Isogeometric Analysis of Integral Equations using Subdivision

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***Abstract*—Isogeometric analysis (IGA) has recently become popular in computational science during the past decade or so. IGA tries to to unify both geometric and field representation; in other words, both the geometry and the fields are represented using the same underlying basis set. However, while the concept of IGA for differential equations is more common, extension to an integral equation framework is significantly more challenging. In this work, we present for the first time, the IGA as applied to integral equations encountered in electromagnetics. The presented approach relies on the subdivision scheme for both geometry and function representation. Results presented attest to the viability of the method.**

I. Introduction

The concept of isogeometric analysis (IGA) on parametric surface (mainly non-unoform rational B-splines or NURBS based surfaces) was introduced in [1] for a class of partial differential equations (PDEs). The attractive features of the IGA include continuity of functions, possibility of $h-$ and $p-$ refinement, mesh-free method, and so on. The main feature of this class of methods is the a unified prescription of both the underlying geometry and the fields/quantities define on the geometry. Due to the nature of basis functions used, this is typically tantamount to imposing higher order continuity for all quantities (geometry and functions). While [1] uses NURBS, it follows that any other geometry interpolation scheme can be used as long as the underlying basis set is suitable for representing both geometry and fields. The past few years have seen significant growth in IGA of PDEs arising from computational mechanics (and related subjects). It has not seen applications to integral equations that are used for time harmonic electromagnetic field analysis. This work presents the first development of IGA for integral equations for electromagnetic analysis; note, as opposed to NURBS, we will use a subdivision surface representation instead.

II. Loop Subdivision and C^2 Basis Function

As opposed to parametric geometry representations such as NURBS, subdivision surfaces are constructed using a recursive refinement of a polygonal mesh such that the resulting limit surface is smooth (up to C^2 almost everywhere. This representation of a surface has seen extensive development in computer graphics and has seen applications to a number of areas of geometry modeling as well as animation. Among the different subdivision surface refinements, equivalent polynomials arising from the Loop subdivision [3] scheme is chosen to interpolate both the scatterer surface and the unknown quantities defined on the surface as well. The focus of this section is to introduce a way to define a basis function with almost global C^2 smoothness, over a triangular mesh that is widely used in computational electromagnetics.

The primal/original triangular mesh, from which the limit surface is extracted, comprises a set of vertices and the connection map. The valence of any vertex denotes the number triangles that are incident on a vertex. There are two types of vertices: (1) regular vertex that has a valence of 6 and (2) irregular vertices with valence not equal to 6. The limit surface can be evaluated at the local parametric space (simplex coordinate system is usually chosen for convenience) through the summation of the weighted box splines [2].

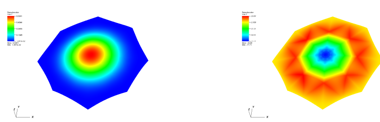
$$\mathbf{S}(u,v) = \sum_{i=1}^{12} N_i(u,v)\mathbf{P}_i \tag{1}$$

where $N_i(u,v)$ is the box spline and $\mathbf{P}_i$ is the position vector of the vertex in the 1-ring of the triangle where the point (u,v) resides [3]. If the 1-ring of the triangle can be found using subdivision refinement, the above expression is valid for any triangle associated with either regular or irregular vertex. It has been shown that the limit surface has continuity of C^2 everywhere except at the irregular vertex. The smoothness of geometry itself is very attractive, since the primal mesh has only C^0 continuity that might confine our choices of basis function to interpolate the field and conduct high-fidelity numerical simulations. Though no explicit basis function is used to interpolate the geometry in subdivision surfaces, one can extract an equivalent basis set associated with the original vertex set in the primal mesh.

It is easier to think of the implicit basis function to represent functions if one writes down an equation to evaluate the "limit function" in a manner similar to Eq.1

$$f(u,v) = \sum_{i=1}^{12} N_i(u,v) w_i. \tag{2}$$

In the primal mesh, one can associate a scalar weight (instead of a vector position) with each vertex. If only one vertex has a weight of unity and other zeros, one would immediately get a smooth (up to 2nd order continuity globally) scalar function. Fig.1a gives an example of the scalar basis function used for formulating isogeometric analysis on top of subdivision-described surface, and the basis function is associated with an irregular vertex of valence 8. To show 2nd order continuity, the surface Laplacian of the scalar function is plotted in Fig.1b.



(a) (b)

Fig. 1: Basis function associated with a vertex with valence of 8. (a) Basis function, (b) Surface Laplacian of the basis function

III. Method of Moments in IGA framework

In the earlier Section, a scalar basis set up to second order continuity was constructed. To apply it to integral equations for electromagnetic scattering, we use a Helmholtz decomposition to define tangential quantities. Therefore, the surface current can be written as

$$\mathbf{J}(u,u) = \sum_{i=1}^{n} \psi_i \nabla_s f_i + \phi_i \hat{n} \times \nabla_s f_i \tag{3}$$

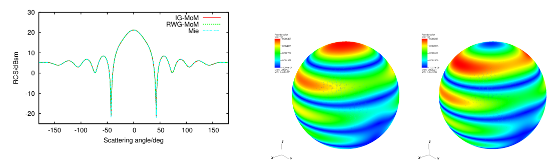
where n is the number of vertices on the primal mesh, ψ_i and ϕ_s are two unknowns for vertex i. Unlike RWG basis function where the degree-of-freedom is associated with an edge, the isogeometric method of moment has unknowns associated with vertex only. For most meshes with vertex valence greater or equal to 6, the number of vertex is only one third the number of edges. It should be noted that the subdivision scheme does not increase the number of unknowns and it only helps to construct basis functions. One can get the linear system through the standard procedure of method of moments.

IV. Results

In this section, some numerical examples are provided to demonstrate the accuracy and feasibility of the proposed IG-MoM. More in-depth analysis will be provided at the conference.

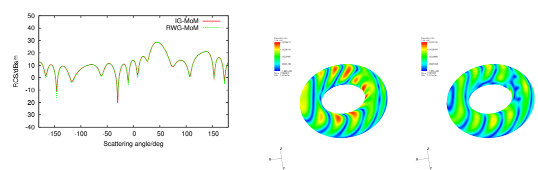
Fig.2a gives the bistatic RCS curves ($\phi = 0°$) of a 1λ sphere obtained by using the proposed IG-MoM (with 5124 unknowns), MoM with RWG basis (RWG-MoM, with 7680 unknowns) and analytical (Mie) approach. The magnitude of both the real and imaginary parts are demonstrated in Fig.2b-2c. Similarly Fig.3 plots the comparison in RCS and the magnitude of currents of a torus of $4\lambda \times 4\lambda \times 1\lambda$. The number of unknowns are 10856 and 16284 respectively.

From the results shown, one can conclude that the accuracy of IG-MoM agrees well with both analytical method and the classic MoM. Great smoothness in the currents can be observed in both examples, owing to the powerful properties of isogeometric analysis.



(a) (b) (c)

Fig. 2: Scattering from a sphere with incidence wave at $\theta = 0°$ and $\phi = 0°$. (a)RCS , (b) Real part and (c) Imaginary part of magnitude of electric currents.



(a) (b) (c)

Fig. 3: Scattering from a torus with the incidence wave at $\theta = 45°$ and $\phi = 0°$. (a) RCS, (b) Real part and (c) Imaginary part of magnitude of electric currents.

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References

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