Last update: October 16, 2023 Original



Continued fractions

Continued fraction is a representation of a real number as a specific convergent sequence of rational numbers. They are useful in competitive programming because they are easy to compute and can be efficiently used to find the best possible rational approximation of the underlying real number (among all numbers whose denominator doesn't exceed a given value).

Besides that, continued fractions are closely related to Euclidean algorithm which makes them useful in a bunch of number-theoretical problems.

Continued fraction representation



Let $a_0, a_1, \ldots, a_k \in \mathbb{Z}$ and $a_1, a_2, \ldots, a_k \geq 1$. Then the expression

$$r=a_0+rac{1}{a_1+rac{1}{\cdots+rac{1}{a_L}}},$$

is called the $\operatorname{\textbf{continued}}$ fraction $\operatorname{\textbf{representation}}$ of the rational number r and is denoted shortly as $r = [a_0; a_1, a_2, \dots, a_k].$

Example

It can be proven that any rational number can be represented as a continued fraction in exactly 2 ways:

$$r = [a_0; a_1, \dots, a_k, 1] = [a_0; a_1, \dots, a_k + 1].$$

Moreover, the length k of such continued fraction is estimated as $k = O(\log \min(p,q))$ for $r = \frac{p}{a}$.

The reasoning behind this will be clear once we delve into the details of the continued fraction construction.

Definition

Let a_0,a_1,a_2,\ldots be an integer sequence such that $a_1,a_2,\cdots \geq 1$. Let $r_k=[a_0;a_1,\ldots,a_k]$. Then the expression

$$r=a_0+rac{1}{a_1+rac{1}{a_2+\dots}}=\lim_{k o\infty}r_k.$$

is called the **continued fraction representation** of the irrational number r and is denoted shortly as $r=[a_0;a_1,a_2,\ldots]$.

Note that for $r=[a_0;a_1,\ldots]$ and integer k, it holds that $r+k=[a_0+k;a_1,\ldots]$.

Another important observation is that $\frac{1}{r}=[0;a_0,a_1,\ldots]$ when $a_0>0$ and $\frac{1}{r}=[a_1;a_2,\ldots]$ when $a_0=0$.

Definition

In the definition above, rational numbers r_0, r_1, r_2, \ldots are called the **convergents** of r.

Correspondingly, individual $r_k=[a_0;a_1,\ldots,a_k]=rac{p_k}{q_k}$ is called the k-th **convergent** of r.

Example

Definition

Let $r_k=[a_0;a_1,\ldots,a_{k-1},a_k]$. The numbers $[a_0;a_1,\ldots,a_{k-1},t]$ for $1\leq t\leq a_k$ are called **semiconvergents**.

We will typically refer to (semi)convergents that are greater than r as **upper** (semi)convergents and to those that are less than r as **lower** (semi)convergents.

Definition

Complementary to convergents, we define the **complete quotients** as $s_k = [a_k; a_{k+1}, a_{k+2}, \ldots]$.

Correspondingly, we will call an individual s_k the k-th complete quotient of r.

From the definitions above, one can conclude that $s_k \geq 1$ for $k \geq 1$.

Treating $[a_0; a_1, \ldots, a_k]$ as a formal algebraic expression and allowing arbitrary real numbers instead of a_i , we obtain

$$r=[a_0;a_1,\ldots,a_{k-1},s_k].$$

In particular, $r=\left[s_{0}
ight]=s_{0}.$ On the other hand, we can express s_{k} as

$$s_k = [a_k; s_{k+1}] = a_k + rac{1}{s_{k+1}},$$

meaning that we can compute $a_k = \lfloor s_k \rfloor$ and $s_{k+1} = (s_k - a_k)^{-1}$ from s_k .

The sequence a_0,a_1,\ldots is well-defined unless $s_k=a_k$ which only happens when r is a rational number.

Thus the continued fraction representation is uniquely defined for any irrational number r.

Implementation

In the code snippets we will mostly assume finite continued fractions.

