Pontifícia Universidade Católica do Paraná
Escola Politécnica

Matemática Discreta – Exercícios - Relações Valor: 1,0 - Prof. Guilherme Schnirmann

Nome: RODRIGO DA SILVA ALVES Data: ________

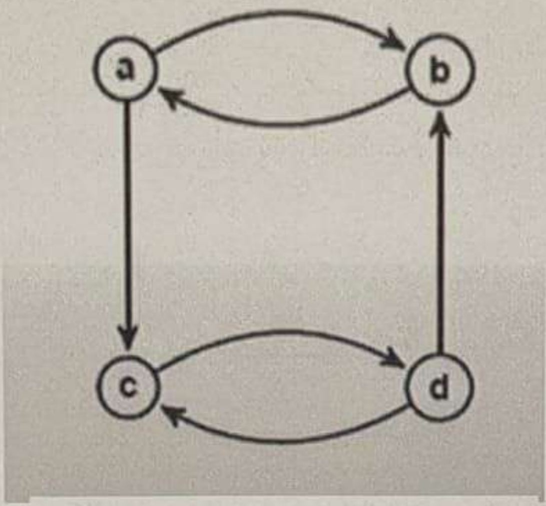
1) Indique quatro pares ordenados (se existirem) que pertencem a cada relação em N a seguir
   a. $x \rho y \leftrightarrow x + y < 7$ = {(1,1) (1,2) (1,3) (1,4)}
   b. $x \rho y \leftrightarrow x = y + 2$ (2,0) (3,1) (4,2) (5,3)
   c. $x \rho y \leftrightarrow 2x + 3y = 10$ (2,2) (5,0)
   d. $x \rho y \leftrightarrow y\ \text{é}\ um\ quadrado\ perfeito$ (1,2) (1,4), 1
2) Classifique cada relação como um-para-um, vários-para-um, um-para-vários ou vários-para-~~um~~.
   a. $S = N; \rho = \{(1,2), (1,4), (1,6), (2,3), (4,3)\}$ 1, n
   b. $S = N; \rho = \{(9,7), (6,5), (3,6), (8,5)\}$ n, 1
   c. $S = N; \rho = \{(12,5), (8,4), (6,3), (7,12)\}$ 1, 1
   d. $S = N; x \rho y \leftrightarrow x = y + 1$ 1, 1
   e. $S = N; x \rho y \leftrightarrow x = 5$ 1, n
3) Sejam ρ_1 e ρ_2 relações binárias em N definias por $x \rho_1 y \leftrightarrow x\ divide\ y$ e $x \rho_2 y \leftrightarrow x \leq 3y$
   Encontre a relação e liste quatro pares ordenados (caso existam) de:
   a) $\rho_1 \cup \rho_2$ {(1,1)(1,3)(2,4)(2,6)}
   b) $\rho_1 \cap \rho_2$
   c) $(\rho_1)'$
   d) $(\rho_2)'$
4) Sejam A = {1,2,3,4}, B = {a,b,c} e $\boldsymbol{\rho}$ e $\boldsymbol{\sigma}$ de $\boldsymbol{A}$ em $\boldsymbol{S}$ definidas por:

$$\boldsymbol{\rho = \{(1, a), (1, b), (2, b), (2, c), (3, b), (4, a)\}}$$

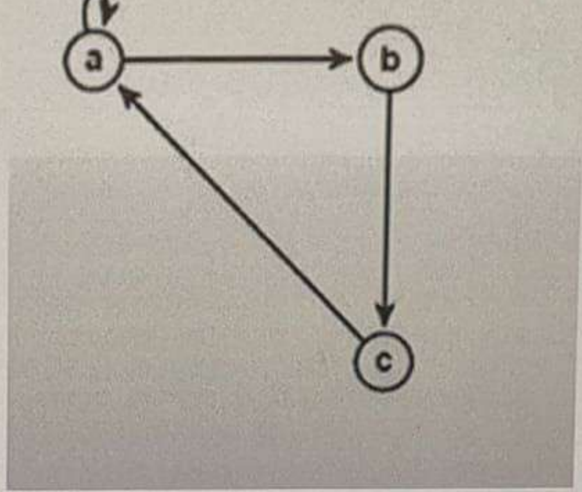
$$\boldsymbol{\sigma = \{(1, b), (2, c), (3, b), (4, b)\}}$$

