6. SHIP HULL FORM DESIGN

At the preliminary stage of ship design it is necessary to evolve a feasible technical design. Here feasible implies a suitable set of dimensions and a balance of masses such that the ship is of adequate size for the intended service, floats upright at the correct draught, achieves the correct speed and has structural integrity. International regulations covering such aspects as freeboard and subdivision will also require consideration at the preliminary stage. One of the major elements of vital importance for a successful ship design is the hull form itself. It affects all the major design elements, including performance, space and function; therefore a sound decision on the hull form characteristics is of utmost importance.

The creation of a fair set of ship lines is an important part of the initial phase of ship design. At the early stages of design, the designer needs to develop from the relatively sparse information available about the desired features of the hull form, a complete set of lines defining the form with sufficient accuracy for subsequent design calculations. The seakeeping, manoeuvring, and speed characteristics, as well as boundaries of the decks, payload capacities, vertical and longitudinal centres of gravity of cargo and fuel oil depend on the hull envelope represented by this plan.

The hull form has to meet a large number of different objectives, including

- Required displacement at the design draught
- Required cargo space and tank capacities
- An LCB position at the design draught which in association with the weights and centres of gravity of the ship and its deadweight items enables the ship to be loaded in a way that will result in satisfactory trim
- Required deck areas to accommodate all aspects of the arrangement
- Features to minimise powering requirements; low resistance, good hull efficiency and an ability to accommodate the propeller with clearences that make vibration unlikely
- Good seakeeping and sufficient manoeuvrability
- KM values at operating draughts which will ensure satisfactory stability when the ship is loaded as intended
- Production friendliness with as much flat plating as can be arranged and with the minimum amount of double curvature in the shell plating
- Aesthetic appeal particularly on passenger ships

There are infinite number of shapes satisfying the displacement equation for any set of values of length, beam, draught, block coefficient, and displacement. The challenge lies in developing an optimum hull form or, at least, one having acceptable performance. Hydrodynamic characteristics are very sensitive to even minor changes in hull form. Therefore, the selection of ship lines requires great care in order to avoid unacceptable results. Although several attempts have been made to apply mathematical theories developed for wave resistance to the development of hull lines, see Inui (1962), Pien (1964) and Goren and Calisal (1988) for example, a fully acceptable procedure does not exist even when the only hydrodynamic consideration is the resistance characteristic in calm water.

6.1. Hull Form Design Parameters

The geometric properties of a ship's hull form can be studied in the following groups:.

- **Sectional Area Curve (SAC)** .
- **Loaded waterline (LWL) or Design waterline (DWL) curve**.
- **Underwater hull form coefficients (C_B, C_M, C_P, C_{WP}, LCB, LCF)**
- **Bow profile and section shape for forebody**
- **Stern profile and section shape for afterbody**
- **Angle of entrance**

In **Figure 6.1**, schematic curves of longitudinal distribution of the sectional area and beam at the loaded waterline are shown. This figure contains useful information concerning the naval architectural hull form parameters. The area under the sectional area curve provides the displaced volume and its longitudinal centre, LCB, whereas the area under the beam curve provides the waterplane area and its longitudinal centre, LCF.

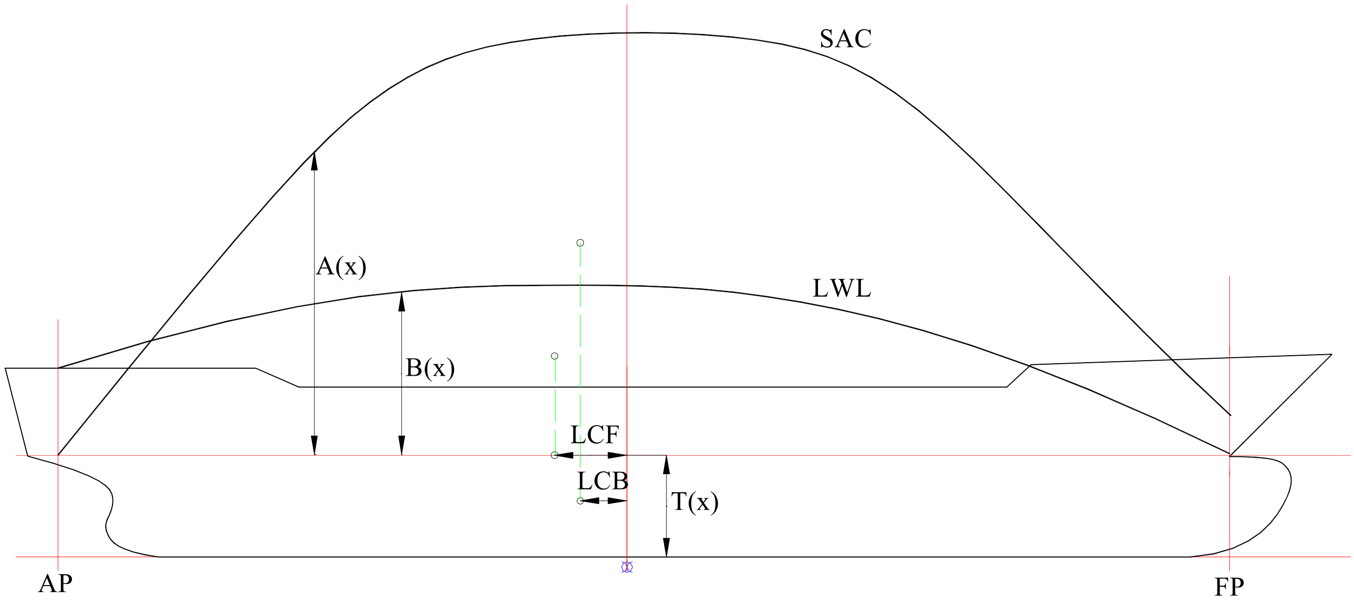


Figure 6.1. Sectional area (SAC) and Loaded waterline (LWL) curves

6.1.1. Block Coefficient (C_B)

The block coefficient is the most basic parameter used to define the fullness of the underwater hull form. Many of the techno economic performance characteristics of a ship are affected by the block coefficient. Consequences of a reduction in block coefficient will, in general, be as follows

- Reduction in the required propulsion power due to the improvements in wavemaking resistance and propulsive efficiency. This may result in a lighter engine and reduced weight.
- Decrease in fuel consumption or higher speed with the same engine
- Increase in steel weight
- Reduced hold capacity
- Improved seakeeping performance with lower motion levels and reduced probability of slamming
- Better directional stability with reduced manoeuvrability
- Slight improvement in static stability characteristics
- Increased building cost due to
  - Greater quantities of curved plates and sections
  - Fewer flat plates with rectangular boundaries
  - The need for a greater variety in plate thickness and size and for more sections
  - More scrap

The upper limit of block coefficient for commercial ships is considered 0.87 beyond which satisfactory flow to the propeller cannot be maintained. There is a long list of empirical formulae for the prediction of block coefficient, which will be noted below.

Alexander :

$$C_B = 1.05 - 1.68F_n$$

$$C_B = k - 0.5\frac{V}{\sqrt{L}} \qquad \text{(V: knot, L: feet)}$$

$$k = 1.04 + 0.08\left(\frac{V}{\sqrt{L}} - 0.5\right) \text{ or}$$

k=1.03 for high speed ships, and
k=1.12 for slow ships

Troost : $C_B = 1.15 - 2.118F_n$

Ayre : $C_B = C - 1.68F_n$

F_n	0.149	0.178	0.208	0.238	0.267	0.297
C	1.04	1.045	1.05	1.06	1.07	1.08

Lindbald : $C_B = 1.10 - 1.68F_n$

Silverleaf/Dawson : $C_B = 1.214 - 2.404F_n$

Van Lammeren : $C_B = 1.137 - 2.02F_n$

Minorsky : $C_B = 1.22 - 2.387F_n$

Telfer : $C_B = 1 - \left[1.263\left(\frac{B}{L} + 1\right)F_n\right]$

Sabit : $C_B = 0.65 + 3.2F_n - 13.6F_n^2$

Katsoulis : $C_B = K\ f\ L^a\ B^b\ T^c\ V^d$ L, B, T (m), V(knot)

a = 0.42 b = -0.3072 c = 0.1721 d = -0.6135 k = 0.8127

Ship type	f
Navy ships	0.91
Car carriers - Ro-Ro	0.97
Refrigerated cargo	0.97
Cargo liners	0.98
General cargo	0.99
Crude oil tankers	0.99
OBO	1.00
Timber ships	1.00
Container ships	1.00
Passenger ships	1.00
Bulk carriers	1.03
Liquefied gas tankers	1.04
Product tankers	1.05
Chemical tankers	1.06
Ferries	1.09

Schneekluth : $C_B = \frac{0.14}{F_n}\frac{L/B + 20}{26}$ $C_B = \frac{0.23}{F_n^{2/3}}\frac{L/B + 20}{26}$

Townsin : $C_B = 0.70 + 0.125\tan^{-1}\left(\frac{23 - 100F_n}{4}\right)$

Barass : $C_B = 1.07 - 1.522F_n$ Supertanker

$C_B = 1.01 - 1.522F_n$ Ro-Ro

Scher and Benford : $C_B = 1.086 - 1.505F_n$ Panamax bulk carrier

($B \leq 32.2$m $T \leq 14.5$m)

Gilfillan-Alexander : $C_B = 0.968 - 0.269\frac{V}{\sqrt{L}}$ V(kn), L(ft) bulk carrier

6.1.2. Prismatic coefficient

The prismatic coefficient represents the distribution of fullness along the length. For low C_P the volume is concentrated towards midships while for high C_P the volume is more uniform along the length. This is a main factor in determining residuary and wave making resistance. The usual approach to determine C_P is to predict C_B and C_M and hence $C_P = C_B / C_M$.

Porricelli : $C_P = 0.917 C_B + 0.073$

Wright : $C_P = 0.96 C_B + 0.04$ Bulk carrier

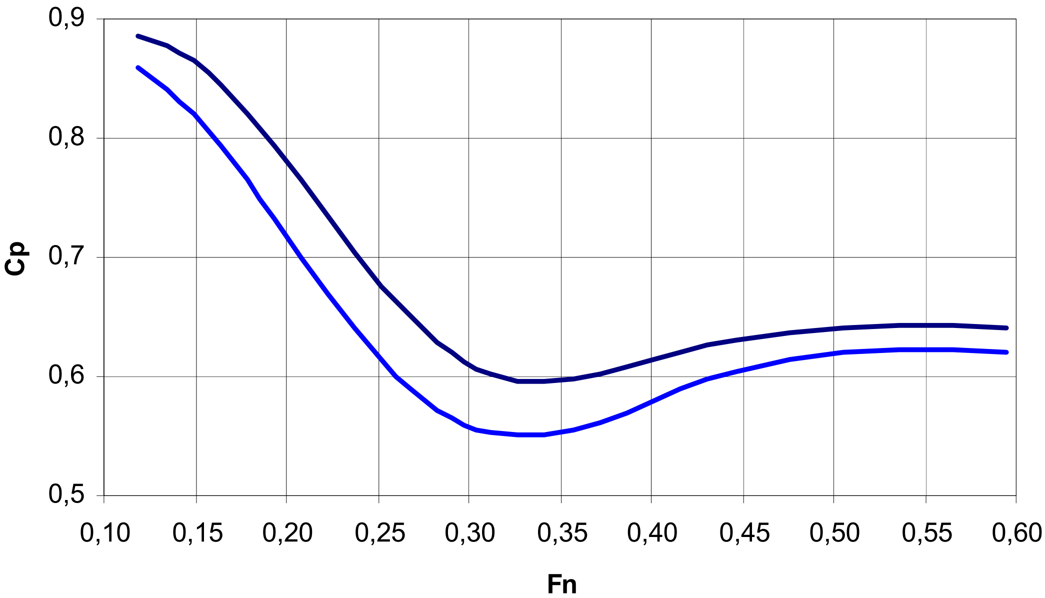


Figure 6.2. Prismatic coefficient as recommended by Saunders

6.1.3. Midship section area coefficient (C_M)

This coefficient represents the fullness of the midship section and is closely related to the resistance characteristics of the ship. An increase in C_M will normally lead to

- Increase in wetted surface area and hence the frictional resistance.
- Increase in the length of entrance which will result in reduced wavemaking resistance
- Increase in the length of run which means a reduction in separation resistance may be expected
- Improved propulsive performance
- Greater roll damping and reduced rolling motion in heavy seas

A formula for C_M in terms of the bilge radius and rise of floor is

$$C_M = 1 - \frac{F\left[\left(\frac{B}{2} - \frac{K}{2}\right) - \frac{R}{\left(\frac{B}{2} - \frac{K}{2}\right)}\right] + 2R\left(1 - \frac{\pi}{4}\right)}{BT}$$

where

R : bilge radius
F : rise of floor
K : width of keel

with no rise of floor this reduces to

$$C_M = 1 - \frac{2R^2\left(1-\frac{\pi}{4}\right)}{BT}$$

There seem to be three motives for keeping the bilge radius small

- The greater resistance to rolling provided by a square bilge
- The easier cargo stowage of a squarer hold
- The finer C_P which will generally reduce the resistance

Schneekluth recommends the following formula for the bilge radius of conventional ship forms

$$R = \frac{BC_K}{\left(\frac{L}{B}+4\right)C_B^2}$$

where

R : bilge radius
L : length between perpendiculars
B : breadth
C_B : block coefficient
C_K : coefficient between 0.5-0.6, in extreme cases 0.4-0.7

An alternative formula is as follows

$$C_M = 1 - \frac{R^2}{2.3299BT}$$

where

R : bilge radius
B : breadth
T : draught

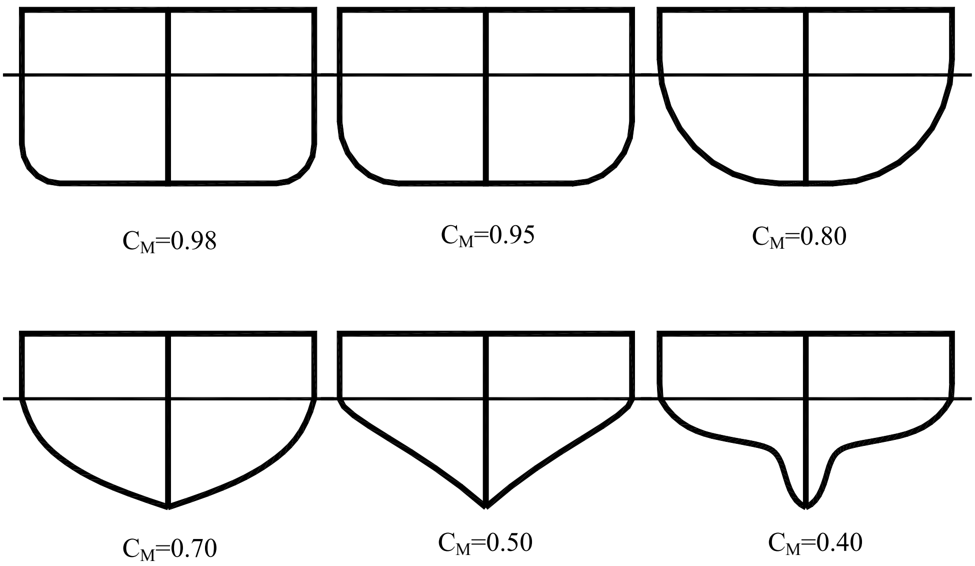


Figure 6.3. Typical midship sections

Some of the empirical formulae, which can be used to estimate the midship section area coefficent, are given below

Van Lammeren :

C_B	0.55	0.60	0.65	0.70	0.75
C_M	0.960	0.976	0.980	0.984	0.987

Van Lammeren : $C_M = 0.9 + 0.1C_B$

Keslen : $C_M = 1.006 - 0.0056C_B^{-3.56}$

HSVA : $C_M = \dfrac{1}{1 + (1 - C_B)^{3.5}}$

Seri 60 : $C_M = \dfrac{C_B}{0.95C_B + 0.045}$ $\quad C_B \geq 0.6$

Seri 60 : $C_M = 0.977 + 0.085(C_B - 0.6)$ $\quad C_B \geq 0.6$

Munro-Smith : $C_M = (-0.4786C_B + 1.445)C_B^{1/3}$

C_B	Van Lammeren	Van lammeren	Keslen	HSVA	Series 60	Series 60	Munro Smith
0.55	0.960	0.955	0.959	0.942	-	-	0.968
0.60	0.976	0.960	0.971	0.961	0.976	0.977	0.977
0.65	0.980	0.965	0.980	0.975	0.981	0.981	0.982
0.70	0.984	0.970	0.986	0.985	0.986	0.986	0.986
0.75	0.987	0.975	0.990	0.992	0.990	0.990	0.987
0.80	-	0.980	0.994	0.996	0.994	0.994	0.986
0.85	-	0.980	0.996	0.999	0.997	0.998	0.983

6.1.4. Waterplane area coefficient (C_{WP})

The waterplane area coefficient significantly affects the resistance, stability and seakeeping characteristics of a ship. There is common tendency to use a high waterplane area coefficient to attain high stability and seakeeping, e.g. passenger ferries. This also has the advantage of providing a large deck area, which is an essential feature for passenger ships and ferries.

