

# Studying Optimal Spilling in the light of SSA

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# Outline

- 1 Introduction
- 2 Formulating “Optimal” Spilling
- 3 A “More Optimal” Formulation
- 4 Experiments
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# Introduction

## Register Allocation

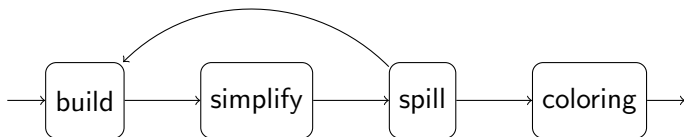
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- Simplified graph coloring based approach

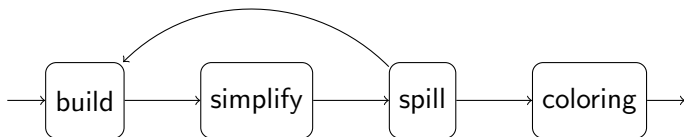


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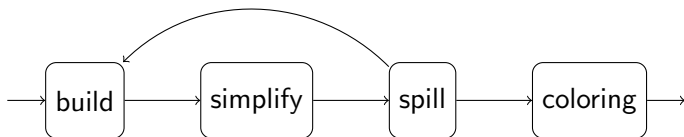
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- ▶ Simplify: Apply Kempe's Algorithm
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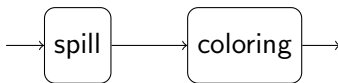
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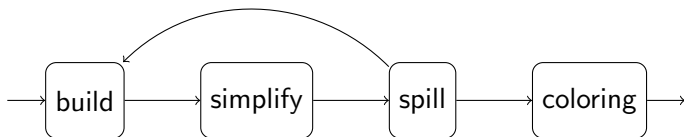


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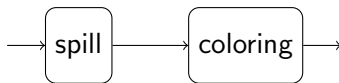
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- ▶ Spill:  $\# \text{live variables} \leq K$  for each program point
- ▶ Coloring: Assign variables colors



# Problematic

## Context

- Decoupled register allocation
  - ▶ Spill
  - ▶ Assignment
- Based on static single assignment (SSA)

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  - ▶ Does SSA help for spilling too?
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## Contributions

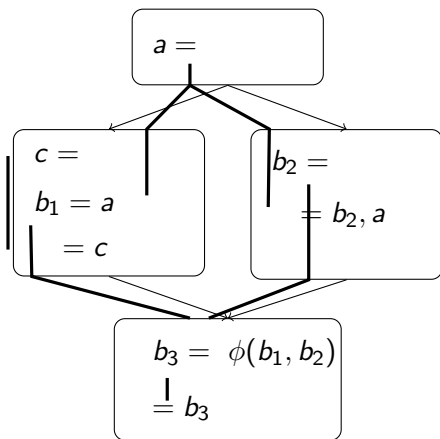
- Provide an exact formulation
- Exploit variable-to-variable copies
- Discuss existing spilling models

# Static Single Assignment (SSA)

SSA provides sufficient split points, unless pre-coloring or aliasing is involved.