   Calcule:
   a) O grafo de $\boldsymbol{\rho}$
   b) O grafo de $\boldsymbol{\sigma}$
   c) A matriz $\boldsymbol{M_\rho}$
   d) A matriz $\boldsymbol{M_\sigma}$
   e) A matriz $\boldsymbol{M_{\rho'}}$
   f) A matriz A matriz $\boldsymbol{M_{\sigma'}}$
   g) A matriz $\boldsymbol{M_{\rho \cap \sigma}}$
   h) A matriz A matriz $\boldsymbol{M_{\rho \cup \sigma}}$
5) Sejam os conjuntos A={2,4,6} e B={1,3,5}, e uma relação de A para B dada por R={(2,1),(4,1),(6,1),(2,3),(6,5)}. Faça a representação por grafo, por conjuntos e por matriz.
6) Seja S={0,1,2,4,6}. Teste se as relações em S dadas a seguir são reflexivas, simétricas, antissimétricas ou transitivas
   a. R={(0,0),(1,1),(2,2),(4,4),(6,6),(0,1),(1,2),(2,4),(4,6)} REFLEXIVA, ANTISIMETRICA
   b. R={(0,1),(1,0),(2,4),(4,2),(4,6),(6,4)} SIMETRICA
   c. R={(0,1),(1,2),(0,2),(2,0),(2,1),(1,0),(0,0),(1,1),(2,2)} ~~REFLEXIVA~~, SIMETRICA, TRANSITIVA
   d. $R = \{(0,0), (1,1), (2,2), (4,4), (6,6), (4,6), (6,4)\}$ SIMETRICA, REFLEXIVA
7) Encontre os fechos reflexivo, simétrico e transitivo para cada uma das relações do exercício 6.
8) Encontre o fecho reflexivo, simétrico e transitivo em cada caso

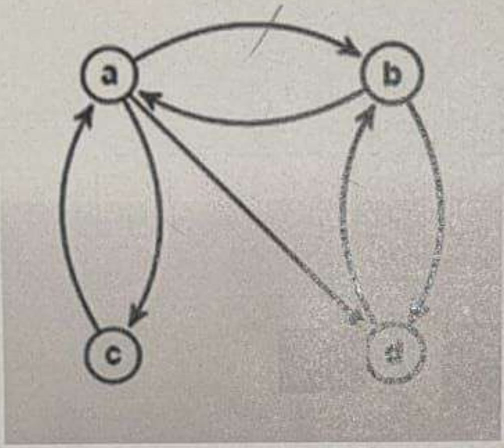
(a)



(b)



(c)



$R = \{(a,b)(a,d)(a,c)(b,a)(b,d)(c,a)(d,b)\}$

$(a,b)(b,a) \to (a,a)$ ✗
$(a,b)(b,d) \to (a,d)$ ✓
$(a,d)(d,b) \to (a,b)$ ✓
$(a,c)(c,a) \to (a,a)$ ✓
$(b,a)(a,b) \to (b,b)$ ✗
$(b,a)(a,d) \to (b,d)$ ✓
$(b,a)(a,c) \to (b,c)$ ✗
$(b,d)(d,b) \to (b,b)$ ✓
$(c,a)(a,b) \to (c,b)$ ✗
$(c,a)(a,d) \to (c,d)$ ✗

$(c,a)(a,c) \to (c,c)$ ✗
$(d,b)(b,a) \to (d,a)$ ✗
$(d,b)(b,d) \to (d,d)$ ✗

$R^*_{T1} = \{(a,a)(b,b)(b,c)(c,b)(c,d)(c,c)(d,a)(d,d)\}$

$(b,c)(c,a) \to (b,a)$ ✓
$(d,a)(a,c) \to (d,c)$ ✗

$R^*_{T2} = \{(d,c)\}$

9) Suponha uma relação R representada pela seguinte matriz:

$$\begin{vmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{vmatrix}$$

1 1 0
1 1 1
0 1 1

Responda se R é reflexiva? Simétrica? Anti-Simétrica? Transitiva? Encontre os fechos dessa relação.