The waterplane area coefficient, together with the block coefficient determines the form of cross sections. Therefore this coefficient should be determine in accordance with the block coefficient and the desired underwater hull form. A higher C_{WP} will usually result in V shaped sections while U shaped sections are typical for low C_{WP} forms.

Some of the empirical formulae to estimate C_{WP} are as follows

Riddlesworth : $C_{WP} = \dfrac{1}{3} + \dfrac{2}{3}C_B$ (normal section)

Schneekluth : $C_{WP} = 0.95C_P + 0.17(1 - C_P)^{1/3}$ (U form)

Schneekluth : $C_{WP} = C_P^{\,2/3}$ (V form)

Schneekluth : $C_{WP} = \dfrac{1}{3} + \dfrac{2}{3}\dfrac{C_B}{\sqrt{C_M}}$ (V form)

: $C_{WP} = \sqrt{C_B} - 0.025$ (V form)

Parsons : $C_{WP} = 0.175 + 0.875C_P$ (single screw, cruiser stern)

Parsons : $C_{WP} = 0.18 + 0.86C_P$ (Seris 60)

Parsons : $C_{WP} = 0.262 + 0.760C_P$ (twin screw, cruisr stern)

Parsons	**:**	$C_{WP} = 0.262 + 0.810C_P$	(small vessel, transom stern)
Eames	**:**	$C_{WP} = 0.44 + 0.52C_P$	(small naval vessel)
Wright	**:**	$C_{WP} = \frac{3}{8} + \frac{5}{8}C_B$	(bulk carrier)
Gilfillan	:	$C_{WP} = 1.265C_B - 0.146$	(bulk carrier)
Porricelli	:	$C_{WP} = 0.325 + 0.702C_B$	(container)
Porricelli	:	$C_{WP} = 0.336 + 0.702C_B$	(RoRo)
Porricelli	:	$C_{WP} = 0.306 + 0.702C_B$	(general cargo, tanker)

C_B	C_M	C_P	$\frac{1}{3}+\frac{2}{3}C_B$	$0.95C_P + 0.17(1-C_P)^{1/3}$	$C_P^{2/3}$	$\frac{1}{3}+\frac{2}{3}\frac{C_B}{\sqrt{C_M}}$	$\sqrt{C_B} - 0.025$
0.55	**0.959**	**0.574**	0.700	0.673	0.691	0.708	0.717
0.60	**0.971**	**0.618**	0.733	0.710	0.726	0.739	0.750
0.65	**0.980**	**0.663**	0.766	0.748	0.760	0.771	0.781
0.70	**0.986**	**0.708**	0.800	0.785	0.794	0.803	0.812
0.75	**0.990**	**0.758**	0.833	0.826	0.831	0.836	0.841
0.80	**0.994**	**0.805**	0.866	0.863	0.865	0.868	0.869
0.85	**0.996**	**0.853**	0.900	0.900	0.899	0.901	0.907

6.1.5. Longitudinal Centre of Buoyancy (LCB)

The position of the centre of buoyancy may be dictated by the disposition of weight and the need to achieve a satisfactory trim, but in most ships it should be governed by a wish to minimise power requirements.

The position of LCB for minimum powering depends mainly on the Froude number and block coefficient. The position of LCB differs for ships with normal and bulbous bows, as the LCB on a form with a bulb will be anything from 0.5-1 % further forward than that of an otherwise very similar form with a normal bow. The position also differs for twin screw ships for which the optimum position is further aft than it is for single screw ships, reflecting the fact that the lines of a twin screw ship can be optimised almost entirely on resistance considerations with little need to consider the flow to the propellers which plays a major part in the design of the stern of a single screw ship.

Figure shows how the position of LCB moves as the C_B changes from unity to a very fine form.

- At C_B=1.00 the LCB must be at amidships
- For a barge the first essential is a swim bow, so at about C_B=0.95 the LCB moves aft to say 1.5% .
- The next improvement to be made to ease movement of the vessel is a swim stern, so at about C_B=0.90 the LCB moves back to amidships
- For the slowest self propelled shipshape vessel the bow is now generally very full spoon shaped and this coupled with the need for good flow to the propeller(s), requiring fining aft means that for a C_B of between 0.90 and 0.75 the LCB is well forward, say about 2.5-3.0% or even 3.5%.
- Once the run has been made such that it provides a satisfactory flow to the propeller, it is only necessary to fine it gradually as the block coefficient is further reduced for ships with higher speeds and powers. The forebody, on the other hand, is where reductions in wavemeking resistance can best be effected and from being markedly fuller than the aft body, the forebody changes to being much finer, with the result that the LCB progressively shifts to a position well aft of amidships.
- Finally for very fine ships there is a tendency for the LCB to return towards amidships.

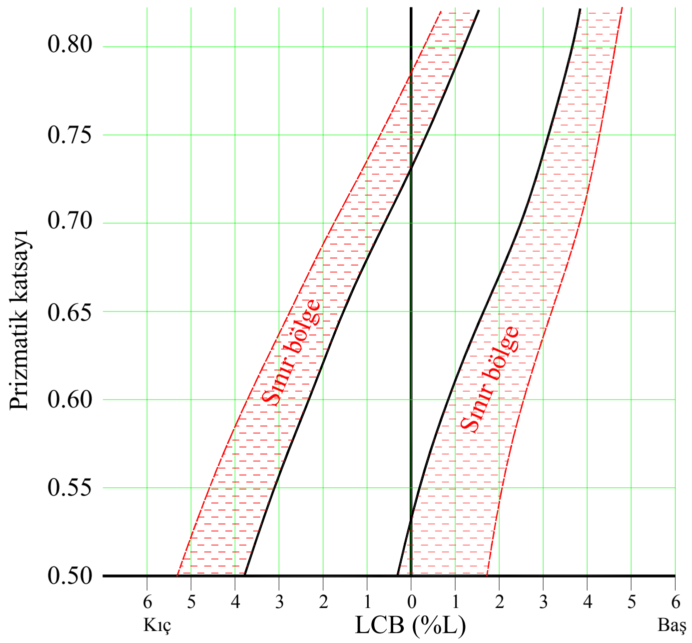


Figure 6.4. Relationship between block coefficient and the position of LCB

Figure 6.5 gives a plot of the optimum range of LCB position for both normal and bulbous bow forms against C_B.

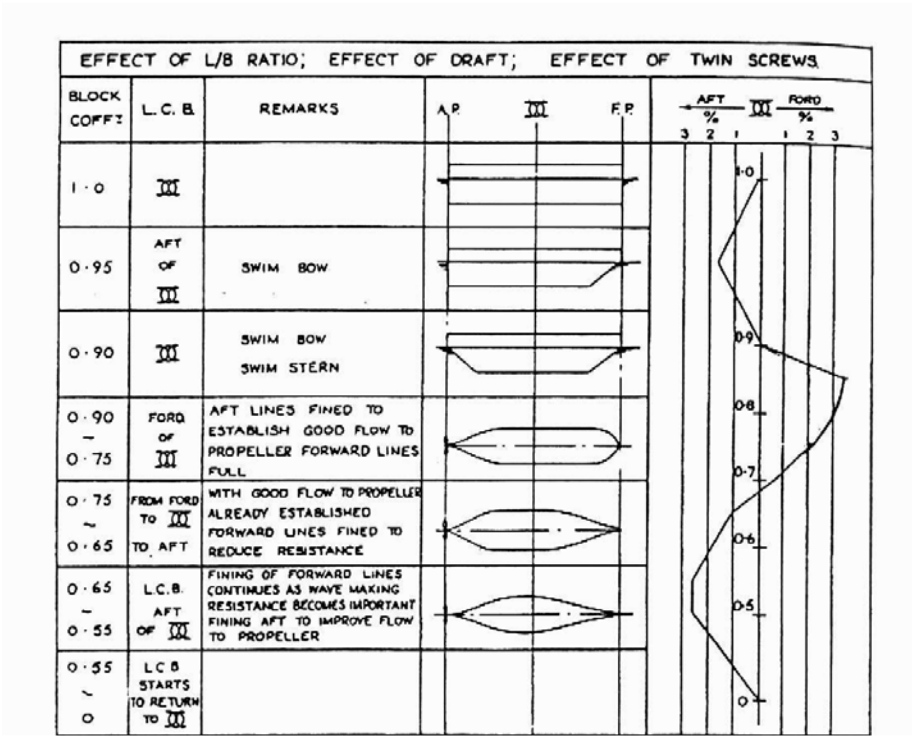


Figure 6.5. Variation of LCB position

Harvald (1983) recommends the following formula based on Froude number

$$LCB = 9.70 - 45F_n \pm 0.8$$

where LCB is a percentage of length and (+) indicates the forward direction

Troost : $$LCB = L[0.5 - (0.175C_P - 0.125)]$$

Gilfillan : $$LCB = (17.5C_P - 12.5)L_{BP}$$ (% L_{BP} – bulk carrier)

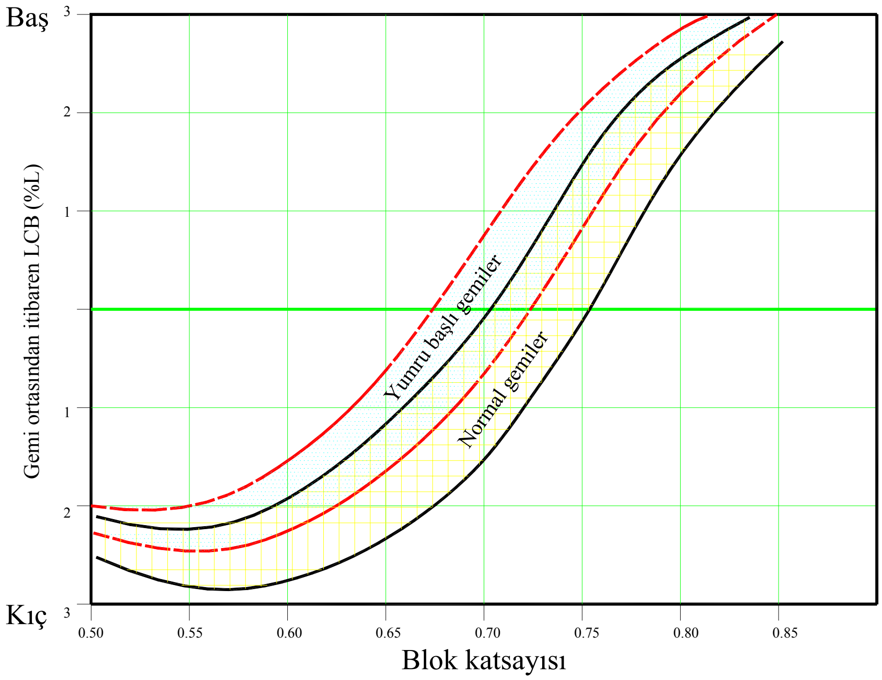


Figure 6.6. LCB position versus block coefficient

6.1.6. Longitudinal centre of flotation (LCF)

Porricelli : $$LCF = 0.5L\left(\frac{V}{160} + 0.914\right)$$ (tanker) V:speed in knots

Porricelli : $$LCF = 0.485L\left(\frac{V}{100} + 0.9\right)$$ (bulk carrier) V:speed in knots

Porricelli : $$LCF = 0.5L\left(\frac{0.95}{V} + 1.03\right)$$ (twin screw – transom stern)

Porricelli : $$LCF = L\left(0.5\left(\frac{V}{135} + 0.924\right) + 0.23\right)$$ (twin screw – cruiser stern)

6.1.7. Bow Profile

The first decision to be taken in relation to the bow is whether to fit a *normal* or a *bulbous* bow. A normal bow is cheaper to manufacture and a bulbous bow should only be fitted if doing so will reduce the resistance and thereby either increase speed or reduce the power required and with it the fuel consumption. The range of Froude numbers and block coefficients at which such an improvement is likely can be summarised as follows:

- The bulbous bow is advantageous for fast ships with C_B values less than 0.625 and F_n greater than about 0.26
- The bulbous bow presents no advantage for ships with C_B values between 0.625 and 0.725 unless these have higher than normal speeds
- The bulbous bow is again advantageous for C_B values between 0.725 and 0.825 but probably not for C_B values over 0.825

At all block coefficients bulbous bows show the best advantage on over driven ships and are often disadvantageous on ships, which are relatively fine for their speeds.

It is generally accepted that bulbous bows can offer their greatest advantage in the ballast condition, particularly on full lined ships with block coefficients in excess of 0.75. In general it appears that if a bulbous bow is not advantageous at the load draught, it will only become advantageous in ballast if the ship is operated at or near its full power giving a speed in ballast at least 10 % or say 2 knots or so, more than the loaded service speed.

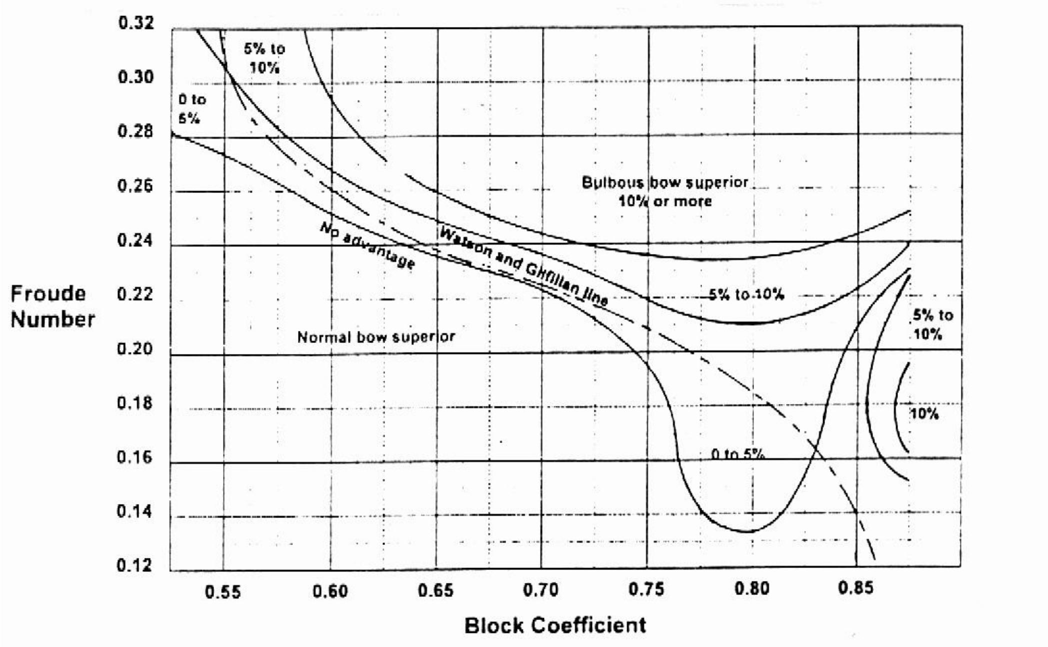


Figure 6.7. Criterion for bulbous bow according to Watson

A bulbous bow will generally help to reducing pitching, but on the other hand it is more likely to cause slamming.

Bulbous bows come in a variety of shapes and sizes . One main division is that between a fully faired bulb and one in which there is a sharp knuckle line between the bulb and a normal bow configuration (added bulb). The added bulb is generally simpler to manufacture and seems, on full lined ships, to give at least as good results as the faired bulb.

The next division is between bulbs which project as rams significantly forward of the for perpendicular, and those with little or no such projection. Ram bows also vary in the vertical positioning of the forward projection, which in some designs commence near the waterline and in others are well submerged.

One of the principal criteria applied to the design of a bulbous bow is the relationship that its sectional area at the fore perpendicular bears to the midship section area.

6.1.8. Forward section forms

To characterize the section form, the letters U and V are used corresponding to the form analogy. In the following table an extreme U section is compared with an extreme V section. It is assumed that both forms have the same sectional area below the loaded waterline.

	U section	V section
Deck area	lower	greater
Initial stability	lower	greater
Wetted surface	lower	greater
Steel weight	lower	greater
Labour	lower	higher
Wave making resistance	lower	higher
Seakeeping	worse	better

6.1.9. Afterbody section forms

Sterns have to be considered in relation to the following roles

- The accommodation of propeller(s) with good clearances that will avoid propeller excited vibration problems
- The provision of good flow to the rudder(s) to ensure both good steering and good course stability
- The termination of the ships waterlines in a way that minimises separation and therefore resistance
- The termination of the ships structure in a way that provides the required supports for the propeller(s) and rudder(s) plus the necessary space for steering gear, stern mooring and towage equipment etc. and is economical to construct

Flow to the propeller

Where the propeller diameter (D) on a single screw ship is of normal size in relation to the draught, i.e. D/T is approximately 0.75, the main consideration is ensuring good flow to the propeller, with a figure of between 28 and 30^0 being about the maximum acceptable slope of a waterline within the propeller disc area.

