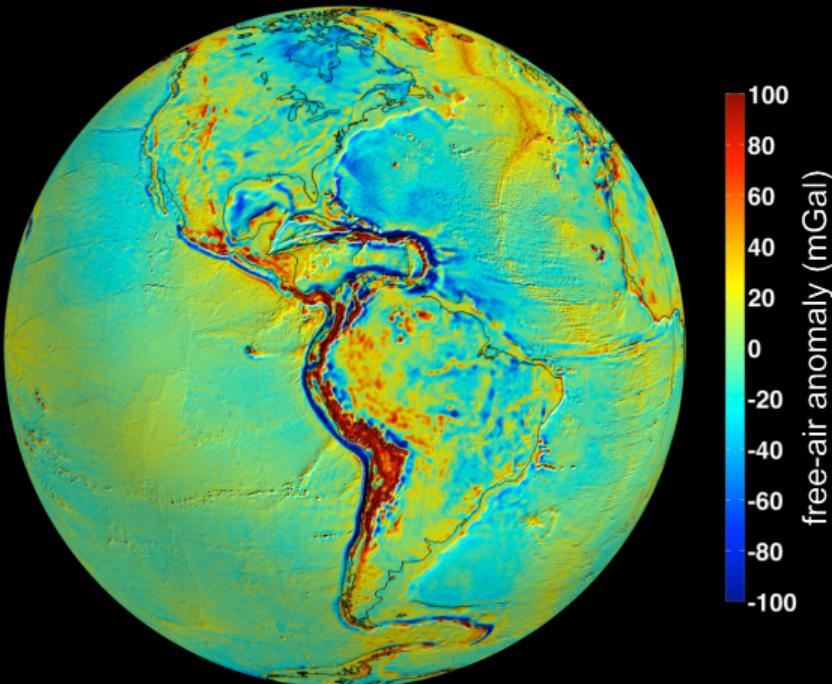


[Fassbinder, Bavarian Academy of Sciences]

# Example: Geoelectrics



# Example: Gravity



[JPL/Caltech GRACE-FO (DT-10)]

# How do we teach?



- ▶ Video Lectures or Plenum (Tuesdays)
- ▶ Exercises & 1-1-Interaction (Thursdays)
- ▶ Applied Exercises (Magnetics, Electrics, Seismics)



## Learning Goals

- ▶ Obtain a broad overview of geophysical methods for sub-surface imaging.
- ▶ Understand the underlying physical principles.
- ▶ Learn how to think logically & quantitatively.



## Expectations

- ▶ Be prepared.
- ▶ Ask questions.
- ▶ Do the work.



## Introduction to Geophysics

Geow-B402-V2

### Instructor Info —

- Reinhard Drews
- Office Hrs: on demand.
- GUZ 3M07/3U03/3F03
- [Website](#)
- [reinhard.drews@uni-tuebingen.de](mailto:reinhard.drews@uni-tuebingen.de)

### Course Info —

- Prereq: None
- Tues & Thurs
- 16:15-18.00
- Online

### Field Exercises —

- In sub-groups of 6
- The field exercises will take place in the field, in groups of 6 students. The exercises will be conducted in English. The fieldwork will be done in the Tübingen area. The exercises will be conducted in English. The fieldwork will be done in the Tübingen area.

### Overview

[Document version Wednesday 9<sup>th</sup> March, 2022 at 16:28:15]

This course provides a broad overview in applied geophysics with a focus on the most common sub-surface imaging techniques: gravimetry, magnetics, geoelectrics, electromagnetic induction, ground-penetrating radar and seismics. We will discuss applications in industry as well as for general scientific questions in the geo- and environmental sciences.

Everything is subject to change with news University regulations regarding the pandemic, but at this stage I anticipate a large in-person component.

### Lecture Format

The lecture is accompanied with three mandatory, hands-on field exercises that will be conducted in small groups. The field measurements take approximately six hours and will be concluded by a joint group report. The lecture format contains frontal lectures on Tuesdays in 3M07, group work on experimental & theoretical exercises on Thursdays in 3U03 and 3F03 and online videos.

### Learning Goals

You should get a broad overview for a number of geophysical methods imaging the sub-surface. You should understand the underlying physical principles, which will enable you to go deeper into specific methods that you may encounter later on. Most importantly you should learn to think straightforwardly, to ask the right questions, and to apply quantitative mathematical methods in problem solving.

### In-class exercises

Exercises are an important part of the Geophysics lecture. They will treat some aspects of the lecture in more detail, but also cover new ground. We expect that you work on the exercises at home and we will discuss questions and solutions interactively together (typically Thursdays). The joint meetings will start with randomly chosen students presenting their approach. It is ok if the full solution is not available at this stage and there will be no interrogation. However, please don't show up unprepared because this will inevitably be awkward.

### Field exercises

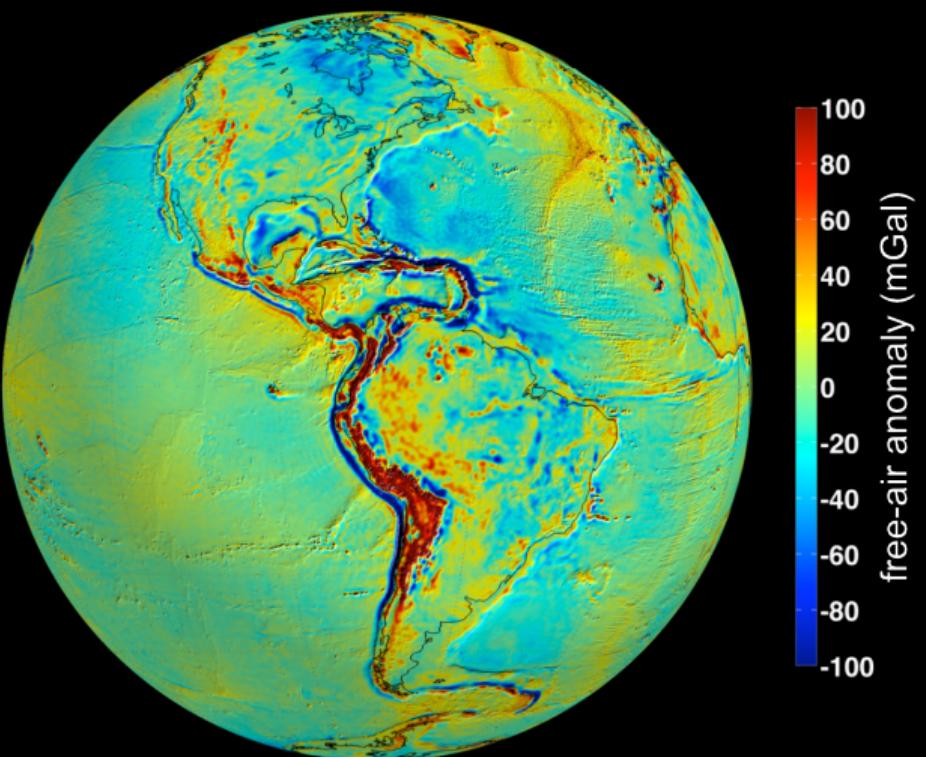
We will conduct field exercises for magnetics, geoelectrics and seismics. This is your maybe once-in-a-lifetime chance to work with professional geophysical equipment. The practical part of the exercises will typically take about six hours. Exercises are mandatory and absence is only permissible with a substantiated excuse approved by the instructor before the exercise takes place. The exercise will then need to be repeated another day. Don't miss the submission deadline of your group reports communicated by the instructor. If you fail, you will have a chance to revise the report.



