CIC0203 - COMPUTAÇÃO EXPERIMENTAL - TA (2021.1 - 35T23)

Conceitos Centrais em Estatística Inferencial

Prof. Jorge H C Fernandes

Conceitos Centrais em Estatística Inferencial

- Fundamentos da Estatística Geral
  - Dados
- Estatística Inferencial
  - P-value – probability of an alpha-error
  - Alpha – level of significance
  - Critical value
  - Test statistic

Test statistic

https://www.youtube.com/watch?v=2fz0STTfIIU

Keys to Understanding (KTUs)

1. A Statistic is a numerical property calculated from Sample data. A Test Statistic is one which has an associated Probability Distribution.

2. There are four commonly used Test Statistics: z, t, F, and χ^2 (Chi-Square). They are used in a variety of tests in Inferential Statistics.

3. A higher value for the Test Statistic tells us that the Sample is likely to be more accurate as a representative of the Population or Process as a whole.

4. The calculated value for a Test Statistic is a point on the horizontal axis of the Test Statistic's Distribution. It marks the boundary for p, the Probability of an Alpha Error.

5. If Test Statistic is $\geq$ Critical Value (this is statistically identical to $p \leq \alpha$), we conclude that there is a Statistically Significant difference, change, or effect. That is, we Reject the Null Hypothesis.

Test statistic

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Keys to Understanding (KTUs)

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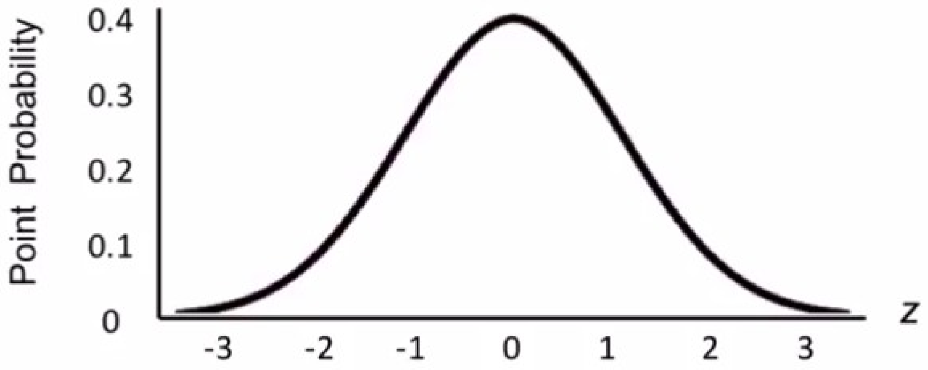
Z-distribution
Normal distribution

The z Distribution



Test statistic

https://www.youtube.com/watch?v=2fz0STTfIIU

2. There are four commonly used Test Statistics: *z*, *t*, *F*, and χ^2 (Chi-Square). They are used in a variety of tests in Inferential Statistics.

Test Statistic	Used for
z	Comparing Proportions, Comparing Means
t	Comparing Means
F	Comparing Variances
χ^2	Comparing Variances, Determining Independence, Determining Goodness of Fit

Test statistic

https://www.youtube.com/watch?v=2fz0STTfIIU

z is the Test Statistic associated with the Standard Normal Distribution

Uses

1. Given a value for x, what is the Probability of exceeding (or being less than) x.
2. With Proportions
3. With Means, when
   - The Population or Process Standard Deviation is known, and
   - The Sample Size is large

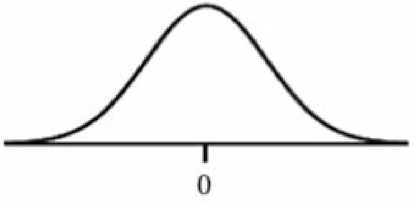
Use z only when you know the population's variance is Known and n is large

Test statistic

https://www.youtube.com/watch?v=2fz0STTfIIU

t is the preferred Test Statistic for use with Means

- The *t*-Distributions are bell-shaped curves.
- *t* self-adjusts for Sample Size
- As *n* grows larger, the t-Distribution converges to the z Distribution.



t-test	Means being compared
1- Sample	Sample Mean to a specified Mean
2-Sample	Means of Samples from 2 different Populations or Processes
Paired	Mean of the differences in pairs of measurements to a Mean of zero

Test statistic

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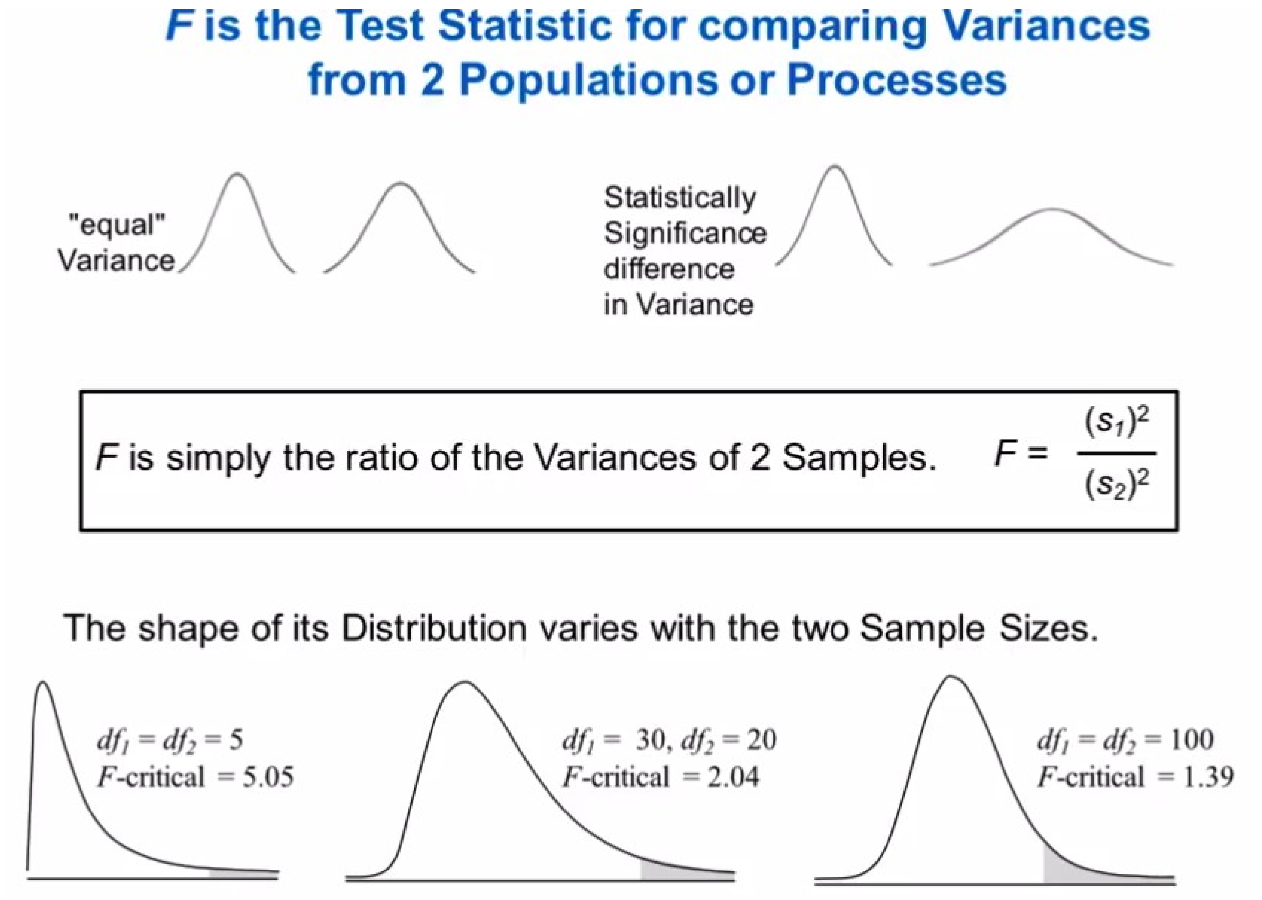