Keeping to such a figure tends, of itself, to force the LCB forward on a full bodied ships.

Lloyds recommended minimum clearances as a fraction of the propeller diameter for a four bladed propeller are

Tip to stern frame arch	: 1.00 K
Stern frame to leading edge at 0.7R	: 1.50 K
Trailing edge to rudder at 0.7R	: 0.12
Tip to top of sole piece	: 0.03

where

$$K = \left(0.1 + \frac{L}{3050}\right)\left(\frac{2.56 C_B P_B}{L^2} + 0.3\right)$$

where P_B : power in kW

The recommended clearance for a four bladed propeller on a twin screw ship, is 1.00 K.

Large propellers

Where the propeller is large in relation to the draught of the ship, a number of options exist:

- The propeller can be fitted in such a position that the lower tip is below the line of the keel. This is common practice on warships, but merchant ship owners have been reluctant to allow this because of possible damage to the propeller in shallow water and possible additional dry docking problems and costs.
- The ship can have a designed trim or raked keel. This is commonly used in small ships, notably tugs and fishing vessels. It is also used for the same reason on warships, even large twin-screw vessels.
- A tunnel type form can be used. This form is successfully used on shallow draught river craft.

Stern lines above the propeller

It is very desirable from a resistance point of view that the stern lines above the propeller should be continued to form a cruiser stern, which is immersed at the operating draughts. A cruiser stern should extend aft sufficiently to cover the rudder but there is no need for there to be any significant immersion at the end of the waterline which may cause eddies particularly if the cruiser stern is terminated by a flat transom as has become fairly general practice in recent years. The top of the rudder should follow the lines of the stern with only the necessary clearance.

Keeping the stern immersion to the desirable waterline position has the added advantage of permitting the greatest possible propeller diameter for a given draught.

In merchant ships transoms were first adopted for cost saving reasons, but once adopted the flat transom concept was progressively developed to provide more deck area for mooring equipment, to provide stowage for a tier of containers or to facilitate moving the accommodation further aft. It was also found that a considerable gain in KM could be obtained by the wider waterlines in the stern.

A transom stern can greatly improve the static stability of a ship by increasing the KM but if advantage is taken of this to permit more top weight, the ship may have inadequate stability when it suffers the big loss in KM which can occur when the stern comes out of water when the ship is pitching in a seaway (broaching).

In warships the transom stern was introduced not for cost cutting reasons but because it improved the hydrodynamic performance giving a less turbulent wake particularly at high speeds. As in merchant ships, the resulting increase in KM was appreciated for stability reasons and the additional deck area because it improved the arrangement. In fact in present warship practice the full midship beam is often maintained right to the transom and from upper deck level to very nearly the waterline. A further development in the sterns of high speed ships is the transom wedge or flap illustrated in Figure. This reduces the high stern wave that used to build up at the stern and thereby reduces resistance.

Table 6.1. Submergence of transom stern

Froude number	**Submergence of transom stern**
<0.3	Above water
≈0.3	Slightly submerged
≈0.5	% 10-15 of draught submerged
>0.5	% 15-20 of draught submerged

6.1.10. Above Water Form

Above the waterline, bows are raked forward largely to conform with the flare of the adjacent sections. Both rake and flare have as one of their objectives reducing both pitching and the amount of water shipped on the fore deck. Appearance and the minimisation of damage caused to the other vessel in a head on collision are further advantages of bow rake.

Care should be taken not to exaggerate flare too much as waves hitting on side of a heavily flared bow can give rise to torsional vibrations and stresses. This is particularly important for fast ships such as container ships and warships. The severity with which the forces generated by the sea can impact on flare has been shown in a number of accidents in which complete bows have been broken off.

6.2. Hull Form Design Methods

A number of different methods of deriving hull shape exist in ship design, and they can be classified as follows:

- The most commonly used approach is to select a previous successful design as the parent hull and to distort it to give the new hull form with desired mix of features. Although thousands of ships have been designed and built, and a great number of ship models have been tested and studied, a thorough understanding of ship hydrodynamics is still lacking. What quality or qualities a good hull form must possess to have superior resistance, seakeeping, propulsive and manoeuvring characteristics are still not quite known. Under such circumstances, a ship designer would normally try to find an existing ship with a good performance record to use as a basis for his new design.

- The use of a particular, successful parent tends to lead to the families of designs that are apparent in the products of most design organisations. There are a number of well-known ship forms such as the Taylor Series, the Series 60, the BSRA Methodical Series et cetera. They are specified in a form, which allows hull offsets to be readily generated for specified hull form parameters. Having selected possible approximate parameters it is possible to use the lines of series forms with similar design parameters as a basis ship in the design studies. Thereafter, theoretical computations and tank test data may be classified according to ship type in the form of design charts or tables based on hull form parameters. However, a review of the literature reveals that there is little systematic hydrodynamic data in the public domain to support the modern ship designer in future hull form development. Most major systematic series date back to 1950s and 1960s and the type and range of hull forms used in these series do not represent those of modern ship forms.

- The designer may develop a rough, faired set of lines without any parent, relying solely on his eye and past experience.

- For simple shapes such as barges the hull forms can be created through the use of geometrical or mathematical equations. For more complicated shapes, direct generation of hull forms is possible with the aid of interactive computer graphics and fairing procedures.

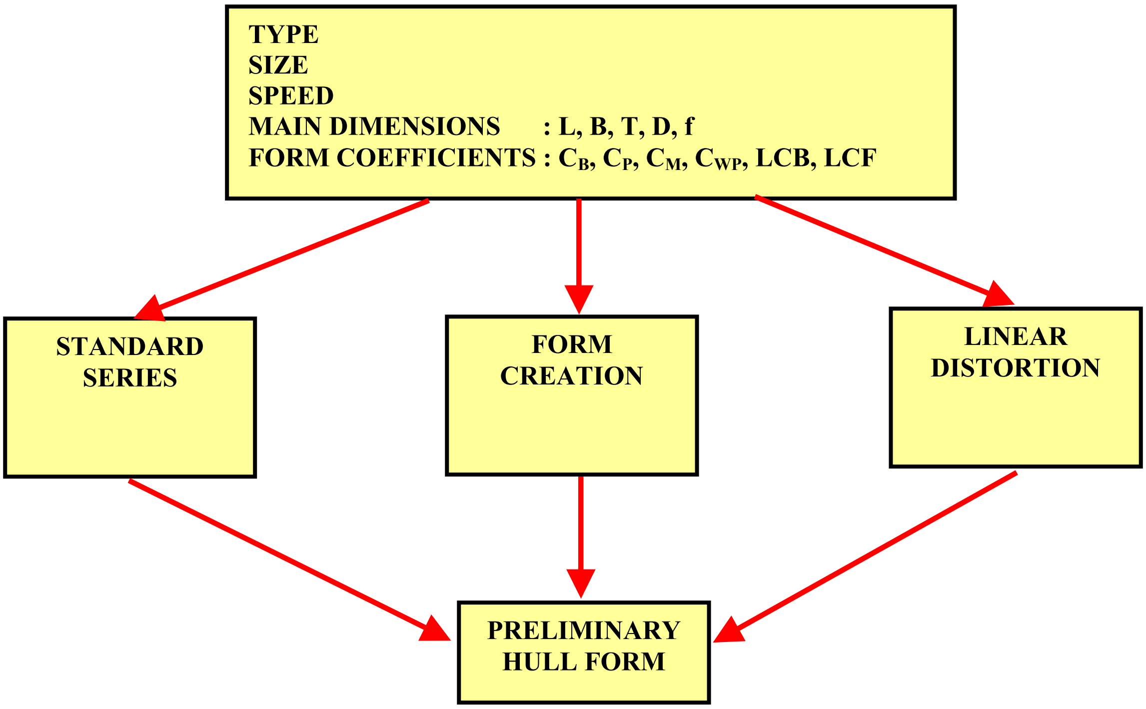


Figure 6.8. Hull form design methods

6.2.1. Standard Series Approach

Over the years several hull form series have been developed for systematic resistance or propulsion tests. Variations from the parent hull(s) are usually developed by linear distortion methods such as **Lackenby (1950)**'s method. Using appropriate series, the designer can obtain a hull form by interpolation for desired parameters. Some of the important standard series with the hull form parameter ranges used is given in **Table 6.1**.

Table 6.1. Parameter Ranges for Some Standard Series

Series	**Series 60**	**Taylor**	**BSRA**	**SSPA**	**MARAD**	**DAWSON Coaster**
Type	Single screw	Twin screw	Single screw	Single screw	Single screw	Single screw
Year						
F_n	0.12 0.30		0.12 0.33	0.16 0.32	0.10 0.21	0.15 0.30
L/B	5.5 8.5		5.33 8.37	6.18 8.35	4.5 6.5	5.5 6.5
B/T	2.5 3.5	2.25 3.75	2.12 3.96	2.1 3.1	3.0 4.5	2.05 2.75
$\dfrac{\Delta}{(0.1L)^3}$	68 302	20 250	114.26 385.64	87.35 220.53	10.43 197.78	106.47 291.62
$\dfrac{L}{\nabla^{1/3}}$	4.56 7.49	4.85 11.26	4.2 6.3	5.06 6.89	5.247 13.992	4.61 6.45
C_B	0.6 0.8	0.444 0.796	0.55 0.85	0.525 0.750	0.800 0.875	0.59 0.75
C_P	0.614 0.805	0.48 0.86	0.570 0.852	0.553 0.762	0.805 0.880	0.607 0.762
LCB (% L)	2.5 Aft 3.5 Fwd	0.0	3.0 Aft 3.5 Fwd	2.0 Aft 0.85 Fwd	2.5 Fwd	3.5 Aft 3.0 Fwd

Series	DeGroot	NPL	Webb	UBC Fishing	BSRA Fishing	Seri 64
Type	Round bilge	Round bilge	Round bilge	Fishing	Fishing	High speed
Year						
F_n		0.5 3.0				
L/B	3.53 10.09	3.33 7.50	3.20 5.75	2.6 4.0	4.3 5.8	8.454 17.734
B/T	2.72 6.58	1.94 10.21	2.3	2 3	2.00 3.50	2 4
$\frac{\Delta}{(0.1L)^3}$		48.20 158.41				
$\frac{L}{\nabla^{1/3}}$		5.65 8.40	3.85 5.22	3.00 4.47	4.35 5.10	8.04 12.40
C_B	0.293 0.560	0.397	0.42 0.53	0.53 0.61	0.53 0.63	0.35 0.55
C_P	0.463 0.791	0.693	0.55 0.70			0.63
LCB	3.09 Aft 11.50 Aft	2.0 Aft 6.5 Aft			2.90 Aft 1.09 Fwd	6.6 Aft

This brief review shows that the standard series approach covers only some of the simpler variations in hull form with respect to the proportions of the main dimensions, fullness, and in some cases longitudinal centre of buoyancy. The range of variation in the series is, of necessity, limited so that the forms that can be deduced from the series are subject to corresponding limitations. For similar reasons there are only few form parameters that can be varied independently while others being dependent variables.

6.2.1.1. Taylor Series

The Taylor standard series was the first major methodical series of ship forms to receive international attention. The series is a result of an evolution of several parent forms patterned after the British armoured cruise Leviathan of the Drake class (1900), a model of which was tested in the US Experimental Model basin in 1902.

The method used to derive the Taylor standard series is essentially a graphical process. The nondimensional offsets are presented as a function of the prismatic coefficient for each waterline.

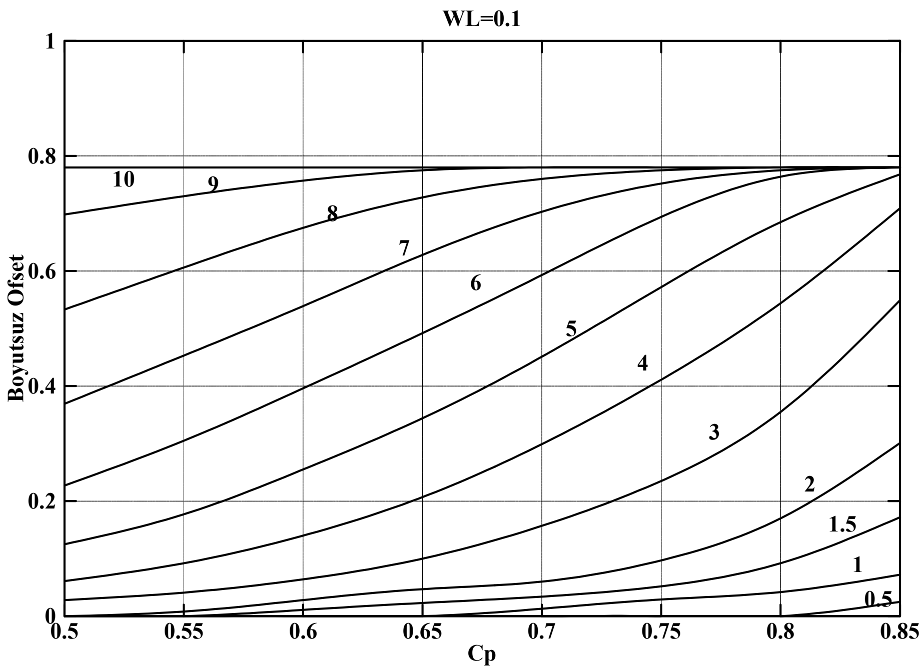


Figure 6.9. Typical Taylor series graph

6.2.2. Lines Distortion Approach

In order to investigate the influence of different form parameters on the performance of a ship the designer needs to derive a new lines plan from a given parent design by modifying some form parameters of the parent design. The approach of linear distortion adresses itself to this task and aims at what amounts to a moderate extrapolation from the parent design by suitable mathematical operations. In principle, the designer should be able to vary any desired hull form parameter while keeping all others constant. The total desired variation may be subdivided into two major steps as principal and secondary parameter variations.

When designing the lines by distorting existing forms it is usually sufficient to design the underwater body and then add the topside in the conventional way

6.2.2.1. Variation of Principal Parameters of Hull Form

To modify length, beam, or draught, the hull offsets may simply be multiplied by corresponding constant expansion or contraction factors. This does not affect the secondary parameters, i.e. C_B, C_M, C_P, C_{WP}, LCB and LCF. However, this variation will alter the displacement. In order to change length, while the displacement is fixed, the midship section area is altered in inverse ratio to the length. The breadth/draught ratio remains constant as well as the displacement, block coefficient and all secondary form parameters. The new main dimensions are:

$$L' = (1+\delta L)L \qquad B' = \frac{B}{\sqrt{1+\delta L}} \qquad T' = \frac{T}{\sqrt{1+\delta L}}$$

Since $\frac{B'}{B}$ and $\frac{T'}{T}$ are equal and $\frac{L'}{L}$ is designer specified, the waterlines and offsets of corresponding stations can be found using standard naval architectural procedures.

With L fixed and B/T changing by a factor of $1+\delta x$, say, the changes in B and T now correspond to

$$B' = \frac{B}{\sqrt{1+\delta x}} \qquad T' = \frac{T}{\sqrt{1+\delta x}}$$

and the corresponding changes in offsets and waterlines are undertaken in the usual manner.

6.2.2.2. Variation of Secondary Parameters of Hull Form

6.2.2.2.1. Swinging the Sectional Area Curve

This method enables to change LCB position by keeping the fullness constant. The sectional area curve of the parent ship represented by solid line ABC and the derived curve represented by the dotted line are illustrated in **Figure 6.8.**

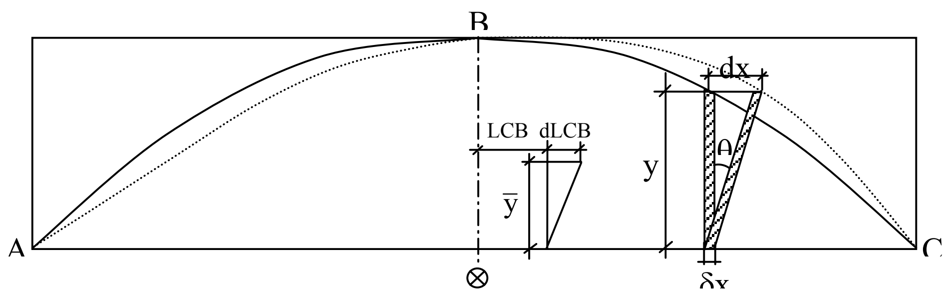


Figure 6.8. Swinging the sectional area curve to change LCB position.

