Multi And Threshold Signatures for Starknet

(Warming up for Lisbon Hackaton)

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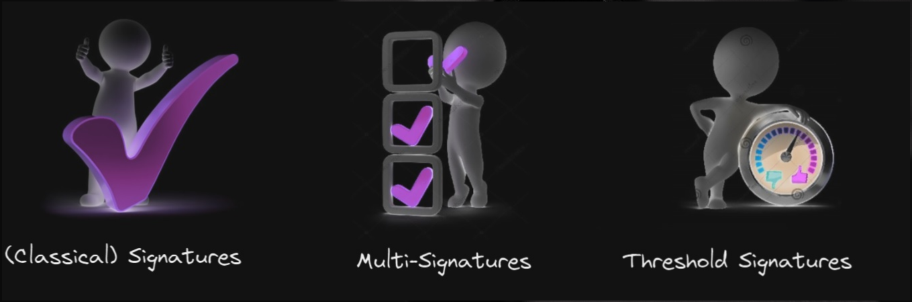
Summary


(Classical) Signatures
Multi-Signatures
Threshold Signatures

Summary

1. Signatures, MultiSigs and ThresholdSigs
   - Basic Concepts
   - Under the hood
2. Use cases
   - Multi factor authentication
   - Voting system
3. Ledger/Starknet Musig2, call for Lisbon

Signatures

A digital signature is a mathematical scheme for verifying the authenticity of digital messages or documents.

Definition ((Classical) Digital Signature)

A signature scheme is a tuple of function:

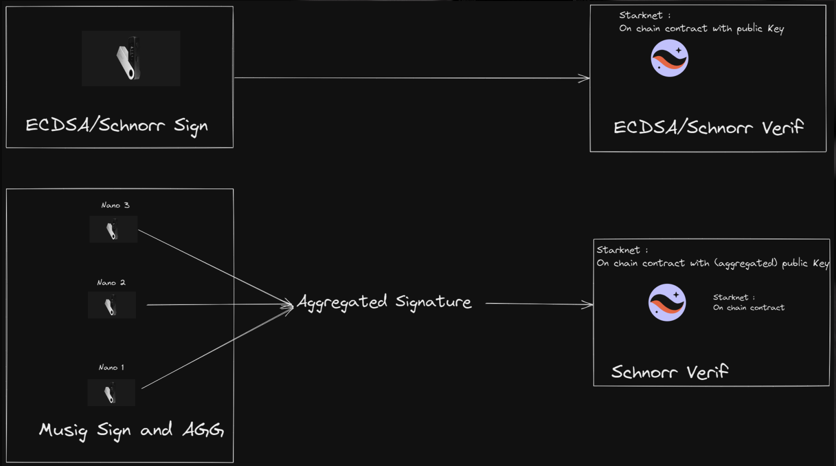
- $Setup$: returns $E(F_p), G, H$
- $KeyGen(E(F_p), G, H, seed)$: returns $(pvk, pubk) = (x, Q)$
- $Sign(x, message)$: returns Sig
- $Verify(Sig, Q)$: returns `true/false`

Most commonly used signature scheme is ECDSA (Bitcoin, Ethereum)

- implemented in Starknet/Cairo (P256, NTT/Stark friendly Starknet Curve)
- available in your favorite sdk Ledger

Multi-signatures

A multi-signature is a digital signature allowing users to *aggregate* their keys in an aggregated public key. The signatures are also aggregated. Verifier API is unchanged.



Multi-signatures

A multi-signature is a digital signature allowing users to *aggregate* their keys in an aggregated public key. The signatures are also aggregated. Verifier API is unchanged.

Definition ((Classical) Digital Signature)

A multisig scheme is a tuple of function:

- $(Setup, Keygen, Verify, Sign)$
- $KeyAgg(Q_1, \dots Q_n)$ returns X
- $SignAgg(Sig_1, \dots, Sig_n)$ returns Sig

Multi-signatures

A multi-signature is a digital signature allowing users to *aggregate* their keys in an aggregated public key. The signatures are also aggregated. Verifier API is unchanged.

Advantages (over naive concatenation/trusted aggregator)

- only one signature over channel (bandwidth consumption)
- no need for a trusted aggregator (no remote private key, own your crypto !)
- no risk of contract failure (don't trust, no don't)
- verifier doesn't need to know the underlying group of users

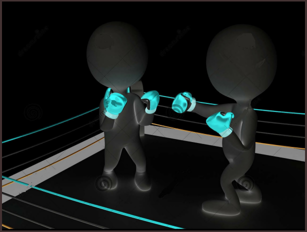
Example: Bitcoin Taproot, BIP340

Multi-signatures

A multi-signature is a digital signature allowing users to *aggregate* their keys in an aggregated public key. The signatures are also aggregated. Verifier API is unchanged.