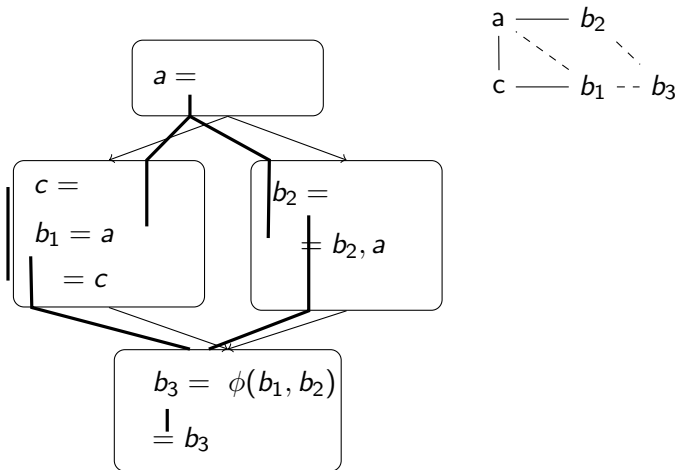
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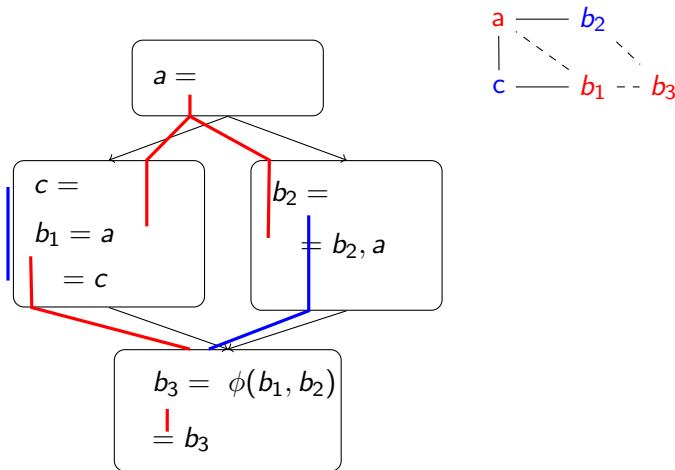
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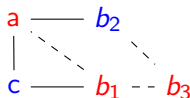
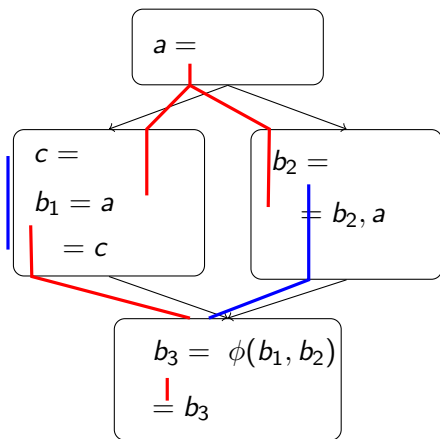
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## Properties

- Every use has at most one reaching definition
- For strict SSA: A definition dominates all its uses.  
I.e. it does not exist a path from the function entry to  $v$ 's use that does not traverse  $v$ 's definition.





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# Existing Approaches

Various 'optimal' approaches:

- Integer Linear Programming (ILP)
  - ▶ Appel & George
  - ▶ related Goodwin & Wilken (ORA)
- Multi-Commodity Network Flow
  - ▶ Koes & Goldstein
- Constrained Min-Cut
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All of these formulations have *surprising* flaws!

# Flaws: Liveness<sup>1</sup>

```
a = ...  
b = a + 1 ⚡  
...  
    = a
```

(a) before spilling

```
a = ...  
b = a + 1 ⚡  
store a  
...  
load a  
    = a
```

(b) ineffective spilling

Problem:

- Variables are either available in memory or register (exclusive)
- Load/store required to change availability
- Artificial interference between a and b

---

<sup>1</sup>Applies to: Appel, Koes; in other form also Goodwin

## Flaws: Spurious Spill Code<sup>2</sup>

```
a = ...  
while(...){  
  if (...)  
    store a  
    ⚡  
    load a  
    = a  
  else  
    = a  
}
```

(a) Koes 1

---

<sup>2</sup>Applies to: Appel, Koes

## Flaws: Spurious Spill Code<sup>2</sup>

<pre>a = ... while(...){   if (...)     store a     ⚡   load a     = a else   = a }</pre>	<pre>a = ... store a while(...){   if (...)     ⚡     load a       = a else   load a     = a   store a }</pre>
(a) Koes 1	(b) Koes 2

---

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a = ...  
store a  
while(...){  
  if (...)  
    ⚡  
    load a  
    = a  
  else  
    load a  
    = a  
  store a  
}
```

(b) Koes 2