## Learning goals today:

- ▶ Understand that gravity methods map sub-surface density variability
- ▶ Understand the gravitational force, its potential field, and one underlying measurement principle.
- ▶ Understand the Earth's geoid and reference ellipsoid

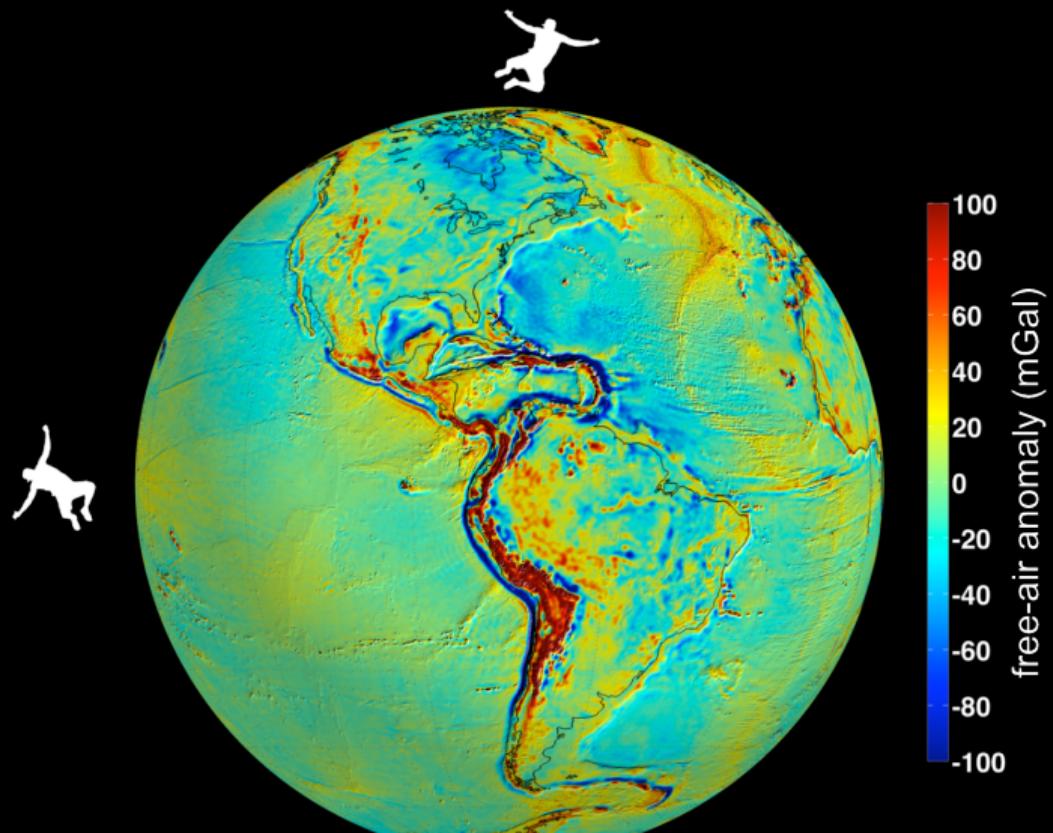
# Example: Global variability



# Example: Global variability



Your mass is constant but your weight is not.



# What is a force?



[Newton (1642-1726) / G. Johnson.]



$$\vec{F} = m\vec{g}$$

$\vec{F}$  : Force (N;  $\text{kg m s}^{-2}$ )

$\vec{g}$  : Acceleration ( $\text{m s}^{-2}$ )

m : Mass (kg)

# The gravitational force



$$\vec{F} = G \frac{mM}{r^2} \hat{r}$$

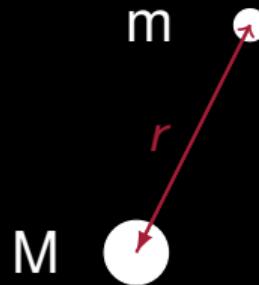
$$G = 6.674 \cdot 10^{-11} \text{ (m}^3\text{kg}^{-1}\text{s}^{-2}\text{)}$$

$\hat{r}$  : unit vector

$r$  : distance between point masses



[Newton (1642-1726) / G. Johnson.]



# Example: The gravitational constant

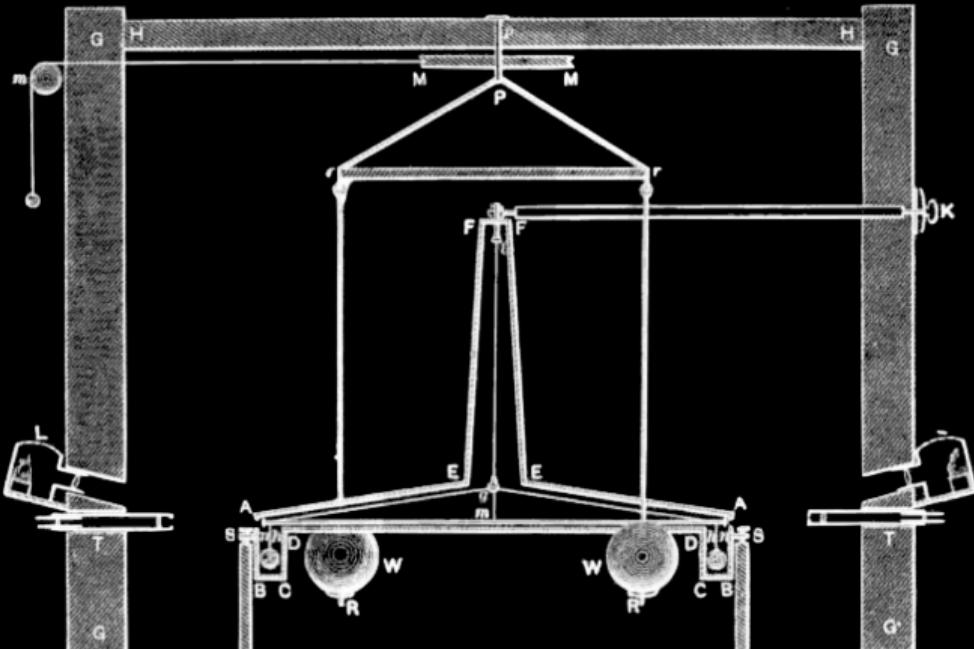
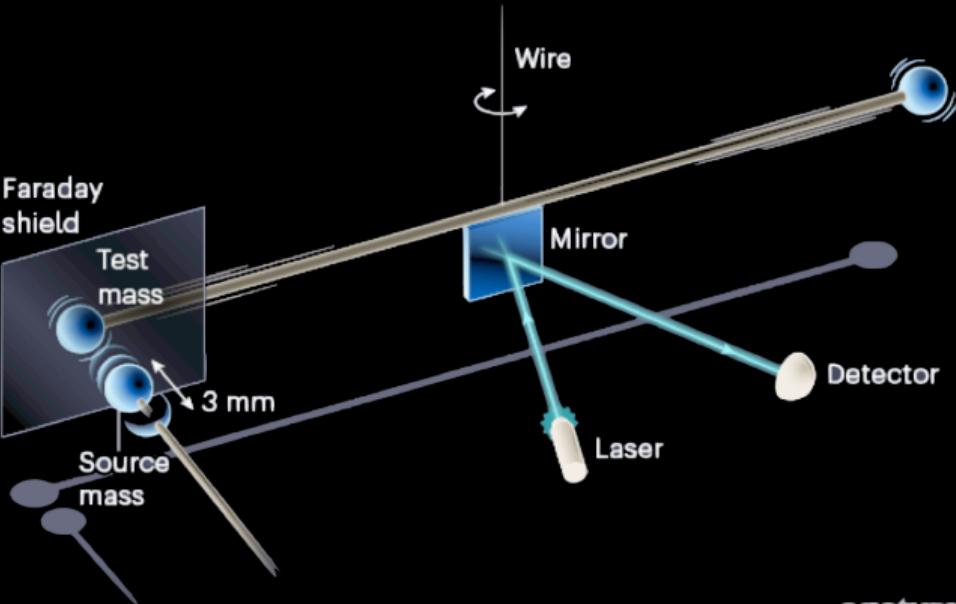