From s_k , the transition to s_{k+1} looks like

$$s_k = \lfloor s_k
floor + rac{1}{s_{k+1}}.$$

From this expression, the next complete quotient s_{k+1} is obtained as

$$s_{k+1} = (s_k - |s_k|)^{-1}.$$

For $s_k = rac{p}{q}$ it means that

$$s_{k+1} = \left(rac{p}{q} - \left\lfloor rac{p}{q}
ight
floor
ight)^{-1} = rac{q}{p - q \cdot \left\lfloor rac{p}{q}
ight
floor} = rac{q}{p mod q}.$$

Thus, the computation of a continued fraction representation for $r=\frac{p}{q}$ follows the steps of the Euclidean algorithm for p and q.

From this also follows that $\gcd(p_k,q_k)=1$ for $\frac{p_k}{q_k}=[a_0;a_1,\ldots,a_k]$. Hence, convergents are always irreducible.

C++

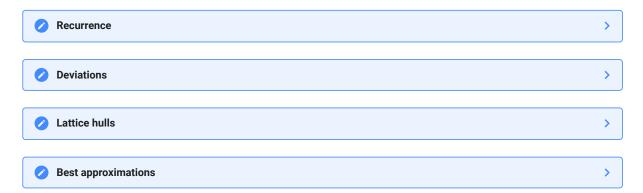
```
auto fraction(int p, int q) {
    vector<int> a;
    while(q) {
        a.push_back(p / q);
        tie(p, q) = make_pair(q, p % q);
    }
    return a;
}
```

Python

```
def fraction(p, q):
    a = []
    while q:
        a.append(p // q)
        p, q = q, p % q
    return a
```

Key results

To provide some motivation for further study of continued fraction, we give some key facts now.



The last fact allows to find the best rational approximations of r by checking its semiconvergents.

Below you will find the further explanation and a bit of intuition and interpretation for these facts.

Convergents

Let's take a closer look at the convergents that were defined earlier. For $r=[a_0,a_1,a_2,\ldots]$, its convergents are

$$egin{aligned} r_0 &= [a_0], \ r_1 &= [a_0, a_1], \ & \dots, \ r_k &= [a_0, a_1, \dots, a_k] \end{aligned}$$

Convergents are the core concept of continued fractions, so it is important to study their properties.

For the number r , its k-th convergent $r_k = rac{p_k}{q_k}$ can be computed as

$$r_k = rac{P_k(a_0,a_1,\ldots,a_k)}{P_{k-1}(a_1,\ldots,a_k)} = rac{a_k p_{k-1} + p_{k-2}}{a_k q_{k-1} + q_{k-2}},$$

where $P_k(a_0,\ldots,a_k)$ is the continuant, a multivariate polynomial defined as

$$P_k(x_0,x_1,\ldots,x_k) = \det egin{bmatrix} x_k & 1 & 0 & \ldots & 0 \ -1 & x_{k-1} & 1 & \ldots & 0 \ 0 & -1 & x_2 & \ldots & dots \ dots & dots & dots & \ddots & 1 \ 0 & 0 & \ldots & -1 & x_0 \ \end{bmatrix}$$

Thus, r_k is a weighted mediant of r_{k-1} and r_{k-2} .

For consistency, two additional convergents $r_{-1}=\frac{1}{0}$ and $r_{-2}=\frac{0}{1}$ are defined.



Implementation

We will compute the convergents as a pair of sequences $p_{-2}, p_{-1}, p_0, p_1, \ldots, p_k$ and $q_{-2}, q_{-1}, q_0, q_1, \ldots, q_k$:

C++

```
auto convergents(vector<int> a) {
    vector<int> p = {0, 1};
    vector<int> q = {1, 0};
    for(auto it: a) {
        p.push_back(p[p.size() - 1] * it + p[p.size() - 2]);
        q.push_back(q[q.size() - 1] * it + q[q.size() - 2]);
    }
    return make_pair(p, q);
}
```

Python

```
def convergents(a):
    p = [0, 1]
    q = [1, 0]
    for it in a:
        p.append(p[-1]*it + p[-2])
        q.append(q[-1]*it + q[-2])
    return p, q
```

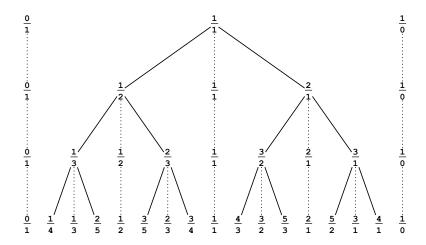
Trees of continued fractions

There are two major ways to unite all possible continued fractions into useful tree structures.