Test statistic

https://www.youtube.com/watch?v=2fz0STTfIIU

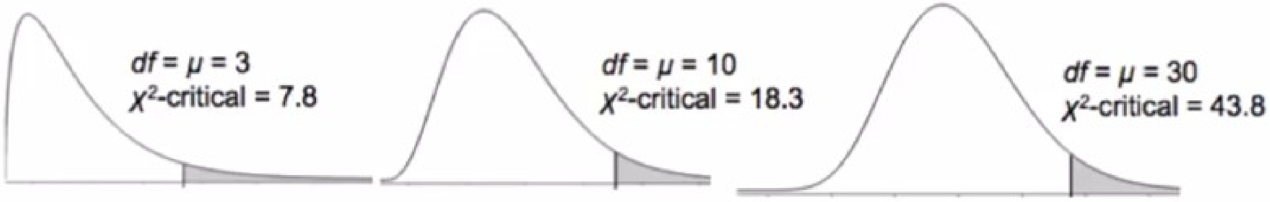
Chi-Square (χ^2) is a very versatile Test Statistic used in 3 Tests.

Chi-Square can be used with different types of data –
Continuous, Discrete/ Count, non-Normal, and Categorical.

<u>Chi-Square Tests</u>

- for the Variance
- for Goodness of Fit
- for Independence

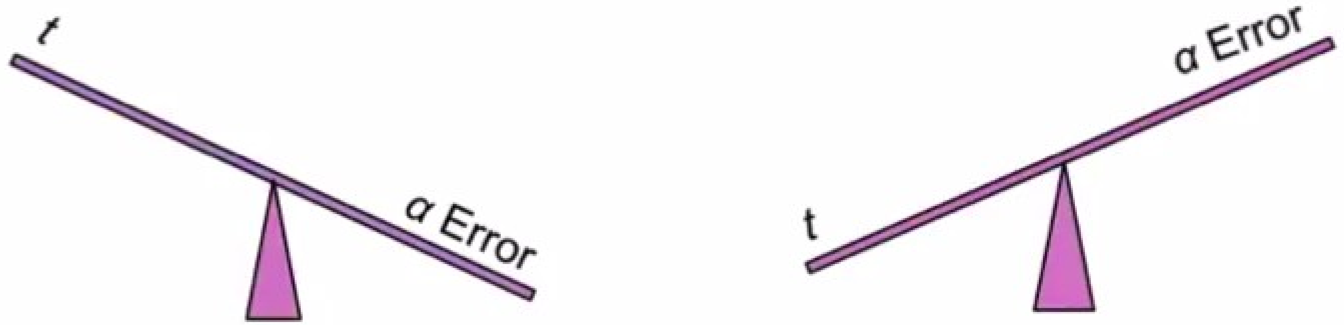
There is a different Distribution for each value of Degrees of Freedom.



Test statistic

https://www.youtube.com/watch?v=2fz0STTfIIU

3. A <u>higher value</u> for the Test Statistic tells us that the Sample is likely to be <u>more accurate</u> as a representative of the Population or Process as a whole.