Fig. 1

Cavendish, PNAS, 1798

# Example: The gravitational constant



©nature

Westphal et al., Nature, 2021

G is the worst known constant in physics. Why?

# Example: Measuring acceleration



$$\vec{F} = m\vec{g}$$

$$\vec{F} = G \frac{mM}{r^2} \hat{r}$$

# Example: Measuring acceleration



$$\vec{F} = m\vec{g}$$

$$\vec{F} = G \frac{mM}{r^2} \hat{r}$$

$$\rightarrow \vec{g} = G \frac{M}{r^2} \hat{r}$$

$$\rightarrow \frac{d^2\vec{x}}{dt^2} = G \frac{M}{r^2} \hat{r}$$

This is a differential equation.

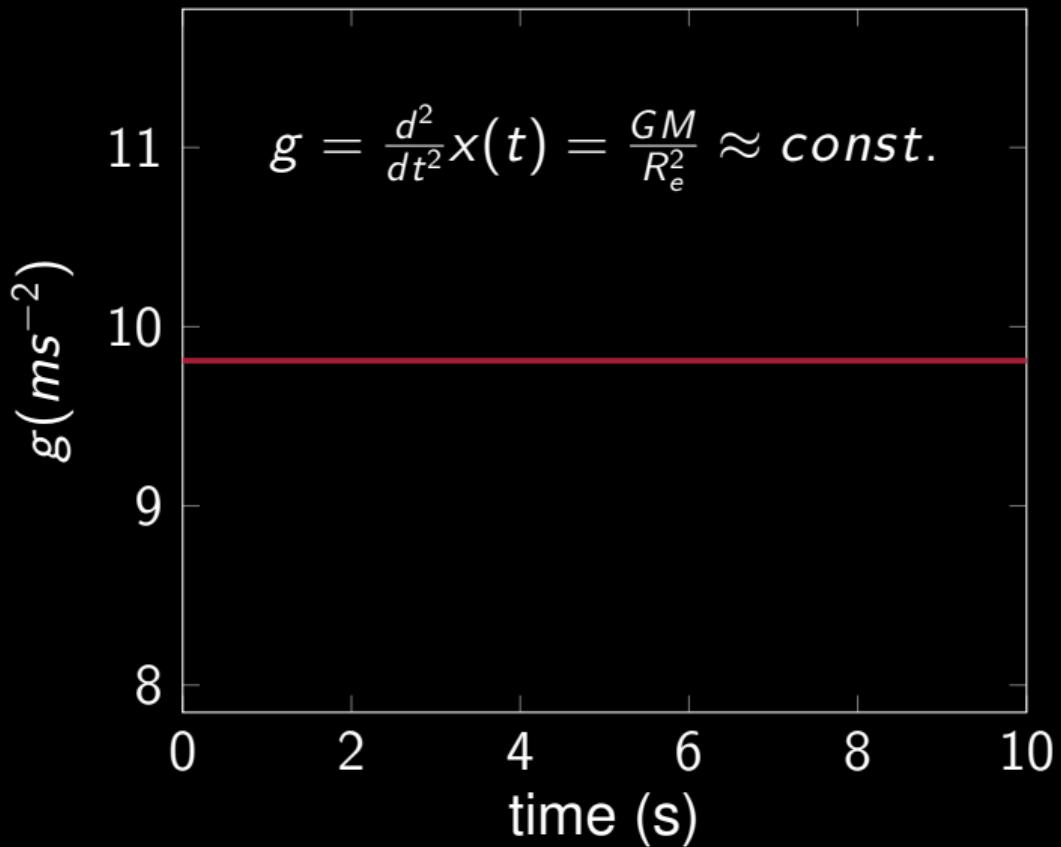
# Example: Measuring acceleration



$$\frac{d^2 \vec{x}}{dt^2} = G \frac{M}{R_E^2} \approx \text{const.}$$

At the Earth's surface ( $R_E$ )  $g$  is close to constant and only vertical. (Later we will see that none of this is quite true).