Stern-Brocot tree

The Stern-Brocot tree is a binary search tree that contains all distinct positive rational numbers.

The tree generally looks as follows:



The image by Aaron Rotenberg is licensed under CC BY-SA 3.0

Fractions $\frac{0}{1}$ and $\frac{1}{0}$ are "virtually" kept on the left and right sides of the tree correspondingly.

Then the fraction in a node is a mediant $\frac{a+c}{b+d}$ of two fractions $\frac{a}{b}$ and $\frac{c}{d}$ above it.

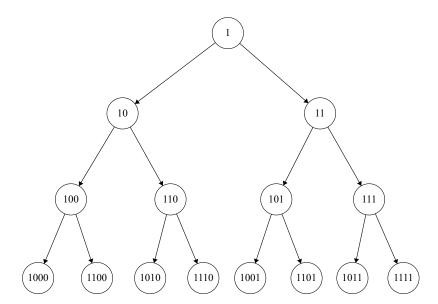
The recurrence $\frac{p_k}{q_k} = \frac{a_k p_{k-1} + p_{k-2}}{a_k q_{k-1} + q_{k-2}}$ means that the continued fraction representation encodes the path to $\frac{p_k}{q_k}$ in the tree. To find $[a_0; a_1, \ldots, a_k, 1]$, one has to make a_0 moves to the right, a_1 moves to the left, a_2 moves to the right and so on up to a_k .

The parent of $[a_0; a_1, \ldots, a_k, 1]$ then is the fraction obtained by taking one step back in the last used direction.

In other words, it is $[a_0;a_1,\ldots,a_k-1,1]$ when $a_k>1$ and $[a_0;a_1,\ldots,a_{k-1},1]$ when $a_k=1$.

Thus the children of $[a_0; a_1, \dots, a_k, 1]$ are $[a_0; a_1, \dots, a_k + 1, 1]$ and $[a_0; a_1, \dots, a_k, 1, 1]$.

Let's index the Stern-Brocot tree. The root vertex is assigned an index 1. Then for a vertex v, the index of its left child is assigned by changing the leading bit of v from 1 to 10 and for the right child, it's assigned by changing the leading bit from 1 to 11:



In this indexing, the continued fraction representation of a rational number specifies the run-length encoding of its binary index.

For $\frac{5}{2} = [2; 2] = [2; 1, 1]$, its index is 1011_2 and its run-length encoding, considering bits in the ascending order, is [2; 1, 1].

Another example is $\frac{2}{5}=[0;2,2]=[0;2,1,1]$, which has index 1100_2 and its run-length encoding is, indeed, [0;2,2]

It is worth noting that the Stern-Brocot tree is, in fact, a treap. That is, it is a binary search tree by $\frac{p}{q}$, but it is a heap by both p and q.

Comparing continued fractions

You're given $A=[a_0;a_1,\ldots,a_n]$ and $B=[b_0;b_1,\ldots,b_m]$. Which fraction is smaller?

Solution

Best inner point

You're given $\frac{0}{1} \leq \frac{p_0}{q_0} < \frac{p_1}{q_1} \leq \frac{1}{0}$. Find the rational number $\frac{p}{q}$ such that (q;p) is lexicographically smallest and $\frac{p_0}{q_0} < \frac{p}{q} < \frac{p_1}{q_1}$.

Solution >

H

GCJ 2019, Round 2 - New Elements: Part 2

You're given N positive integer pairs (C_i, J_i) . You need to find a positive integer pair (x, y) such that $C_i x + J_i y$ is a strictly increasing sequence.

Among such pairs, find the lexicographically minimum one.