Test statistic

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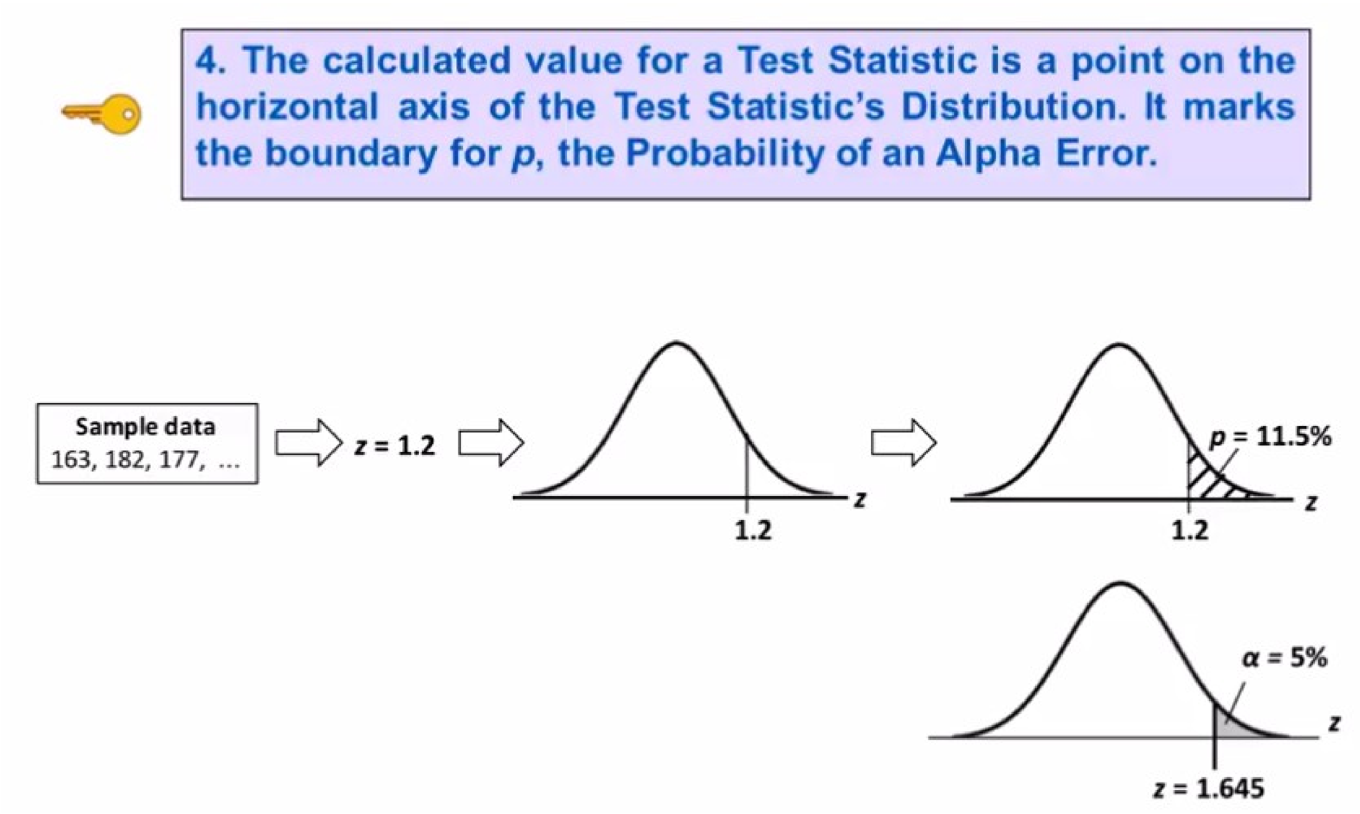

Test statistic

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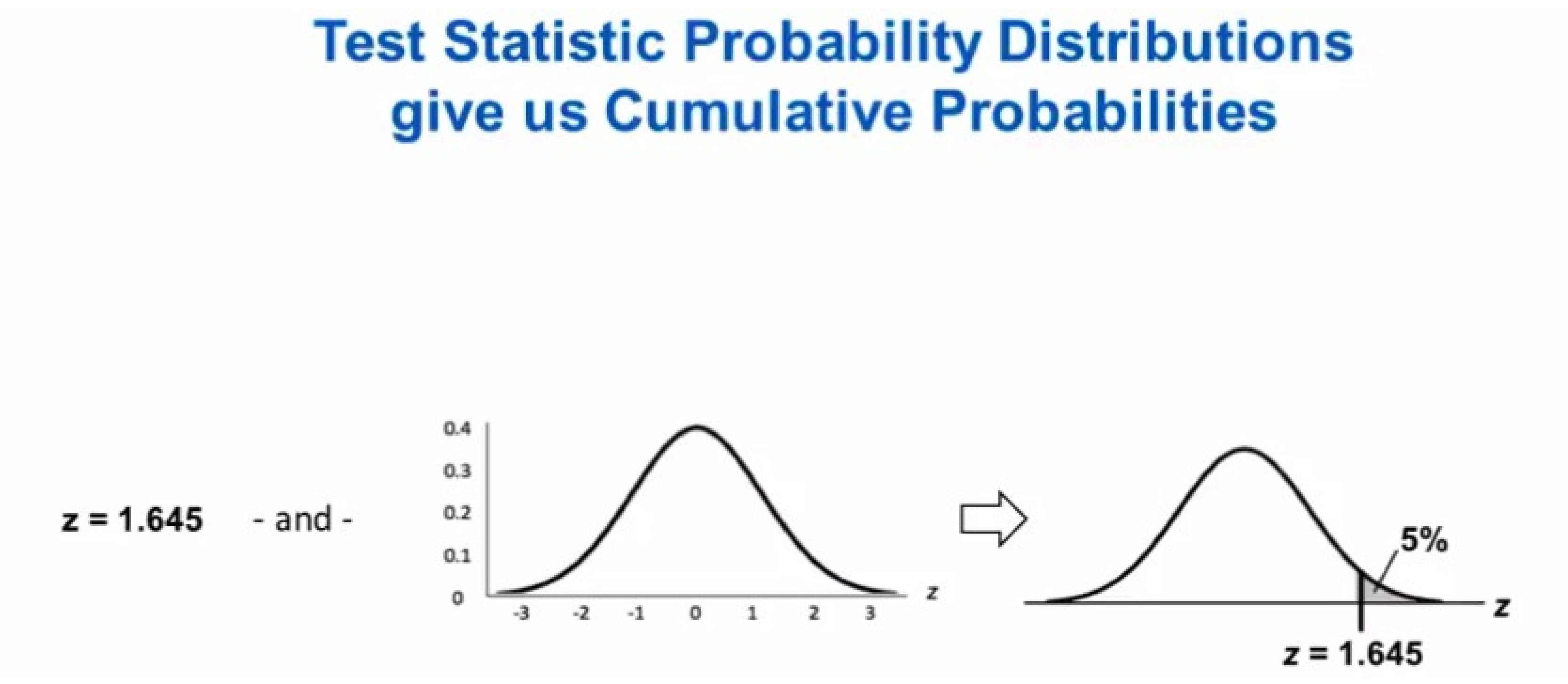
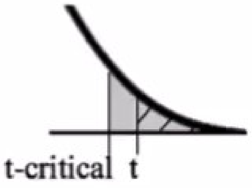


Test statistic

https://www.youtube.com/watch?v=2fz0STTfIIU

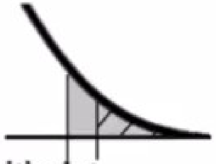
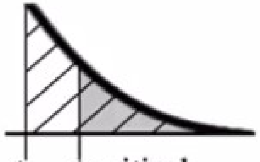
5. If Test Statistic is ≥ Critical Value (this is statistically identical to $p \leq \alpha$), **we conclude that there is a Statistically Significant difference, change, or effect.** That is, we Reject the Null Hypothesis.