```
a = ...  
store a  
load a  
while(...){  
  if (...)  
    store a  
    ⚡  
    load a  
    = a  
  else  
    = a  
}
```

(c) Koes 3

---

<sup>2</sup>Applies to: Appel, Koes



# Limitations

## Limitations of existing approaches:

- Rematerialization
  - ▶ Appel & George: None
  - ▶ related Goodwin & Wilken: Simple and partial
  - ▶ Koes & Goldstein: Simple and partial
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- Support of SSA
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We design a new model

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# Our Formulation - Concepts

Express spilling using ILP:

- Availability around program points
  - ▶ Available in memory / in register
  - ▶ Non-exclusive!!
- Actions on program points
  - ▶ Load, store, rematerialization, ...
- Propagation along points
  - ▶ Along edges in the control flow graph (CFG)
  - ▶ Between operations within basic blocks  
accounting for uses/definitions

# Our Formulation - Features

## Basic

- Load/Store placement
- Simple rematerialization

This model can emulate all existing approaches.

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## Extended

- Features of basic model
- Copy/SSA handling
- Generalized rematerialization

The extended model is able to emulate the basic one.

# SSA Specificities

$\phi$ -Operations represent implicit copies:



$$a = \phi(b, c)$$

$$e = \phi(b, d)$$


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$$(a_\phi, e_\phi) = (b, b) \quad (a_\phi, e_\phi) = (c, d)$$

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(b) Transform  $\phi$ -operations

Simple approach: spilling as if not under SSA form.



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Example: Appel & George with and without SSA:

- Spill cost:
  - ▶ 5% worse on average under SSA
  - ▶ Best improvement: 2%
  - ▶ Worst case: 50%

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⇒ Copies force variables to be in register.

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  - ▶ Coalesce memory slots afterwards
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  - ▶ Repair when memory slots cannot be shared
- Pessimistic:
  - ▶ Replace  $\phi$ -operations by copies
