

# Cryptographic Toolbox for Privacy Preserving with Applications to Self Sovereignty

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**Abstract.** Self-sovereign identity (SSI) is an approach to digital identity that gives individuals control of their digital identities. In the Web3 framework, this **Decentralized Identity** is referred as DID [?]. In the literature, the dedicated Cryptographic solution is referred as Anonymous Credentials (AC) [?], [?]. It is a complex protocol that requires the use of several Privacy-Preserving protocols. The aim of this memo is to provide an overview of existing DIDs frameworks and the underlying cryptographic mechanisms in order to give an input to reflexion about how SSI could benefit to Web3 and be enforced by Ledger products and solution.

**keywords:** Decentralized Identifiers, Credential, Zero knowledge, Cryptographic Commitments

## Introduction

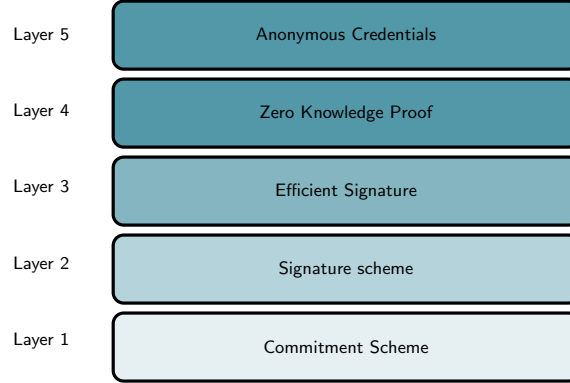
**Decentralized Identifiers.** Self Sovereign Identity (SSI) : is a paradigm shift from centralized and trusted Credential issuing to a ‘user-centric’ management of the identity. According to [?], Decentralized identifiers (DIDs) are a new type of identifier that enables verifiable, decentralized digital identity. A DID refers to any subject (e.g., a person, organization, thing, data model, abstract entity, etc.) as determined by the controller of the DID. In contrast to typical, federated identifiers, DIDs have been designed so that they may be decoupled from centralized registries, identity providers, and certificate authorities. Specifically, while other parties might be used to help enable the discovery of information related to a DID, the design enables the controller of a DID to prove control over it without requiring permission from any other party. DIDs are URIs that associate a DID subject with a DID document allowing trustable interactions associated with that subject.

**Anonymous Credentials.** In traditional electronical authentication, user identities are readily available to service providers. These latter can exchange collected data about any particular user among themselves. Anonymous credentials aims at mitigating such privacy breaches and giving the user control over his data. Informally, as explained in [?], a user acts under an arbitrary number of unlinkable pseudonyms rather than under his identity. In anonymous credentials, these access rights are described by attributes. A service provider can issue a credential to a user, which is parameterized with attributes. These attributes can, for example, encode access rights to a service or some user data. The user can then prove possession of a credential to the same or to other service providers in a privacy-preserving way. This process is called showing a credential.

The first efficient realization of anonymous credentials was first issued by Camenish in [?] and [?] which demonstrated how to build such protocol using the following primitives:

- a commitment scheme (the analogue of digital envelop),
- a signature scheme,
- an efficient signature scheme (a protocol to obtain a signature on a committed value without revealing the value to the signer),
- a zero knowledge proof of knowledge (ZKPoK, NIZKP).

The three lasters are the real algorithmic stake because they are the major bottleneck.



**Fig. 1.** The cryptographic stack for AC construction

**Contributions** The efficiency of the design of ZKP and efficient schemes relies on the cunning imbrication of the commitment and signatures with specific properties (structure preserving, use of homomorphic additive properties, etc.). This note tries to give an overview of those imbrications and the evolution of the state of the art to its current shape. We acknowledge the previous work of [?] and section D.5.2.1 of Prometheus project [?] as a starting point of the redaction of this note and our comprehension. The note is structured as follows:

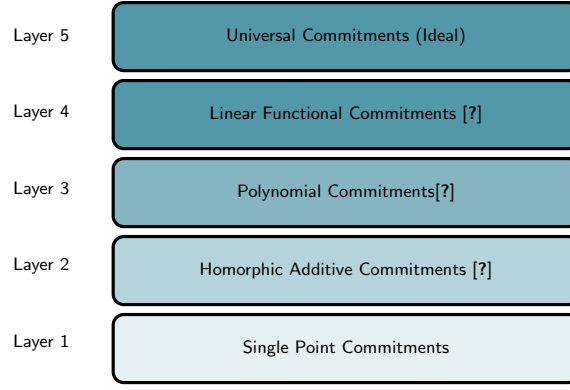
- the first section gives an overview of the cryptographic primitives used for AC constructions (figure 1),
- the second section provides a state of the art of existing Anonymous Credentials in the wild with fair Comparizon,
- the last section provides a survey of existing project and applications in  $\mathbb{R}$ Ledger that would benefit of such privacy preserving mechanisms.

