Sudoku Solver

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Sudoku

- 9 x 9 gird
- Each column, row and 3 x 3 sub-grid should contain all numbers from 1 – 9
- Only admit exactly 1 solution

							1	
					2			3
			4					
						5		
4		1	6					
		7	1					
	5					2		
				8			4	
	3		9	1				

Algorithms

- Simple search without Heuristics
- Search with Minimum Remaining Value (MRV) Heuristic
- SAT encoding

SAT Encoding

- There is at least one number in each entry:

$$\bigwedge_{x=1}^{9} \bigwedge_{y=1}^{9} \bigwedge_{z=1}^{9} S_{xyz}$$

- Each number appears at most once in each row:

$$\bigwedge_{y=1}^{9} \bigwedge_{z=1}^{9} \bigwedge_{x=1}^{8} \bigwedge_{i=x+1}^{9} \neg S_{xyz} \vee \neg S_{iyz}$$

- Each number appears at most once in each column:

$$\bigwedge_{x=1}^{9} \bigwedge_{z=1}^{9} \bigwedge_{y=1}^{8} \bigwedge_{i=y+1}^{9} \neg S_{xyz} \vee \neg S_{xiz}$$

- Each number appears at most once in each 3×3 sub-grid:

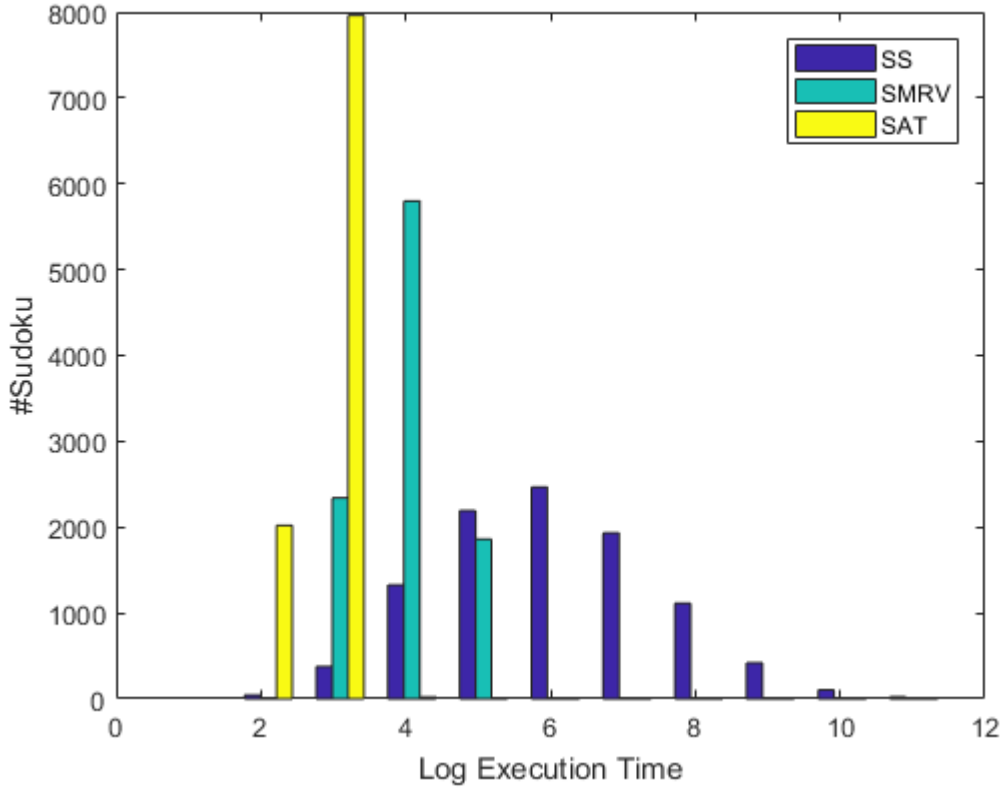
$$\bigwedge_{z=1}^{9} \bigwedge_{i=0}^{2} \bigwedge_{j=0}^{2} \bigwedge_{x=1}^{3} \bigwedge_{y=1}^{3} \bigwedge_{k=y+1}^{3} \neg S_{(3i+x)(3j+y)z} \vee \neg S_{(3i+x)(3j+k)z}$$