  - ▶ Copies are virtual operations
  - ▶ Propagate locations through a subset of copies
  - ▶ Coalesce remaining memory slots afterwards

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# Experiments

## Setup:

- Production compiler for STMicroelectronics ST2xx VLIW
  - ▶ 4-way parallel
  - ▶ 32KB direct mapped I-cache
  - ▶ 32KB 4-way set associative D-cache
  - ▶ 1 load/store per cycle
  - ▶ 3 cycles load-use delay
- Restricted to 8 registers
- SPEC2000 and EEMBC v1.1 benchmarks
- IBM CPLEX 12.2 with 1000s time limit

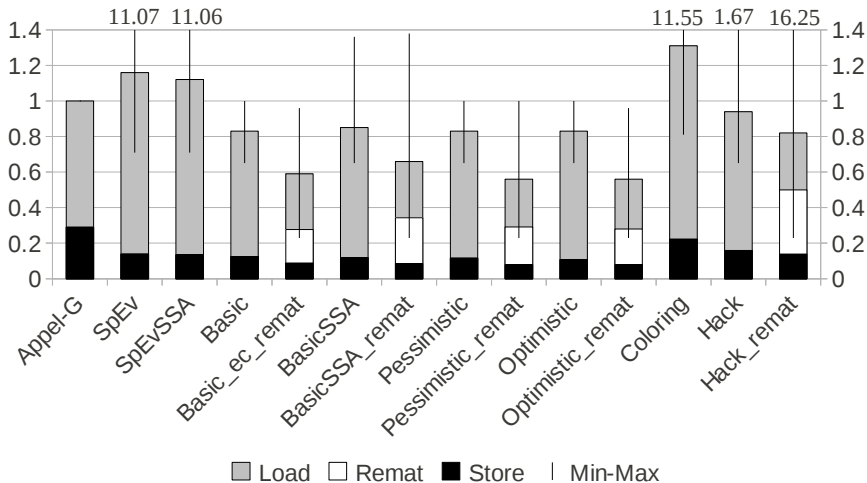
## Experiments (2)

### Configurations:

Appel-G.	Appel and George's ILP Formulation
Coloring	Heuristic using iterated register coalescing
SpEv	Basic formulation emulating spill everywhere
Basic	Our basic formulation
BasicSSA	Naive handling of SSA
Pessimistic	Extended formulation, pessimistic coalescing sets
Optimistic	Extended formulation, optimistic coalescing sets
SpEv_ssa	Emulation of spill everywhere under SSA
Hack	Hack's SSA-based spilling heuristic

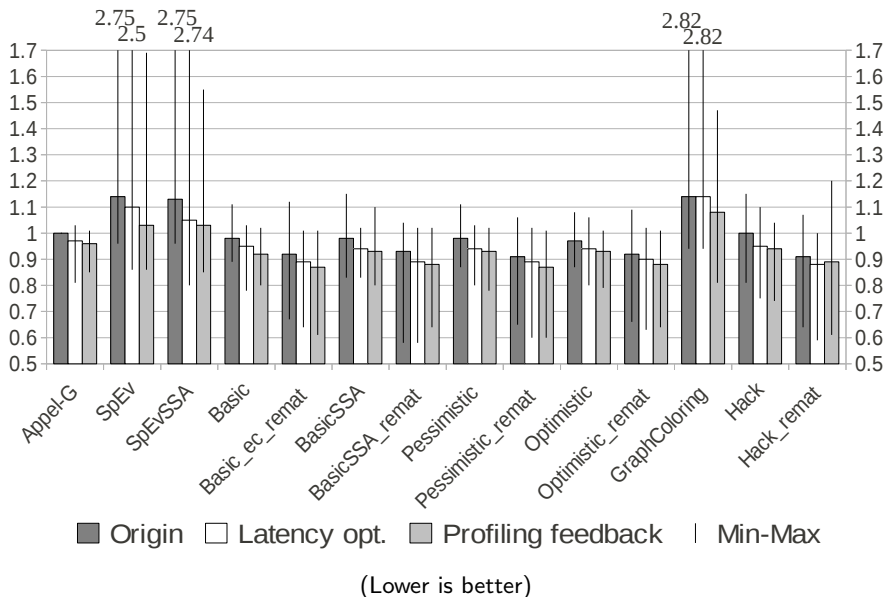


# Spill Costs (EEMBC)

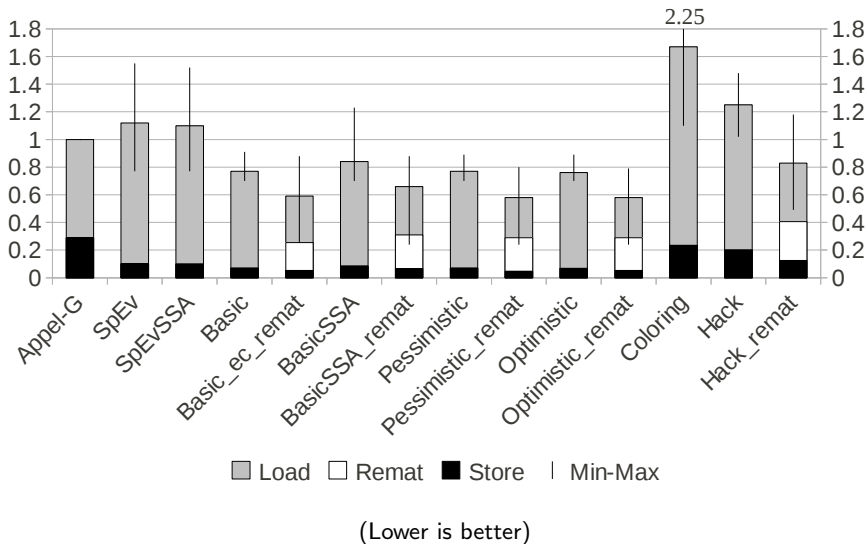


(Lower is better)

# Runtime (EEMBC)



# Spill Costs (SPEC)



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  - ▶ Ignoring  $\phi$ s gives unpredictable behavior

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- SSA form complicates matters
  - ▶ Parallel  $\phi$ -semantics and memory coalescing
  - ▶ Ignoring  $\phi$ s gives unpredictable behavior
- Placement of spill code is important
  - ▶ Spill costs alone are a bad metric
  - ▶ State of pipeline and memory subsystem have to be considered

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$$(c) \text{ **Spill**}$$

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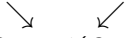


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(a) **SSA form**


$$\begin{aligned} @_a &= \phi(@_b, @_c) \\ @_e &= \phi(@_b, @_d) \end{aligned}$$

(b) **Spill**

$$\begin{aligned} v_1 &= ld @_b \\ @_ {ac} &= st \ v_1 \\ v_2 &= ld @_b \\ @_ {ed} &= st \ v_2 \end{aligned}$$


(c) **Coalescing and repairing**

# Partial Rematerialization Support

