Homework 1: Theoretical and Numerical Analysis

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Abstract: This task explores the theoretical and numerical verification of diffraction effects in freeform beam shaping optics, based on the composite method outlined in Z. Feng et al. Using the Angular Spectrum Method (ASM), we analyze wave propagation from a Gaussian beam through a freeform optical surface. Results confirm significant diffraction effects, demonstrating the need for wave optics corrections in freeform optical design.

1. Introduction

Beam shaping optics play a critical role in modern optical systems, enabling applications from laser fusion to medical imaging. This homework involves the design of a freeform optical surface to transform a Gaussian beam into a prescribed irradiance pattern—an image of A. J. Fresnel. The composite method by Z. Feng et al. is used, which integrates geometric optics with wave optics corrections through the Monge–Ampère equation and Iterative Fourier Transform Algorithms (IFTA). This study verifies diffraction effects using the Angular Spectrum Method (ASM).

2. Theoretical Framework

2.1. Gaussian Beam Description

The initial Gaussian beam is described by the intensity distribution:

$$I(x, y) = \exp\left(-2\left(\frac{x^2}{w_x^2} + \frac{y^2}{w_y^2}\right)\right),$$

where $w_x = w_y = 5.12\,\mathrm{mm}$ defines the beam waist. The beam propagates along the z-axis, with a wavelength $\lambda = 532\,\mathrm{nm}$.

2.2. Wave Propagation: Angular Spectrum Method

Wave propagation is modeled using ASM. The output field $U(x', y', z)$ at a distance z is calculated as:

$$U(x', y', z) = \mathcal{F}^{-1}\left[H(f_x, f_y, z) \cdot \mathcal{F}[U(x, y, 0)]\right],$$

where:

- $\mathcal{F}$ and $\mathcal{F}^{-1}$ are the Fourier transform and its inverse.
- $H(f_x, f_y, z) = \exp\left(ikz\sqrt{1-\lambda^2(f_x^2+f_y^2)}\right)$ is the transfer function.
- $k = \frac{2\pi}{\lambda}$ is the wavenumber.

2.3. Diffraction Effects and Freeform Surface

Diffraction effects blur the irradiance distribution produced by the geometrical optics-based freeform surface. Wave optics simulation refines this prediction, accounting for Fresnel diffraction:

$$I(x', y') = |U(x', y', z)|^2.$$

3. Numerical Implementation

3.1. Simulation Parameters

The following parameters were used:

- Aperture size: $10.24 \times 10.24\,\mathrm{mm}$.
- Propagation distance: $z = 200\,\mathrm{mm}$.
- Grid resolution: 512×512, with step size $dx = 0.02\,\mathrm{mm}$.

3.2. Python Simulation

The Angular Spectrum Method was implemented in Python, and the intensity distributions were computed. The incident Gaussian beam and the output irradiance at the target plane are shown in Figure 1.

4. Results and Analysis

The results demonstrate significant diffraction effects in the output irradiance pattern. The original Gaussian beam profile was transformed, but the image of A. J. Fresnel was blurred, validating the theoretical predictions.

5. Conclusion

This task verified diffraction effects in freeform beam shaping optics using the Angular Spectrum Method. The wave optics corrections revealed the limitations of geometrical optics in high-precision irradiance control. Future work could explore iterative optimization methods for improving the accuracy of freeform surface designs.

References

[1] Z. Feng, B. D. Froese, and R. Liang, "Composite method for precise freeform optical beam shaping," Appl. Opt. 54, 9364-9369 (2015).

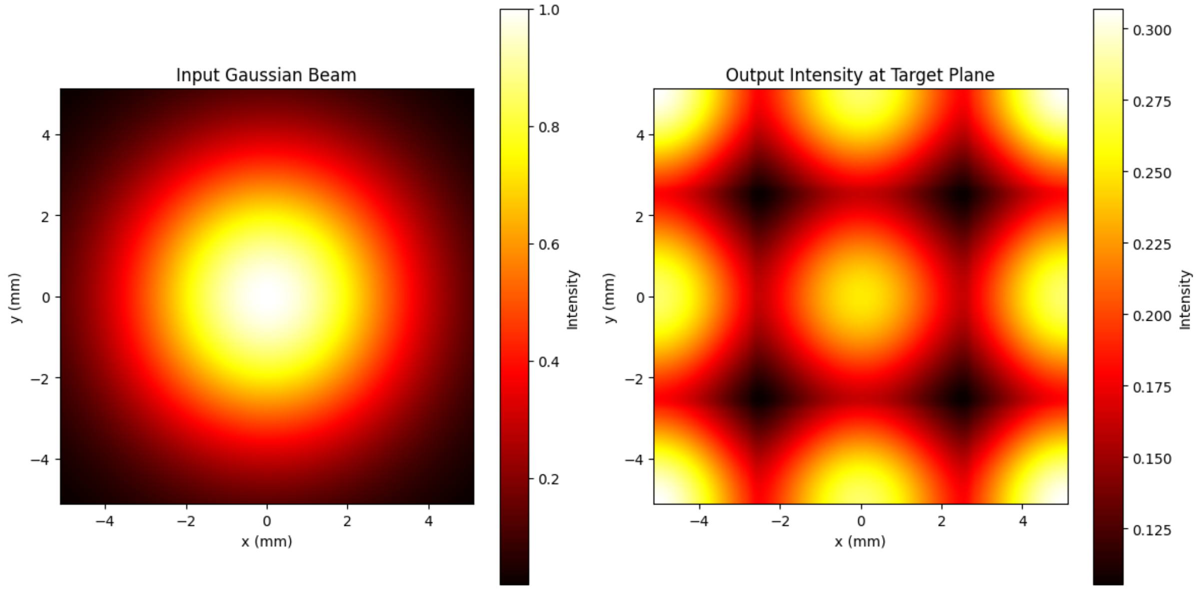

Figure 1: Left: Input Gaussian beam intensity. Right: Output intensity at the target plane, showing diffraction effects.