

Home Work 2

- 1) a) Let the number of frames sent per frame be n bits

Given, the probability of failure as p (probability of success $1-p$)

Also, mentioned that probability of success in the next transmission as 1

Thus, for the total number of bits sent we need to sum up both of the above two cases

Number of bits sent when there is a success = $(1-p) * n$

Number of bits sent in case of failure in the first attempt but success in the next attempt = $p * 2n$
(as we resend n bits in case of failure)

Total number of bits sent = $(1-p) * n + p * 2n = 1 + p*n$

Let the total time for travel (Round Trip Time) be t

Throughput is defined as the number of bits sent per second.

Throughput = number of bits sent/RTT

$$= (1 + p * n)/t$$

- b) In this case, the probability of success in successive transmission is p (not 1 as in previous case)

Number of bits sent initially when there is a success = $(1-p) * n$

Number of bits sent in the first transmission when there is a failure in the initial attempt
= $(p * (1-p) * (n + n))$

Number of bits sent in the second transmission when there is a failure in the first attempt
= $(p * p * (1-p) * (n + n + n))$

Thus, total number of bits sent = $(1-p) * n + (p * (1-p) * (n + n)) + (p * p * (1-p) * (n + n + n)) + \dots$
= $(n * (1-p)) (1 + 2*p + 3*p*p + 4*p*p*p + \dots)$

Time taken for Round trip time = t

Throughput = $(n * (1-p)) (1 + 2*p + 3*p*p + 4*p*p*p + \dots)/t$

- 2) The four orthogonal code vectors are

$$A = [1 \ 2 \ 3 \ 4];$$

$$B = [-1 \ -4 \ 3 \ 0];$$

$$C = [-8 \ 2 \ 0 \ 1];$$

$$D = [-6 \ -102 \ -138 \ 156];$$

i.e., $A \cdot B = B \cdot C = C \cdot D = A \cdot D = B \cdot D = A \cdot C = 0$

Sender for transmitting $1 \rightarrow 1$ and for $0 \rightarrow -1$

For ex, if he wishes to send $(1 \ 0 \ 1 \ 0)$ to $(A \ B \ C \ D)$ he uses the following code vector:

$$1 * A + (-1) * B + 1 * C + (-1) * D$$

$$\begin{aligned} \text{Code Vector}(V) &= 1 * [1 \ 2 \ 3 \ 4] + (-1) * [-1 \ -4 \ 3 \ 0] + 1 * [-8 \ 2 \ 0 \ 1] + (-1) * [-6 \ -102 \ -138 \ 156] \\ &= [0 \ 110 \ 138 \ -151] \end{aligned}$$

All of A, B, C, D are transmitted the Code Vector. Each of them decode their value by doing a dot product of their respective vectors to the Code Vector. If the value is greater than zero, they decode the bit as 1, else they decode it as 0

A decodes bit sent to him by doing a dot product of his vector with Code Vector: $A(\text{dot})V$
 $= [1 \ 2 \ 3 \ 4] (\text{dot product}) [0 \ 110 \ 138 \ -151]$
 $= 30 > 0$
 $= 1$

B decodes bit sent to him by doing a dot product of his vector with Code Vector: $B(\text{dot})V$
 $= [-1 \ -4 \ 3 \ 0] (\text{dot product}) [0 \ 110 \ 138 \ -151]$
 $= -26 < 0$
 $= 0$

C decodes bit sent to him by doing a dot product of his vector with Code Vector: $C(\text{dot})V$
 $= [-8 \ 2 \ 0 \ 1] (\text{dot product}) [0 \ 110 \ 138 \ -151]$
 $= 69 > 0$
 $= 1$

D decodes bit sent to him by doing a dot product of his vector with Code Vector: $D(\text{dot})V$
 $= [-6 \ -102 \ -138 \ 156] (\text{dot product}) [0 \ 110 \ 138 \ -151]$
 $= -53820 < 0$
 $= 0$

Orthogonality comes in handy when decoding the bits by doing dot product

$A(\text{dot})V = A(\text{dot})(+1) * (A) + A(\text{dot})(-1) * (B) + A(\text{dot})(+1) * (C) + A(\text{dot})(-1) * (D)$
 $= A(\text{dot})(+1) * A$ (The rest all dot products are zero by orthogonality)
 $= (+1) A * A$

As $A * A$ is always greater than zero, the sign of $A(\text{dot})V$ gives us the code that sender wished to send. This can be proven similarly for the other users B, C, D.

Also, if the vectors are not orthogonal, it would be difficult to decode what sender wished to send as there would be values from the other dot products in $A(\text{dot})V$ calculation.