É REFLEXIVA PORQUE TODA A DIAGONAL PRINCIPAL SÃO 1s

É SIMETRICA, PORQUE A MATRIZ TRANSPOSTA É IGUAL

NÃO É ANTI-SIMETRICA, PORQUE JÁ É SIMETRICA

NÃO É TRANSITIVA, PORQUE TEMOS $(a,b),(b,c)$, MAS NÃO TEMOS (a,c)

$R = \{(a,a),(a,b),(b,a)(b,b)(b,c),(c,b),(c,c)\}$

$(a,a)(a,b) \to (a,b)$ ✓
$(a,b)(b,a) \to (a,a)$ ✗
$(a,b)(b,b) \to (a,b)$ ✓
$(a,b)(b,c) \to (a,c)$ ✗
$(b,a)(a,a) \to (b,a)$ ✓

$(b,b)(b,a) \to (b,a)$ ✓
$(b,b)(b,c) \to (b,c)$ ✓
$(b,c)(c,b) \to (b,b)$ ✓
$(b,c)(c,c) \to (b,c)$ ✓
$(c,b)(b,a) \to (c,a)$ ✗
$(c,b)(b,b) \to (c,b)$ ✓
$(c,b)(b,c) \to (c,c)$ ✓

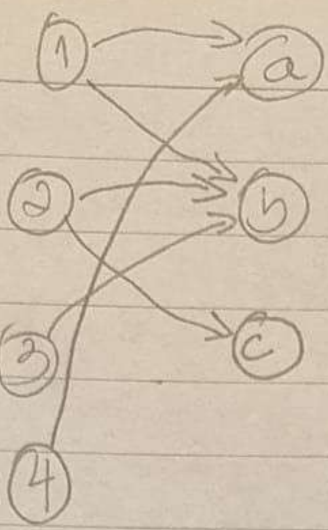
$R^*_{T1} = \{\cancel{(a,a)}, (a,c)(c,a)\}$

$(c,c)(c,b) \to (c,b)$ ✓
$(a,a)(a,b) \to (a,b)$ ✓
$(a,a)(a,c) \to (a,c)$ ✓
$(a,c)(c,b) \to (a,b)$ ✓

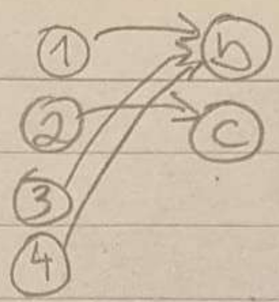
$(c,a)(a,b) \to (c,b)$ ✓

$R^*_T = R^{*1}_T \cup R$

4a)



b)



c) $M_\rho = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

d) $M_\sigma \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

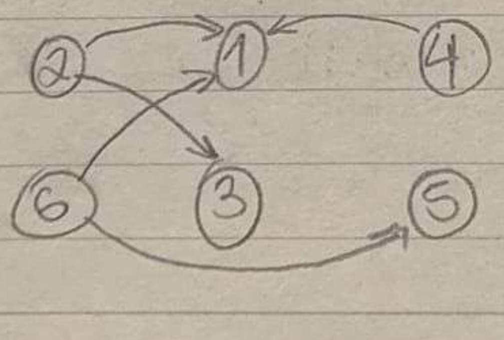
e) $M_{\rho'} \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$

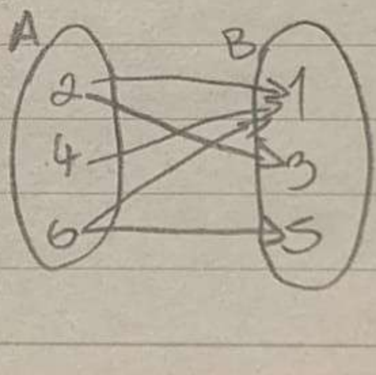
f) $M_{\sigma'} \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$

g) $M_{\rho \cap \sigma} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

h) $M_{\rho \cup \sigma} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$

5)

