Homework 2 Solutions

Introduction to Computational Optics

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Contents

Exercise 7.3

(a) Coherent and Incoherent Cutoff Frequencies

Problem Statement: We are given the parameters:

- Distance from the exit pupil to the image plane: $z_{XP} = 50\,\text{mm}$
- Wavelength of light: $\lambda = 0.5\,\mu\text{m}$
- Half-widths of the rectangular exit pupil: $w_x = 1\,\text{mm}$ and $w_y = 0.5\,\text{mm}$

The coherent cutoff frequencies in the x and y directions are defined as:

$$f_{0x} = \frac{w_x}{\lambda z_{XP}}, \quad f_{0y} = \frac{w_y}{\lambda z_{XP}}$$

The incoherent cutoff frequencies are double the coherent values:

$$2f_{0x} = \frac{2w_x}{\lambda z_{XP}}, \quad 2f_{0y} = \frac{2w_y}{\lambda z_{XP}}$$

Calculations:

$$f_{0x} = \frac{1}{0.5 \times 10^{-6} \times 50 \times 10^{-3}} = 40,000\,\text{cycles/mm}$$

$$f_{0y} = \frac{0.5}{0.5 \times 10^{-6} \times 50 \times 10^{-3}} = 20,000\,\text{cycles/mm}$$

$$2f_{0x} = 80,000\,\text{cycles/mm}, \quad 2f_{0y} = 40,000\,\text{cycles/mm}$$

Interpretation: These results show that the spatial frequency bandwidth of the imaging system is higher along the x-direction due to the larger width of the exit pupil in that direction.

(b) Diffraction-Limited Coherent Image Simulation

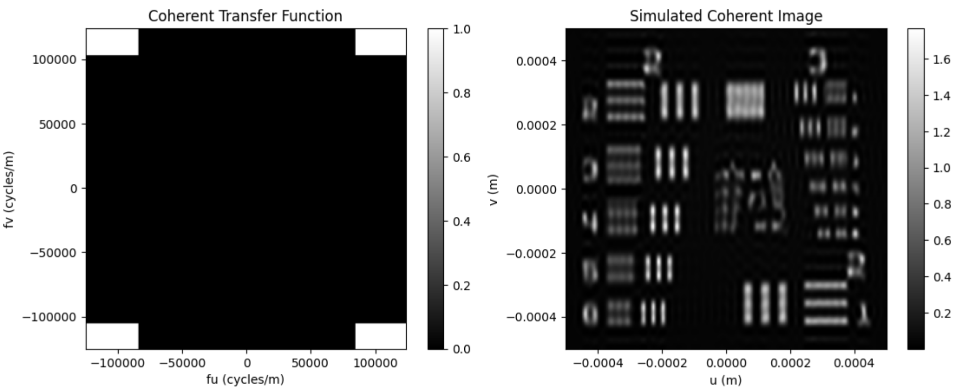

Figure 1: Transfer function and the resulting coherent image simulation

(c) Diffraction-Limited Incoherent Image Simulation

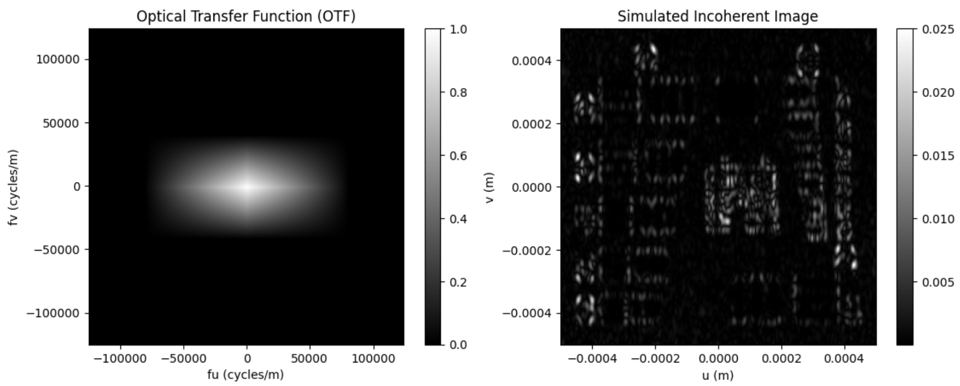

Figure 2: Transfer function (OTF) and the simulated incoherent image

(d) Spatial Resolution Difference

Analysis: The rectangular aperture introduces anisotropic cutoff frequencies:

- The higher cutoff frequency in the x-direction means better resolution along this axis.
- Conversely, the lower cutoff frequency in the y-direction results in reduced resolution along this axis.

This difference is apparent in the resulting images, where features along the y-direction may appear blurred compared to those along the x-direction.