Let us consider a thin vertical strip of δx. The area of the element is δxy. Longitudinal transfer of moment of the strip due to swinging is

$$dM = \delta x y \frac{1}{2} dx = \frac{1}{2} y^2 \tan\theta \delta x$$

$$M = \tan\theta \frac{1}{2} \Sigma y^2 \delta x$$

Vertical Moment : $\frac{1}{2} \Sigma y^2 \delta x = A \bar{y}$

Longitudinal Moment : $A \bar{y} \tan\theta = AdLCB$

$$\tan\theta = \frac{dLCB}{\bar{y}}$$

where

θ : Required angle of shift for adjusting the LCB position
A : total area under the sectional area curve ABC, indicating fullness
$\bar{y}$: The vertical centroid of sectional area curve
dLCB : required change in LCB position

Once the new positions of transverse sections are determined the modified offsets can be obtained directly from the waterlines plan of the parent form.

Example 6.1: Consider a parabolic form, L=100 m long and B=10 m beam floating with a draught of T=10 m

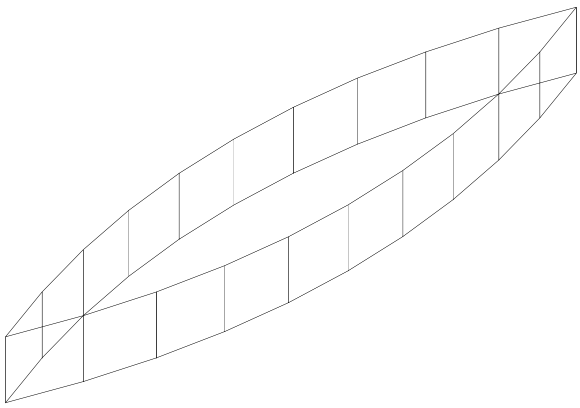
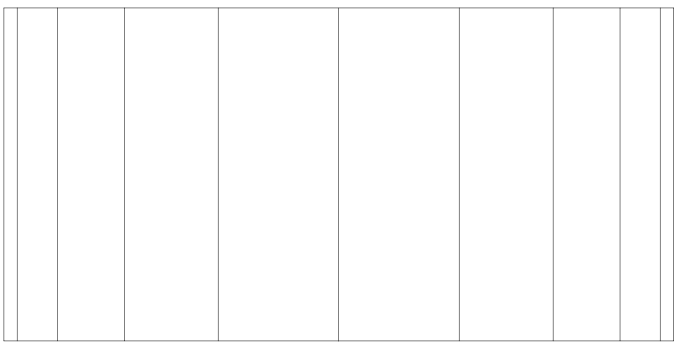


Figure 6.9. Parabolic form



6.10. Body plan for the Parabolic form

The half breadth values at equally spaced stations are given in the following table

Station	0	1	2	3	4	5	6	7	8	9	10
Breadth	0	3.6	6.4	8.4	9.6	10	9.6	8.4	6.4	3.6	0
Sectional area	0	36	64	84	96	100	96	84	64	36	0

Due to the fore-aft symmetry the original position of LCB is amidships. The modified form has a position of LCB 5% L forward of midships.

First, the sectional area curve of the original form should be evaluated as shown in the following table

Station	Area	SM	Product	Area^2	SM	Product
0	0	1	0	0	1	0
1	36	4	144	1296	4	5184
2	64	2	128	4096	2	8192
3	84	4	336	7056	4	28224
4	96	2	192	9216	2	28432
5	100	4	400	10000	4	40000
6	96	2	192	9216	2	18432
7	84	4	336	7056	4	28224
8	64	2	128	4096	2	8192
9	36	4	144	1296	4	5184
10	0	1	0	0	1	0

$\Sigma_1 = 2000$ $\qquad\qquad$ $\Sigma_2 = 160064$

Displacement volume $\qquad V = \frac{s}{3}\Sigma_1 = 6666.666\text{m}^3$

Moment $\qquad M = \frac{s}{6}\Sigma_2 = 266773.333\text{m}^4$

Height of centroid $\qquad \bar{y} = \frac{M}{V} = 40\text{m}$

Then the angle of shift of stations is

$$\tan\theta = \frac{dLCB}{\bar{y}} = \frac{5}{40} = 0.125$$

The amount of shift for each station can be calculated as follows

Station	0	1	2	3	4	5	6	7	8	9	10
dx	0	4.5	8.0	10.5	12.0	12.5	12.0	10.5	8.0	4.5	0

The modified sectional area curve is shown in **Figure 6.11.** The ordinates of the new sectional area curve can be read off as follows

Station	0	1	2	3	4	5	6	7	8	9	10
Area	0	25.278	48.176	67.610	83.327	94.428	99.760	97.556	84.991	56.864	0

Hydrostatic properties of the new form can be calculated as shown in the following table

Station	Area	SM	Product	MC	Product
0	0	1	0	5	0
1	25.278	4	101.112	4	404.448
2	48.176	2	96.352	3	289.056
3	67.610	4	270.440	2	540.880
4	83.327	2	166.654	1	166.654
5	94.428	4	377.712	0	0
6	99.760	2	199.520	-1	-199.520
7	97.556	4	390.224	-2	-780.448
8	84.991	2	169.982	-3	-509.946
9	56.864	4	227.456	-4	-909.824
10	0	1	0	-5	0

$\Sigma_1 = 1999.452$ $\qquad\qquad$ $\Sigma_2 = -998.7$

Displacement volume $V = \frac{s}{3}\Sigma_1 = 6664.840m^3$

Difference in volume $dV = \frac{6666.666 - 6664.840}{6666.666} \Rightarrow \%0.03$

LCB $LCB = s\frac{\Sigma_2}{\Sigma_1} = 4.995m$ başa

Difference in LCB $dLCB = \frac{4.995 - 0}{100} \Rightarrow \%4.995$

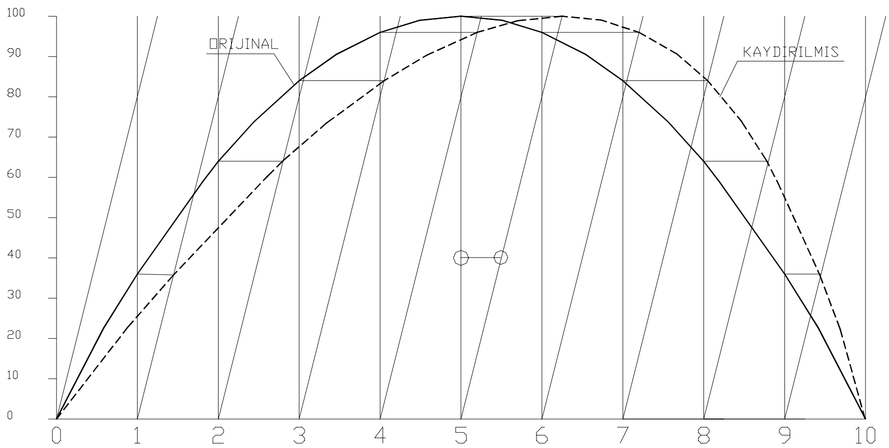


Figure 6.11. Sectional area curves for the original and modified forms

The required shift of LCB is achieved with almost constant displacement. The body plan and a 3D view of the new form are shown in **Figures 6.12** and **6.13** respectively.

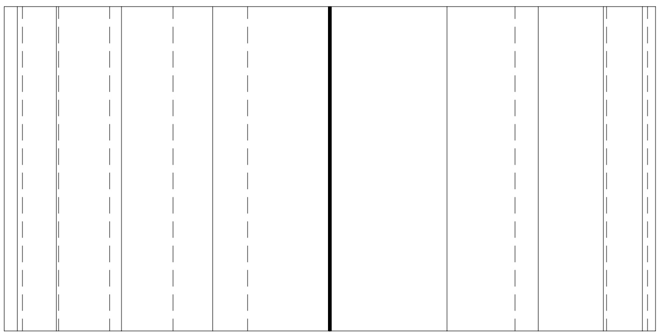


Figure 6.12. Body plan of the modified form

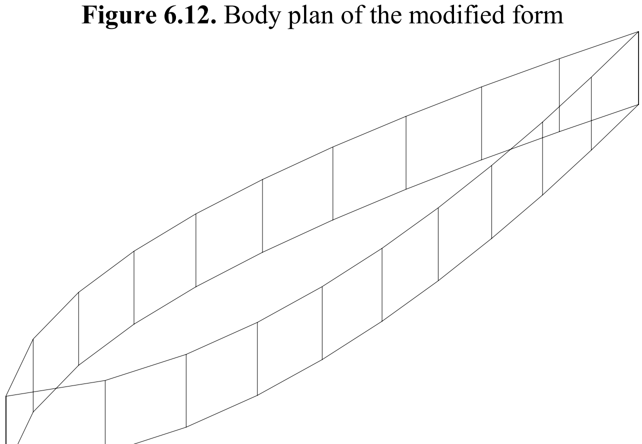


Figure 6.13. 3D view of the modified form

<u>Example 6.2.</u> A mathematically defined (Wigley) form, L=16 m long, B=1.6 m wide and T=1 m deep is defined by the following surface equation

$$y(x,z) = \frac{B}{2}\left\{1-\left(\frac{2x}{L}\right)^2\right\}\left\{1-\left(\frac{z}{T}\right)^2\right\}$$

x : Longitudinal distance from midships (positive forward)
y : half breadth at (x,z)
z : height (positive downwards)

Note that the form has fore-aft symmetry as shown in **Figure 6.14.** The body plan is shown in **Figure 6.15.**

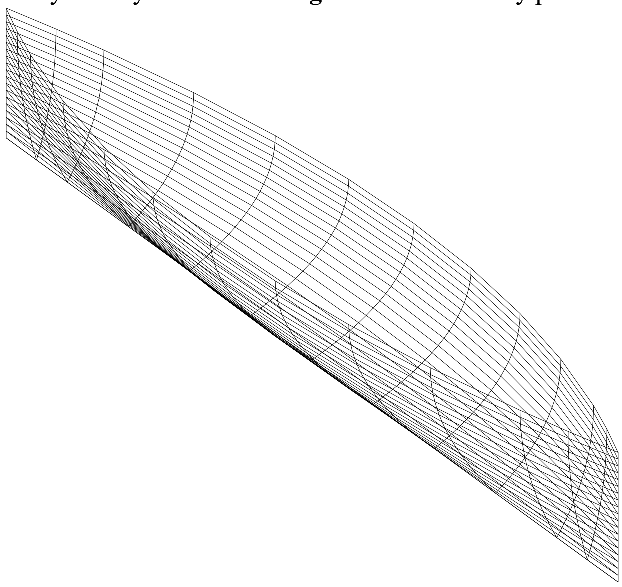


Figure 6.14. General view of Wigley form

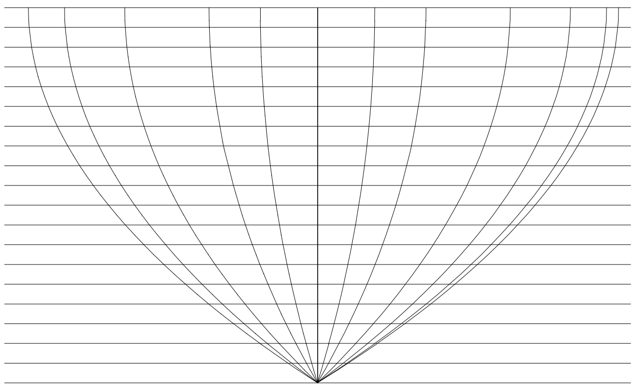


Figure 6.15. Body plan of Wigley form

The position of LCB is required to shift aft by 7.5 %L.

The following table indicates the maximum area and half breadth values

Station	0	0.5	1	2	3	4	5	6	7	8	9	9.5	10
Half bradth	0	0.152	0.288	0.512	0.672	0.768	0.8	0.768	0.672	0.512	0.288	0.152	0
Area	0	0.2027	0.3841	0.6826	0.896	1.024	1.067	1.024	0.896	0.6826	0.3841	0.2027	0

First the general hydrostatic properties of the original form must be calculated as shown in the following table.

Station	Area	SM	Product	Area^2	SM	Product
0	0	1/2	0	0	1/2	0
0.5	0.2027	2	0.4054	0.041	2	0.082
1	0.3841	3/2	0.5762	0.147	3/2	0.221
2	0.6826	4	2.7304	0.466	4	1.864
3	0.8960	2	1.7920	0.803	2	1.606
4	1.0240	4	4.0960	1.049	4	4.196
5	1.0670	2	2.1340	1.138	2	2.276
6	1.0240	4	4.0960	1.049	4	4.196
7	0.8960	2	1.7920	0.803	2	1.606
8	0.6826	4	2.7304	0.466	4	1.864
9	0.3841	3/2	0.5762	0.147	3/2	0.221
9.5	0.2027	2	0.4054	0.041	2	0.082
10	0	1/2	0	0	1/2	0
			$\Sigma_1 = 21.33$			$\Sigma_2 = 18.214$

Displacement volume $V = \frac{s}{3}\Sigma_1 = 11.376\text{m}^3$

Moment $M = \frac{s}{6}\Sigma_2 = 4.857\text{m}^4$

Height of centroid $\bar{y} = \frac{M}{V} = 0.427\text{m}$

Then the angle of shift is $\tan\theta = \frac{dLCB}{\bar{y}} = \frac{6.8-8}{0.427} = -2.81$. The required shift of stations are shown in the following table

Station	0	0.5	1	2	3	4	5	6	7	8	9	9.5	10
dx	0	-0.57	-1.08	-1.92	-2.52	-2.88	-3.0	-2.88	-2.52	-1.92	-1.08	-0.57	0

The modified sectional area curve is shown in **Figure 6.16**.

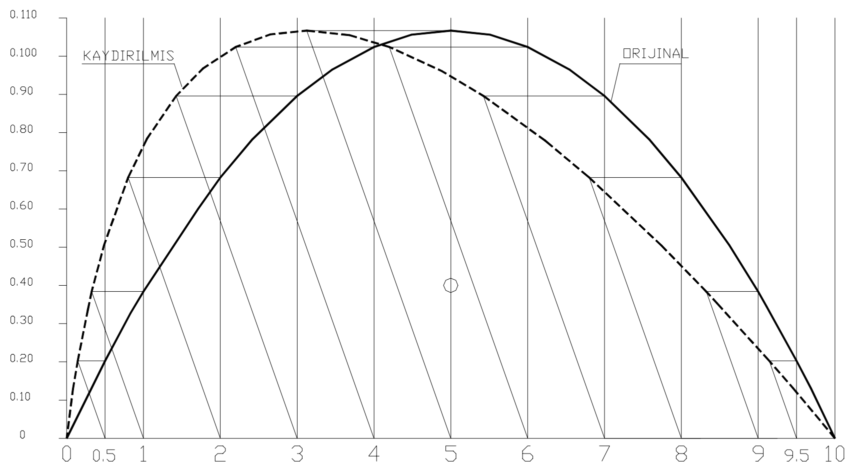


Figure 6.16. original and modified sectional area curves

New sectional area ordinates are read off from the figure as follows

Station	0	0.5	1	2	3	4	5	6	7	8	9	9.5	10
Area	0	0.515	0.767	1.00	1.067	1.038	0.948	0.814	0.647	0.452	0.236	0.119	0

Hydrostatic properties of the new form are calculated as shown in the following table

Station	Area	SM	Product	MC	Product
0	0	1/2	0	5	0
0.5	0.515	2	1.030	4.5	4.635
1	0.767	3/2	1.151	4	4.604
2	1.000	4	4.000	3	12.00
3	1.067	2	2.134	2	4.268
4	1.038	4	4.152	1	4.152
5	0.948	2	1.896	0	0
6	0.814	4	3.256	-1	-3.256
7	0.647	2	1.294	-2	-2.588
8	0.452	4	1.808	-3	-5.424
9	0.236	3/2	0.354	-4	-1.416
9.5	0.119	2	0.238	-4.5	-1.071
10	0	1/2	0	5	0
			Σ_1=21.313		Σ_2=15.904

Dsplacement volume $V = \frac{s}{3}\Sigma_1 = 11.367\text{m}^3$

Difference in volume $dV = \frac{11.376 - 11.367}{11.376} \Rightarrow \%0.08$

LCB $LCB = s\frac{\Sigma_2}{\Sigma_1} = 1.194\text{m}$ kıça

Difference in LCB $dLCB \Rightarrow \%7.46$

As indicated by the results the required shift is achived with good accuracy. In order to obyain the offsets for the modified form stations are moved by dx and offsets are read off from the waterlines plan. The body plan and a 3D view of the modified form are shown in **Figures 6.17** and **6.18** respectively.

