

# Computational Photography

- \* Study the basics of computation and its impact on the entire workflow of photography, from capturing, manipulating and collaborating on, and sharing photographs.



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# Digital Images: Into the Frequency Domain

- \* Images are samples of light information stored in array of pixels.



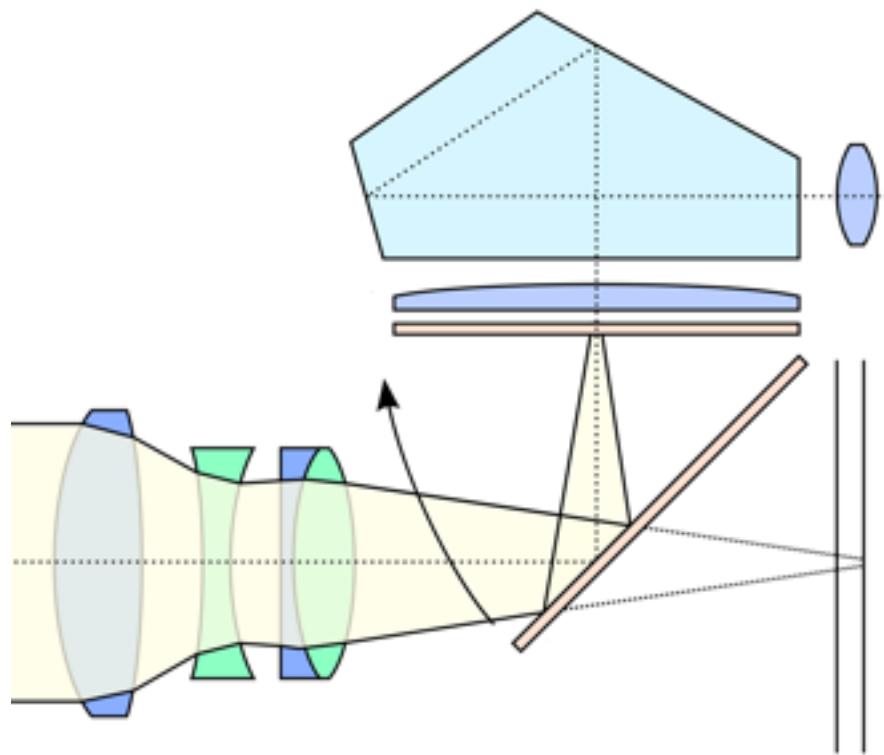
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## Lesson Objectives

1. Using sines and cosines to reconstruct a signal
2. The Fourier Transform
3. Frequency Domains for a Signal
4. Three properties of Convolution relating to Fourier Transform