Exercise 7.4

(a) Coherent and Incoherent Cutoff Frequencies

Problem Statement: We are given:

- Distance from the exit pupil to the image plane: $z_{XP} = 50\,\text{mm}$
- Wavelength of light: $\lambda = 0.5\,\mu\text{m}$
- Radii of the annular exit pupil: $w_o = 1\,\text{mm}$ (outer), $w_i = 0.5\,\text{mm}$ (inner)

The coherent cutoff frequency is:

$$f_0 = \frac{w_o}{\lambda z_{XP}} - \frac{w_i}{\lambda z_{XP}}$$

The incoherent cutoff frequency is:

$$2f_0 = 2\left(\frac{w_o}{\lambda z_{XP}} - \frac{w_i}{\lambda z_{XP}}\right)$$

Calculations:

$$f_0 = \frac{1}{0.5 \times 10^{-6} \times 50 \times 10^{-3}} - \frac{0.5}{0.5 \times 10^{-6} \times 50 \times 10^{-3}} = 20,000\,\text{cycles/mm}$$

$$2f_0 = 2 \times 20,000 = 40,000\,\text{cycles/mm}$$

Interpretation: The annular aperture removes the central (low-frequency) components, resulting in a band-pass behavior where only mid-range spatial frequencies contribute to the image.

(b) Diffraction-Limited Coherent Image Simulation

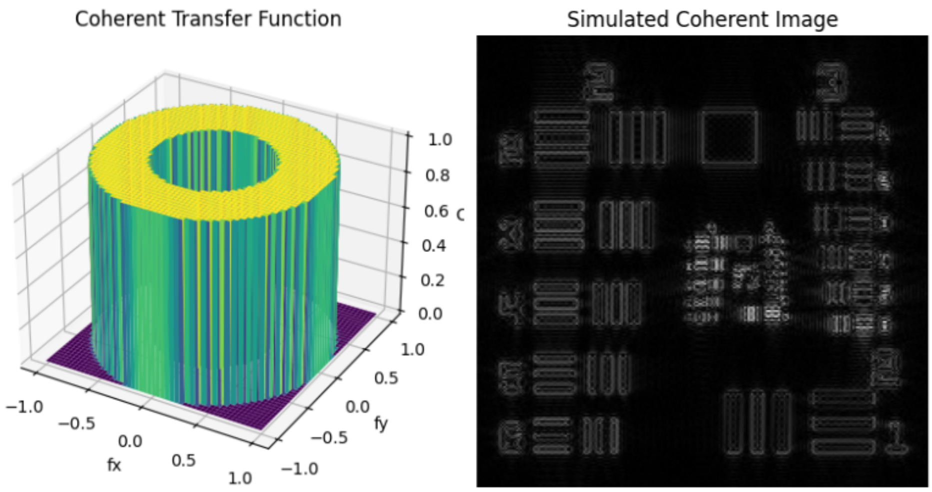

Figure 3: Transfer function and the simulated coherent image

(c) Diffraction-Limited Incoherent Image Simulation

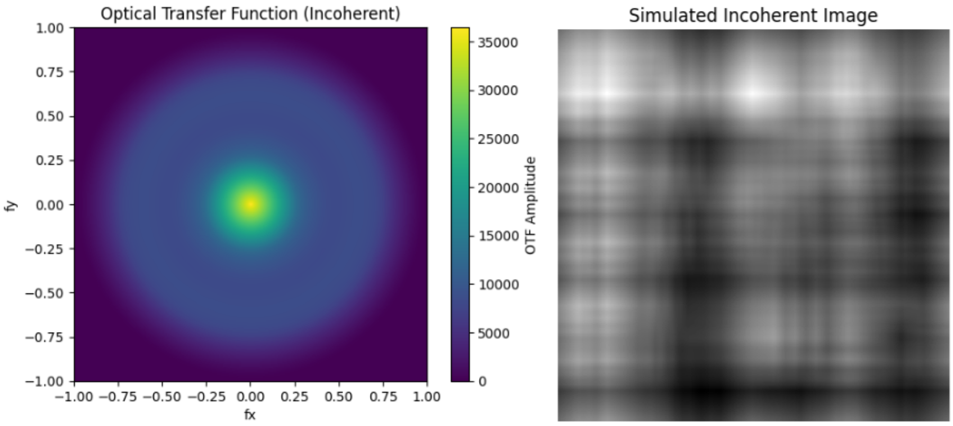

Figure 4: Surface plot of the OTF and the simulated incoherent image

(d) Effect of Losing DC and Low Spatial Frequencies

Analysis:

- Removing the DC and low spatial frequencies causes a significant reduction in overall brightness and smooth intensity variations.
- The resulting coherent image emphasizes high-frequency features, such as edges and fine structures, but can appear less natural and harder to interpret.
- This effect is a direct consequence of the annular aperture's band-pass nature.

References

References

[1] D. G. Voelz, *Computational Fourier Optics: A MATLAB Tutorial*, Chapter 7: Imaging and Diffraction-Limited Imaging Simulation, SPIE Press, Bellingham, WA, 2011. ISBN: 9780819486474.
DOI: https://doi.org/10.1117/3.899758.