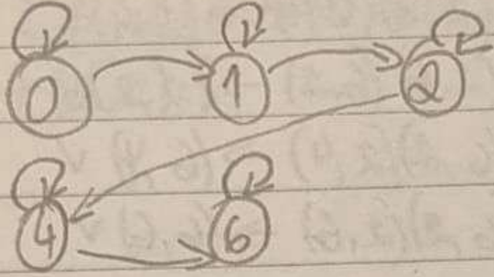




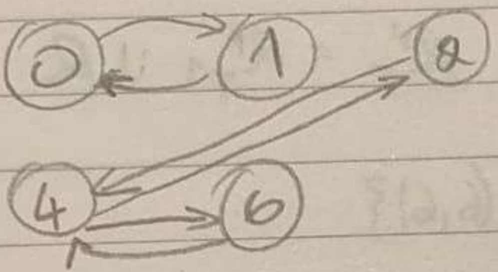
5) matriz

$$M_R = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

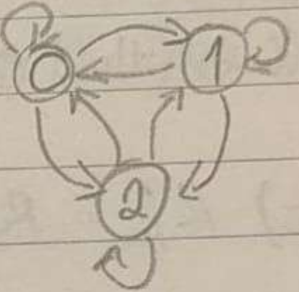
6) a)



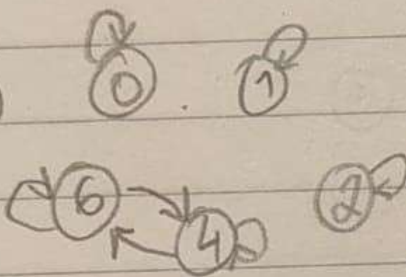
b)



c)



d)



7) a) $(0,0)(0,1) \to (0,1)$ ✓ $\quad R^*_{T1} = \{(0,2)(1,4)(2,6)\}$

$(1,1)(1,2) \to (1,2)$ ✓ $\quad (0,2)(2,4) \to (0,4)$ ×

$(2,2)(2,4) \to (2,4)$ ✓ $\quad (0,2)(2,6) \to (0,6)$ ×

$(4,4)(4,6) \to (4,6)$ ✓ $\quad (1,4)(4,6) \to (1,6)$ ×

$(0,1)(1,1) \to (0,1)$ ✓

$(0,1)(1,2) \to (0,2)$ × $\quad R^*_{T2} = \{(0,4)(0,6)(1,6)\}$

$(1,2)(2,4) \to (1,4)$ × $\quad (0,4)(4,6) \to (0,6)$ ✓

$(2,4)(4,6) \to (2,6)$ ×

$$R^*_T = R^*_{T1} \cup R^*_{T2} \cup R$$

$$R^*_S = R \cup \{(1,0)(2,1)(4,2)(6,4)\}$$

b) $R^*_R = R \cup \{(0,0), (1,1), (2,2), (4,4), (6,6)\}$

$(0,1)(1,0) \to (0,0)$ x $\quad R^*_{T1} = \{(0,0)(1,1)(2,2)(2,6)(4,4)(6,2)(6,6)\}$
$(1,0)(6,1) \to (1,1)$ x
$(2,4)(4,2) \to (2,2)$ x $\quad (2,6)(6,4) \to (2,4)$ ✓
$(2,4)(4,6) \to (2,6)$ x $\quad (2,6)(6,2) \to (2,2)$ ✓
$(4,2)(2,4) \to (4,4)$ x $\quad (6,2)(2,4) \to (6,4)$ ✓
$(4,6)(6,4) \to (4,4)$ ✓ $\quad (6,2)(2,6) \to (6,6)$ ✓
$(6,4)(4,2) \to (6,2)$ x
$(6,4)(4,6) \to (6,6)$ x $\quad R^*_T = R^*_{T1} \cup R$

c) $R^*_R = R \cup \{(4,4)(6,6)\}$

d) $R^*_T = R$

$(4,6)(6,4) \to (4,4)$ ✓
$(6,4)(4,6) \to (6,6)$ ✓

8) a) $R^*_R = R \cup \{(a,a)(b,b)(c,c)(d,d)\}$
$R^*_S = R \cup \{(c,a)(b,d)\}$
$R^*_T = R \cup \{(b,c)(a,d)(d,a)(a,a)\}$

b) $R^*_R = R \cup \{(b,b)(c,c)\}$
$R^*_S = R \cup \{(b,a)(c,b)(a,c)\}$
$R^*_T = R \cup \{(a,c)(b,a)(b,b)(c,b)\}$

c) $R^*_R = R \cup \{(a,a)(b,b)(c,c)(d,d)\}$

$R^*_S = R \cup \{(d,a)\}$

$R^*_T = R \cup R^*_{T1} \cup R^*_{T2}$

$R^*_{T1} = \{(a,a)(b,b)(b,c)(c,b)(c,d)(c,c)(d,a)(d,d)\}$

$R^*_{T2} = \{(d,c)\}$