

### Expectations for Exercises

Exercises are an important part of the Geophysics lecture. They will treat some aspects of the lecture in more detail, but also cover new ground. We expect that you work on the exercises at home and we will discuss questions and solutions interactively together. Questions that are marked with 'Extra' are not required but geared to stir your further interest. We will surely support you if you tackle those as well.

## 2 Exercises for Gravity Method 2

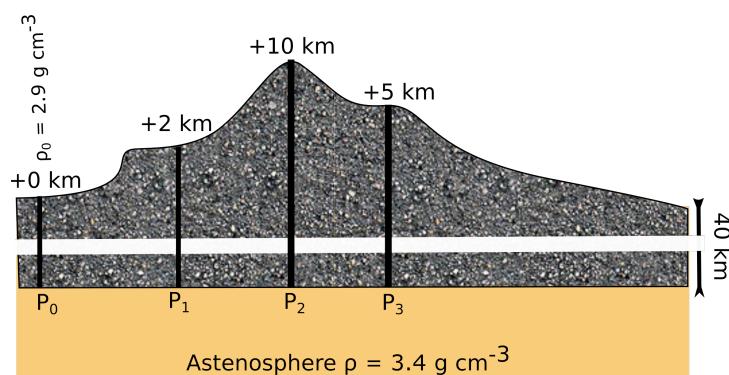
**Version:** April 26, 2022

**Context:** Videos Introduction & Gravity 01, Gravity 02

**Timing:** All gravity exercises should be completed by Thursday in week 3.

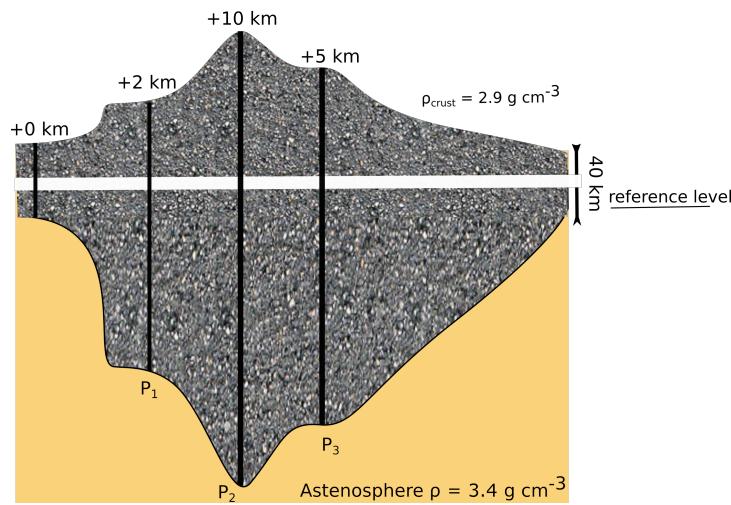
### 2.1 Airy and Pratt hypothesis for mountain ranges

(a)



The figure above illustrates a crust with inhomogeneous density and a mountain range floating on the asthenosphere. Consider this as an idealised case in which every vertical slice is locally balanced (i.e. everything is in hydrostatic equilibrium which reduces to an effective 1D problem). Calculate the required densities in the vertical slices at  $P_1 - P_3$ . (Tip: Below the crust the pressure is equal everywhere  $P_1 - P_3$ )

(b)



The figure above illustrates a crust with a homogenous density and a mountain change floating on the asthenosphere. Consider this as an idealised case in which every vertical slice is locally balanced (i.e. everything is in hydrostatic equilibrium which reduces to an effective 1D problem). Calculate the thickness differences between  $P_1 - P_3$ .

(c) Draw an approximate profile for the free-air and the Bouger anomalies. How would the free-air anomaly profile change if the mountain change is not in hydrostatic equilibrium? Which conclusions regarding the temporal evolution of the mountain chain would you draw from that? In which areas along this profile do you think is the assumption of local hydrostatic equilibrium most unlikely and how would this be reflected in the free-air anomaly?

## 2.2 Forward modelling and non-uniqueness in potential field methods

### Matlab (or Python)

Basic programming (Matlab/Python/R) will likely be part of your study experience when you move to MSc level courses. It is a useful skill to have, but here we do not cover any introduction. What we do is that we start with codes that need little user interaction to give you a feel for what programming can be about. In order to run this exercise you should have a working Matlab version on your Computer, please follow the installation instructions provided by the ZDV. Alternatively, we can also give you a laptop for the joint meeting.

In order to predict how any kind of object will appear in a gravity survey, we need to solve the volume integral:

$$\vec{g}(r) = G \int \frac{1}{r^3} \rho(r) \vec{r} dV$$

which simplifies slightly to:

$$g_z(r) = G \int \frac{1}{r^2} \cos(\phi) \rho(r) dV = G \int \frac{z}{r^3} \rho(r) dV$$

because often only the vertical component is of interest (same as in Ex 1.1). However, the problem remains complicated as the integration bounds depend on the object's geometry and the integral needs to be solved for every  $r$  along the gravimetry profile. Some solutions for special shapes you already know (e.g. sphere,

bouger plate). Here we use the solution for a rectangular prism which fortunately others have already calculated for us (*Nagy 1966, Geophysics VOL. XXX, SO. 2*). Using this solution, we can build up more complicated shapes out of individual prisms.

In the specific model applied individual prisms are defined with their widths in the horizontal ( $w_x$ ,  $w_y$ ) and the vertical ( $w_z$ ), together with the positions in the subsurface. The key is that the position coordinates ( $d_{x1}$ ,  $d_{x2}$ ,  $d_{y1}$ ,  $d_{y2}$ ,  $d_{z1}$ ,  $d_{z2}$ ) need to be prescribed relative to the measurement position which changes along the profile. The expected anomaly is then calculated based on the analytical solution.

- (a) This exercises uses Matlab. However, only minimal Matlab skills are required to follow along. Download the files *Gravimetry02\_ForwardModelling.m* and *gravprism.m* into the same folder on your computer. Check out case 1 which simulates a rectangular object in the subsurface. Change it's location and size so that you know what is going on.
- (b) Switch to case 2. This one treats the combined effect of two prisms. See what's different compared to case 1. Play around with positions to see what is going on.
- (c) Switch to case 3. This one treats the individual effects of two prisms meaning that it doesn't sum them up. This one will not run until you fill out the parts marked with XXX. Use this case to illustrate that multiple situations in the sub-surface (e.g. a shallow prism with low density contrast vs. a lower prism with larger density contrast) can result in similar anomalies. This is an important finding. Forward models are often not unique, and therefore your interpretation won't be either. This situation occurs in many geophysical situations. Remember that.

### 2.3 Some general questions to reflect on

1. Why does the mean sea level follow the shape of the Geoid?
2. How can you describe the gravitational attraction between two point masses if none of them is located in the origin of the coordinate system applied?
3. How does an equipotential line change by crossing an area of (a) mass deficit, and (b) mass excess?
4. Why do we have Earth & Ocean tides? To understand the principle focus on the Moon's effect only.
5. Discuss whether the sun or the moon is more important for tides.
6. How does deglaciation support the idea of a ductile asthenosphere on top of which the continental plates 'float'?