The note has a minor contribution in the generalization of functional commitment to linear function described in [?] to ideal Universal commitment. Unfortunately the practical (in term of complexities) committed functions are however quite limited.

## 1 State of the Art

### 1.1 Commitments

**First examples** A commitment scheme emulates a publicly observed sealed envelope; it allows a party to commit to a message  $m$  so that this message is not revealed



**Fig. 2.** Evolution of Commitment schemes

until a later moment when the commitment is opened and the receiver gets convinced that the message was indeed  $m$ . Two important security properties of commitment schemes are called **hiding** and **binding**. The first property requires that no information about the committed message is revealed to an observer. The second property means that the committing party cannot alter the message after committing to it. The first well known way to commit is to commit to the hash of a value. In this trivial scheme, the value of the envelope  $m$  is committed though its hash  $h(m)$ . It is possible for the Prover to prove the knowledge of  $m$  by providing its commitment. This solution is classically used in online Password authentication, which can be considered as a zero knowledge proof of the value  $m$ . In [?], the author describes a commitment scheme with **homomorphic additive property**. Pedersen Vector Commitment is a direct corollary of this additive feature. He also provides the first description of a Verifiable Secret Sharing Scheme, which use the proposed commitment as a tool to provide a verification aspect to the well known Shamir Secret Sharing Scheme [?] (which is in fact a diverted way to use a Reed Solomon Code). This scheme introduce the idea of an “**Opening function**”, formalized later on. Indeed while committing to  $x_1$  and  $x_2$ , it is possible to reveal later the value  $x_1 + x_2$ , without revealing separately the  $x_i$ . This property is used in Monero to hide transaction amount with blinding factors. It has also been used in BulletProof for a **range proof system**. *TODO : hunt for more Pedersen Commitment uses in the litterature.*

<b>Pedersen Commitment</b> <ul style="list-style-type: none"> <li>– <math>Setup(1^\kappa, t) : \langle G, Q \rangle \leftarrow^{\\$} E(F_p)</math></li> <li>– <math>Commit(x, r) : \mathcal{C}(x, r) = xG + rQ</math></li> <li>– <math>Addition : \sum \mathcal{C}(x_i, r_i) : \mathcal{C}(\sum x_i, \sum r_i)</math></li> </ul>
<b>Pedersen Vector Commitment</b> <ul style="list-style-type: none"> <li>– <math>Setup(1^\kappa, t) : \langle G_1, \dots, G_n, Q \rangle \leftarrow^{\\$} E(F_p)</math></li> <li>– <math>Commit(\vec{x}, r) : \mathcal{C}(\vec{x}, r) = \sum x_i G_i + rQ</math></li> </ul>

**Fig. 3.** Pedersen Commitments [?]

**Definitions.** Now that some insight with basic constructions have been provided, the definition of a Universal Commitment is given. It appears as a natural extension of the Functional Linear Commitment of [?] to any function.

**Ideal Universal Commitment**

- $Setup(1^\kappa, t)$  : takes security parameter  $\kappa$  and additional parameters  $t$  then output an algebraic structure, optionally some  $\langle P_k, S_k \rangle$  elements.
- $Commit(S_K, r)$  : outputs commitment  $\mathcal{C}$  to element  $r$  and (optionally) auxiliary information **aux**
- $Open(S_K, r)$  output element  $r$
- $Witness(S_K, r, i)$  output  $\langle i, f(i, r), w_i \rangle$  where  $w_i$  is a witness of the evaluation of  $f$  in  $i$  relatively to the committed element  $r$ .
- $VerifyOpen(S_K, \mathcal{C}, i, r, f(i, r))$
- $VerifyEval(P_K, i, f(i, r), w_i)$

**Fig. 4.** Ideal Universal Commitments Formalization (this note)

Scheme	Commit	Open	Witness	f
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**Table 1.** Instantiation of known commitment schemes in the Universal Commitment framework (TBD)

**Polynomial Commitments** Polynomial commitments are used to provide exact range proof. The idea is to commit a Polynomial that is evaluated to 0 in all elements of the interval. It is done by committing a polynomial  $P = (\prod x_i - a_i)Q$  where  $Q$  is used for security.