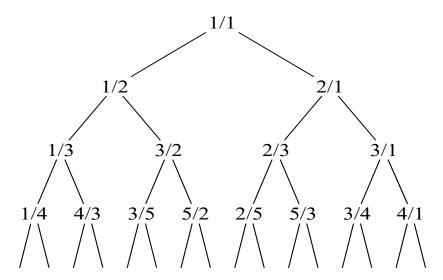
Solution

>

Calkin-Wilf tree

A somewhat simpler way to organize continued fractions in a binary tree is the Calkin-Wilf tree.

The tree generally looks like this:



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In the root of the tree, the number $\frac{1}{1}$ is located. Then, for the vertex with a number $\frac{p}{q}$, its children are $\frac{p}{p+q}$ and $\frac{p+q}{q}$.

Unlike the Stern-Brocot tree, the Calkin-Wilf tree is not a binary *search* tree, so it can't be used to perform rational binary search.

In the Calkin-Wilf tree, the direct parent of a fraction $\frac{p}{q}$ is $\frac{p-q}{q}$ when p>q and $\frac{p}{q-p}$ otherwise.

For the Stern-Brocot tree, we used the recurrence for convergents. To draw the connection between the continued fraction and the Calkin-Wilf tree, we should recall the recurrence for complete quotients. If $s_k = \frac{p}{q}$, then $s_{k+1} = \frac{q}{p \mod q} = \frac{q}{p-\lfloor p/q \rfloor \cdot q}$.

On the other hand, if we repeatedly go from $s_k=\frac{p}{q}$ to its parent in the Calkin-Wilf tree when p>q, we will end up in $\frac{p\mod q}{q}=\frac{1}{s_{k+1}}$. If we continue doing so, we will end up in s_{k+2} , then $\frac{1}{s_{k+3}}$ and so on. From this we can deduce that:

1. When $a_0>0$, the direct parent of $[a_0;a_1,\ldots,a_k]$ in the Calkin-Wilf tree is $rac{p-q}{q}=[a_0-1;a_1,\ldots,a_k]$.

- 2. When $a_0=0$ and $a_1>1$, its direct parent is $rac{p}{q-p}=[0;a_1-1,a_2,\ldots,a_k]$
- 3. And when $a_0=0$ and $a_1=1$, its direct parent is $rac{p}{q-p}=[a_2;a_3,\ldots,a_k].$

Correspondingly, children of $rac{p}{q}=[a_0;a_1,\ldots,a_k]$ are

1.
$$rac{p+q}{q}=1+rac{p}{q}$$
 , which is $[a_0+1;a_1,\ldots,a_k]$,

2.
$$rac{p}{p+q}=rac{1}{1+rac{p}{2}}$$
 , which is $[0,1,a_0,a_1,\ldots,a_k]$ for $a_0>0$ and $[0,a_1+1,a_2,\ldots,a_k]$ for $a_0=0$.

Noteworthy, if we enumerate vertices of the Calkin-Wilf tree in the breadth-first search order (that is, the root has a number 1, and the children of the vertex v have indices 2v and 2v+1 correspondingly), the index of the rational number in the Calkin-Wilf tree would be the same as in the Stern-Brocot tree.

Thus, numbers on the same levels of the Stern-Brocot tree and the Calkin-Wilf tree are the same, but their ordering differs through the bit-reversal permutation.

Convergence

For the number r and its k-th convergent $r_k = \frac{p_k}{q_k}$ the following formula stands:

$$r_k = a_0 + \sum_{i=1}^k rac{(-1)^{i-1}}{q_i q_{i-1}}.$$

In particular, it means that

$$r_k - r_{k-1} = rac{(-1)^{k-1}}{q_k q_{k-1}}$$

and

$$p_k q_{k-1} - p_{k-1} q_k = (-1)^{k-1}.$$

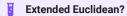
From this we can conclude that

$$\left|r-rac{p_k}{q_k}
ight| \leq rac{1}{q_{k+1}q_k} \leq rac{1}{q_k^2}.$$

The latter inequality is due to the fact that r_k and r_{k+1} are generally located on different sides of r, thus

$$|r-r_k| = |r_k-r_{k+1}| - |r-r_{k+1}| \le |r_k-r_{k+1}|.$$

Detailed explanation



You're given $A,B,C\in\mathbb{Z}$. Find $x,y\in\mathbb{Z}$ such that Ax+By=C.