<u>Areas under the curve (right tail)</u> Fail to Reject Region: α, the Rejection Region: p:	 $t \geq$ t-critical $p \leq \alpha$ (p is entirely within the Rejection Region)
Null Hypothesis	Reject
Any difference, change, or effect observed in the Sample data is:	Statistically Significant

Test statistic

https://www.youtube.com/watch?v=2fz0STTfIIU

Conversely, if Test Statistic is < Critical Value (this is statistically identical to $p > \alpha$), **we conclude that there is <u>not</u> a Statistically Significant difference, change, or effect.** That is, we Fail to Reject the Null Hypothesis.

<u>Areas under the curve (right tail)</u> Fail to Reject Region: ▭ α, the Rejection Region: ▭ (shaded) p: ▭ (hatched)	 t-critical t	 t t-critical
	$t \geq t$-critical $p \leq \alpha$ (p is entirely within the Rejection Region)	$t < t$-critical $p > \alpha$ (p extends into the Fail-to-Reject Region)
Null Hypothesis	Reject	Fail To Reject
Any difference, change, or effect observed in the Sample data is:	Statistically Significant	<u>Not</u> Statistically Significant

Alpha, critical value, and test statistic, and how they work together
https://www.youtube.com/watch?v=SAYGehjRZ8E

Alpha, critical value, and test statistic, and how they work together
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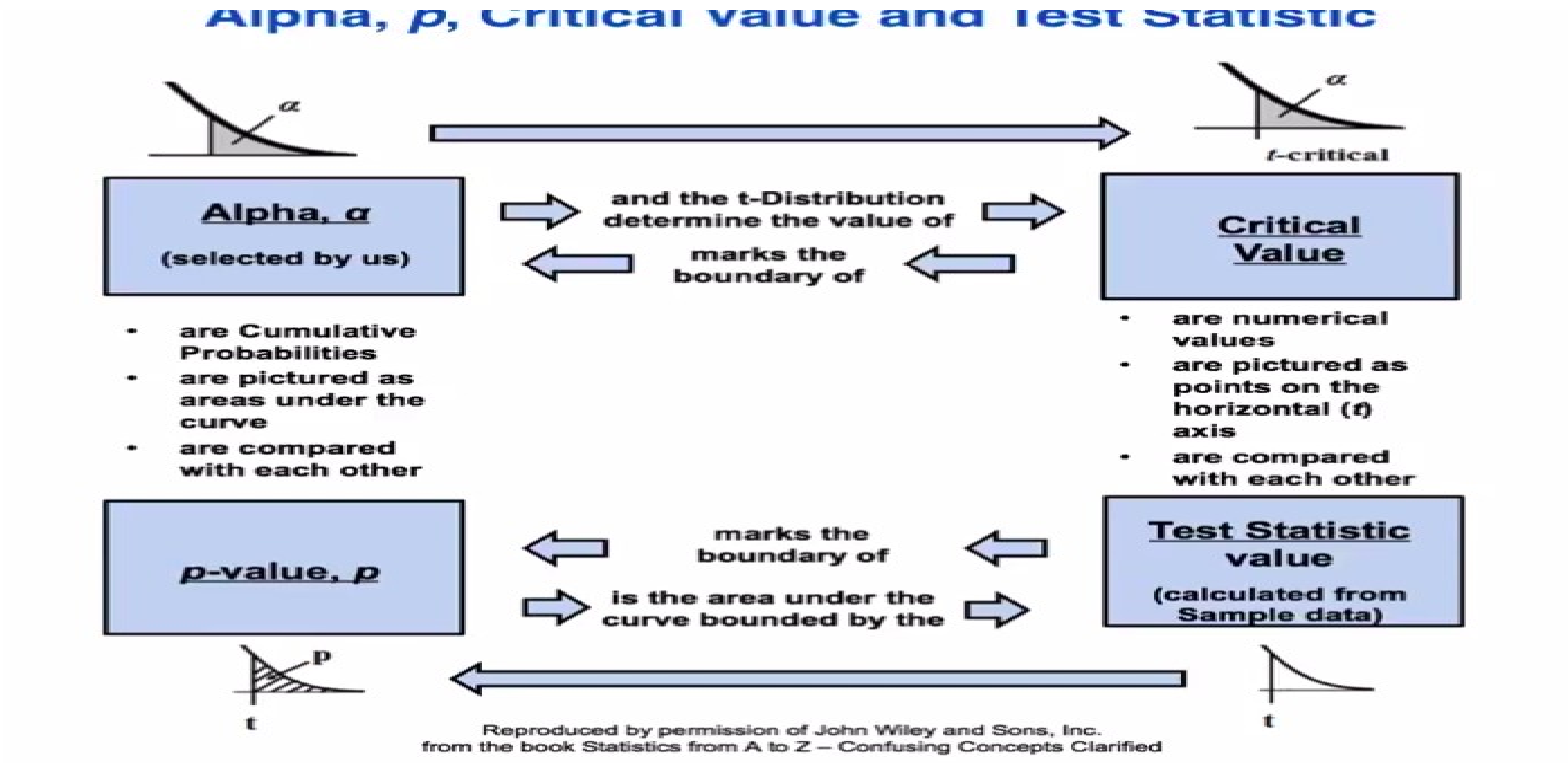


Alpha, critical value, and test statistic, and how they work together https://www.youtube.com/watch?v=SAYGehjRZ8E

- Alpha and p are cumulative probability, and are pictured as an area under a curve of the distribution of the test statistic
- Alpha is selected by the tester
- Alpha error is a false positive
- P is the probability of an alpha error
- If p <= a the reject the null hypothesis

Alpha, critical value, and test statistic, and how they work together https://www.youtube.com/watch?v=SAYGehjRZ8E