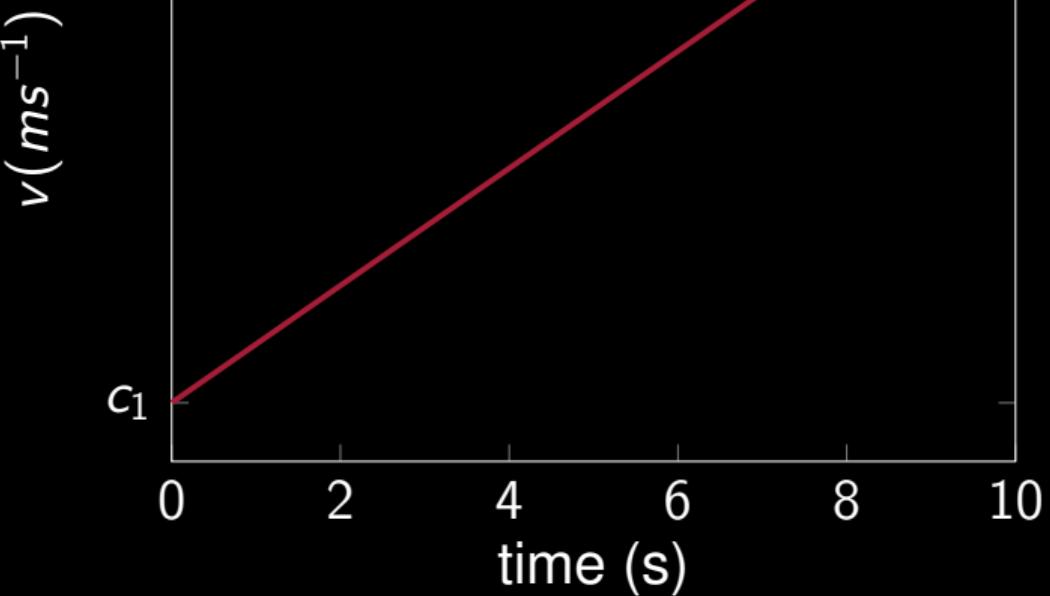
# Example: Measuring acceleration



# Example: Measuring acceleration



$$v = \int g dt = \frac{d}{dt} x(t) = \frac{GM}{R_e^2} t + c_1$$

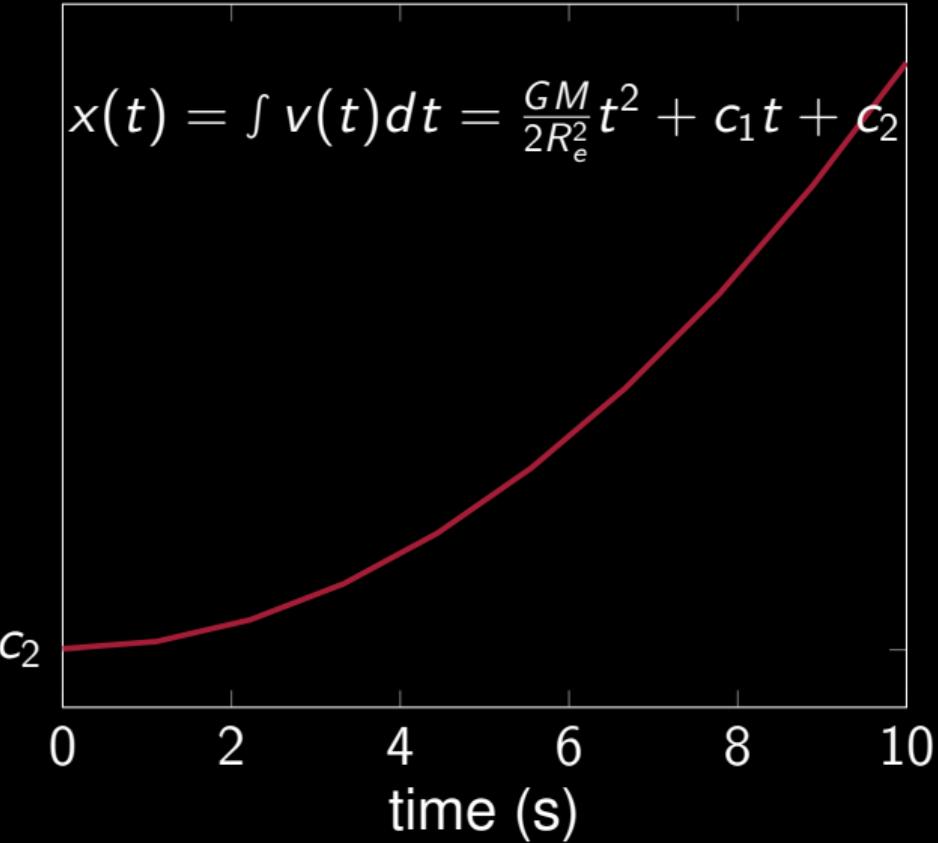


# Example: Measuring acceleration



$$x(t) = \int v(t)dt = \frac{GM}{2R_e^2}t^2 + c_1t + c_2$$

$x(m)$





$$x(t) = \frac{GM}{2R_e^2}t^2 + c_1t + c_2$$

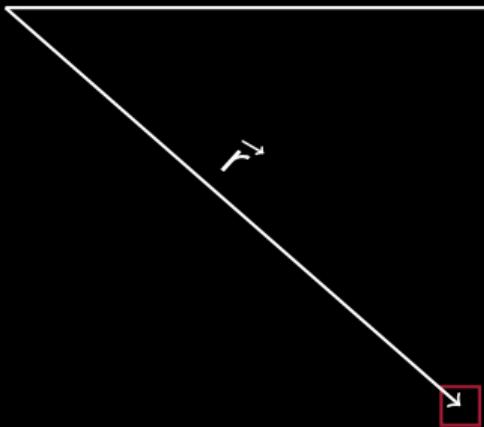
- ▶ Setting, e.g.,  $c_1 = 0$  (initial velocity) and  $c_2 = 0$  (initial position) is quite convenient.
- ▶ This is the principal of a free-fall gravimeter.



- ▶ Thanks to the Greeks we know the radius  $R_E$  for the Earth. However, its mass was unknown for a while.
- ▶ Go ahead and determine the mass of the Earth  $M$  with your Smartphone!
- ▶ There is an important first-order finding in Earth Sciences that you can (re-) discover. Which one?

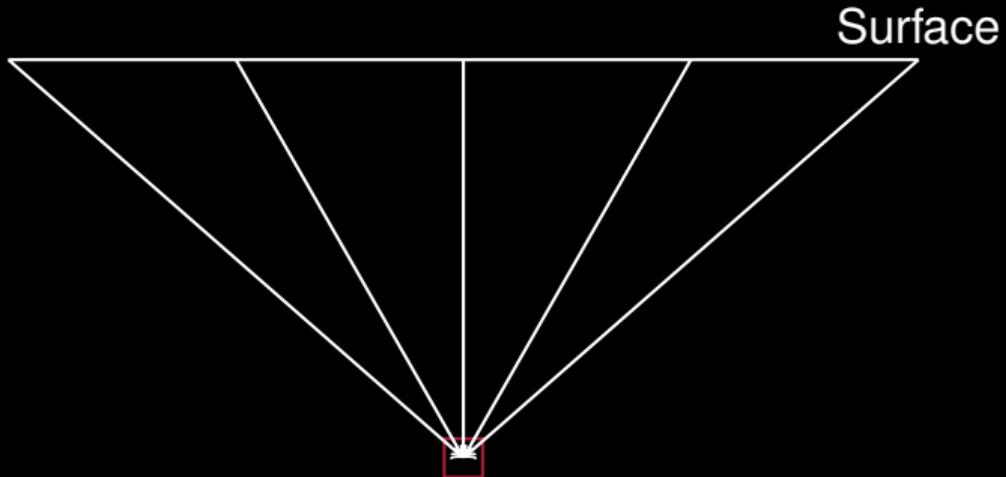


Surface



$$\vec{F} = G \frac{dM}{r^2} \hat{r}$$

For a small mass  $dM$  the point mass approximation holds.