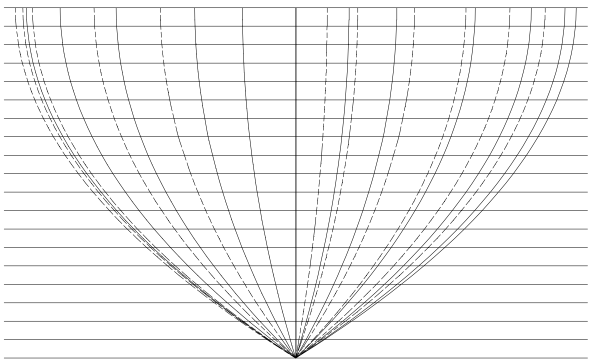


Figure 6.17. Body plan of the modified Wigley form

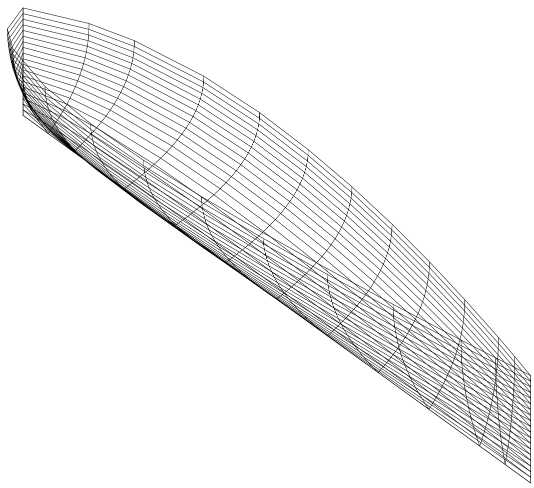


Figure 6.18. 3D view of the modified form

Example 6.3 : A cargo ship has the following dimensions; L=120 m, B=18 m , T=8 m and D=12 m. The offsets of the ship are given in the following table. The body plan is shown in **Figure 6.19**. The position of LCB is required to move aft by 3 %.

	0	**0.5**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**9.5**	**10**
WL0 z=0m	0.000	0.153	1.305	4.023	6.039	6.813	6.813	6.813	6.372	4.437	1.548	0.351	0.000
WL0.5 z=1m	0.000	0.486	2.565	6.246	8.199	8.658	8.658	8.658	8.370	6.570	3.519	2.169	1.314
WL1 z=2m	0.000	0.639	2.871	6.894	8.613	8.991	8.991	8.991	8.829	7.317	4.032	2.556	1.710
WL2 z=4m	0.000	0.774	3.294	7.596	8.865	9.000	9.000	9.000	8.982	8.037	4.473	2.583	1.647
WL3 z=6m	0.000	2.061	4.752	8.082	8.964	9.000	9.000	9.000	9.000	8.298	4.788	2.376	0.774
WL4 z=8m	2.925	5.022	6.741	8.550	9.000	9.000	9.000	9.000	9.000	8.415	5.157	2.439	0.063
WL5 z=10m	5.112	6.957	8.136	8.865	9.000	9.000	9.000	9.000	9.000	8.586	5.823	3.276	0.630
WL6 z=12m	6.642	8.253	8.865	8.982	9.000	9.000	9.000	9.000	9.000	8.757	6.867	5.058	2.367

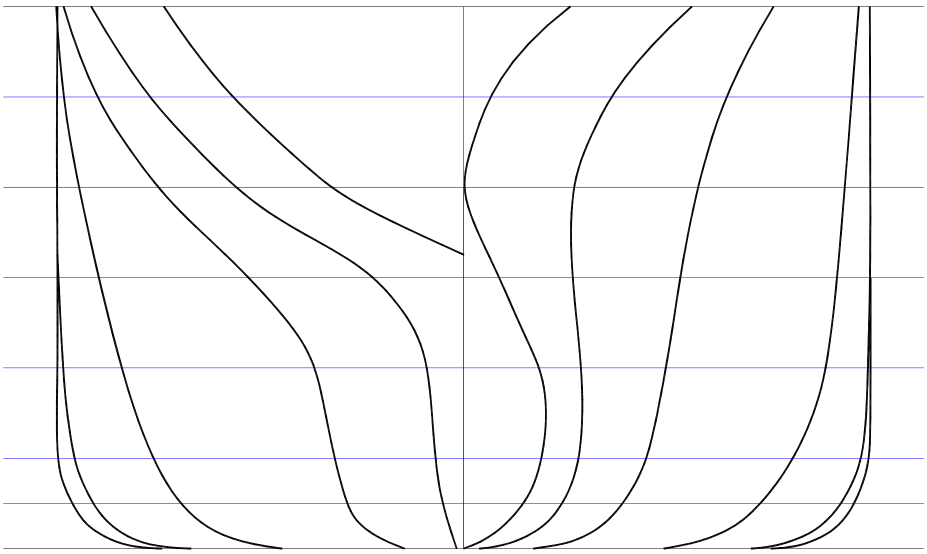


Figure 6.19. Body plan of the basis ship

Solution :

First the sectional areas upto the draught level are calculated as shown in the following table.

Sta	0	0.5	1	2	3	4	5	6	7	8	9	9.5	10
Half Breadth	2.925	5.022	6.741	8.550	9.000	9.000	9.000	9.000	9.000	8.415	5.157	2.439	0.063
Area	4.6165	23.386	60.072	117.02	137.57	141.07	141.07	141.07	139.57	121.93	68.01	37.04	17.882

Then, basic hydrostatic properties of the basis form, including the vertical centroid of the sectional area curve has to be calculated. This can be achieved by numerical integration rules as shown in the following table.

Sta	Area	SM	Product	MC	Product	Area^2	SM	Product
0	4.6165	1/2	2.3083	5	11.54125	21.312	1/2	10.656
0.5	23.3857	2	46.771	4.5	210.4713	546.8910	2	1093.782
1	60.0722	3/2	90.108	4	360.4332	3608.669	3/2	5413.004
2	117.0212	4	468.085	3	1404.254	13693.96	4	54775.84
3	137.5697	2	275.139	2	550.2788	18925.42	2	37850.84
4	141.07261	4	564.290	1	564.2904	19901.48	4	79605.92
5	141.07261	2	282.145	0	0	19901.48	2	39802.96
6	141.07261	4	564.290	-1	-564.2904	19901.48	4	79605.92
7	139.5754	2	279.151	-2	-558.3016	19481.29	2	38962.58
8	121.9288	4	487.715	-3	-1463.145	14866.63	4	59466.52
9	68.0098	3/2	102.015	-4	-408.0588	4625.330	3/2	6937.995
9.5	37.0407	2	74.0814	-4.5	-333.3663	1372.013	2	2744.026
10	17.8823	1/2	8.9412	5	-44.70575	319.7766	1/2	159.888

$\Sigma_1=3245.04$ $\Sigma_2=-270.6$ $\Sigma_3=406429.9$

Displacement volume $\nabla = \frac{s}{3}\Sigma_1 = 12980.2\,\text{m}^3$

Block coefficient $C_B = \frac{\nabla}{LBT} = \frac{12980.2}{120\times18\times8} = 0.751$

Longitudinal centre of buoyancy $LCB = s\frac{\Sigma_2}{\Sigma_1} = 12\frac{270.6}{3245.04} = 1\,\text{m}$ forward

Vertical moment $M = \frac{s}{6}\Sigma_2 = 812859.9\text{m}^4$

Vertical centroid of SAC $z = \frac{M}{V} = 62.623$ m

Then $\tan\theta = \frac{dLCB}{z} = \frac{3.6}{62.623} = 0.0575$. The amount of shift for each station, dx=ytanθ, are shown in the following table.

Sta	0	0.5	1	2	3	4	5	6	7	8	9	9.5	10
dx	0.266	1.350	3.467	6.753	7.939	8.141	8.141	8.141	8.054	7.036	3.925	2.137	1.032

The original and modified sectional area curves are shown in the following figure.

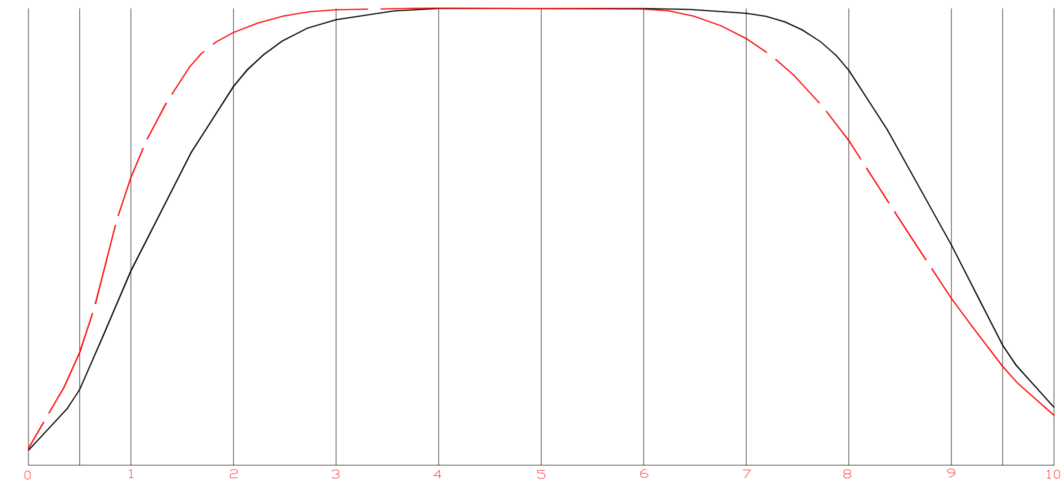


Figure 6.20. The basis and modified sectional area curves

Sectional area values for the variant form can be read off from the sectional area curve as follows

Sta	0	0.5	1	2	3	4	5	6	7	8	9	9.5	10
A	5.175	34.802	88.949	133.65	140.67	141.05	141.05	140.87	131.76	100.24	51.508	30.477	15.418

Basic hydrostatic characteristics of the variant hull forms can be calculated as follows

Sta	Area	SM	Product	MC	Product
0	5.175	1/2	2.588	5	12.938
0.5	34.802	2	69.604	4.5	313.218
1	88.949	3/2	133.424	4	533.694
2	133.651	4	534.604	3	1603.812
3	140.667	2	281.334	2	562.668
4	141.051	4	564.204	1	564.204
5	141.051	2	282.102	0	0.000
6	140.865	4	563.460	-1	-563.460
7	131.756	2	263.512	-2	-527.024
8	100.240	4	400.960	-3	-1202.880
9	51.508	3/2	77.262	-4	-309.048
9.5	30.477	2	60.954	-4.5	-274.293
10	15.418	1/2	7.709	5	-38.545
			Σ_1= 3241.716		Σ_2= 675.284

Displacement volume $$\nabla = \frac{s}{3}\Sigma_1 = 12966.864\,m^3$$

Difference in displacement $$d\nabla = \frac{12980.2 - 12966.864}{12980.2} \Rightarrow \%0.1$$

Block coefficient $$C_B = \frac{\nabla}{LBT} = \frac{12966.864}{120\times 18\times 8} = 0.750$$

LCB $$LCB = s\frac{\Sigma_2}{\Sigma_1} = 2.5\,m \quad \text{aft}$$

Difference in LCB $$dLCB = \frac{2.5+1}{120} \Rightarrow \%2.9$$

These results indicate that the required position of LCB is achieved with negligible difference in displacement. In order to obtain the new hull form each waterline needs to be varied in a similar manner as the sectional area curve. For example, the modified WL 2 is shown in **Figure 6.21**. The offsets of the modified form are read off from the waterlines. The body plan of the modified form is shown in **Figure 6.22**.

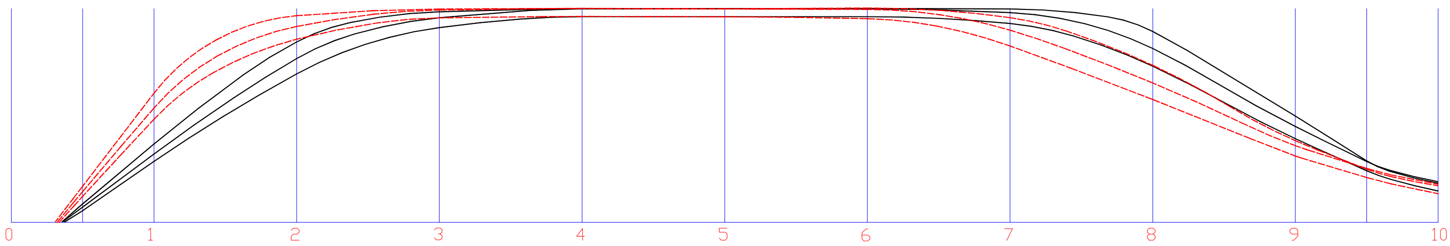


Figure 6.21. Original and modified Waterlines.

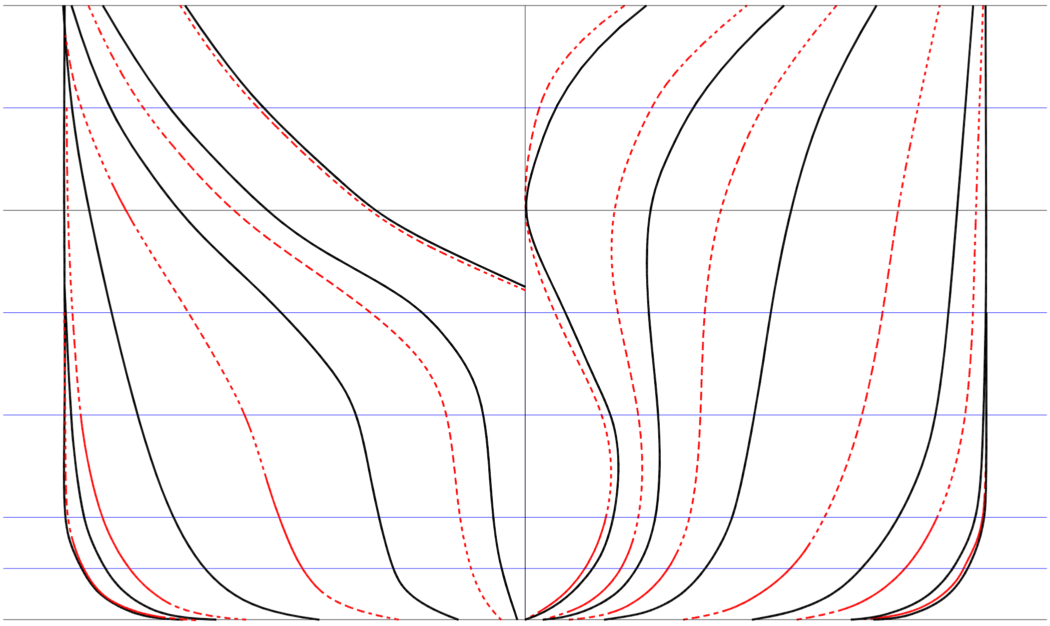


Figure 6.22. Original and modified forms

An obvious disadvantage of this method is that the extent of parallel middle body cannot be controlled. Also the midship section characteristics may alter. However, for small changes in the position of LCB this may not be a serious problem.

6.2.2.2.2. One Minus Prismatic Method

The basis of this method is to adjust the sectional area curve of the basic ship form by contracting or expanding the entrance and run and reducing or increasing the parallel middle body length as necessary. New offsets can be obtained directly from the parent design. The new form is subjected either to expansion or contraction depending on the desired form characteristics. These adjustments are likely to influence some of the geometric particulars like LCB (longitudinal centre of buoyancy) position, C_P (prismatic coefficient) and the extent of the parallel middle body in both the fore and after bodies.

The curve in **Figure 6.23** represents the sectional area curve of the parent ship for one half of the body. For convenience, the terms given in this study are not separated for entrance and run, and valid for both halves of the ship. It is necessary to consider this half body and maximum sectional area ordinate as equal to unity. Therefore, the area under the curve becomes numerically equal to the prismatic coefficient of the half body and the added slice represents the change in C_P, indicated by δC_P.

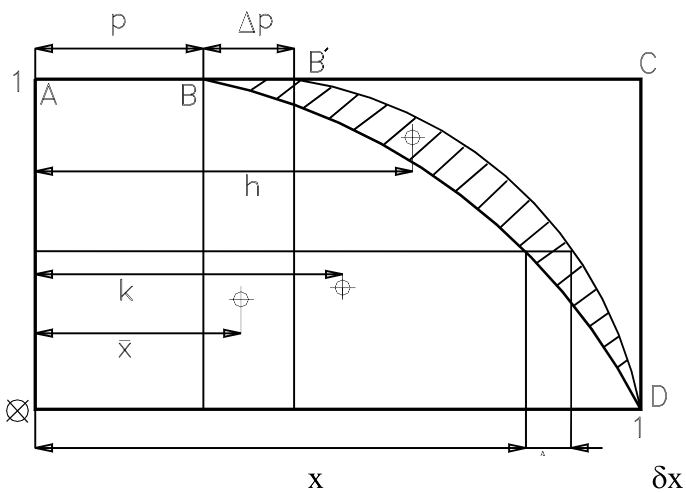


Figure 6.23. Geometrical derivation of shifting function.