```
a = remat
while(...){
    = a
    ⚡
}
```

(a) Origin

```
a = remat
while(...){
    = a
    ⚡
    a = remat
}
```

(b) Partial support

```
while(...){
    a = remat
    = a
    ⚡
}
```

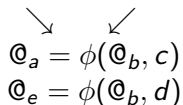
(c) Optimal



# Partial SSA Support: Ebner et al.

- No particular constraints on  $\phi$ -operations.
- Deal with  $\phi$ -operations with mixed type of operands.
- $\Rightarrow$  Repairing cost not in the model.

Example:



The diagram shows two equations stacked vertically. Above the first equation,  $@_a = \phi(@_b, c)$ , there is a downward-pointing arrow. Above the second equation,  $@_e = \phi(@_b, d)$ , there is an upward-pointing arrow. Both arrows point to the  $@_b$  operand in their respective equations.

$$\begin{aligned} @_a &= \phi(@_b, c) \\ @_e &= \phi(@_b, d) \end{aligned}$$

# Program Point and ILP Variables

store a		$\rho_{p,a} = ?$	$\mu_{p,a} = ?$
$l_{p,a} = ?$	$s_{p,a} = ?$	$\bullet p$	
load a		$\bar{\rho}_{p,a} = 1$	$\bar{\mu}_{p,a} = ?$
$b = a + 1$			

(a) A program point and its ILP variables

		$\bullet p$	$\bar{\rho}_{p,a} = 1$	$\bar{\mu}_{p,a} = ?$
	$b = a + 1$		$\geq$	$\geq$
$\rho_{q,b} = 1$	$\mu_{q,b} = 0$	$\bullet q$	$\rho_{q,a} = ?$	$\mu_{q,a} = ?$

(b) Program points surrounding an instruction

# Emulating other Approaches

Constraints to emulate Appel & George:

$$(\text{Appel}) \quad \bar{\mu}_{p,v} + \bar{\rho}_{p,v} = 1$$

Alternatively:

$$(\text{Appel}_I) \quad l_{p,v} + \bar{\mu}_{p,v} \leq 1 \qquad (\text{Appel}_s) \quad s_{p,v} + \bar{\rho}_{p,v} \leq 1$$

Constraints to emulate Koes & Goldstein:

$$(\text{Appel}_s) \quad s_{p,v} + \bar{\rho}_{p,v} \leq 1$$

# Discussion

- Huge gains in spill costs
  - ▶ Compared to 'optimal' techniques
  - ▶ Mostly due to elimination of stores
- Dynamic metrics
  - ▶ Lower cache miss rates
  - ▶ Lower number of loads/stores ( $-20\%$ )
  - ▶ Lower number of operations executed ( $-8\%$ )
  - ▶ Lower number of instruction bundles
- Marginal improvements in actual runtime
  - ▶ Costs of stores 'over-weighted'
  - ▶ Costs of secondary effects are missing (**pipeline**, cache, code layout)

# Optimal Coalescing

```
a = ...  
b = ...  
⚡  
    = b  
if (...)  
    ⚡  
endif  
c =  $\phi$ (a, b)  
⚡
```

(a) Original

```
store a at @c  
store b at @b  
⚡  
load b  
    = b  
if (...)  
    ⚡  
    mem_dup c = b  
endif  
c =  $\phi$ (a, b)
```

(b) Optimistic/pessimistic

```
store a at @c  
store b at @b  
⚡  
load b  
    = b  
if (...)  
    store b at @c  
    ⚡  
endif  
c =  $\phi$ (a, b)
```

(c) Optimal