Alpha, p, Critical Value and Test Statistic

	Alpha, α	p	Critical Value of Test Statistic	Test Statistic value
What is it?	a Cumulative Probability		a value of the Test Statistic	
How is it pictured?	an area under the curve of the Distribution of the Test Statistic		a point on the horizontal axis of the Distribution of the Test Statistic	
Boundary	Critical Value marks its boundary	Test Statistic value marks its boundary	Forms the boundary for Alpha	Forms the boundary for p
How is its value determined?	Selected by the tester	area bounded by the Test Statistic value	boundary of the Alpha area	calculated from Sample Data
Compared with	p	α	Test Statistic value	Critical Value of Test Statistic
Statistically Significant/ Reject the Null Hypothesis if	$p \leq \alpha$		Test Statistic value ≥ Critical Value e.g., $z \geq$ z-critical	

Alpha, critical value, and test statistic, and how they work together https://www.youtube.com/watch?v=SAYGehjRZ8E

- Critical value of a test statistic and Test statistic value are numerical values. A point in the horizontal axis of the probability distribution
- Test statistic is calculated from sample data

Alpha and beta errors

https://www.youtube.com/watch?v=6gfSa8HFEXY

- In any inferential statistical analysis
- Type i error = alpha error
- Type ii error = beta error

Alpha and beta errors

https://www.youtube.com/watch?v=6gfSa8HFEXY

Keys to Understanding

1. **There is a risk of an Alpha** (aka Type I) **Error or a Beta** (aka Type II) **Error in any Inferential Statistical analysis.**

2. **(Comparison table of Alpha Error vs Beta Error)**

3. **There is a tradeoff between Alpha and Beta Errors.**

4. **To reduce both Alpha and Beta Errors, increase the Sample Size.**

Alpha and beta errors

https://www.youtube.com/watch?v=6gfSa8HFEXY

Tradeoffs between Alpha and Beta Errors.

Situation	Consequence of an Alpha Error (False Positive)	Consequence of a Beta Error (False Negative)	Wise choice for level of risk	
			Alpha Error (risk of False Positive)	Beta Error (risk of False Negative)
Airport Security	Detain an innocent person as a terrorist	Let a terrorist on board	higher	lower
Inspect critical components for jet engine	Reject a good component	Engine failure	higher	lower
Inspect painting on the underside of a wheelbarrow	Cost of a reject	Customer will probably not notice or care	lower	higher

Alpha and beta errors

https://www.youtube.com/watch?v=6gfSa8HFEXY

1. There is a risk of an Alpha (aka Type I) **Error or a Beta** (aka Type II) **Error in any Inferential Statistical analysis.**

Descriptive Statistics: we have all the data for the entire universe we wish to observe.

So we can calculate properties like the Mean or Standard Deviation

Inferential Statistics: we don't have all the data, we only have a Sample of the data.

- We can only calculate a Sample Statistic like the Sample Mean or Standard Deviation
- As a result, there will always be chance for error -- the difference between the Sample Statistic and the actual value of the property for the universe of data.
- This is called the Sampling Error

Alpha and beta errors

https://www.youtube.com/watch?v=6gfSa8HFEXY

2.	**Alpha Error (False Positive)**	**Beta Error (False Negative)**
	I saw a unicorn. 	**Smoking doesn't cause cancer.** 
What it is	**The error of concluding that there is something** – a difference, or a change, or an effect – **when, in reality, there is not.**	**The error of concluding that there is nothing** – no difference, or no change, or no effect – **when, in reality, there is.**
In Hypothesis Testing	The error of Rejecting the Null Hypothesis when it is true.	The error of Failing to Reject the Null Hypothesis when it is false.
Also known as	Type I Error, Error of the First Kind Colloquially: **False Positive**, False Alarm, Crying Wolf	Type II Error, Error of the Second Kind, **False Negative**
Found in:	Hypothesis Testing and Confidence Levels, *t*-tests, ANOVA, ANOM, etc.	
Example: in blood tests	Indicate a disease in a healthy person.	Fail to find a disease that exists.
Probability of the error	p	β (Beta)

Distribuições estatísticas
o que são?
como são usadas?
quais usar?

Distribuições estatísticas
Situações de uso
https://www.youtube.com/watch?v=Tum2xEaMsEs

Distribution	Data	Probability Curve
Binomial Hypergeometric Poisson	Discrete	Discrete
Exponential Normal *t*	Continuous	Continuous
F Chi-Square	Both	Continuous

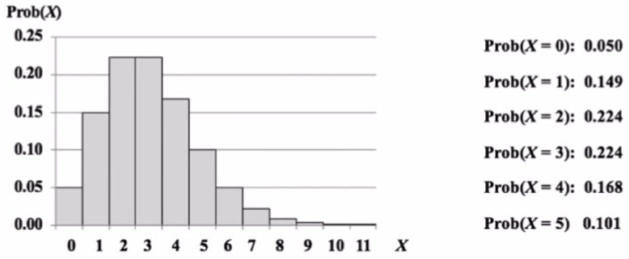
Tipos de medidas estatísticas

- Tendência central
  - Média, moda, mediana
- Variação/ dispersão
  - Desvio padrão, variação, range
- Forma
  - Desvio e kurtose

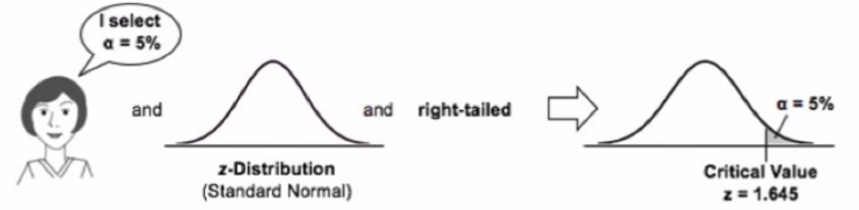
Distribuições estatísticas

https://www.youtube.com/watch?v=Tum2xEaMsEs

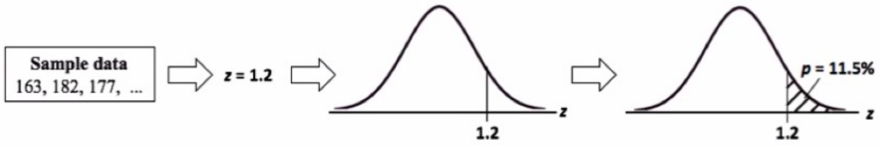
- Discretas
  - Binomial, hipergeométrica, poisson
    - Indica a probabilidade de ocorrência de um resultado