**Polynomial Commitment**

- (Trusted) Setup :  $(g, g^\alpha, g^{\alpha^2}, \dots, g^{\alpha^d}), (h, h^\alpha, h^{\alpha^2}, \dots, h^{\alpha^d})$
- Com(p,r) :  $(\prod_{j=0}^d (h^{\alpha^j})^{p_j}) \cdot (\prod_{j=0}^d (g^{\alpha^j})^{r_j}), p(x) = \sum p_j x^j, r(x) = \sum r_j x^j$
- Open: output  $(p, r)$
- VerifyPoly : output  $\mathcal{C} \stackrel{?}{=} g^{p(\alpha)} \cdot h^{r(\alpha)}$
- Witness :
- VerifyEval :

**Fig. 5.** Polynomial Commitment [?]

## Functional Linear Commitments

## Applications

Scheme	Required for	Description
Pedersen	VSS Monero BulletProof Credentials	Verifiable Secret Sharing Transaction amount hiding Range Proof [?], [?]
Polynomial C.	eVSS	Strongly Verifiable Secret Sharing Range Proof Credentials Zk-Snark
Functional Linear		

**Table 2.** protocols build on top of described commitments schemes

## 1.2 Efficient Signatures

**Schnorr Signature** Schnorr signatures [?] are almost as old as ECDSA, which was selected mainly because Schnorr used to be covered by patents (now expired), limiting its adoption. Due to some different favorable algebraic properties, it is feasible to convey additive properties into the computation. Doing so, combining SS [?] with Schnorr provide threshold signature, combining with Groth signatures, it enable Sibling Signatures leading to AC [?]. It leads to Multisignature by enabling key aggregation ([?]). The diversity of such ZK-friendly constructions is very high. This richness is such that the Bitcoin community (prudent among blockchain ones) is waiting for its adoption through BIP340 [?].

**Groth Signature** Groth signature [?] is a structure-preserving signature that sign vectors of group elements.

**Multi, Aggregate and Threshold signatures** In a multi-signature scheme [?], public keys of a group of users are aggregated such that verifiers are given a signature and an aggregate public key and do not learn whether the signature was created by a single signer or by a group of signers. This leads to possible uses that may benefit to the privacy of users.

**Ring Signatures** A ring signature is a scheme allowing to sign messages on behalf of a group (including thyself), without revealing its identity.

**Linkable Signatures** Linkability is an additional property to ring signature allowing to detect if two messages have been issued by the same entity, learning nothing more about its identity.

## Ring CT

## 1.3 Zero knowledge Proofs

A zero-knowledge protocol or proof (ZKP) is a method by which one party (the prover) can prove to another party (the verifier) that a given statement is true while no information is learned except the fact that the statement is indeed true. As for

commitment this property covers a wild area, from proving knowledge of a Password, a private key (a signature is a ZKP), to general purpose circuit. The complexity of the proof depends on the richness of the proofs that can be done with the protocol.

**Groth NIZK argument** Groth’s constant size NIZK argument is based on constructing a set of polynomial equations and using pairings to efficiently verify these equations.

### Commit-and-prove ZKsnarks

Scheme	Required for	Description
Groth16	Tornado Nova Zcash v1	Transaction hiding (Mixer) Coin ownership
Stark		

**Table 3.** protocols build on top of ZKP schemes

### LegoSnark: gadget for Commitments

## 2 Anonymous Credentials/DIDs

This section describes some existing DIDs frameworks with pointers to the underlying cryptographic mechanisms. The last subsection focus on [?] which proposes a construction providing a decentralized approach for multi-issuers DIDs.

### 2.1 Existing SSI

	[?]	[?]	Sovrin [?]	IPv8	YL20	Uport
Control	✓	✓	✓	✓	✓	✓
Data Minimization	✓	✓	✓	✓	✓	×
Commitment Type						
Signature Type	[?]					
ZKP type	[?]	[?]	[?]	[?]+[?]		-
Issuer Anonymity Optimization	✓	×	×	×	×	×

**Table 4.** Comparizon of existing DIDs (completed from[?])

### 2.2 A multi-issuer scheme

## 3 Applications for Ledgers

In this section we provide a list of Ledgers services which could benefit to an improvement of the privacy using some element of the “toolbox” (sorry Data Team).

