Networker's guide to decibels

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1 Motivation

Why shall we bother with logarithms rather than just use linear values? Numbers in the logarithmic domain have the great advantage of being easy to manipulate.

- Multiply numbers: add their logarithms
  $log(10 \cdot 20) = log(10) + log(20)$
- Divide numbers: subtract their logarithms
  $log(\frac{10}{20}) = log(10) - log(20)$

1.1 Example

Multiply 100 with 12122. Either by using linear numbers or their logarithmic representation.

1.1.1 Linear

$$100 \cdot 12122 = 13358444 \tag{1}$$

1.1.2 Logarithmic

$$\begin{aligned} log_{10}(100 \cdot 12122) &= log_{10}(100) + log_{10}(12122) \\ &= 2 + log_{10}(12122) \end{aligned} \tag{2}$$

To get the result of $2 + log_{10}(12122)$ one has to use a lookup table, so called logarithm tables, or use a calculator. The advantage of this scheme over multiplying the numbers directly is, that these logarithm tables are relatively small compared to tables which would contain all possible multiplications.

2 Conversion

$$N = 10^{dB/10}$$

$$dB = 10 \cdot log_{10}(N)$$

N a linear number

dB Measure on a logarithmic scale

2.1 Examples

An antenna has a gain factor of 100, i.e it does amplify the output power 100 times. We can also say it has a 20-dB gain, because:

$$10 \cdot log_{10}(100) = 10 \cdot 2 = 20dB$$

2.2 Attenuation and Gain

To model signal loss, the most natural way is to explain the loss by multiplying the orignal signal with a factor. For instance, if the sender sends the signal with power $log_{10}(N)$ the receiver receives the signal with power $c \cdot log_{10}(N)$. If the receiver only receives a hundreth of it, this would be $\frac{1}{100} \cdot log_{10}(N)$, or expressed with logarithms a loss of $20dB$. A loss simply has a negative sign (remember division by subtracting logarithms). Now you should see that this makes mental maths very easy.

2.3 Calculation scheme

You might wonder how you calculate the dB values, such as the $log_{10}(12122)$ in the example above. The answer is: you simply do not. Manufacturers usually already list characteristics with dB values. For example if you buy an antenna, there is a figure which tells you the gain factor in decibels. The calculation scheme is usually start-off your calculations with dB values and do the conversion back to linear values as late as possible , but this is usually not needed at all.

2.4 Link model / Link budget

Transmission paths are ususally modelled as one-way paths. This means a sending station transmits (TX) to a receiving station, which receives (RX) the signal. Simple as that. For back communication in the opposite direction, it follows the same scheme, only the roles change. Thus, the communication paths are regarded as independent. Once a signal is generated in the sender, it passes several stages until it arrives at the receiver. By passing these stages it either experiences gain or attenuation. For example a stage in a wireless network is the propagation through air. In wired networks propagation through copper or fibre. These stages are modelled as a sum of logarithms. Common stages are:

Modulator output P_{TX} the raw signal generated by a device (dBm)

TX antenna gain G_{AntTX} gain of the signal by the transmitting antenna (dB)

Path loss PL the attenuation that occurs on the air (dB). This is usually frequency dependent (dB).

RX antenna gain G_{AntRX} gain of the signal by the receiving antenna (dB)

To be a little bit more precise, there are some other losses involved. For example in the connection cable from the modulator output to the antenna. Also every plug adds to the attenuation. Manufacturers usually list these attenuations in the datasheets.

2.4.1 Link budget

Now you can do your path calculations by simply add up all your gains and attenuation values. The resulting equation is called the link-budget. For example:

$$P_{TX} + G_{AntTX} \quad - PL \quad + G_{AntRX} = P_{RX} \tag{3}$$

$$\underbrace{20dBm + 3dB}_{\text{power of sender}} \quad \underbrace{-80dB}_{\text{path loss}} \quad \underbrace{+3dB}_{\text{gain of receiver}} = \underbrace{-54dBm}_{\text{received power}} \tag{4}$$

This is where another important concept comes in place: the receiver sensitivity. This is the lowest dB level at which the receiver is able to successfully receive the signal. For example if the receiver sensitivity is $-90dBm$, the receiver would be able to successfully receive the signal calculated in Equation 4 (-54 is bigger than -90).

3 dBm

Whereas dB values only express a ratio, dBm express a ratio to a fixed base of $1mW$. For example, if you have a sender that has an output power P_{TX} of 1 Watt, it has a dBm output value of:

$$dBm = 10 \cdot log_{10}(\frac{P_{TX}}{1mW}) = 10 \cdot log_{10}(\frac{1000mW}{1mW}) = 30dBm \tag{5}$$

4 dBi

Like dBm, dBi is a value with a fixed base. But the base is not a fixed number, it is a theoretical concept. dBi are used to express antenna gains. The base reference is the behaviour of an isotropic antenna. Isotropic simply means its transmission characteristic is the same in all directions. You can conceive it as transmitting as a sphere around the antenna. Isotropic antennas are ideal in terms of equal power distribution, but this such an antenna does not exist. dBi measure the antenna gain compared to this ideal antenna. Note that the maximal output power of an antenna P_{TX} is usually given by regulations. So how can you achieve a wider range with the given power? Simply direct the beam in the particular direction you need it, and do not send it in other directions. By this you bring together more power, but only in one direction. The directionality loss is the trade-off.