-
- Contínuas, associadas com estatística de teste
  - z- (normal), t-, f- e chi-quadrado
    - Dado um valor da probabilidade cumulativa, alpha, apresenta um valor crítico (ex: z) para a estatística do teste, a partir da qual a hipótese nula é refutada
    -
    -



- Contínuas
  - Dado um valor calculado para a estatística do teste, nos dá a probabilidade cumulativa, o valor p



Distribuições estatísticas/Distribuições de probabilidades
Qual usar?

https://www.youtube.com/watch?v=Tum2xEaMsEs

			Distribution
Continuous Data, Continuous Distribution			
Compare 2 Means			
	Population or Process Standard Deviation is not known		t
	Population or Process Standard Deviation is known.		
		Sample Size < 30	t
		Sample Size ≥ 30	t or Normal (z)
Compare Variances			
	Two Sample Variances		F
	Sample Variance to specified Variance		Chi-Square
Involves time to an event or between events			Exponential
Discrete/ Count Data, Discrete Distribution "What is the Probability of ...?"			
Occurrences are counted			Poisson
Units are counted			
	Sampling Without Replacement		Hypergeometric
	Sampling With Replacement, and other criteria met		Binomial
Discrete/ Count Data, Continuous Distribution			
Compare Observed to Expected Counts			Chi-Square
Compare 2 or more Proportions			Chi-Square
Compare 2 Proportions			Normal (z)

Chi-square test statistics and its distributions

Chi-square (x^2) statistics – and its distributions
https://www.youtube.com/watch?v=RJMNkzuxOA4

Keys to Understanding (KTUs)

1. **Chi-Square, χ^2, is a Test Statistic** which is **very versatile in the types of data it can handle:** Discrete, Continuous, non-Normal, Categorical.

2. **There is a different Chi-Square Distribution for each value of Degrees of Freedom** (df).
   - In each case, **the Distribution's Mean is equal to the Degrees of Freedom (μ = df).**
   - **For larger values of Degrees of Freedom, the Distributions move to the right, they become more symmetrical, Critical Values increase** (move to the right), **the Variances increase** (the Spread becomes wider).

3. Furthermore, **for All Chi-Square Distributions:**
   - the **Mode = df – 2** (for df ≥ 3)
   - the **Variance = 2df**; **Range: χ^2 = 0 to Infinity**
   - they **approach, but never touch the horizontal axis** as they extend to the right.
   - they **are not symmetrical** – they are skewed toward the right tail

4. Since Chi-Square Distributions are not symmetrical, **there are two different Critical Values for a 2-sided Chi-Square test.**

5. **Chi-Square is used in Inferential Statistics to analyze Variances via 3 different Chi-Square Tests: for the Variance, for Independence, and for Goodness of Fit.**

Chi-square (x^2) statistics – and its distributions
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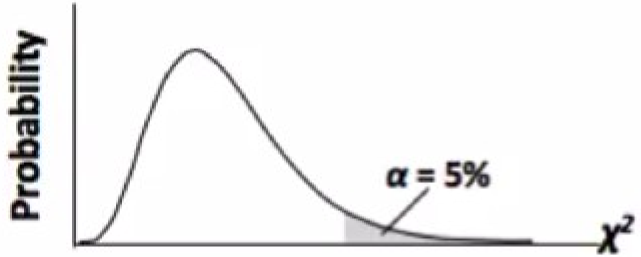
1. Chi-Square, χ^2, is a Test Statistic which is **very versatile in the types of data it can handle:** Discrete, Continuous, non-Normal.



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Chi-Square – the Test Statistic and Its Distributions

Keys to Understanding (KTUs)

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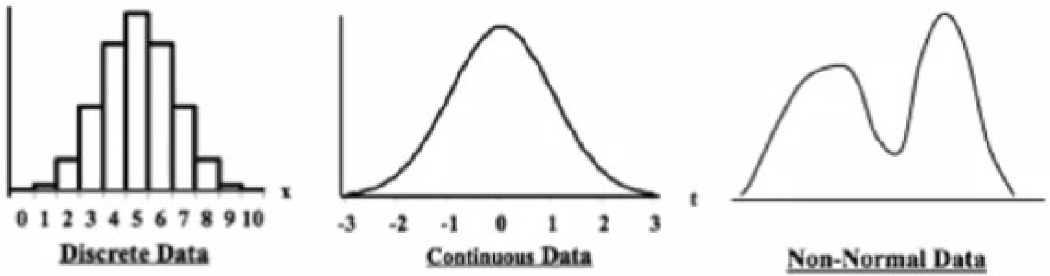

Categorical Variable → Gender; ↓ Ice Cream

Ice Cream	Gender: f	m
chocolate	60	20
vanilla	25	30
strawberry	15	50

Chi-square (x^2) statistics – and its distributions
https://www.youtube.com/watch?v=RJMNkzuxOA4

2. There is a different Chi-Square Distribution for each value of Degrees of Freedom (df).

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Chi-Square Test	df	Explanation
for Goodness of Fit	n - 1	n: # bins, columns
for Independence	(r – 1)(c – 1)	r and c: # of rows & columns
for Variance	n - 1	n: Sample Size

Chi-square (x^2) statistics – and its distributions
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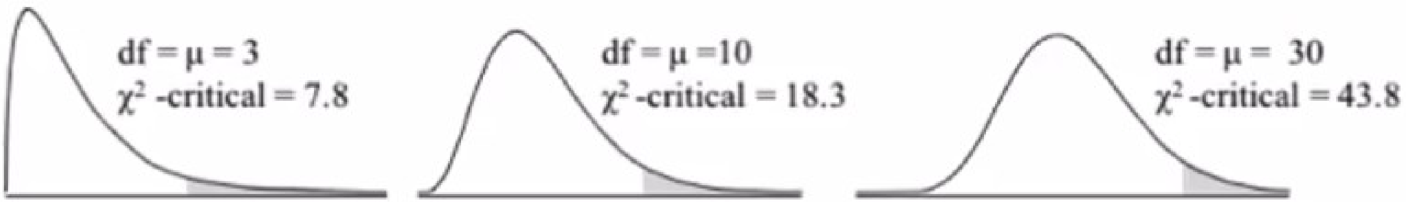