	RuleMaking			Operation		Security*			
	Network	Registry	Specification	Network	Registry	Pro	Per	Int	Conf
did:btc	● □	● □	○	● □	● □	+	+	+	±
did:v1	● <sup>†</sup>	● <sup>†</sup>	○	● <sup>†</sup>	● <sup>†</sup>	-	±	±	+ <sup>†</sup>
did:ethr	● □	N/A <sup>†</sup>	●	● □	● ■	+	+	+	-
did:sov	● ■	● ■	●	● ■	● ■	-	±	+	±
did:web	●	●	○	● □	● □	-	-	-	±
did:peer	●	●	○	● <sup>†</sup>	● <sup>†</sup>	±	-	-	+

\* Security - Pro: protection Per: persistence Int: integrity Conf: confidentiality

● fully decentralized ● partially decentralized ○ centralized N/A<sup>†</sup> not applicable

Required resources: ■ modest □ substantial

<sup>†</sup> Not clear or well defined how method satisfies criteria at time of writing

**Table 5.** Comparizon of existing DIDs (source : [?])

### 3.1 Endorsement.

During Ledger genuine check of a Nano device on the Ledger Live, the public key is send to the BackEnd, then a “challenge-response” between Ledger services and Nano is performed. Using a Zero-knowledge proof instead of sending the public key would reduce the information that Ledger could store about links between a device and a host (smartphone or labtop). This could be achieved using a Group Signatures as described in this document instead. The backend only learns that it is indeed a Nano that is connecting to the Ledger Live application, without learning the device public key.

### 3.2 Protect.

In Protect, the seed element (input to the BIP32 protocol, the core of all user security) is stored via a threshold mechanism which is a VSS (verifiable Secret Sharing scheme) described in [?]. While the scheme is secure under the assumption that the gathering of shares is, it could be a concern if not. The scheme is as secure as the strongest back up provider access control. In the Pedersen VSS, if an attacker success in obtaining one of the share, which is protected by KYC in Protect, the share is known, unless performing a resharing. In a ZKP-VSS scheme [?], the attacker learns nothing if the reconstruction fails.

### 3.3 Device ID: Airdrop

One of the application of the Device Identifier (which is a very risky initiative in term of Privacy) is to perform Airdrops to the owner of nanos. As for the genuine check, the sender learns the device public key. Using a linkable signature [?] would provide the desired service, without having the sender learning nothing except the eligibility to the service.

### 3.4 DIDs

#### DIDs Nano and Host Side

Issue Name	Description	# (Link)
Computational Delegation of pairings	Hybrid Architecture for efficient Nano PBC	
Strongly verifiable secret sharing	eVSS for Protect	
Musig2	Schnorr Multisignature	
Nano with Group Signature endorsement	Group signature for Ledger endorsement	31
Linkable signature for Airdrop	Linkable signature for Anonymous Airdrop	30

**Table 6.** Innovation issues related to Privacy Preserving

#### DIDs Back End Side

## 4 Conclusion

In this note we provided an overview of some privacy preserving tools and principles. We also provide a non exhaustive list of applications that could immediately benefit from the described mechanisms. Most of those building blocks rely on the use of computation of pairing over elliptic curves.

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# Annexes

## A Issues and Roadmap

## B Implementation of Pairings

### B.1 Pairings used in blockchain framework

### B.2 Hybrid Architecture for Nano

As a constrained device, the Implementation of pairings only on the device may The pairings are a bilinear map

$$e : (G_1 \times G_2) \rightarrow \mathbb{F}_{p^k},$$

where  $G_1$  and  $G_2$  are elliptic curves define over  $\mathbb{F}_p$  and  $\mathbb{F}_{p^d}$  and  $\mathbb{F}_{p^k}$  some prime field extension of degree  $k$ . The pairing  $e$  is homomorphic such that  $e(aP, bQ) = e(P, Q)^{ab}$ . Using this property it is possible to blind the computation using the following algorithm, inspired by the well known Coron countermeasure:

- randomly select  $a$  and  $b$  in  $\mathbb{F}_q$  ( $q$  being the order of the curves)
- send  $aP$  and  $bQ$  to the host
- host compute  $e(aP, nQ)$  send back results
- delegator (Nano) computes  $e(aP, bQ)^{-ab}$

Note that if element of  $G_2$  are public elements, it is possible to avoid the Implementation of elliptic curve over extension fields.

## C Group Signature for Endorsement

## D Linkable Signature for Anonymous Airdrop

## E Strongly Verifiable Secret Sharing for Seed Protection

## F Schemes and Assumptions

Scheme	Reference	Assumptions	Pairings (Y/N)

**Table 7.** Cryptographic schemes and underlying Assumptions and Tools