Solution

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Linear fractional transformations

Another important concept for continued fractions are the so-called linear fractional transformations.

Definition

A linear fractional transformation is a function $f:\mathbb{R} o\mathbb{R}$ such that $f(x)=rac{ax+b}{cx+d}$ for some $a,b,c,d\in\mathbb{R}$.

A composition $(L_0 \circ L_1)(x) = L_0(L_1(x))$ of linear fractional transforms $L_0(x) = \frac{a_0x + b_0}{c_0x + d_0}$ and $L_1(x) = \frac{a_1x + b_1}{c_1x + d_1}$ is itself a linear fractional transform:

$$\frac{a_0\frac{a_1x+b_1}{c_1x+d_1}+b_0}{c_0\frac{a_1x+b_1}{c_1x+d_1}+d_0} = \frac{a_0(a_1x+b_1)+b_0(c_1x+d_1)}{c_0(a_1x+b_1)+d_0(c_1x+d_1)} = \frac{(a_0a_1+b_0c_1)x+(a_0b_1+b_0d_1)}{(c_0a_1+d_0c_1)x+(c_0b_1+d_0d_1)}.$$

Inverse of a linear fractional transform, is also a linear fractional transform:

$$y = \frac{ax+b}{cx+d} \iff y(cx+d) = ax+b \iff x = -\frac{dy-b}{cy-a}.$$

DMOPC '19 Contest 7 P4 - Bob and Continued Fractions

You're given an array of positive integers a_1, \ldots, a_n . You need to answer m queries. Each query is to compute $[a_l; a_{l+1}, \ldots, a_r]$.

Solution

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Linear fractional transformation of a continued fraction

Let $L(x)=rac{ax+b}{cx+d}$. Compute the continued fraction representation $[b_0;b_1,\ldots,b_m]$ of L(A) for $A=[a_0;a_1,\ldots,a_n]$.

This allows to compute $A+rac{p}{q}=rac{qA+p}{q}$ and $A\cdotrac{p}{q}=rac{pA}{q}$ for any $rac{p}{q}$

Solution

Continued fraction arithmetics

Let $A=[a_0;a_1,\ldots,a_n]$ and $B=[b_0;b_1,\ldots,b_m]$. Compute the continued fraction representations of A+B and $A\cdot B$.

Solution

>

Definition

A continued fraction $x=[a_0;a_1,\ldots]$ is said to be **periodic** if $x=[a_0;a_1,\ldots,a_k,x]$ for some k.

A continued fraction $x=[a_0;a_1,\ldots]$ is said to be **eventually periodic** if $x=[a_0;a_1,\ldots,a_k,y]$, where y is periodic.

For $x=[1;1,1,\ldots]$ it holds that $x=1+\frac{1}{x}$, thus $x^2=x+1$. There is a generic connection between periodic continued fractions and quadratic equations. Consider the following equation:

$$x = [a_0; a_1, \dots, a_k, x].$$

On one hand, this equation means that the continued fraction representation of x is periodic with the period k+1.

On the other hand, using the formula for convergents, this equation means that

$$x=\frac{p_kx+p_{k-1}}{q_kx+q_{k-1}}.$$

That is, x is a linear fractional transformation of itself. It follows from the equation that x is a root of the second degree equation:

$$q_k x^2 + (q_{k-1} - p_k) x - p_{k-1} = 0.$$

Similar reasoning stands for continued fractions that are eventually periodic, that is $x=[a_0;a_1,\ldots,a_k,y]$ for $y=[b_0;b_1,\ldots,b_k,y]$. Indeed, from first equation we derive that $x=L_0(y)$ and from second equation that $y=L_1(y)$, where L_0 and L_1 are linear fractional transformations. Therefore,

$$x=(L_0\circ L_1)(y)=(L_0\circ L_1\circ L_0^{-1})(x).$$

One can further prove (and it was first done by Lagrange) that for arbitrary quadratic equation $ax^2 + bx + c = 0$ with integer coefficients, its solution x is an eventually periodic continued fraction.