Chi-square (x^2) statistics – and its distributions
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3. Furthermore, **for All Chi-Square Distributions:**

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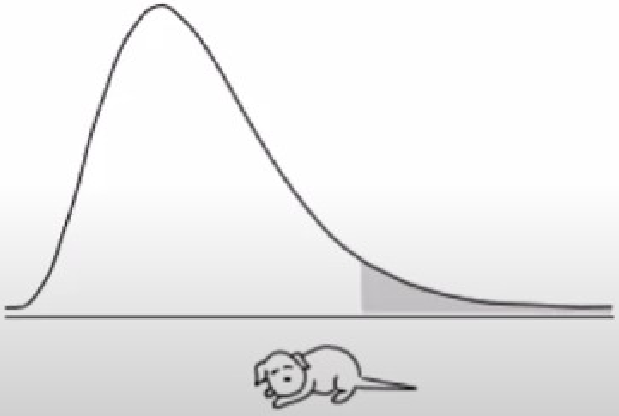

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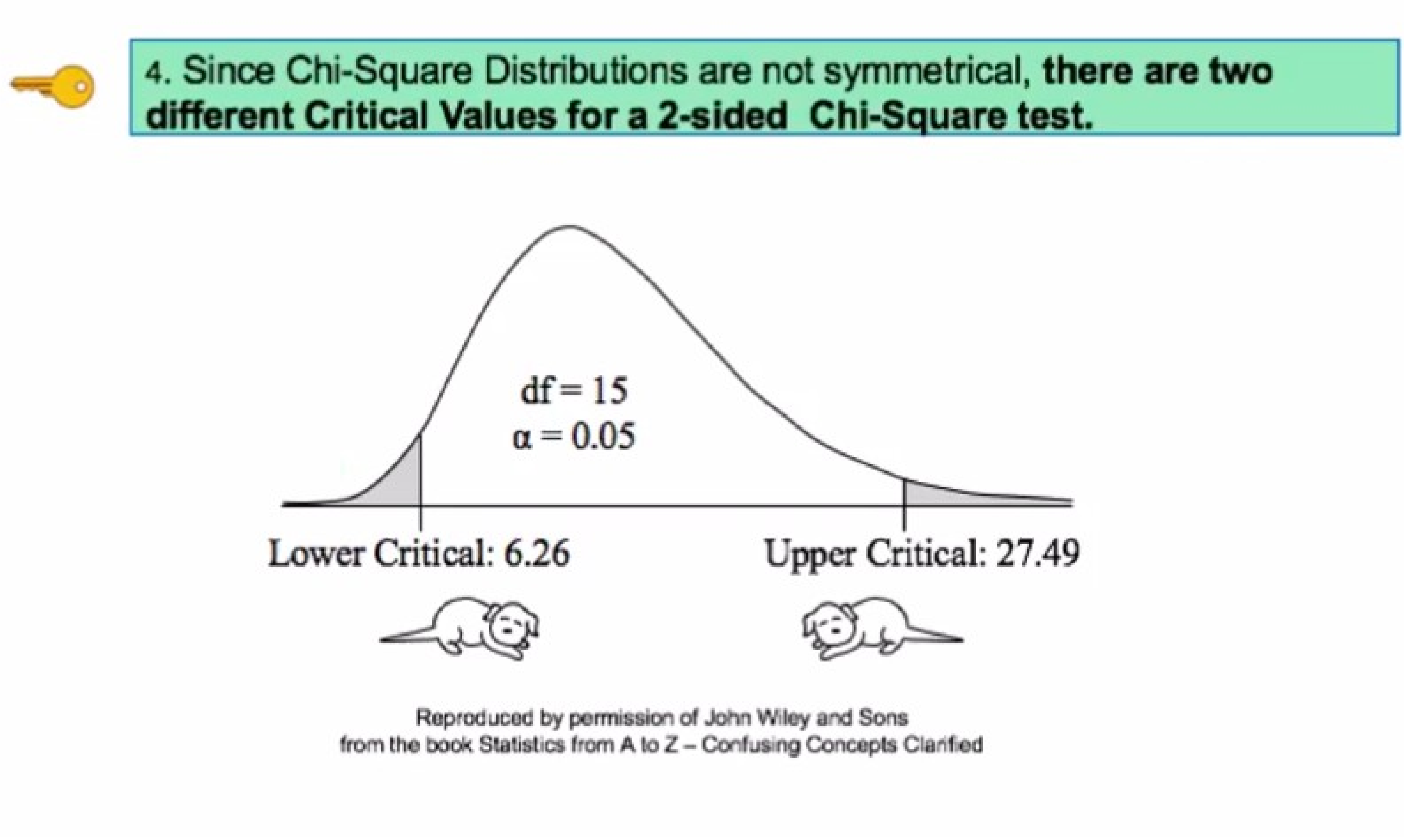
Chi-square (x^2) statistics – and its distributions
https://www.youtube.com/watch?v=RJMNkzuxOA4



Chi-square (x^2) statistics – and its distributions https://www.youtube.com/watch?v=RJMNkzuxOA4

5. Chi-Square is used in Inferential Statistics to analyze Variances via 3 different **Chi-Square Tests**: **for the Variance,** for Independence, and for Goodness of Fit.

We Want to Compare	the Statistic	Test to Use
the Calculated value of a Sample Statistic with a value we specify	**Variance**	**Chi-Square Test for the Variance**
	Mean	1-Sample t-test
Calculated values of Statistics from 2 different Samples	**Variance**	**F-test**
	Mean	2-Sample t-test

Chi-square (x^2) statistics – and its distributions
https://www.youtube.com/watch?v=RJMNkzuxOA4

5. Chi-Square is used in Inferential Statistics to analyze Variances via 3 different Chi-Square Tests: for the Variance, **for Independence**, and for Goodness of Fit.