According to **Figure 6.23** the linear shift of dimensionless sectional area ordinates (δx) is obtained by using the proportionality of the areas before and after the distortion procedure. Hence, δx is derived as follows:

$$\frac{BB'D}{BCD} = \frac{\delta x}{1-x} \qquad \Rightarrow \qquad \delta x = \frac{\delta C_P}{1-C_P}(1-x)$$

It can clearly be seen from **Figure 6.23** that the variation of C_P produces inevitable changes in the parallel middle body length (p), according to the modifications of the entrance and run, e.g., at x = p

$$\delta p = \frac{\delta C_P}{1-C_P}(1-p)$$

In order to change the total prismatic coefficient (C_P) and/or the longitudinal centre of buoyancy (LCB), the required changes in fore and afterbodies must be determined. This can be achieved by taking moments as follows

$$\delta C_{PF} = \frac{2\left[\delta C_P(h_a + LCB) + 2\delta LCB(C_P + \delta C_P)\right]}{(h_f + h_a)}$$

$$\delta C_{PA} = \frac{2\left[\delta C_P(h_f - LCB) - 2\delta LCB(C_P + \delta C_P)\right]}{(h_f + h_a)}$$

where

C_P : prismatic coefficient of the parent form
δC_P : the required change in prismatic coefficient
LCB : the distance of the LCB in the parent form (positive forward)
δLCB : the required shift of the LCB
δC_{PF} : the change in forebody prismatic coefficient
δC_{PA} : the change in afterbody prismatic coefficient
h_f : centroid of the added area in the forebody
h_a : centroid of the added area in the afterbody

The exact values of levers h_f and h_a can be calculated by the following relations

$$h_f = \frac{C_{PF}(1-2\bar{x}_f)}{1-C_{PF}} + \frac{\delta C_{PF}}{2(1-C_{PF})^2}\left[1-2C_{PF}(1-\bar{x}_f)\right]$$

$$h_a = \frac{C_{PA}(1-2\bar{x}_a)}{1-C_{PA}} + \frac{\delta C_{PA}}{2(1-C_{PA})^2}\left[1-2C_{PA}(1-\bar{x}_a)\right]$$

where C_{PF} and C_{PA} are the prismatic coefficients for fore and afterbodies, and $\bar{x}_f$ and $\bar{x}_a$ are the centroids of the original fore and afterbodies. However, since δC_{PF} and δC_{PA} are not known h_f and h_a cannot be determined exactly and the second term may be ignored, i.e.,

$$h_f = \frac{C_{PF}(1-2\bar{x}_f)}{1-C_{PF}}$$

$$h_a = \frac{C_{PA}(1-2\bar{x}_a)}{1-C_{PA}}$$

This technique of form distortion is useful and relatively simple to apply but there are some restrictions, which are:

- The parallel middle body length and the prismatic coefficient cannot be varied independently,
- The prismatic coefficient of the fore and aft halves can not be adjusted,
- The process cannot be applied to some types of forms, e.g., ships which has no parallel middle body,

- There is limitations in the range of longitudinal shift of sections,
- The maximum longitudinal shift of sections is restricted to the ends.

An application of this variation procedure is shown in **Figure 6.24** where the block coefficient is increased by 5% .

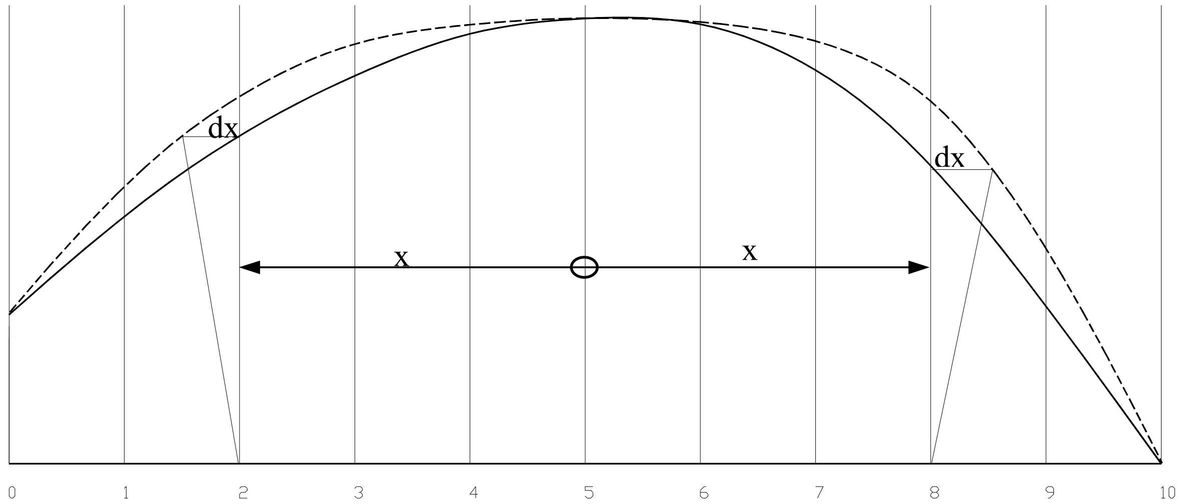


Figure 6.24

Example 6.4 : As a first example consider a parabolic form L=100 m long, B=10 m beam, floating with a draught of T=10 m. A 3D view and the body plan area shown in the following figures.

The breadth and area of each section are given in the following table. The prismatic coefficient is required to increase by 10 % while the location of LCB remaining constant.

Station	0	1	2	3	4	5	6	7	8	9	10
Breadth	0	3.6	6.4	8.4	9.6	10	9.6	8.4	6.4	3.6	0
Area	0	36	64	84	96	100	96	84	64	36	0

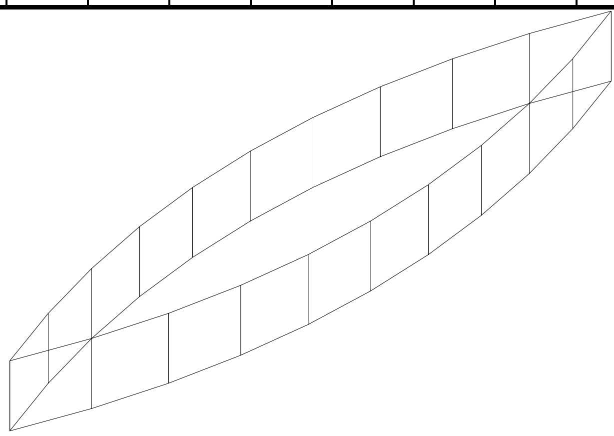


Figure 6.25. Parabolic form general view

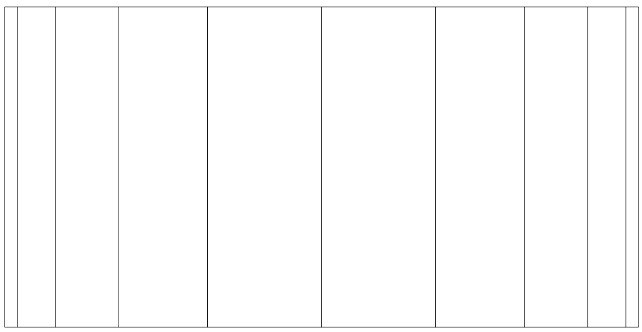


Figure 6.26. Parabolic form body plan

First, the original values of C_P and LCB has to be determined. Due to fore-aft symmetry it is clear that the position of LCB amidships.

Station	Area	SM	Product
0	0	1	0
1	36	4	144
2	64	2	128
3	84	4	336
4	96	2	192
5	100	4	400
6	96	2	192
7	84	4	336
8	64	2	128
9	36	4	144
10	0	1	0
			Σ_1= 2000

Displacement volume $$V = \frac{s}{3}\Sigma_1 = 6666.666 m^3$$

Prismatic coefficient $$C_P = C_B = \frac{\nabla}{LBT} = \frac{6666.666}{100 \times 10 \times 10} = 0.666$$

Change in prismatic coefficient $$dC_P = 0.1 \times C_P = 0.0666$$

The required shift of each station is determined by using $dx = \frac{dC_P}{1 - C_P}(\frac{L}{2} - x)$ as follows

Station	0	1	2	3	4	5	6	7		9	10
	0	2	4	6	8	10	8	6	4	2	0

The result of the movement of stations is shown in **Figure 6.27**. The modified values of SAC are read off from the curve as follows.

Station	0	1	2	3	4	5	6	7	8	9	10
Area	0	43.847	74.962	93.798	100	100	100	93.798	74.962	43.847	0

The volume, prismatic coefficient and LCB values for the modified form are calculated in the following table

Station	Area	SM	Product	MC	Product
0	0	1	0	5	0
1	43.847	4	175.388	4	701.552
2	74.962	2	149.924	3	449.772
3	93.798	4	375.192	2	750.384
4	100	2	200	1	200
5	100	4	400	0	0
6	100	2	200	-1	-200
7	93.798	4	375.192	-2	-750.384
8	74.962	2	149.924	-3	-449.772
9	43.847	4	175.388	-4	-701.552
10	0	1	0	-5	0
			Σ_1 = 2201.008		Σ_2 = 0

Displacement volume $$V = \frac{s}{3}\Sigma_1 = 6664.840 m^3$$

Prismatic coefficent $$C_P = C_B = \frac{\nabla}{LBT} = \frac{7336.693}{100 \times 10 \times 10} = 0.733$$

LCB $$\mathrm{LCB} = s\frac{\Sigma_2}{\Sigma_1} = 0\,\mathrm{m}$$

The results indicate that the required change in prismatic coefficent is obtained while the position of LCB kept constant. The body plan and a 3D view of the new form are shown in **Figures 6.28** and **6.29**, respectively.

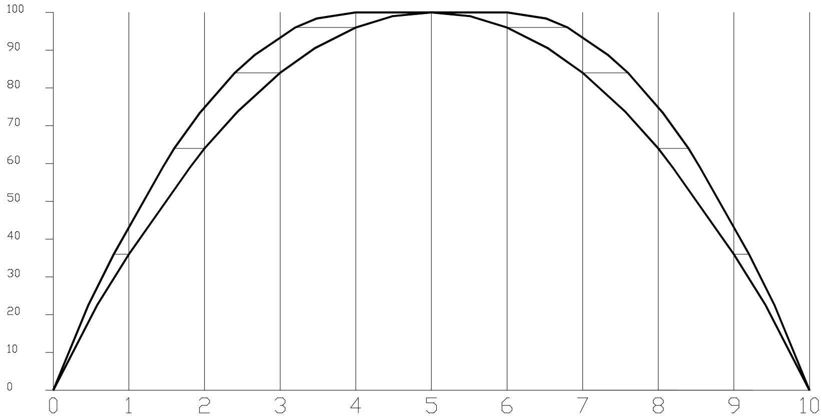


Figure 6.29. Original and modified sectional area curves

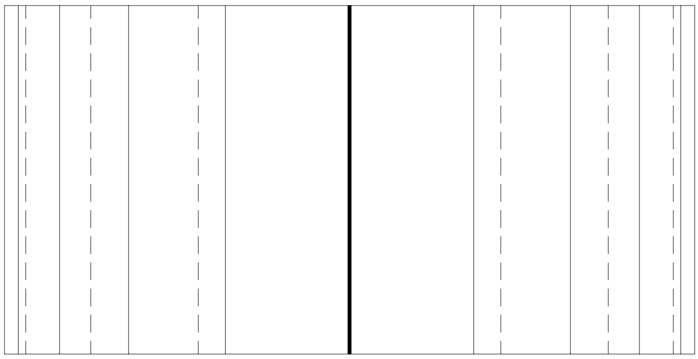


Figure 6.30. Body plan of the original and modified forms

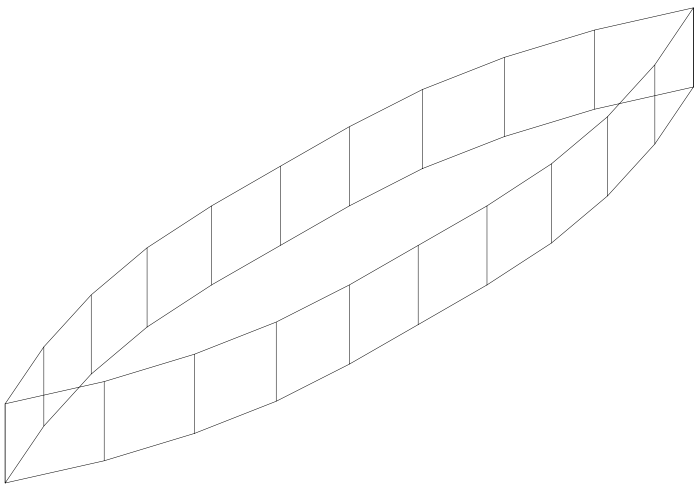


Figure 6.31. 3D view of the modified form

Example 6.5. A cargo ship has the following dimensions. L=120 me, B=18 m, T=8 m and D=12 m. The offsets are given in the following table. The body plan is shown in **Figure 6.32**. The prismatic coefficient is require tı increase by 2% while the position of LCB is shifted forward by 1% L.

	0	0.5	1	2	3	4	5	6	7	8	9	9.5	10
WL0 z=0m	0.000	0.153	1.305	4.023	6.039	6.813	6.813	6.813	6.372	4.437	1.548	0.351	0.000
WL0.5 z=1m	0.000	0.486	2.565	6.246	8.199	8.658	8.658	8.658	8.370	6.570	3.519	2.169	1.314
WL1 z=2m	0.000	0.639	2.871	6.894	8.613	8.991	8.991	8.991	8.829	7.317	4.032	2.556	1.710
WL2 z=4m	0.000	0.774	3.294	7.596	8.865	9.000	9.000	9.000	8.982	8.037	4.473	2.583	1.647
WL3 z=6m	0.000	2.061	4.752	8.082	8.964	9.000	9.000	9.000	9.000	8.298	4.788	2.376	0.774
WL4 z=8m	2.925	5.022	6.741	8.550	9.000	9.000	9.000	9.000	9.000	8.415	5.157	2.439	0.063
WL5 z=10m	5.112	6.957	8.136	8.865	9.000	9.000	9.000	9.000	9.000	8.586	5.823	3.276	0.630
WL6 z=12m	6.642	8.253	8.865	8.982	9.000	9.000	9.000	9.000	9.000	8.757	6.867	5.058	2.367

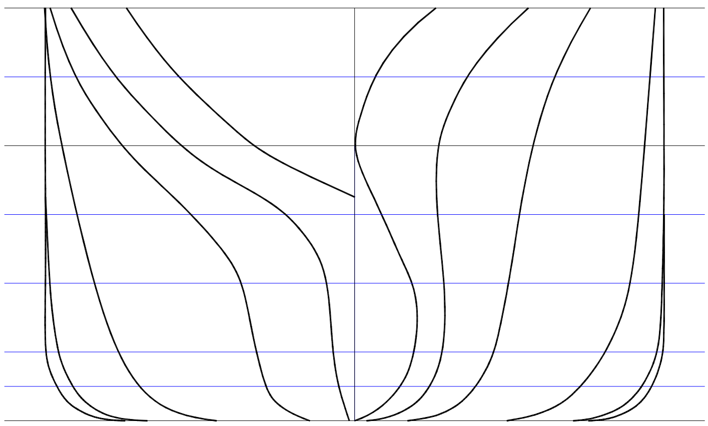


Figure 6.32. Body plan of the parent form

Half breadth and sectional areas of the parent form are given in the following table.