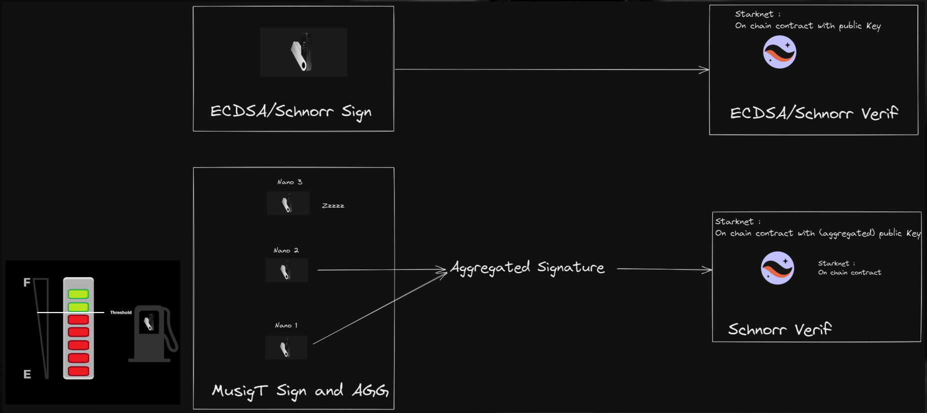
drawback

- increased computational complexities for signers
- requires (off chain) communications between signers
- computation in 2 rounds (Sig1, Sig2)



Threshold-signatures

A (k, n) threshold signature (TS-Sig) is a digital signature allowing a subset (threshold) of k users from n to *aggregate* a signature .



Threshold-signatures

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Definition ((Classical) Digital Signature)

A multisig scheme is a tuple of function:

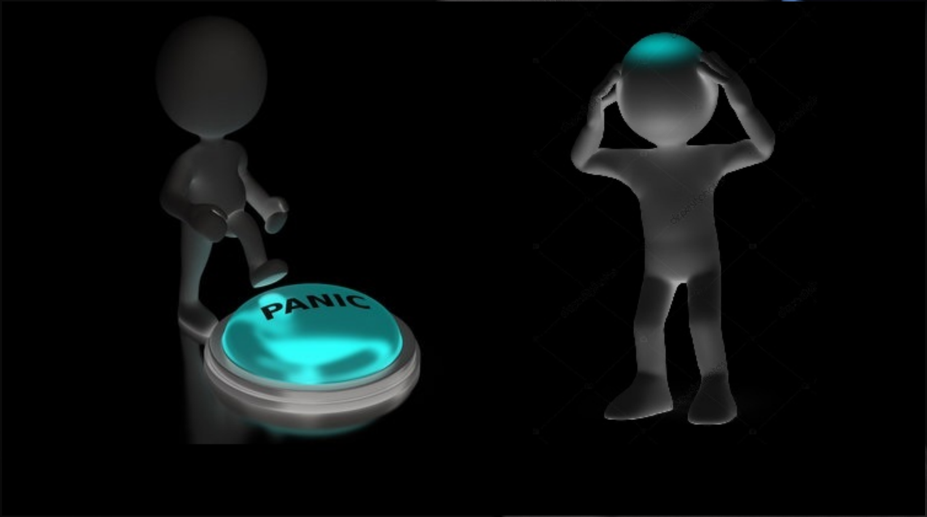
- $(Setup, Verify, Sign)$
- $DistributedKeygen$,
- $KeyAgg(Q_1, \dots Q_n)$ returns X
- $SignAgg(Sig_1, \dots, Sig_n)$ returns Sig

Advantages (over naive concatenation/trusted aggregator)

- All of multisig ($k = n$ is equivalent)
- More flexibility in access policy

Example: FROST(Blockstream)

Disclaimer



EC-Schnorr and ECDSA

SetUP() : Pick a curve with parameters (p, a, b, Gx, Gy, q) (weierstrass equations and formulaes).

Operation	Schnorr	ECDSA
KeyGen	$Q = xG$	$Q = xG$
Nonce*	k	k
Ephemeral	$R = kG$	$R = kG$
Hash	$e = H(m\|\|R)$	$e = H(m)$
Sign	$s = k - xe$ $Sig = (R, s)$	$s = k^{-1}(e + xr)$ $Sig = (r, s)$
Verif	$R' = sG + eQ$ Accept if R'=R	$r' = (es^{-1}G + rs^{-1}Q)_x$ Accept if r'=r

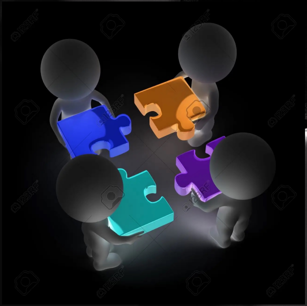
(nonce generation may use RFC6979 for misuse resistance)*

Musig2: using Schnorr additive properties

Schnorr s part is linear in (k, x) and linearity is cool:

$$\begin{aligned} s(k, x_1) + Sig(k, x_2) &= s(k, x_1 + x_2), & \forall x = x_1 + x_2 \\ s(k_1, x) + Sig(k_2, x) &= s(k, x), & \forall k = k_1 + k_2 \end{aligned}$$

(while ECDSA has degree two monomial in (k, x))



Linearity allow homomorphic additions. Idea: split X into $X = \sum a_i X_i$, k into $k = \sum k_i$.