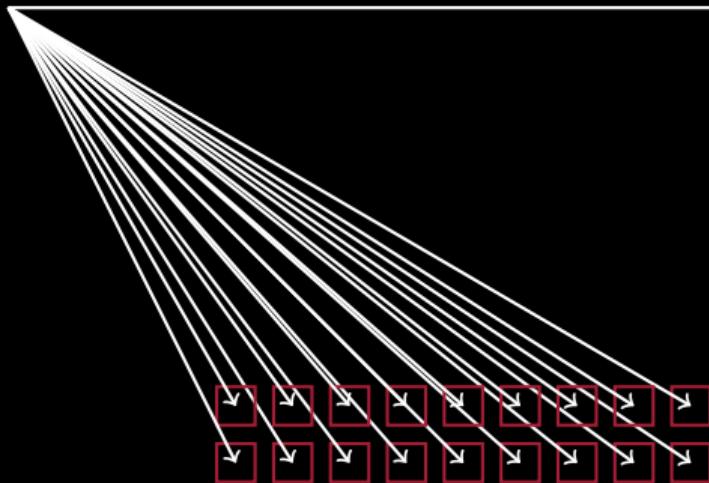


$$\vec{F} = G \frac{dM}{r^2} \hat{r}$$

Profiling across a sub-surface target results in a gravity anomaly ( $\rightarrow$  Exercises).



Surface



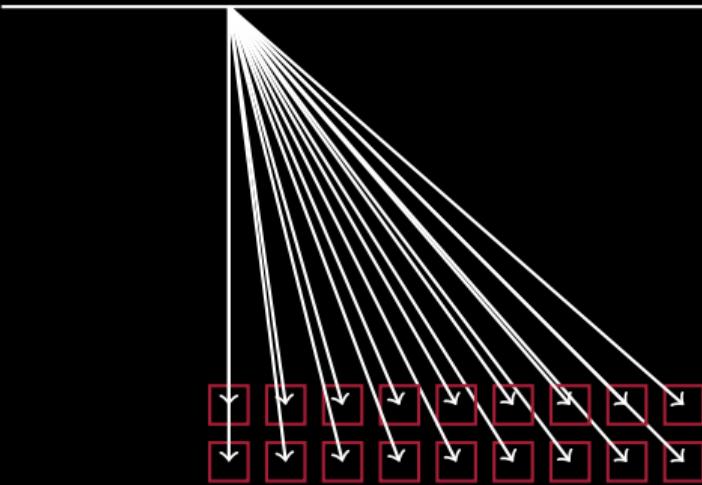
$$\vec{F}(\vec{r}) = \sum_i G \frac{dM_i}{r_i^2} \hat{r}_i$$

For  $i$  point masses the effect adds up.

# Beyond point masses



Surface

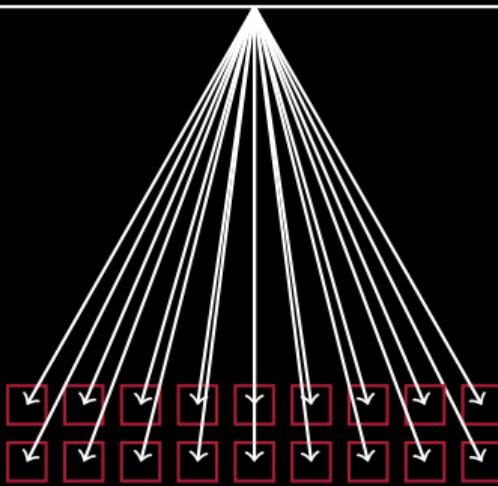


$$\vec{F}(\vec{r}) = \sum G \frac{dM_i}{r_i^2} \hat{r}_i$$

# Beyond point masses

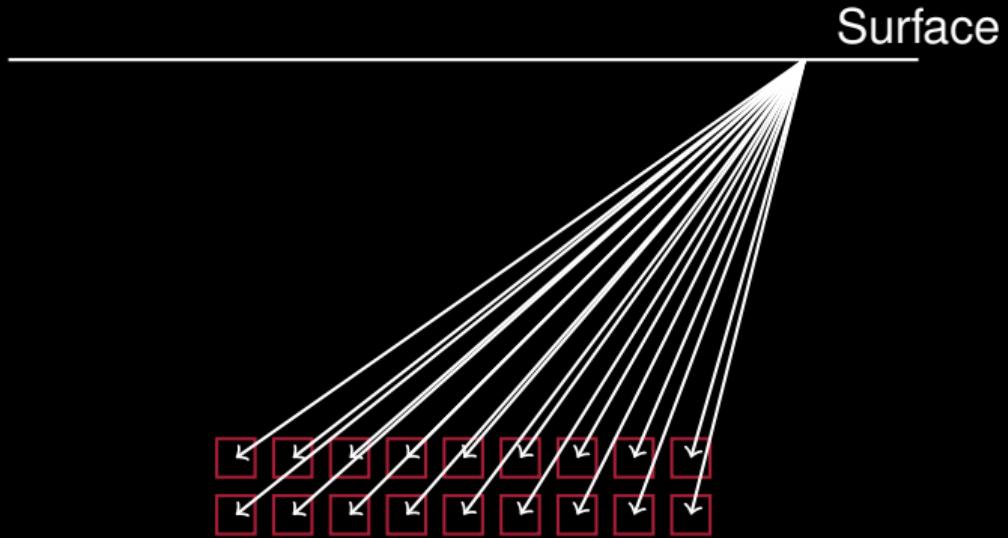


Surface



$$\vec{F}(\vec{r}) = \sum G \frac{dM_i}{r_i^2} \hat{r}_i$$

# Beyond point masses



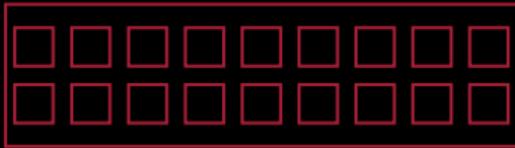
$$\vec{F}(\vec{r}) = \sum G \frac{dM_i}{r_i^2} \hat{r}_i$$



## Surface

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$$\vec{F}(\vec{r}) = G \int \rho \frac{1}{r^2} \hat{r} dV$$



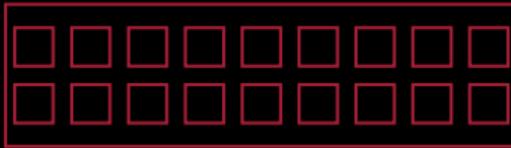
The summation can be replaced by an integration over a volume enclosing a continuous density.