Sta	0	0.5	1	2	3	4	5	6	7	8	9	9.5	10
b	2.925	5.022	6.741	8.550	9.000	9.000	9.000	9.000	9.000	8.415	5.157	2.439	0.063
A	4.6165	23.386	60.072	117.02	137.57	141.07	141.07	141.07	139.57	121.93	68.01	37.04	17.882

Basic hydrostatic characteristics of the parent form are calculated as follows

Sta	Area	SM	Product	MC	Product
0	4.6165	1/2	2.30825	5	11.54125
0.5	23.3857	2	46.7714	4.5	210.4713
1	60.0722	3/2	90.1083	4	360.4332
2	117.0212	4	468.0848	3	1404.254
3	137.5697	2	275.1394	2	550.2788
4	141.0726	4	564.2904	1	564.2904
5	141.0726	2	282.1452	0	0
6	141.0726	4	564.2904	-1	-564.2904
7	139.5754	2	279.1508	-2	-558.3016
8	121.9288	4	487.7152	-3	-1463.145
9	68.0098	3/2	102.0147	-4	-408.0588
9.5	37.0407	2	74.0814	-4.5	-333.3663
10	17.8823	1/2	8.94115	5	-44.70575
			$\Sigma_1 = 3245.04$		$\Sigma_2 = -270.6$

Displacement volume $\nabla = \frac{s}{3}\Sigma_1 = 12980.2\,m^3$

Block coefficient $C_B = \frac{\nabla}{LBT} = \frac{12980.2}{120 \times 18 \times 8} = 0.751$

Midship section coefficient $C_M = \frac{A_M}{BT} = \frac{141.0726}{18 \times 8} = 0.98$

Prismatic coefficient $C_P = \frac{C_B}{C_M} = \frac{0.751}{0.98} = 0.767$

Longitudinal centre of buoyancy $LCB = s\frac{\Sigma_2}{\Sigma_1} = 12\frac{270.6}{3245.04} = 1\,m$ (%0.8333 L) forward

C_{PF}, C_{PA}, $\overline{x}_f$ and $\overline{x}_a$ are calculated in the following table.

Sta	A	A'	SM	Product	MC	Product
0	4.6165	0.0327	1/2	0.0164	5	0.0818
0.5	23.3857	0.1651	2	0.3301	4.5	1.4855
1	60.0722	0.4258	3/2	0.6387	4	2.5549
2	117.0212	0.8295	4	3.3180	3	9.9541
3	137.5697	0.9752	2	1.9503	2	3.9007
4	141.0726	1.0000	4	4.0000	1	4.0000
5	141.0726	1.0000	1	1.0000	0	0.0000
			$\Sigma_1 =$	11.2536	$\Sigma_3 =$	21.9771
5	141.0726	1.0000	1	1.0000	0	0.0000
6	141.0726	1.0000	4	4.0000	1	4.0000
7	139.5754	0.9894	2	1.9788	2	3.9575
8	121.9288	0.8643	4	3.4572	3	10.3716
9	68.0098	0.4821	3/2	0.7231	4	2.8925
9.5	37.0407	0.2626	2	0.5251	4.5	2.3631
10	17.8823	0.1268	1/2	0.0634	5	0.3169
			$\Sigma_2 =$	11.7476	$\Sigma_4 =$	23.9017

Afterbody prismatic coefficent $C_{PA} = \frac{0.2}{3} \times \Sigma_1 = 0.7502$

Forward prismatic coefficient $C_{PF} = \frac{0.2}{3} \times \Sigma_2 = 0.7832$

Centroid of afterbody $\overline{x}_a = 0.2 \times \frac{\Sigma_3}{\Sigma_1} = 0.3906$

Centroid of forebody $\overline{x}_f = 0.2 \times \frac{\Sigma_4}{\Sigma_2} = 0.4069$

h levers are calculated as follows:

$$h_f = \frac{C_{PF}(1 - 2\overline{x}_f)}{1 - C_{PF}} = \frac{0.7832 \times (1 - 2 \times 0.4069)}{1 - 0.7832} = 0.6727$$

$$h_a = \frac{C_{PA}(1 - 2\overline{x}_a)}{1 - C_{PA}} = \frac{0.7502 \times (1 - 2 \times 0.3906)}{1 - 0.7502} = 0.6571$$

The required changes in the prismatic coefficents of fore and afterbody are as follows

$$\delta C_{PF} = \frac{2[\delta C_P (h_a + LCB) + 2\delta LCB(C_P + \delta C_P)]}{(h_f + h_a)}$$

$$\delta C_{PF} = \frac{2[0.02 \times 0.767 \times (0.6571 + 0.00833) + 2 \times 0.01 \times (0.767 + 0.02 \times 0.767)]}{0.6571 + 0.6727} = 0.03888$$

$$\delta C_{PA} = \frac{2[\delta C_P(h_f - LCB) - 2\delta LCB(C_P + \delta C_P)]}{(h_f + h_a)}$$

$$\delta C_{PA} = \frac{2[0.02 \times 0.767 \times (0.6727 - 0.00833) - 2 \times 0.01 \times (0.767 + 0.02 \times 0.767)]}{0.6571 + 0.6727} = -0.00820$$

Then the required change of stations are

$$dx_A = \frac{dC_{PA}}{1 - C_{PA}}(\frac{L}{2} - x) = \frac{-0.00820}{1 - 0.7502}(\frac{L}{2} - x)$$

	Afterbody						
	0	0.5	1	2	3	4	5
x	0	6	12	24	36	48	60
dx	0	-0.1968	-0.3936	-0.7872	-1.1808	-1.5744	-1.968
	0	5.8032	11.6064	23.2128	34.8192	46.4256	58.032

$$dx_F = \frac{dC_{PF}}{1 - C_{PF}}(\frac{L}{2} - x) = \frac{0.03888}{1 - 0.7832}(\frac{L}{2} - x)$$

	Forebody						
	5	6	7	8	9	9.5	10
x	60	72	84	96	108	114	120
dx	10.758	8.6064	6.4548	4.3032	2.1516	1.0758	0
x’	70.758	80.6064	90.4548	100.3032	110.1516	115.0758	120

The original and modified sectional area curves are shown in **Figure 6.33**. In order to obtain the modified hull form each waterline is to be shifted by dx as shown in **Figure 6.34.** New offsets then can be read of from the waterlines and the modified form is shown in **Figure 6.35.**

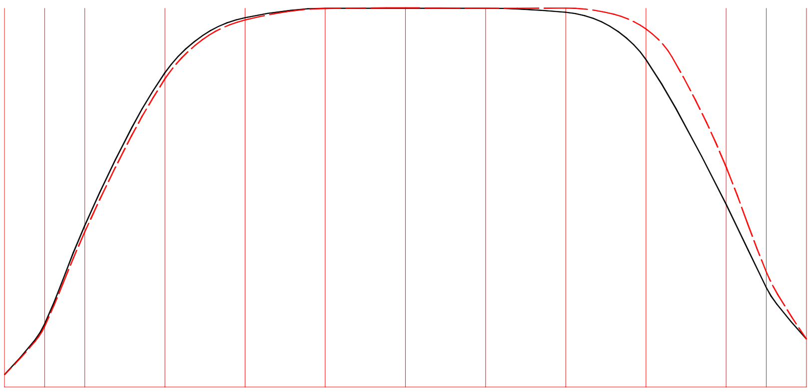


Figure 6.33. Original and modified sectional area curves

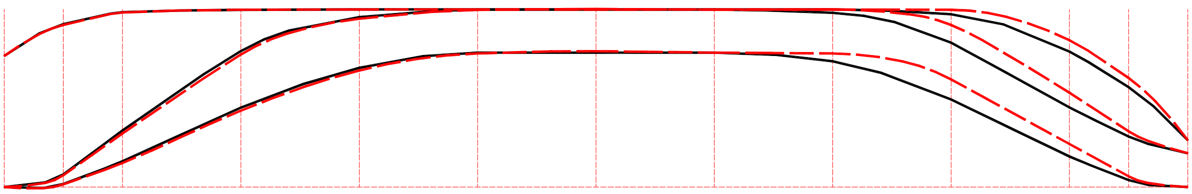


Figure 6.34. Original and modified waterlines

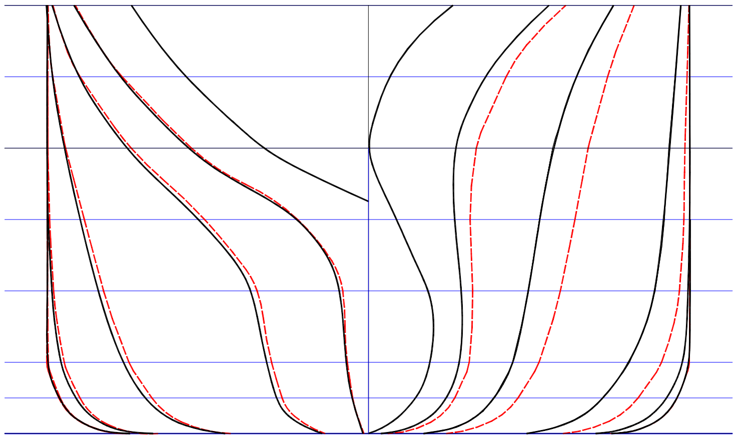


Figure 6.35. Original and modified body plans

6.2.3. Lines Creation Approach

Linear distortion methods such as the Lackenby's method as described above, enables the designer to derive a series of hull forms by a systematic change in the locations of the stations at which the offsets are given. That is, the shape of the sections remains the same as in the parent hull, but they are moved forward or aft in some manner so that the curve of sectional areas changes. However, in many cases a parent hull form may not be available therefore the hull form may have to be created from scratch. In such cases a lines creation approach may be adopted to generate a feasible hull form. The lines creation procedures could be applied in the following order

- Estimation of main dimensions
- Estimation of hull form parameters
- Evaluation of a sectional area curve and a design waterline
- Creation of section lines

The first two elements of the process are dealt with in the first parts of these chapters. Having estimated the main dimensions and hull form parameters a sectional area curve and a design waterline may be obtained by using suitable polynomials.

For example, let us assume that the sectional area curve is represented by a fifth degree polynomial in the following manner

$$A'(x') = a_0 + a_1x' + a_2x'^2 + a_3x'^3 + a_4x'^4 + a_5x'^5$$

where

$$A'(x) = \frac{A}{B.T.C_M} \qquad x' = \frac{x}{L} \qquad LCB' = \frac{LCB}{L}$$

The boundary conditions may be applied as follows

$$A'(0) = A'_{tr} \quad \rightarrow \; a_0 = A'_{tr}$$

$$A'(1) = 0$$

$$A'(0.5) = 1$$

$$\frac{dA}{dx}\Big|_{0.5} = 0 \qquad \int_0^1 A'(x')dx' = C_P \qquad \int_0^1 A'(x')\,x\,dx' = C_P\,(0.5 - \frac{LCB'}{100})$$

We have 6 unknowns and 6 equations. By using the condition $x' = 0.5$, a_1 and a_2 can be obtained in terms of other unknown parameters.

$$A'(0) = A'_{tr} \quad \rightarrow \quad a_0 = A'_{tr}$$

$$A'(1) = 0 \quad \rightarrow \quad A'_{tr} + a_1 + a_2 + a_3 + a_4 + a_5 = 0$$

$$A'(0.5) = 1 \quad \rightarrow \quad A'_{tr} + \frac{1}{2}a_1 + \frac{1}{4}a_2 + \frac{1}{8}a_3 + \frac{1}{16}a_4 + \frac{1}{32}a_5 = 1$$

$$\frac{dA}{dx}\Big|_{0.5} = 0 \quad \rightarrow \quad a_1 + a_2 + \frac{3}{4}a_3 + \frac{1}{2}a_4 + \frac{5}{16}a_5 = 0$$

$$\int_0^1 A'(x')dx' = C_P \quad \rightarrow \quad A'_{tr} + \frac{1}{2}a_1 + \frac{1}{3}a_2 + \frac{1}{4}a_3 + \frac{1}{5}a_4 + \frac{1}{6}a_5 = C_P$$

$$\int_0^1 A'(x')\, x\, dx' = C_P(0.5 - \frac{LCB'}{100})$$

$$\rightarrow \quad \frac{1}{2}A'_{tr} + \frac{1}{3}a_1 + \frac{1}{4}a_2 + \frac{1}{5}a_3 + \frac{1}{6}a_4 + \frac{1}{7}a_5 = C_P(0.5 - \frac{LCB'}{100})$$

The following system of equations must be solved

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1/2 & 1/4 & 1/8 & 1/16 & 1/32 \\ 1 & 1 & 3/4 & 1/2 & 5/16 \\ 1/2 & 1/3 & 1/4 & 1/5 & 1/6 \\ 1/3 & 1/4 & 1/5 & 1/6 & 1/7 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{bmatrix} = \begin{bmatrix} -A'_{tr} \\ 1 - A'_{tr} \\ 0 \\ C_P - A'_{tr} \\ (0.5 - LCB'/100)C_P - 1/2A'_{tr} \end{bmatrix}$$

The unknown coefficients for the case of $C_P = 0.7$, $LCB' = 2$, $A'_{tr} = 0.2$ are as follows:

$a_1 = 3.56$

$a_2 = -6.32$

$a_3 = 9.28$

$a_4 = -11.2$

$a_5 = 4.48$

Then the equation of the sectional area curve is

$$A'(x') = 0.2 + 3.56x' - 6.32x'^2 + 9.28x'^3 - 11.2x'^4 + 4.48x'^5$$

This polynomial representation is shown in the following figure

The existence of a parallel middle body will require further conditions. For instance two more conditions representing the forward and aft portions of the parallel middle body will result ina 7th degree polynomial as follows

$$A'(x') = a_0 + a_1 x' + a_2 x'^2 + a_3 x'^3 + a_4 x'^4 + a_5 x'^5 + a_6 x'^6 + a_7 x'^7$$

The following boundary conditions may be used

1. $A'(0) = A'_{ayna} \quad \rightarrow \; a_0 = A'_{ayna}$
2. $A'(1) = 0$
3. $A'(p_A) = 1$
4. $A'(p_F) = 1$
5. $\dfrac{dA'}{dx} \,|\, p_A = 0$
6. $\dfrac{dA'}{dx} \,|\, p_F = 0$
7. $\int_0^1 A'(x')dx' = C_P$
8. $\int_0^1 A'(x')\,x\,dx' = C_P(0.5 - \dfrac{LCB'}{100})$

Where p_A and p_F represent the forward and aft portions of the parallel middle body. Then we have eight unknowns and eight conditions.

$$A'(0) = A'_{ayna} \quad \rightarrow \quad a_0 = A'_{ayna}$$

$$A'(1) = 0 \quad \rightarrow \quad A'_{ayna} + a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7 = 0$$

$$A'(p_A) = 1$$

$$\rightarrow \quad A'_{ayna} + p_A a_1 + (p_A)^2 a_2 + (p_A)^3 a_3 + (p_A)^4 a_4 + (p_A)^5 a_5 + (p_A)^6 a_6 + (p_A)^7 a_7 = 1$$

$$A'(p_F) = 1$$

$$\rightarrow \quad A'_{ayna} + p_F a_1 + (p_F)^2 a_2 + (p_F)^3 a_3 + (p_F)^4 a_4 + (p_F)^5 a_5 + (p_F)^6 a_6 + (p_F)^7 a_7 = 1$$

$$\frac{dA}{dx} \,|\, p_A = 0$$

$$\rightarrow \quad a_1 + 2p_A a_2 + 3(p_A)^2 a_3 + 4(p_A)^3 a_4 + 5(p_A)^4 a_5 + 6(p_A)^5 a_6 + 7(p_A)^6 a_7 = 0$$

$$\frac{dA}{dx} \,|\, p_F = 0$$

$$\rightarrow \quad a_1 + 2p_F a_2 + 3(p_F)^2 a_3 + 4(p_F)^3 a_4 + 5(p_F)^4 a_5 + 6(p_F)^5 a_6 + 7(p_F)^6 a_7 = 0$$

$$\int_0^1 A'(x')dx' = C_P \quad \rightarrow \quad A'_{ayna} + \frac{1}{2}a_1 + \frac{1}{3}a_2 + \frac{1}{4}a_3 + \frac{1}{5}a_4 + \frac{1}{6}a_5 + \frac{1}{7}a_6 + \frac{1}{8}a_7 = C_P$$

$$\int_0^1 A'(x')\,x\,dx' = C_P(0.5 - \frac{LCB'}{100})$$

$$\rightarrow \quad \frac{1}{2}A'_{ayna} + \frac{1}{3}a_1 + \frac{1}{4}a_2 + \frac{1}{5}a_3 + \frac{1}{6}a_4 + \frac{1}{7}a_5 + \frac{1}{8}a_6 + \frac{1}{9}a_7 = C_P(0.5 - \frac{LCB'}{100})$$

The unknown coefficents may be obtained by solving the following system of equations.