Quadratic irrationality

Find the continued fraction of $lpha=rac{x+y\sqrt{n}}{z}$ where $x,y,z,n\in\mathbb{Z}$ and n>0 is not a perfect square.



Tavrida NU Akai Contest - Continued Fraction

You're given x and k, x is not a perfect square. Let $\sqrt{x}=[a_0;a_1,\ldots]$, find $\frac{p_k}{q_k}=[a_0;a_1,\ldots,a_k]$ for $0\leq k\leq 10^9$.



>

Geometric interpretation

Let $ec{r}_k=(q_k;p_k)$ for the convergent $r_k=rac{p_k}{q_k}$. Then, the following recurrence holds:

$$\vec{r}_k = a_k \vec{r}_{k-1} + \vec{r}_{k-2}.$$

Let $\vec{r}=(1;r)$. Then, each vector (x;y) corresponds to the number that is equal to its slope coefficient $\frac{y}{x}$.

With the notion of pseudoscalar product $(x_1;y_1) imes (x_2;y_2)=x_1y_2-x_2y_1$, it can be shown (see the explanation below) that

$$s_k = -rac{ec{r}_{k-2} imesec{r}}{ec{r}_{k-1} imesec{r}} = \left|rac{ec{r}_{k-2} imesec{r}}{ec{r}_{k-1} imesec{r}}
ight|.$$

The last equation is due to the fact that r_{k-1} and r_{k-2} lie on the different sides of r, thus pseudoscalar products of $ec{r}_{k-1}$ and $ec{r}_{k-2}$ with $ec{r}$ have distinct signs. With $a_k=\lfloor s_k
floor$ in mind, formula for $ec{r}_k$ now looks like

$$ec{r}_k = ec{r}_{k-2} + \left\lfloor \leftert rac{ec{r} imes ec{r}_{k-2}}{ec{r} imes ec{r}_{k-1}}
ightert
ight
floor ec{r}_{k-1}.$$

Note that $ec{r}_k imes r = (q;p) imes (1;r) = qr - p$, thus

$$a_k = \left \lfloor \left \lfloor rac{q_{k-1}r - p_{k-1}}{q_{k-2}r - p_{k-2}}
ight
floor.$$



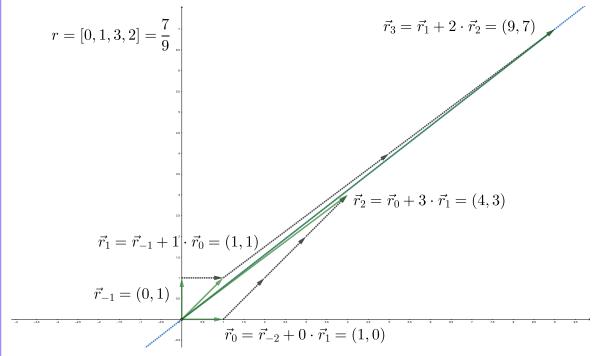
Explanation

Nose stretching algorithm

Each time you add \vec{r}_{k-1} to the vector \vec{p} , the value of $\vec{p} \times \vec{r}$ is increased by $\vec{r}_{k-1} \times \vec{r}$.

Thus, $a_k = \lfloor s_k \rfloor$ is the maximum integer number of \vec{r}_{k-1} vectors that can be added to \vec{r}_{k-2} without changing the sign of the cross product with \vec{r} .

In other words, a_k is the maximum integer number of times you can add \vec{r}_{k-1} to \vec{r}_{k-2} without crossing the line defined by \vec{r} :



Convergents of $r=\frac{7}{9}=[0;1,3,2]$. Semiconvergents correspond to intermediate points between gray arrows.

On the picture above, $\vec{r}_2=(4;3)$ is obtained by repeatedly adding $\vec{r}_1=(1;1)$ to $\vec{r}_0=(1;0)$.