## Surface

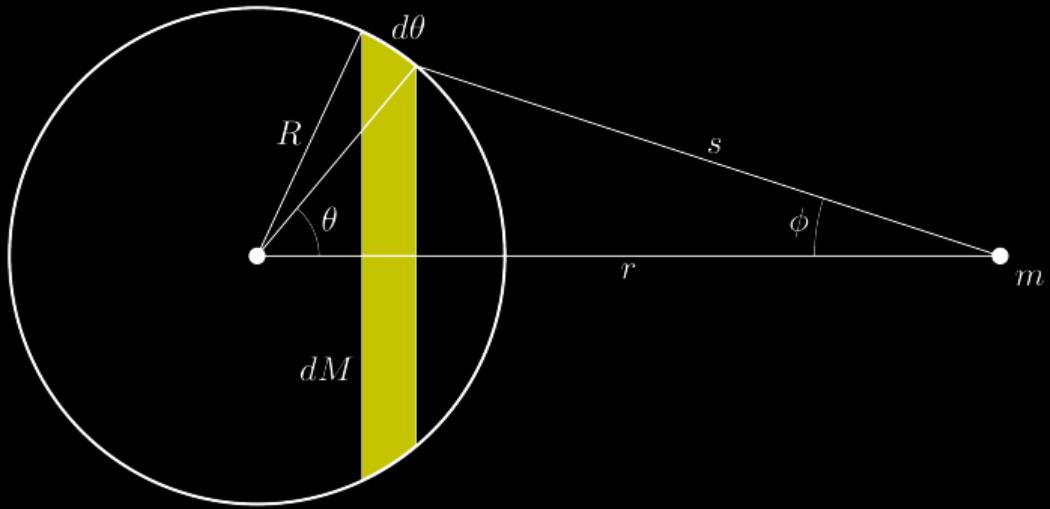
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$$\vec{F}(\vec{r}) = G \int \rho \frac{1}{r^2} \hat{r} dV$$



The integration is a triple integral. Integration limits and coordinates depend on the viewpoint. Example is a Bouger plate, in general not easy to solve ( $\rightarrow$  Exercises).

# Example: Shell



[Xaononl CC BY-SA 4.0]

Newton's shell theorem solves the volume integral inside and outside spherical objects (→ Ex.-Discussion)



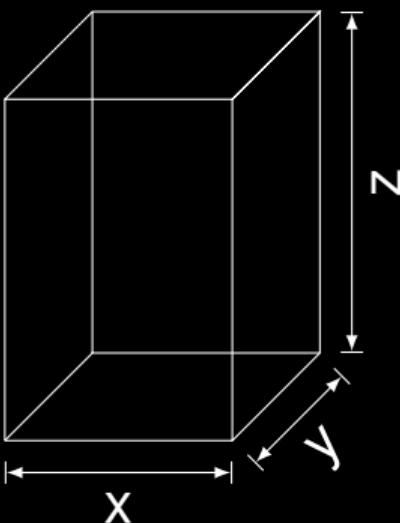
- ▶ The field outside a shell is the same as the one from an equivalent point mass
- ▶ The field inside a shell is zero. Everywhere.



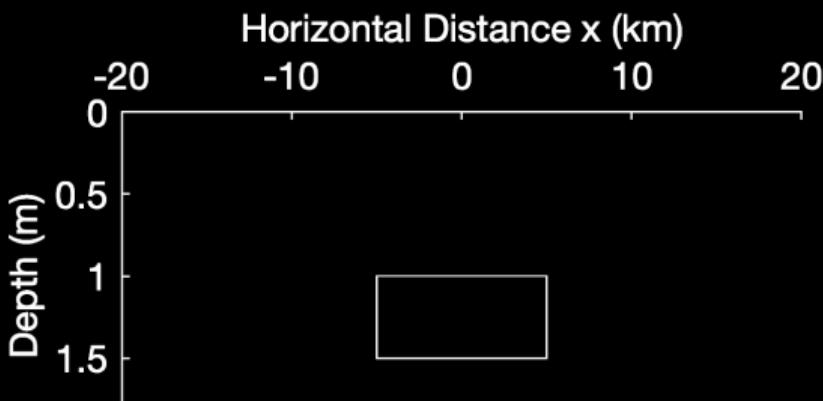
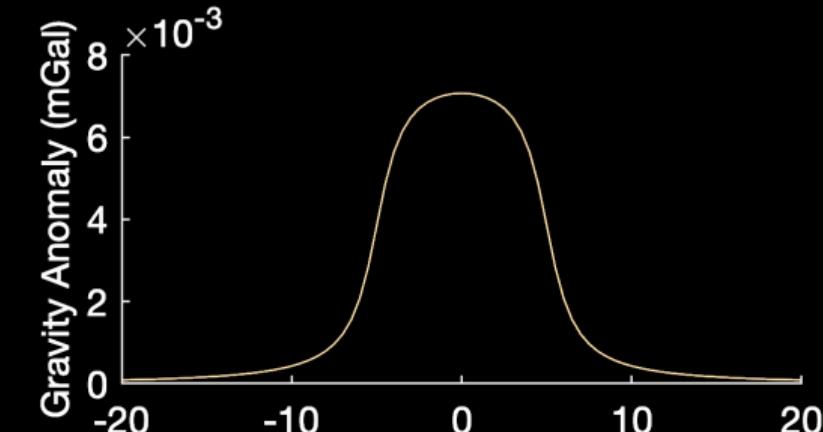
- ▶ There are analytical solutions for other shapes (e.g., Nagy 1966 for Prism).

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Surface



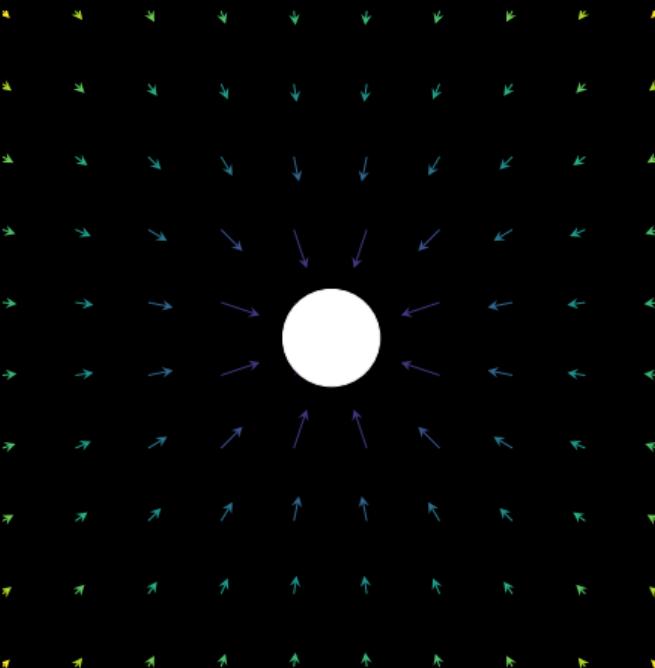
# Numerical forward modelling ( $\rightarrow$ Ex)