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ p_A & p_A^2 & p_A^3 & p_A^4 & p_A^5 & p_A^6 & p_A^7 \\ p_F & p_F^2 & p_F^3 & p_F^4 & p_F^5 & p_F^6 & p_F^7 \\ 1 & 2p_A & 3p_A^2 & 4p_A^3 & 5p_A^4 & 6p_A^5 & 7p_A^6 \\ 1 & 2p_F & 3p_F^2 & 4p_F^3 & 5p_F^4 & 6p_F^5 & 7p_F^6 \\ 1/2 & 1/3 & 1/4 & 1/5 & 1/6 & 1/7 & 1/8 \\ 1/3 & 1/4 & 1/5 & 1/6 & 1/7 & 1/8 & 1/9 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_7 \end{bmatrix} = \begin{bmatrix} -A'_{ayna} \\ 1-A'_{ayna} \\ 1-A'_{ayna} \\ 0 \\ 0 \\ C_P - A'_{ayna} \\ (0.5-\frac{LCB'}{100})C_P - \frac{1}{2}A'_{ayna} \end{bmatrix}$$

Example 4.2. Generate a sectional area curve for $C_P = 0.8$, $LCB' = 2$, $A'_{ayna} = 0.2$. The ship has a symmetrical parallel middle body with a length of 20% of ship's length, in which case p_A=0.4 and p_F=0.6

The sectional area curve is assumed to be represented by a 8th degree polynomial for which the unknown coefficients are calculated as follows

a_1 = 6.65245

a_2 = -25.51791

a_3 = 74.64714

a_4 = -184.73191

a_5 = 301.93469

a_6 = -258.16271

a_7 = 84.97825

The nondimensional sectional area curve is shown in the following figure

Figure 6.36. Sectional area curve

This method can easily be applied to design waterline, in which case C_P and LCB are replaced by C_{WP} and LCF. Provided that sectional area and sectional beam of sections along the length are determined a body plan can be obtained by a graphical method as shown in **Figure 6.37.**

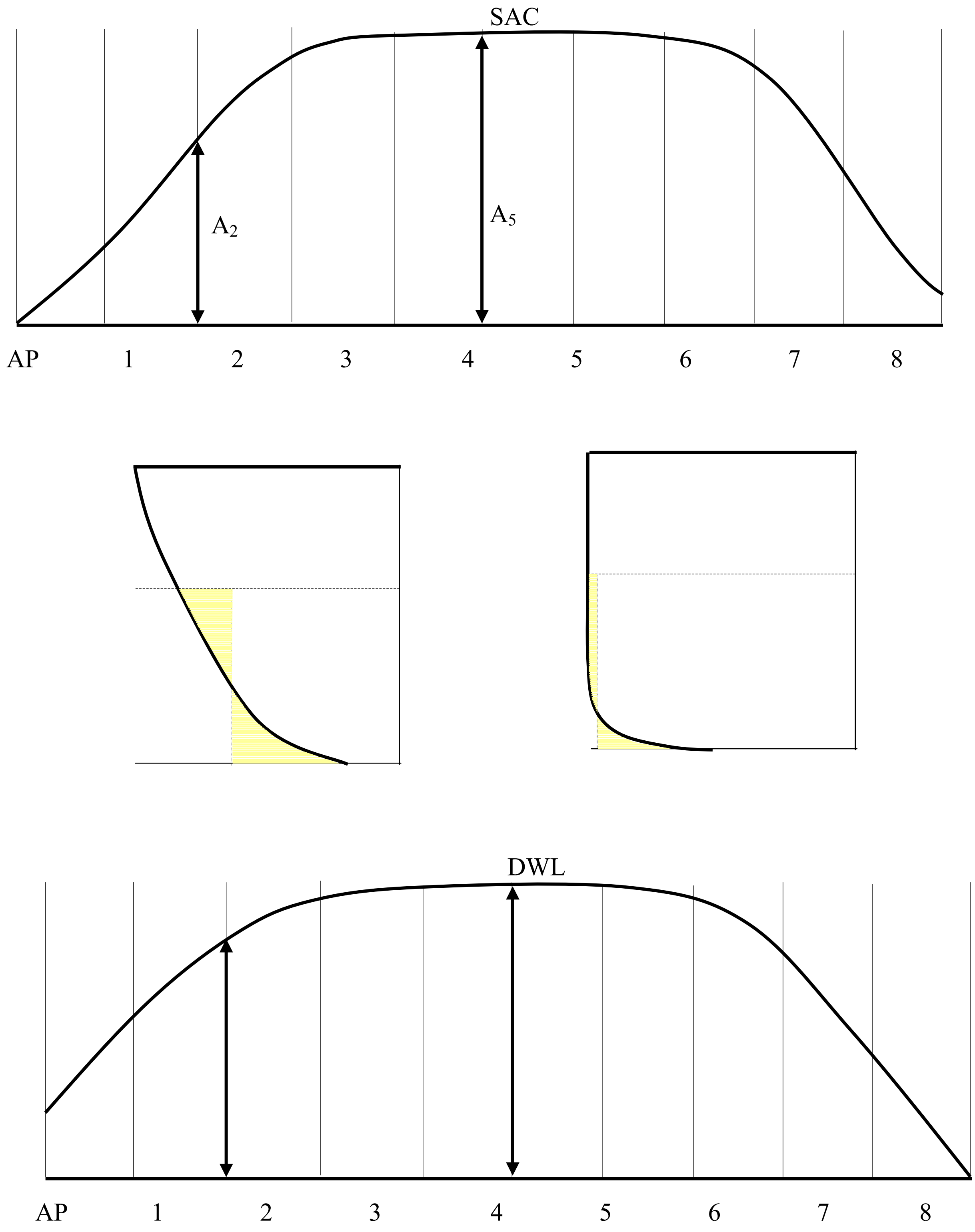


Figure 6.37. Lines creation approach

Under the assumption that a sectional form can be defined by a limited set of parameters, Lackenby's linear distortion method can also be applied to design waterline curve in a similar way it is applied to the sectional area curve.

Therefore, given the desired changes in LCB, C_{WP} and LCF, the designer can, for each section, calculate sectional beam, draught, and area. This information is sufficient for defining, by conformal mapping transformations, a concrete underwater section shape, namely a Lewis section.

The suitability of conformal mapping techniques for the approximate representation of underwater sections of ships has been well known at least since **Lewis (1929)** adopted this approach to calculate the added mass of ships. Lewis used mapping functions with three terms and hence three free parameters, which enabled him to control the beam, draught, and area of the section. Evidently, by including more terms and free coefficients in the mapping transformation it is possible to control more features of section shape. **Von Kerzcek** and **Tuck** **(1969)** demonstrated that it took only relatively few terms in the mapping function to obtain very close

approximations to a broad range of realistic ship section shapes.

Main advantage of conformal mapping is that the sections generated are represented by a single set of mathematical equation which greatly simplifies certain hydrostatic and hydrodynamic calculations. Moreover, the section shapes generated by conformal mapping tend to be automatically fair in the manner of streamline shapes and do not exhibit rapid oscillations.

However, the designer may want to control the section character in the direction of more U-shaped or V-shaped sections, keeping the other section parameters the same. This is feasible within the scope of the conformal mapping method by introducing extra parameters. One drawback of section shapes derived by mapping for the circle is that the tangent to the section at the waterline is vertical, making it difficult to represent sections with flare. This can be avoided by using polynomials for the above water form, matching the underwater form at a point slightly below the waterline.

The fundamental problem to be solved in applying the conformal mapping technique is the determination of the mapping function corresponding to a given sectional curve. Unfortunately, this problem cannot be solved in closed form but in some simple, special cases. In general, approximate methods and numerical techniques should be used. A succesful choice of the mapping technique is primarily dependent upon the desired accuracy and the kind of data used in defining the sectional curve. such data can be classified into three categories:

- A set of points on the curve,
- A number of local properties, other than points, of the curve; e.g. the slope of the curvature at given points, and
- A number of global properties of the curve; e.g. the area, breadth, draught, and various moments with respect to the axes.

When the problem is to accurately represent an actual section shape, first two categories of data are used. However, in the hull form variation, the problem is to be able to create a section shape using a small number of parameters to be used in hydrodynamic calculations. several families of conformal mapping functions have been developed, most convenient ones being the two-parameter family introduced by **Lewis (1929)** and the three-parameter family by **Landweber** and **Macagno (1959).**

Limiting the problem to conventional ship sections only and assuming a mirror image about both axis of symmetry and the free surface, that is, symmetry about both the x and y axes, the mapping function can be reduced to,

$$z = \sum_{n} c_n \zeta_n \qquad -\infty < n < \infty$$

or

$$z = c_0 \zeta + \sum_{n=2} c_{n-1} \zeta^{1-n}$$

where z and ζ are the complex variables,

$$z = x + iy \qquad \text{and} \qquad \zeta = \xi + i\eta = Re^{i\theta}$$

and the coefficients a_n are real. Due to the symmetry with respect to the both axes, only a single quadrant need be considered for each section as illustrated in Figure

Figure 6.38. Mapping a Ship Section to a Circle

In the ζ -plane, for a unit circle where the radius of the circle is 1, and

$$\zeta = e^{i\theta} = \cos\theta + i\sin\theta$$

Substituting {\bf (8.22)} into {\bf (8.23)} and separeting real and imaginary parts, one gets

$$\begin{aligned} x(\theta) &= a_0 \cos\theta + a_1 \cos\theta + a_3 \cos 3\theta + ... + a_{2N-1} \cos(2N-1)\theta \\ y(\theta) &= a_0 \sin\theta - a_1 \sin\theta - a_3 \sin 3\theta - ... - a_{2N-1} \sin(2N-1)\theta \end{aligned}$$

where the a_is are called mapping coefficients and varying θ from 0 to π the section contour can be described.

The two parameter mapping introduced by Lewis (1929) utilizes the first three terms of the equations

$$\begin{aligned} x(\theta) &= a_0 \cos\theta + a_1 \cos\theta + a_3 \cos 3\theta \\ y(\theta) &= a_0 \sin\theta - a_1 \sin\theta - a_3 \sin 3\theta \end{aligned}$$

Let b denote the half-beam of the section at the waterline and T the draught. Since x=b when θ =0 and y=T when $\theta = \pi/2$, from equation {\bf (8.24)}, the half-beam and draught are obtained as follows

$$\begin{aligned} b &= x(\theta = 0) = a_0 + a_1 + a_3 \\ T &= y(\theta = \frac{\pi}{2}) = a_0 - a_1 + a_3 \end{aligned}$$

Sectional area of the contour is given by,

$$S = \int_0^{\pi/2} x(\theta) \frac{dy(\theta)}{d\theta} d\theta$$

which becomes

$$S = \frac{\pi}{2}\left(a_0^2 - a_1^2 - 3a_3^2\right)$$

Given the sectional half-beam, draught, and area, the mapping coefficients can be determined as follows

$$\begin{aligned} a_0 &= \frac{1}{2}(b+T) - a_3 \\ a_1 &= \frac{1}{2}(b-T) \\ a_3 &= -\frac{1}{4}(b+T) + \frac{1}{4}\sqrt{(b+T)^2 + 8\left(bT - \frac{2}{\pi}S\right)} \end{aligned}$$

where b, T, S are the half-beam, draught and the sectional area, respectively.

Half beam – draught ratio $\lambda = \frac{b}{T}$

Sectional area coefficent $\sigma = \frac{S}{2bT}$

Then

$$a_0 = \frac{1}{2}(\lambda + 1) - a_3$$

$$a_1 = \frac{1}{2}(\lambda - 1)$$

$$a_3 = -\frac{1}{4}(\lambda + 1) + \frac{1}{4}\sqrt{(\lambda + 1)^2 + 8\lambda\left(1 - \frac{4}{\pi}\sigma\right)}$$

It can be seen that the slope of the section is infinite at the waterplane (vertical tangent at $\theta = 0$), and zero at the keel (horizontal tangent at $\theta = \pi / 2$). This is a consequence of the corresponding properties of the unit circle, the conformality of the mapping, and the regular character of the mapping function on the section contour. Therefore, sections with flare in the waterplane cannot be represented by conformal mapping. One way of getting around this difficulty is instead of mapping all the way from $\theta = 0$ to $\theta = \pi / 2$, to carry it out starting from a fixed value of θ, say from $\theta = \pi / 20$ to $\theta = \pi / 2$. At this starting point which is slightly below the waterline, conformal mapping equation can be matched with the offset slope and possibly the curvature of above water polynomial.

The viability of a lines creation method based on conformal mapping depends on the range of shapes that can be created by this technique. For Lewis forms, Von Kerzcek and Tuck (1969) have determined the limits of feasible practicle shapes and dicussed how the sections degenerate into tunneled, bulbous, reentrant forms as shown in Figure .

The reentrant form is characterized by the property that x<0 and/or y>0 for some θ in the fourth quadrant. Equations {\bf (8.24)} and {\bf (8.30)}, and the requirement that $x \geq 0$ and $y \leq 0$ for $0 \leq \theta \leq \pi / 2$ yields the inequalities

$$x \geq 0 \qquad \Rightarrow \qquad a_3 \leq \frac{\lambda}{4}$$

$$y \geq 0 \qquad \Rightarrow \qquad a_3 \leq \frac{1}{4}$$

Thus to avoid reentrant forms we take {\bf (8.38)} if $\lambda \leq 1$ and {\bf (8.39)} if $\lambda > 1$. In terms of the half-beam/draught ratio and sectional area coefficient we have,

$$\sigma \geq \frac{3\pi}{32}(2 - \lambda) \qquad \text{and} \qquad \sigma \geq \frac{3\pi}{32}\left(2 - \frac{1}{\lambda}\right)$$

Forms on the borderline between the rentrant and non-renetrant categories are cusped at the waterline or the keel.

The next two categories are conventional forms, in which x and y are monotone functions of θ, and bulbous forms, with x not monotone. A third category is the tunneled form which are similar to bulbous forms turned clockwise through 90 degrees. The definition of conventional forms requires that

$$\sigma \leq \frac{3\pi}{32}\left(3 + \frac{\lambda}{4}\right) \qquad \text{and} \qquad \sigma \leq \frac{3\pi}{32}\left(3 + \frac{1}{4\lambda}\right)$$

Örnek 4.3. Form karakteristikleri aşağıda verilen geminin su altı formunu temsil eden ofset değerlerinin eldesi istenmektedir.

L : 100 m
B : 20 m
T : 10 m
C_M : 0.98
C_P : 0.75
LCB : % 2 L kıç
C_{WP} : 0.85
LCF : % 4 L kıç.
Kıç kesit alan oranı : 0.2
Kıç genişlik oranı : 0.7

Öncelikle uygun dereceden polinomlar seçilerek kesit alan eğrisi ve dizayn su hattı eğrisi aşağıdaki şekillerde görülen şekilde oluşturulur.

Şekil 6.39. Örnek gemiye ait en kesit alanları ve yüklü su hattı eğrileri

Herbir kesitteki yarı genişlik ve kesit alanı değerleri ile su çekimi değerlerinden yararlanılarak eşit aralıklı 20 keside ait kesit formları aşağıdaki şekilde görüldüğü şekilde elde edilir.

Şekil 6.40. Örnek gemiye ait kesit formları

Exercise. Form karakteristikleri aşağıda verilen geminin su altı formunu temsil eden ofset değerlerinin eldesi istenmektedir.

L : 100 m
B : 20 m
T : 10 m
Kıç kesit alan oranı : 0.2
Kıç genişlik oranı : 0.7

KOD	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
C_M	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98
C_P	0.60	0.62	0.64	0.66	0.68	0.70	0.72	0.74	0.76
LCB	2%L aft	2%L aft	2%L aft	2%L aft	2%L aft	2%L aft	2%L aft	2%L aft	2%L aft
C_{WP}	0.70	0.70	0.70	0.70	0.70	0.70	0.70	0.70	0.70
LCF	2%L aft	2%L aft	2%L aft	2%L aft	2%L aft	2%L aft	2%L aft	2%L aft	2%L aft

KOD	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**
C_M	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98
C_P	0.60	0.62	0.64	0.66	0.68	0.70	0.72	0.74	0.76

LCB	2%L aft	2%L aft	2%L aft	2%L aft	2%L aft	2%L aft	2%L aft	2%L aft	2%L aft
C_{WP}	0.72	0.74	0.76	0.78	0.80	0.82	0.84	0.86	0.88
LCF	2%L aft	2%L aft	2%L aft	2%L aft	2%L aft	2%L aft	2%L aft	2%L aft	2%L aft

KOD	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**
C_M	0.95	0.95	0.95	0.95	0.95	0.95	0.95	0.95	0.95
C_P	0.60	0.62	0.64	0.66	0.68	0.70	0.72	0.74	0.76
LCB	2%L aft	2%L aft	2%L aft	2%L aft	2%L aft	2%L aft	2%L aft	2%L aft	2%L aft
C_{WP}	0.70	0.70	0.70	0.70	0.70	0.70	0.70	0.70	0.70
LCF	2%L aft	2%L aft	2%L aft	2%L aft	2%L aft	2%L aft	2%L aft	2%L aft	2%L aft

KOD	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**
C_M	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98
C_P	0.60	0.62	0.64	0.66	0.68	0.70	0.72	0.74	0.76
LCB	1%L aft	1%L aft	1%L aft	1%L aft	1%L aft	1%L aft	1%L aft	1%L aft	1%L aft
C_{WP}	0.70	0.70	0.70	0.70	0.70	0.70	0.70	0.70	0.70
LCF	2%L aft	2%L aft	2%L aft	2%L aft	2%L aft	2%L aft	2%L aft	2%L aft	2%L aft