

Obtener todas las raíces en forma polar de:

$$z^4 = \left(\cos\left(\frac{\pi}{6}\right) + i \operatorname{sen}\left(\frac{\pi}{6}\right) \right)^{33}$$

Por la regla de De Moivre:

$$\left(\cos\left(\frac{\pi}{6}\right) + i \operatorname{sen}\left(\frac{\pi}{6}\right) \right)^{33} = \cos\left(\frac{33\pi}{6}\right) + i \operatorname{sen}\left(\frac{33\pi}{6}\right) = \cos\left(\frac{11\pi}{2}\right) + i \operatorname{sen}\left(\frac{11\pi}{2}\right)$$

$$\cos\left(\frac{11\pi}{2}\right) = 0, \operatorname{sen}\left(\frac{11\pi}{2}\right) = i(-1) = -i$$

$z^4 = -i$, que corresponde gráficamente al punto (0,-1) que tiene módulo de una unidad y ángulo de -90 grados que son $-\frac{\pi}{2} \text{ rad}$. Ergo $z^4 = 1_{-\frac{\pi}{2}}$

Hallar las cuatro raíces de $z^4 = 1_{-\frac{\pi}{2}}$.

- $\omega_0: \cos\left(\frac{-\frac{\pi}{2}+2\cdot0\pi}{4}\right) + i \operatorname{sen}\left(\frac{-\frac{\pi}{2}+2\cdot0\pi}{4}\right) = \cos\left(-\frac{\pi}{8}\right) + i \operatorname{sen}\left(-\frac{\pi}{8}\right) = 1_{-\frac{\pi}{8}}$
- $\omega_1: \cos\left(\frac{-\frac{\pi}{2}+2\cdot1\pi}{4}\right) + i \operatorname{sen}\left(\frac{-\frac{\pi}{2}+2\cdot1\pi}{4}\right) = \cos\left(\frac{3\pi}{8}\right) + i \operatorname{sen}\left(\frac{3\pi}{8}\right) = 1_{\frac{3\pi}{8}}$
- $\omega_2: \cos\left(\frac{-\frac{\pi}{2}+2\cdot2\pi}{4}\right) + i \operatorname{sen}\left(\frac{-\frac{\pi}{2}+2\cdot2\pi}{4}\right) = \cos\left(\frac{7\pi}{8}\right) + i \operatorname{sen}\left(\frac{7\pi}{8}\right) = 1_{\frac{7\pi}{8}}$
- $\omega_3: \cos\left(\frac{-\frac{\pi}{2}+2\cdot3\pi}{4}\right) + i \operatorname{sen}\left(\frac{-\frac{\pi}{2}+2\cdot3\pi}{4}\right) = \cos\left(\frac{11\pi}{8}\right) + i \operatorname{sen}\left(\frac{11\pi}{8}\right) = 1_{\frac{11\pi}{8}}$