# Vector fields



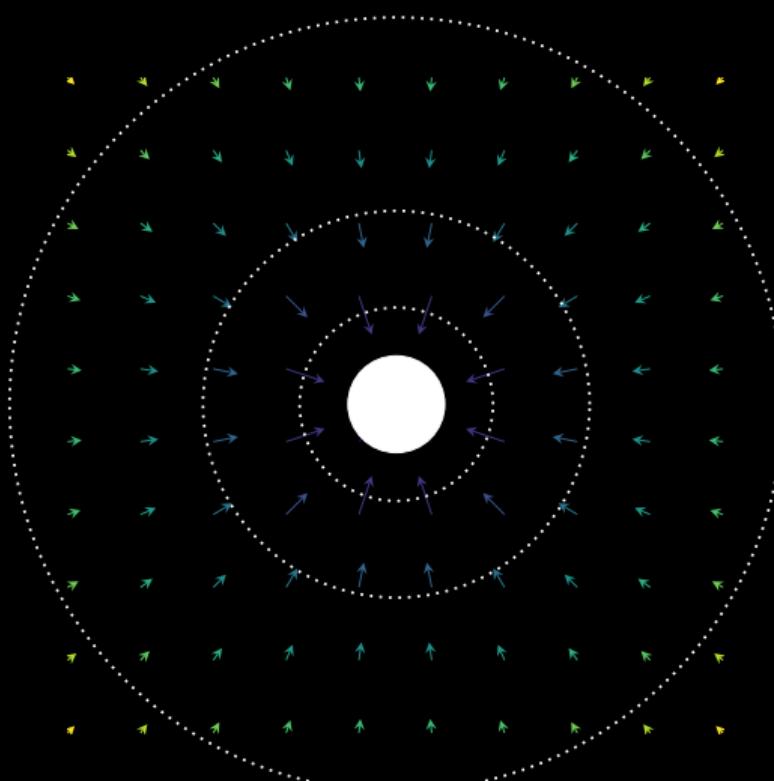
$$\vec{g} = G \frac{M}{r^2} \hat{r}$$



# Potential Field

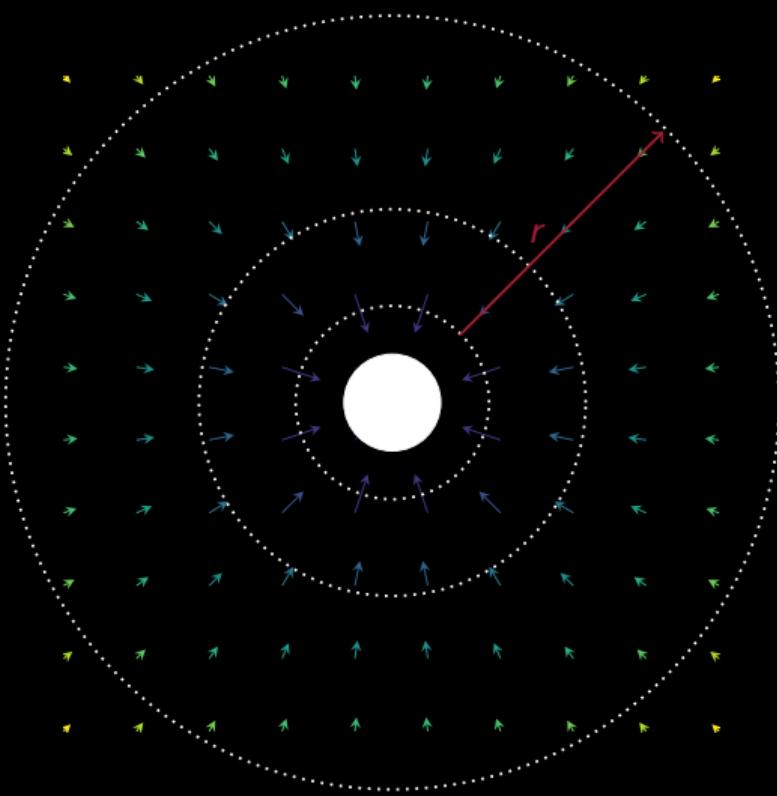


$$\vec{g} = G \frac{M}{r^2} \hat{r}$$





What is the amount of work required?





$$\begin{aligned} U(r) &= - \int_{\infty}^r \vec{g} d\vec{r} \\ &= - \int_{\infty}^r g dr \\ &= -GM \int_{\infty}^r \frac{1}{r^2} dr \\ &= -GM \left[ -\frac{1}{r} \right]_{\infty}^r \\ &= GM \frac{1}{r} \end{aligned}$$

Potential for a point mass.

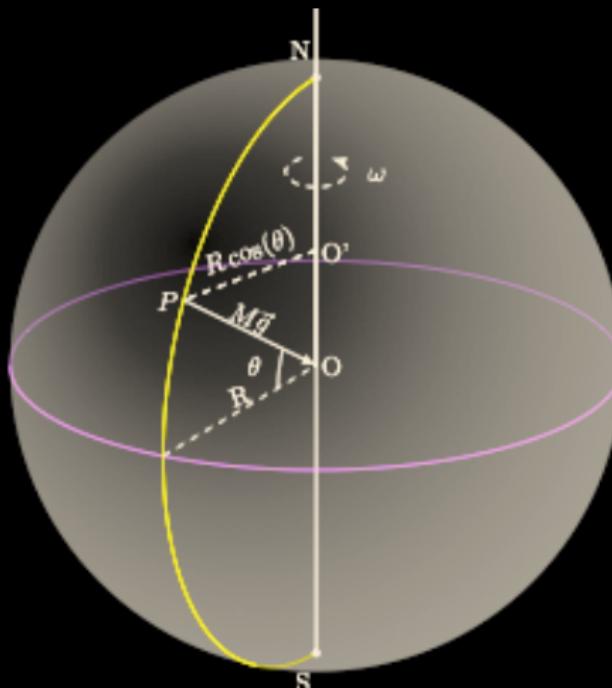


$$\vec{g}(r) = -\nabla U(r)$$

- ▶ It is sometimes easier to calculate the potential of an anomaly and to infer the acceleration via the gradient.
- ▶ Equipotential lines are perpendicular to the field direction.
- ▶ Equipotential lines are in general NOT lines of equal field strength (cf. with down-hill slope force in landscape)



The Earth's rotation minimizes gravitational acceleration at the equator. At the poles it does nothing.





Centripetal acceleration at P perpendicular to rotation axis parallel to O'-P:

$$g_r = \omega^2 R \cos(\theta)$$

Centripetal acceleration at P perpendicular to rotation axis parallel to O'-P:

