A1

① $(\lambda z.\, z\ (\lambda x.\, x)(\lambda x.\, \lambda_y.\, \lambda z.\, z\ (x\ y)\ x))\ (\lambda x.\, \lambda y.\, x)\ (\lambda z.\lambda s.\, s(s\ z))$

$\to_\beta\ z\ (\lambda x.\, x)\ (\lambda x.\lambda_y.\, \lambda z.\ z\ (x\ y)\ x)[z := (\lambda x.\, \lambda_y.\, x)]\ (\lambda z.\lambda s.\, s\ (s\ z))$

$\to\ (\lambda x.\lambda_y.\, x)\ (\lambda x.x)\ (\lambda x.\, \lambda_y.\, \lambda z.\, z\ (x\ y)\ x)\ \ (\lambda z.\, \lambda s.\ s(s\ z))$

$\to_\beta (\lambda_y.\, x)[x := \lambda x.x]\ \ (\lambda x.\, \lambda_y.\, \lambda z.\ z\ (x\ y)\ x)\ \ (\lambda z.\lambda s.\, s(s\ z))$

$\to\ (\lambda y.\, \lambda x.\, x)\ \ (\lambda x.\, \lambda y.\, \lambda z.\ z(x\ y)\ x)\ \ (\lambda z.\lambda s.\ s\ (s\ z))$

$\to_\beta (\lambda x.\, x)\ \ (\lambda z.\, \lambda s.\, s\ (s\ z))$

$\to_\beta\ x\ [x := \lambda z.\, \lambda s.\ s\ (s\ z)]$

$\to\ \lambda z.\, \lambda s.\ s\ (s\ z)$

②

$\lambda x.\, x\ ((\lambda z.\, \lambda s.\ s\ (s\ z))\ (\lambda y.\ y)\ (\lambda_y.y))$

$\to_\beta \lambda x.\, x\ ((\lambda s.\ s\ (s\ z))[z := \lambda_y.y]\ (\lambda_y.y))$

$\to \lambda x.\, x\ ((\lambda s.\ s\ (s\ (\lambda_y.y)))\ (\lambda_y.y))$

$\to_\beta \lambda x.x\ ([\ s(s\ (\lambda_y.y))][s := (\lambda_y.y)])$

$\to \lambda x.x\ ((\lambda_y.y)\ ((\lambda_y.y)\ (\lambda_y.y)))$

$\to \lambda x.x\ (\lambda_y.y)$

A2

① 1. $C_5 := \lambda f x.\ f(f(f(f(f\ x))))$

②

$A_+ C_2\ C_3 = (\lambda x y p q.\ x\ p\ (y\ p\ q))\ \ (\lambda f x.\ f(f\ x))\ \ (\lambda f x.\ f(f(f\ x)))$

$\to_\beta (\lambda y p q.\ x\ p\ (y\ p\ q))[x := (\lambda f x.\ f(f\ x))\ \ (\lambda f x.\ f(f(f\ x)))]$

$= (\lambda y p q.\ (\lambda f x.\ f(f\ x))\ p\ (y\ p\ q))\ \ (\lambda f x.\ f(f(f\ x)))$

$\to_\beta (\lambda p q.\ (\lambda f x.\ f(f\ x))\ p\ (y\ p\ q))[y := (\lambda f x.\ f(f(f\ x)))]$

$= (\lambda p q.\ (\lambda f x.\ f(f\ x))\ p\ ((\lambda f x.\ f(f(f\ x)))\ p\ q))$

$\to_\beta \lambda p q.\ (\lambda x.\ f(f\ x))[f := p]\ ((\lambda f x.\ f(f(f\ x)))\ p\ q))$

$= \lambda p q.\ (\lambda x.\ p\ (p\ x))\ ((\lambda f x.\ f(f(f\ x)))\ p\ q))$

$\to_\beta \lambda p q.\ (p\ (p\ x))[x := ((\lambda f x.\ f\ (f\ (f\ x)))\ p\ q))]$

$= \lambda p q.\ p\ (p((\lambda f x.\ f(f(f\ x)))\ p\ q)$

$\to_\beta \lambda p q.\ p\ (p((\lambda x.\ f(f(f\ x)))[f := p]\ q)$

$= \lambda p q.\ p(p((\lambda x.\ p(p(p\ x)))\ q)$

$= \lambda p q.\ p\ (p\ (p\ (p\ (p\ q))))\ \Rightarrow C_5$

$Arg_5 = (\lambda x\, y\, z.\, x(y\, z))\ (\lambda f x.\, f x)\ (\lambda f x.\, f^5\, x)$

$= (\lambda y\, z.\, (\lambda f x.\, f x)(y\, z))\ (\lambda f x.\, f^5\, x)$

$= (\lambda z.\, (\lambda f x.\, f x)\ ((\lambda f x.\, f^5\, x)\, z))$

$= (\lambda z.\, (\lambda x.\, ((\lambda f x.\, f^5\, x)\, z)\, x))$

$= (\lambda z.\, (\lambda x.\, ((\lambda x.\, z^5\, x)\, x)))$

$= (\lambda z.\, (\lambda x.\, (z^5\, x)))$

$= (\lambda z\, x.\, z^5\, x)$