When it is not possible to further add \vec{r}_1 to \vec{r}_0 without crossing the y=rx line, we go to the other side and repeatedly add \vec{r}_2 to \vec{r}_1 to obtain $\vec{r}_3=(9;7)$.

This procedure generates exponentially longer vectors, that approach the line.

For this property, the procedure of generating consequent convergent vectors was dubbed the **nose stretching algorithm** by Boris Delaunay.

If we look on the triangle drawn on points \vec{r}_{k-2} , \vec{r}_k and $\vec{0}$ we will notice that its doubled area is

$$|\vec{r}_{k-2} \times \vec{r}_k| = |\vec{r}_{k-2} \times (\vec{r}_{k-2} + a_k \vec{r}_{k-1})| = a_k |\vec{r}_{k-2} \times \vec{r}_{k-1}| = a_k.$$

Combined with the Pick's theorem, it means that there are no lattice points strictly inside the triangle and the only lattice points on its border are $\vec{0}$ and $\vec{r}_{k-2} + t \cdot \vec{r}_{k-1}$ for all integer t such that $0 \le t \le a_k$. When joined for all possible k it

means that there are no integer points in the space between polygons formed by even-indexed and odd-indexed convergent vectors.

This, in turn, means that \vec{r}_k with odd coefficients form a convex hull of lattice points with $x \geq 0$ above the line y = rx, while \vec{r}_k with even coefficients form a convex hull of lattice points with x > 0 below the line y = rx.

Definition

These polygons are also known as **Klein polygons**, named after Felix Klein who first suggested this geometric interpretation to the continued fractions.

Problem examples

Now that the most important facts and concepts were introduced, it is time to delve into specific problem examples.

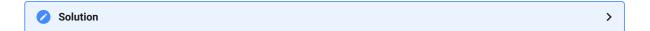
Convex hull under the line

Find the convex hull of lattice points (x;y) such that $0 \le x \le N$ and $0 \le y \le rx$ for $r = [a_0;a_1,\ldots,a_k] = \frac{p_k}{q_k}$.

✓ Solution >

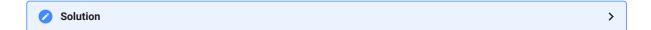
Timus - Crime and Punishment

You're given integer numbers A, B and N. Find $x \geq 0$ and $y \geq 0$ such that $Ax + By \leq N$ and Ax + By is the maximum possible.



June Challenge 2017 - Euler Sum

 $\text{Compute } \sum_{x=1}^N \lfloor ex \rfloor \text{, where } e=[2;1,2,1,1,4,1,1,6,1,\dots,1,2n,1,\dots] \text{ is the Euler's number and } N \leq 10^{4000}.$



NAIPC 2019 - It's a Mod, Mod, Mod, Mod World

Given p, q and n, compute $\sum_{i=1}^{n} [p \cdot i \mod q]$.





Library Checker - Sum of Floor of Linear

Given N, M, A and B, compute $\sum\limits_{i=0}^{N-1} \lfloor \frac{A \cdot i + B}{M} \rfloor$.



Solution





OKC 2 - From Modular to Rational

There is a rational number $rac{p}{q}$ such that $1 \le p, q \le 10^9$. You may ask the value of pq^{-1} modulo $m \sim 10^9$ for several prime numbers m. Recover $\frac{p}{q}$.

Equivalent formulation: Find x that delivers the minimum of $Ax \mod M$ for $1 \le x \le N$.



Solution

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Practice problems

- UVa OJ Continued Fractions
- ProjectEuler+ #64: Odd period square roots
- Codeforces Round #184 (Div. 2) Continued Fractions
- Codeforces Round #201 (Div. 1) Doodle Jump
- Codeforces Round #325 (Div. 1) Alice, Bob, Oranges and Apples
- POJ Founder Monthly Contest 2008.03.16 A Modular Arithmetic Challenge
- 2019 Multi-University Training Contest 5 fraction
- SnackDown 2019 Elimination Round Election Bait
- Code Jam 2019 round 2 Continued Fraction

Contributors:

adamant-pwn (99.91%) jatingaur18 (0.09%)