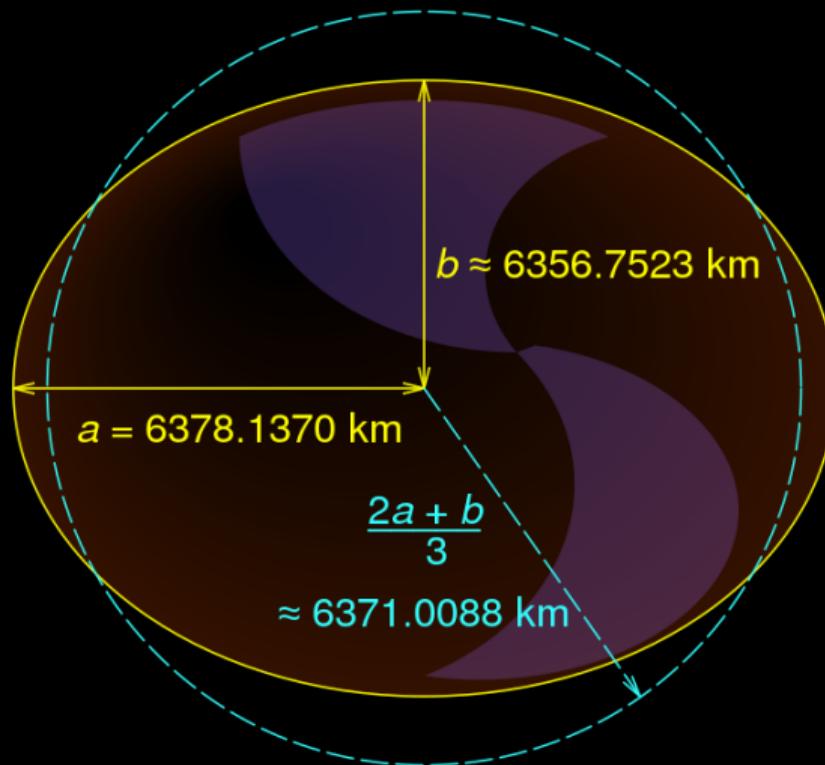
$$g_{r,proj.} = \omega^2 R \cos^2(\theta)$$

Angular Frequency:  $\omega$

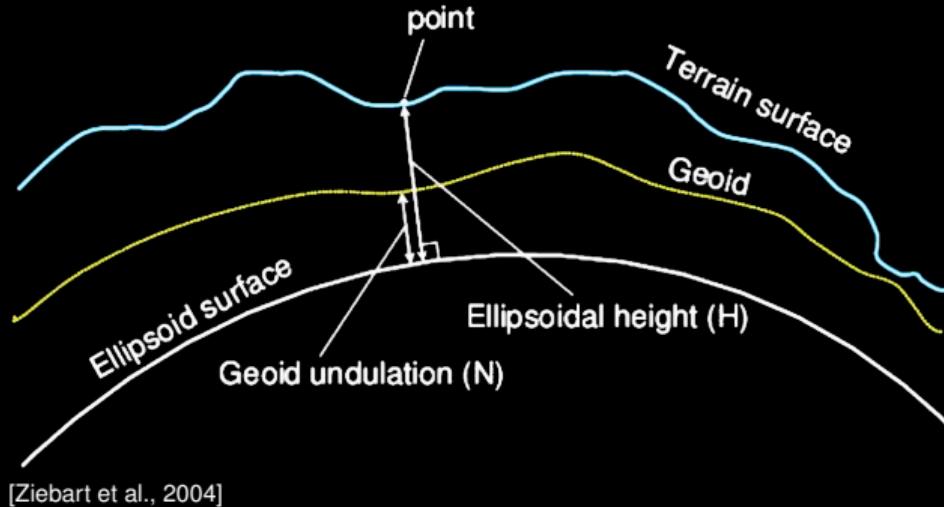
Angular Velocity:  $\vec{\nu}_r = \vec{\omega} \times \vec{R} \cos(\theta)$

Angular Acceleration:  $\vec{g}_r = \dot{\vec{\nu}}_r = \vec{\omega} \times \vec{\omega} \times \vec{R} \cos(\theta)$

# An ellipsoidal Earth



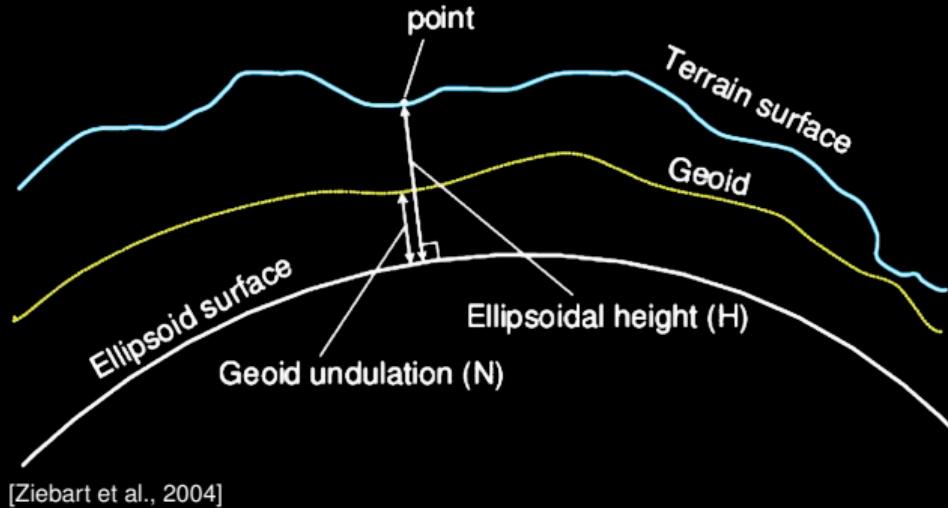
# An ellipsoidal Earth



[Ziebart et al., 2004]

- ▶ Geoid is a real-world equipotential line approximating sea level.
- ▶ It is referenced to the geometric ellipsoid.

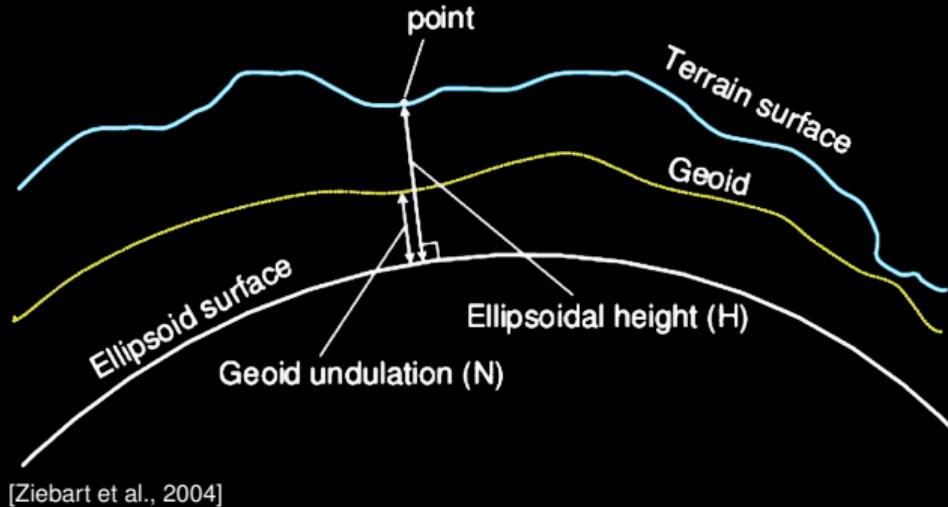
# An ellipsoidal Earth



[Ziebart et al., 2004]

- ▶ The reference of elevation is a constant source of confusion.
- ▶ The geoid defines the local vertical direction.

# An ellipsoidal Earth



[Ziebart et al., 2004]

- ▶ Upwarping of geoid indicates mass excess.
- ▶ Downwarping of geoid indicates mass deficit.



## Learning goals today:

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