Musig2: using Schnorr additive properties

Operation	Schnorr	Insec_Musig
KeyGen	$X = xG$	$X_i = x_iG$
KeyAgg	-	X=$\sum_{i=0}^{n-1} a_iX_i$
Nonce*	k	k_i
Ephemeral	$R = kG$	$R_i = k_iG$
Aggregate R	-	$R = (\sum_{i=0}^{n-1} a_i.k_i).G = k.G$
Hash	$e = H(m\|\|R)$	$e = H(m\|\|R)$
Sign	$s = k - xe$	$s_i = k_i - a_ix_ie$
Aggregate s	-	$s = \sum s_i = k - xe$

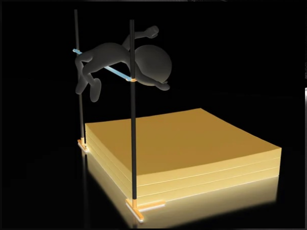
Musig2: using Schnorr additive properties

Musig2 uses a vectorial nonce of length μ, injected in previous Insec_Musig scheme.

Operation	Schnorr	Musig2
KeyGen	$X = xG$	$X_i = x_iG$
KeyAgg	-	X$=\sum_{i=0}^{n-1} a_iX_i$
Nonce*	k	$\vec{k_i} = (k_{i1}, \ldots, k_{i\mu})$
Ephemeral	$R = kG$	$\vec{R_i} = \vec{k_i}G$
Hash Nonce	-	$b = H(X\|\|R_0 \ldots R_\mu\|\|m)$
Aggregate R	-	$R = \sum_{j=1}^{\mu} b^{j-1}(\sum_{i=0}^{n-1} a_i.k_i).G = k.G$
Hash	$e = H(m\|\|R)$	$e = H(m\|\|R)$
Sign	$s = k - xe$	$s_i = (\sum_{j=1}^{\mu} k_ijb^{j-1}) - a_ix_ie$
Aggregate s	-	$s = \sum s_i = k - xe$

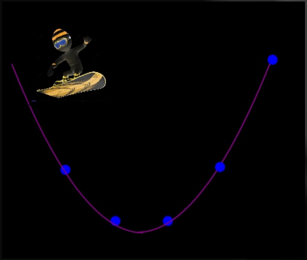
Musig2: Thresholdisation Principle

Thresholdisation use the principle of Shamir's secret sharing scheme , which is in fact a reed solomon erasure code.
Goal: Given enough shares, it is possible to reconstruct the initial value.



Musig2: Thresholdisation Principle

Lagrange interpolation enables to switch from points to polynomial coefficients using the following formulaes:



$$l_j(x) = \prod_{m \neq j} \frac{x - x_m}{x_j - x_m}.$$

$$L(x) = \sum_{j=0}^{k} P(x_j) l_j(x).$$

The transformation L from $(P_0 \ldots P_k)$ to $(a_0 \ldots a_k)$ is a linear transformation in x.

Sidenote: This is closely related to the principle of FRI used in starks.

Musig2: Thresholdisation Principle

Key ideas:

- interprete aggregated secret key as a polynomial P of degree k,
- each share (user secret key) is a point of the polynomial,
- blind the computation in the curve domain to perform the aggregation only handling public elements,
- replace '$\sum_{i=0}^{n}$' in previous scheme by Lagrange polynomials,
- some more steps are necessary (commitments) to avoid cheating.

Read FROST for full description.

Use cases

Multi factor authentication to Starknet Contract

Implement enhanced policy access to assets.

Access Policy

- Low amount: Host (hot wallet) only
- High amount: Host (smartphone) + HW wallet (Nano)

Voting system

Reduce risk and complexity of a contract implementing a voting system. A vote is adopted only though a valid TS-Sig.

Gnosis advanced

- implement a threshold voting system, with $k = \frac{n}{2}$

Ledger/Starknet Musig2

Specificities of Musig2 Starknet (ease Prover/Verifier computations)

- Uses Pedersen Hash as core hash function
- Uses Starknet curve as elliptic domain
- Implement x-only verification

The implementation is a generic one (takes hash and elliptic domain as SetUp parameter). The aim is to overlap with B340 if selecting P256k1 and SHA256 instead (TBD).

Ledger/Starknet Musig2

Current state:

- Schnorr verification available in Cairo,
- High-level simulation in Sagemath of full protocol (Sign/Verify for a pool of users)
- Musig2 implementation on top of a virtualization layer (only integrating bolos for now)
- currently working on thresholdization

C Library (Nano Signer)

SCAN ME

Cairo Code (Contract Verifier)

SCAN ME

Starknet Hackaton

Join the team for:

- Front end integration (wallet, current development over Argent) over verifier (Cairo) or signer (C)
- Integration of a different accelerator/library in the virtualization layer (C)
- Contribute to the threshold version (Sagemath/C)

Questions ?

C Library

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Slides

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Cairo&Sage

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