		Gender	
		f	m
Ice Cream	chocolate	60	20
	vanilla	25	30
	strawberry	15	50

Variable

values of the Variable

Counts

values of the Variable

Counts

Chi-square (x^2) statistics – and its distributions
https://www.youtube.com/watch?v=RJMNkzuxOA4

5. Chi-Square is used in Inferential Statistics to analyze Variances via 3 different Chi-Square Tests: for the Variance, for Independence, and **for Goodness of Fit.**

	Monday	Tuesday	Wed.	Thursday	Friday	Saturday
Expected Frequencies	102.5	102.5	102.5	102.5	246	164
Observed Counts	98	112	91	102	244	160

Chi-square (x^2) test for variance

Chi-square (x^2) test for variance
https://www.youtube.com/watch?v=MnnOK0I2PLE

	Chi-Square Test for the Variance	F-test
Compares	Variance from 1 Population or Process to a Specified Variance.	Variances from 2 Populations or Processes
Analogous t-test (comparing Means)	1-Sample	2-Sample
Data restriction	**none**	**roughly Normal**

Chi-square (x^2) test for variance
https://www.youtube.com/watch?v=MnnOK0I2PLE

6- Step Procedure
for performing the Chi-Square Test for the Variance

Step 1: If Hypothesis Testing is being used, we would state the **Null Hypothesis (H_0)**

Type of Test	Null Hypothesis
2-sided	Population or Process Variance = Specified Variance
1-sided, left tailed	Population or Process Variance ≥ Specified Variance
1-sided, right tailed	Population or Process Variance ≤ Specified Variance

Chi-square (x^2) test for variance
https://www.youtube.com/watch?v=MnnOK0I2PLE

Step 2: **Select a** value for the **Significance Level** (most commonly, $\alpha = 0.05$).

Step 3: **Collect a Sample of data**, size ***n*.**

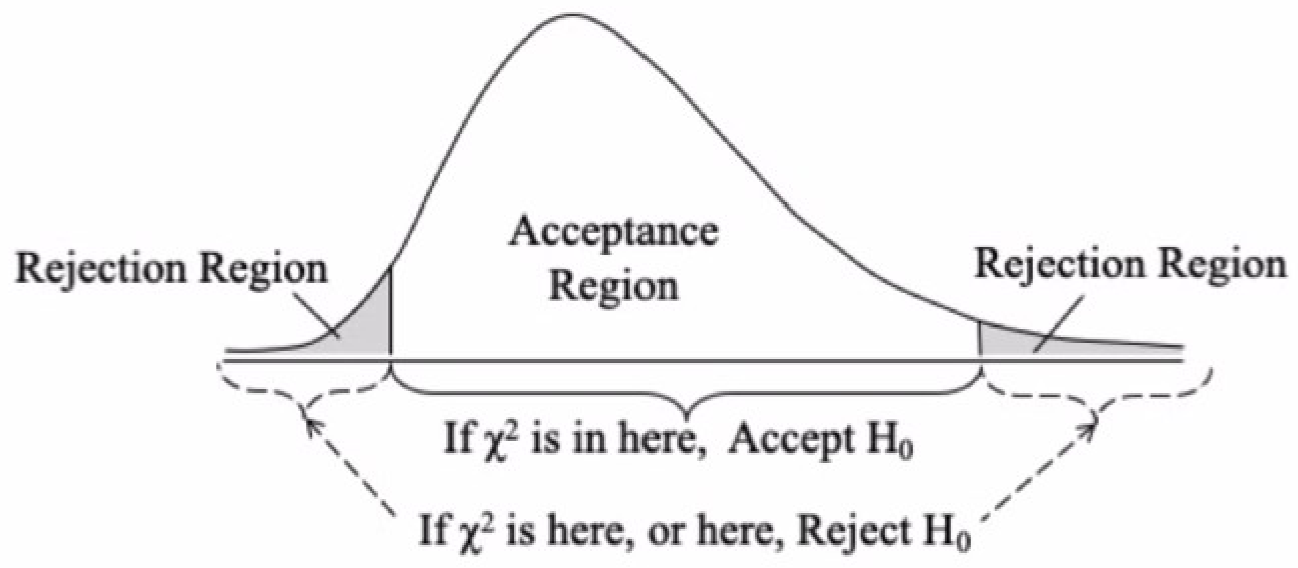
Step 4: **Determine the Degrees of Freedom; df** = ***n* – 1.**

Step 5: **Calculate the Critical Value(s)**

- 2-sided test would have 2 different Critical Values

Chi-square (x^2) test for variance
https://www.youtube.com/watch?v=MnnOK0I2PLE

Step 6: **Compare** either χ^2 **to** χ^2**-critical(s) or** compare ***p*** **to** α.



Distribuições estatísticas/Distribuições de probabilidades
Qual usar?
https://www.youtube.com/watch?v=Tum2xEaMsEs

- T-statistics – t-test
  - Used for comparation of means
    - 1-sample t-test
    - 2 -sample t-test
    - Paired
- F-distribution – f-test
- Chi-square distribution – chi-square test

Distribuições estatísticas

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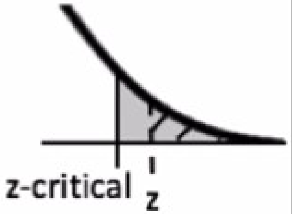
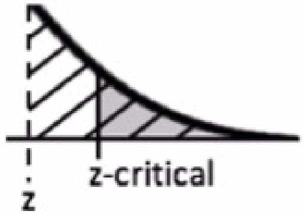
Areas under the curve (right tail) α: p:		
	$p < \alpha$ $z >$ z-critical	$p > \alpha$ $z <$ z-critical
The observation from the Sample data is an accurate estimate for the Population or Process as a whole.	True	False
Null Hypothesis	Reject	Accept (Fail to Reject)
The observed difference, change, or effect is:	Statistically Significant	due to chance alone

f-Distribution

- O teste de hipótese com duas populações (1 e 2) compara duas variâncias
  - N_1 = número de amostras da população 1
  - N_2 = número de amostras da população 2
  - S_1^2 = variância da amostra da população 1
  - S_2^2 = variância da amostra da população 2
- Estatística do teste
  - F = s_1^2/s_2^2
- Graus de liberdade (DoF) = número de amostras -1
- DoD numerador = n_1 – 1
- DoD denominador = n_2 - 1

f-Distribution