Assume a case where vectors are not orthogonal but are independent

$A = [1 \ 1]$, $B = [2 \ 3]$

$A(\text{dot})B$ is not equal to 0

Let's assume that sender wishes to send 1 to A, 0 to B

As per the above calculations he would send $(1) * A + (-1) * B$

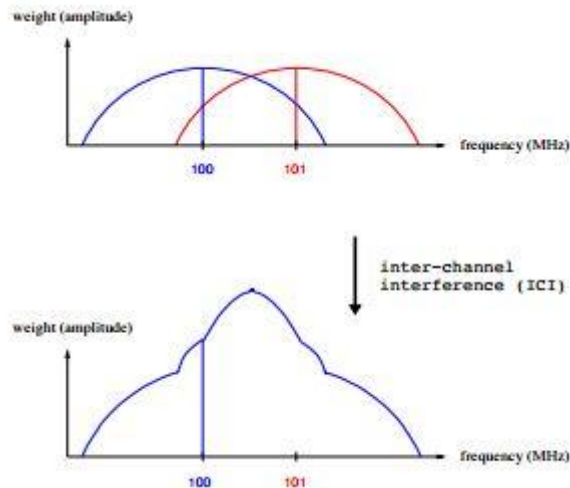
Code Vector (V): $[1 \ 1] + (-1)[2 \ 3] = [-1 \ -2]$

A decodes by: $A(\text{dot})V = [1 \ 1] \cdot [-1 \ -2] = -3 < 0$

Here, A assumes the bit sent as 0 which is not the case

Thus, decoding fails if the vectors are not orthogonal as there is interference from other vectors.

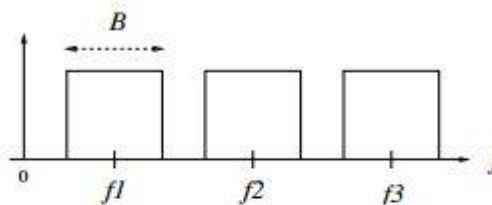
This is similar to the issue when sending bits using amplitude modulation (AM) of electromagnetic waves modeled as complex sinusoids as there is an interference of waves which leads to distortion – Inter Channel Distortion

*Inter Channel Interference*

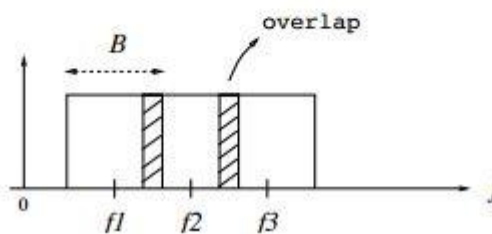
One solution to solve the above problem is to set a guard band between two frequencies, i.e., to put the carrier frequencies far apart so that there is no interference. The obvious problem with this approach would be the limit on the number of carrier frequencies that could be squeezed in a given frequency range.

Another solution would be to go for Orthogonal Frequency Division Multiple Access (OFDMA) where the sinusoids that are used for sending EM waves are mutually orthogonal. This solves the ICI problem (no interference between two different channels).

FDM:



OFDM:

*OFDM vs FDM*

- 3) Human auditory system is sensitive from 20Hz- 20kHz frequency range. And, the human speech frequency range lies in 300-3300Hz range. The Amplitude Modulated (AM radio) carrier frequencies are in the frequency range 535-1605 kHz. Each carrier frequency is assigned a bandwidth of 10kHz frequencies. This intrinsic bandwidth frequency limitation of AM waves makes it suitable only for transmitting waves within 10kHz range.

AM carrier waves are capable of transmitting waves within the speech range (4kHz) and hence they are preferably used for transmitting speech over the music which require much higher frequency range. There is also the problem of interference (atmospheric – tall building/metal structures, electrical noise – electric motors/lightning) that comes into play when using AM waves that generally hinders clear signal transmission. Also, there is no protection from the disturbance created by lightning, electrical equipment, and other sources of signal pollution. These are the primary reasons that make AM waves suitable only for speech transmissions.

As a communication system, AM radio uses radio waves for information transmission. An AM receiver detects amplitude variations in radio waves at a particular frequency. It then detects changes in the signal to amplify a speaker or an ear phone. AM radio is broadcasted on several frequency bands Long Frequency (LF), Medium Frequency (MF), Small Frequency (SF) with each serving different purposes. MF waves are the ones that everyone uses as AM radio (MF waves behave differently at night and day- at night they have much higher range when compared to day).

FDMA on the other hand doesn't have such small band width restrictions (much better would be OFDMA). They operate on EM waves and use amplitude, frequency and phase modulation for information transmissions. They have analog to digital, digital to analog converters that help the transmission process. Instead of broadcasting they are used for transmitting different messages to multiple users at a single time using their respective frequencies.

If the law mandated conversion of AM radio signals to digital it implies a better transmission of information through the signals. Digital sound technology was developed to reduce noise, electric disturbances that are otherwise present in AM/FM technologies. Digital which is much better than either of these thus would make the sound transmitted much better to listen to. This technology which is now in the developing stage has a lot of advance features that make it non receptive to the AM receivers used way back. Hence, the AM receivers in the car and home stereos need to be replaced to support such infrastructure and bandwidths.

Though Digital Audio Broadcasting (DAB) is better when compared to analogous systems there are some disadvantages that are associated with it – one of which is the audio quality. Broadcasters using DAB have the capability to squeeze in more stations than recommended which in general is done by minimizing the bit rate – reducing it to the lowest level of sound quality that humans can tolerate. Thus, though we have DAB up and running, it is not necessarily guaranteed that we will have better audio quality.