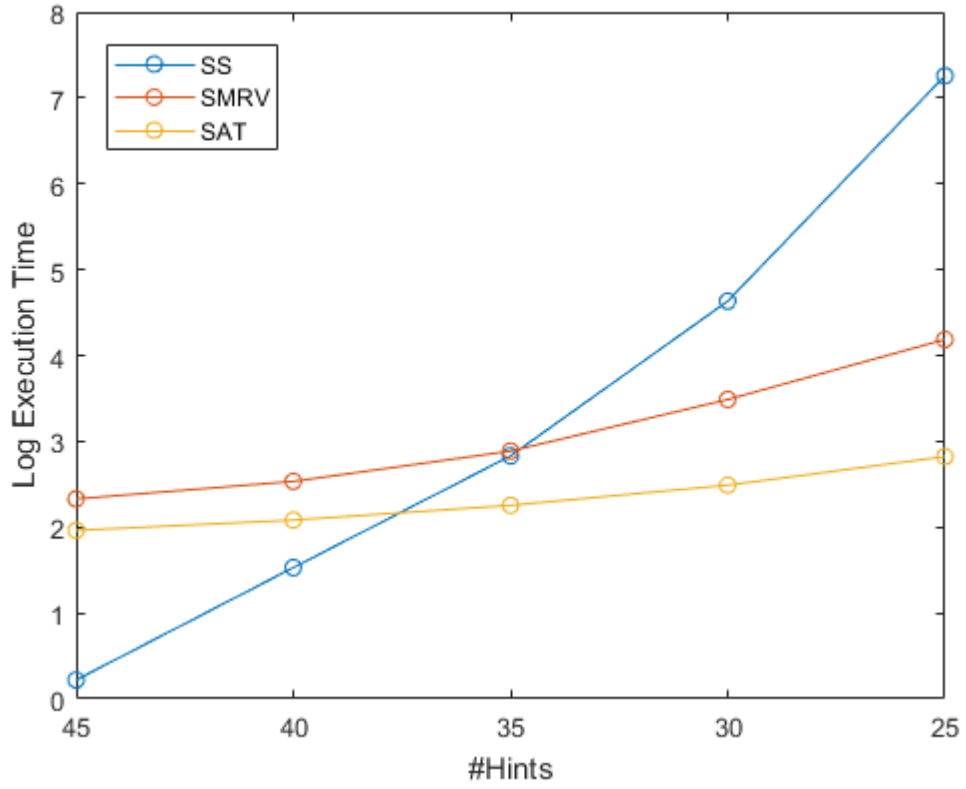
$$\bigwedge_{z=1}^{9} \bigwedge_{i=0}^{2} \bigwedge_{j=0}^{2} \bigwedge_{x=1}^{3} \bigwedge_{y=1}^{3} \bigwedge_{k=x+1}^{3} \bigwedge_{l=1}^{3} \neg S_{(3i+x)(3j+y)z} \vee \neg S_{(3i+k)(3j+l)z}$$

Inference Techniques

- Resolution: $(x_i \vee \alpha) \wedge (\neg x_i \vee \beta) \Rightarrow (\alpha \vee \beta)$
- 2 restricted forms of resolution: Unit Propagation and Failed Literal Rule.

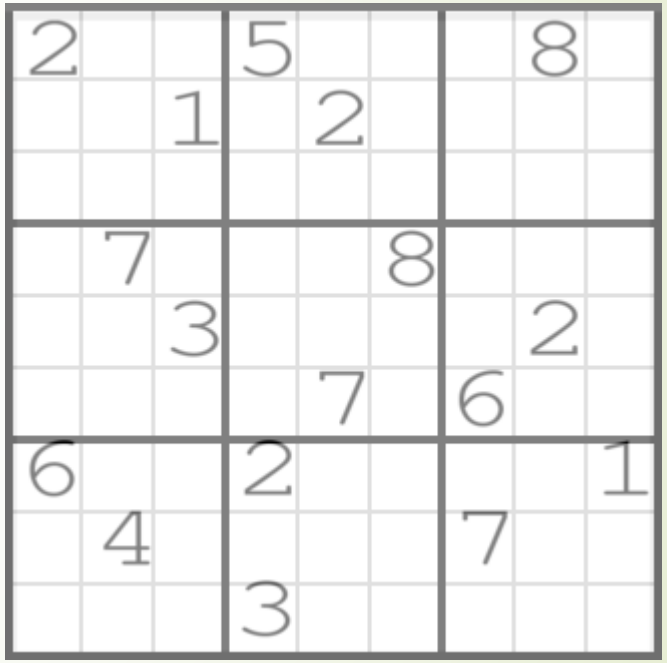
Results


SS
SMRV
SAT
#Sudoku
Log Execution Time
0
1000
2000
3000
4000
5000
6000
7000
8000
0
2
4
6
8
10
12


SS
SMRV
SAT
Log Execution Time
#Hints
0
1
2
3
4
5
6
7
8
45
40
35
30
25

An Evil Sudoku

- Search based methods cannot return solution within reasonable time.
- SAT encoding find solution in 130 milliseconds.



Conclusion

- SAT encoding is extremely fast.
- The inference techniques will try to exploit every inference to reduce search space.
- It is always recommend transferring Finite Domain CSPs to SAT problem.