5 Antennas

As soon as it comes to wireless signal transmission one has to consider beam forming and radiation patterns. As opposed to the propagation direction in cables, which is given. As a simplification you can imagine electromagnetic waves used by wireless technology just as form of non visible light. There exist two types of antennas: omnidirectional and directional. The third type, the isotropic antenna, is only a hypothetical model and does not exist in the real world (except the sun). For detailed information on the different antenna types and radiation patterns have a look at this article on antennas from Cisco

6 EIRP

Equivalent isotropically radiated power. Is the *dBm* value that is actually leaving the antenna. One has to take into account the P_{TX} of the sender, any attenuation by cables, plugs, amplifiers and the antenna gain. This is usually the value that is given by radio regulations. To comply with this regulations lies in the responsibility of the device operator. The output power of a transmitter is usually adjustable to help to compensate for this losses that occur on the way to the antenna.

6.1 isotropic

The radio waves are radiated with the same intensity in all directions. Imagine this as a sphere.

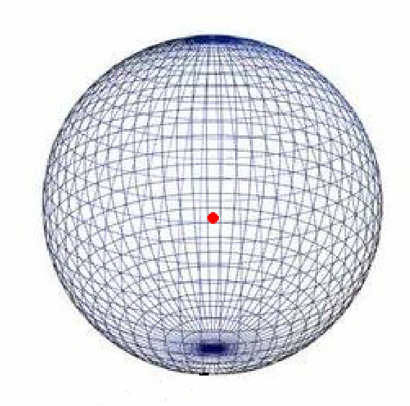

Figure 1: Isotropic radiation

6.2 omnidirectional

Omnidirectional antennas radiate signals uniformly in one plane in all directions. Imagine this as a Donut. The radiation in the elevation pane is not as big as in the azimut pane.

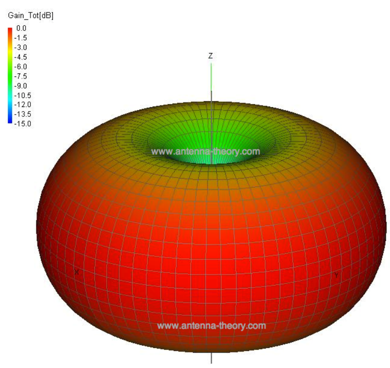

Figure 2: Omnidirectional radiation

6.3 directional

Directional antennas radiate their power especially in one direction. Usually the beam is formed like a baseball bat in the desired direction.

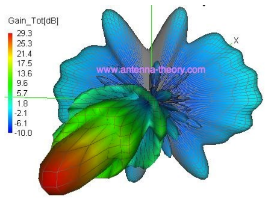

Figure 3: Directional radiation

7 Summary

dB Expresses a factor

dBm Expresses power in milliwats

dBi Antenna gain compared to a hypothetically lossless isotropic antenna

8 Exercises

Convert 4W into *dBm*

$$\begin{aligned} 4W &= 4000\ mW \\ 10 \cdot log_{10}(4000) &= 36\ dBm \end{aligned} \tag{6}$$

Convert 8W into *dBm*

$$\begin{aligned} 8W &= 8000\ mW \\ 10 \cdot log_{10}(8000) &= 39\ dBm \end{aligned} \tag{7}$$

Link budget calculation

Consider the following situation: A receiver has a P_{TX} of $20dBm$, with an directional antenna attached at both the receiver and transmitter with a gain of $+5dBi$, the atmospheric loss is $20dB$, and the receiver's sensitivity is $-140dB$. Which signal strength would the receiver receive? And would it be able to successfully decode the signal?

$$\begin{aligned} P_{RX} &= 20\ dBm + 5\ dBi - 80\ dB + 5\ dBi \\ P_{RX} &= -50\ dBm \end{aligned} \tag{8}$$

$-50\ dB$ is way bigger than $-140\ dB$, this means the receiver can successfully decode the signal.

9 Typical figures

Power level	**Power**	**Application**
80 dBm	100 kW	FM Radio station
60 dBm	1000 W	Microwave oven
33 dBm	2 W	Maximum output from a GSM850/900 mobile phone
23 dBm	200 mW	EEE 802.11n Wireless LAN in 5 GHz subband
20 dBm	100 mW	IEEE 802.11b/g Wireless LAN in the 2.4 GHz band Bluetooth Class 1
10 dBm	10 mW	Bluetooth Low Energy LoRa

Table 1: Typical power levels (source: wikipedia.org)