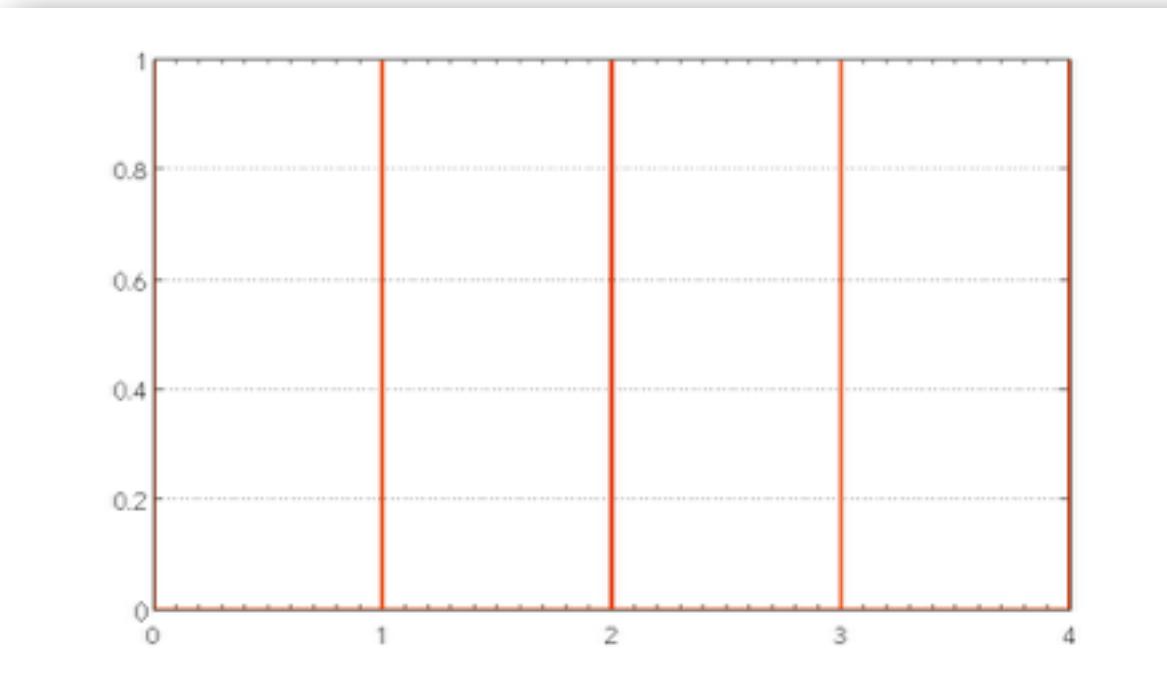
# Recall: Images and Camera



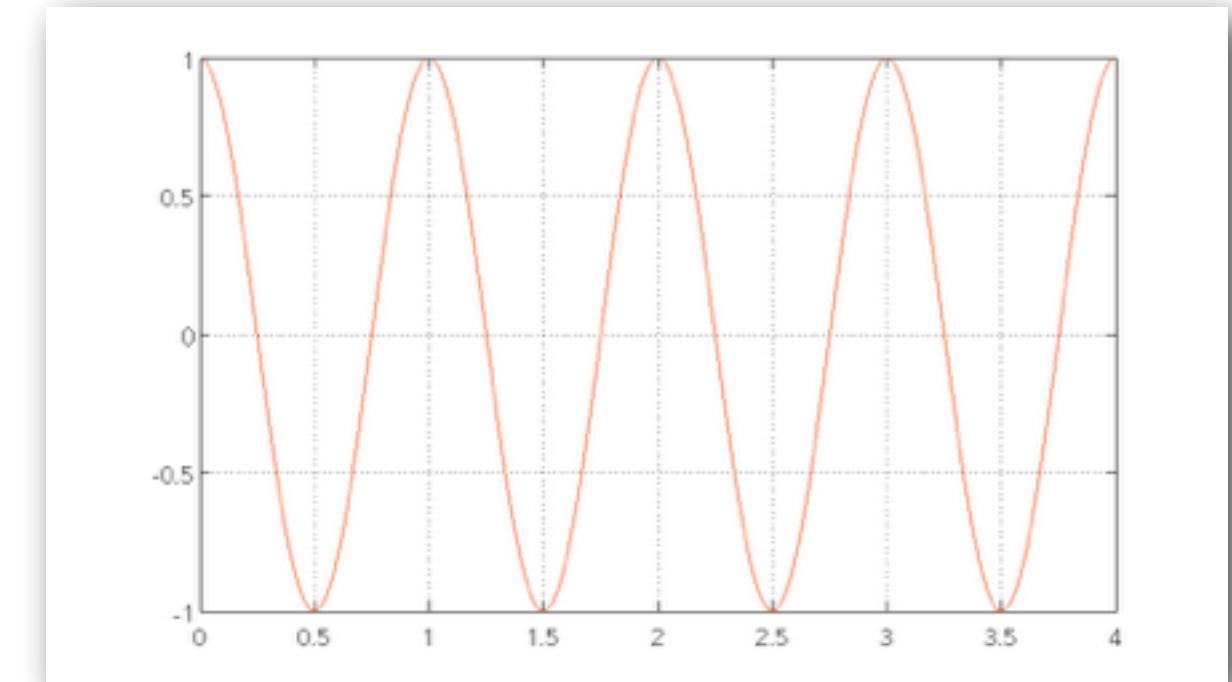
- \* Rays of Light go through a Camera / Optics / Lens
- \* Aperture, Shutter, and Film Sensitivity
- \* Sensor to Convert Light to Digital Information
- \* Process the Image

# Reconstructing a Signal

Target Signal:  $f^T(t)$



$f_1$



Repeating Impulse Function

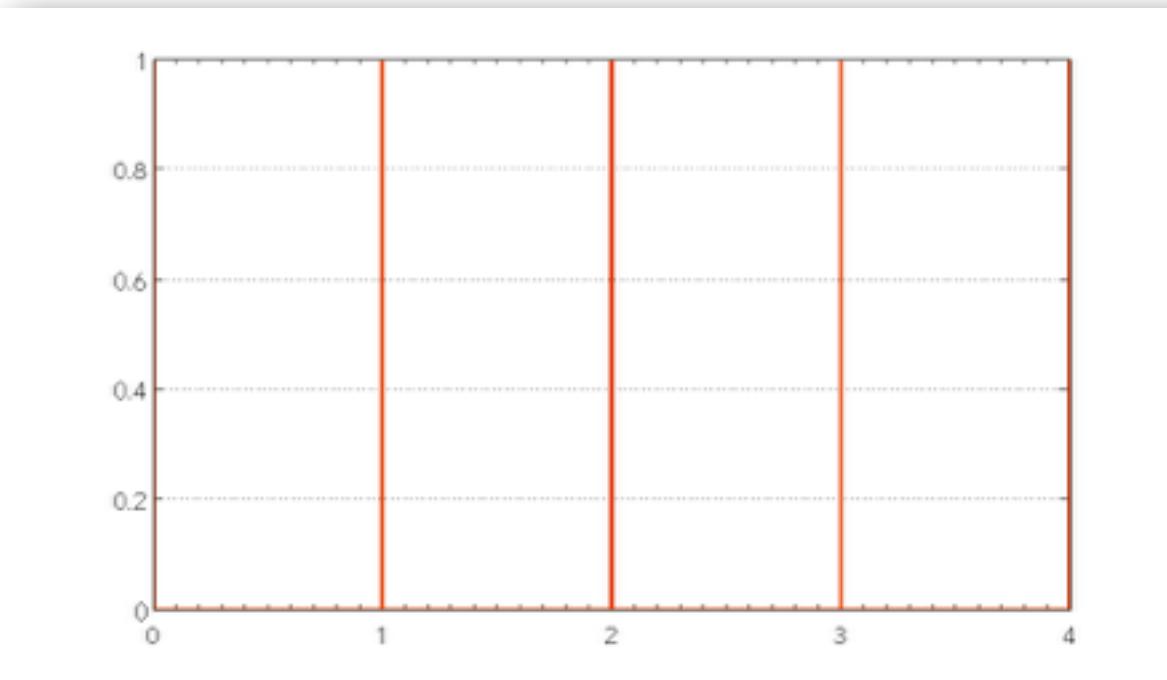
\* Basic building block

$$f(t) = A \cos(nwt)$$

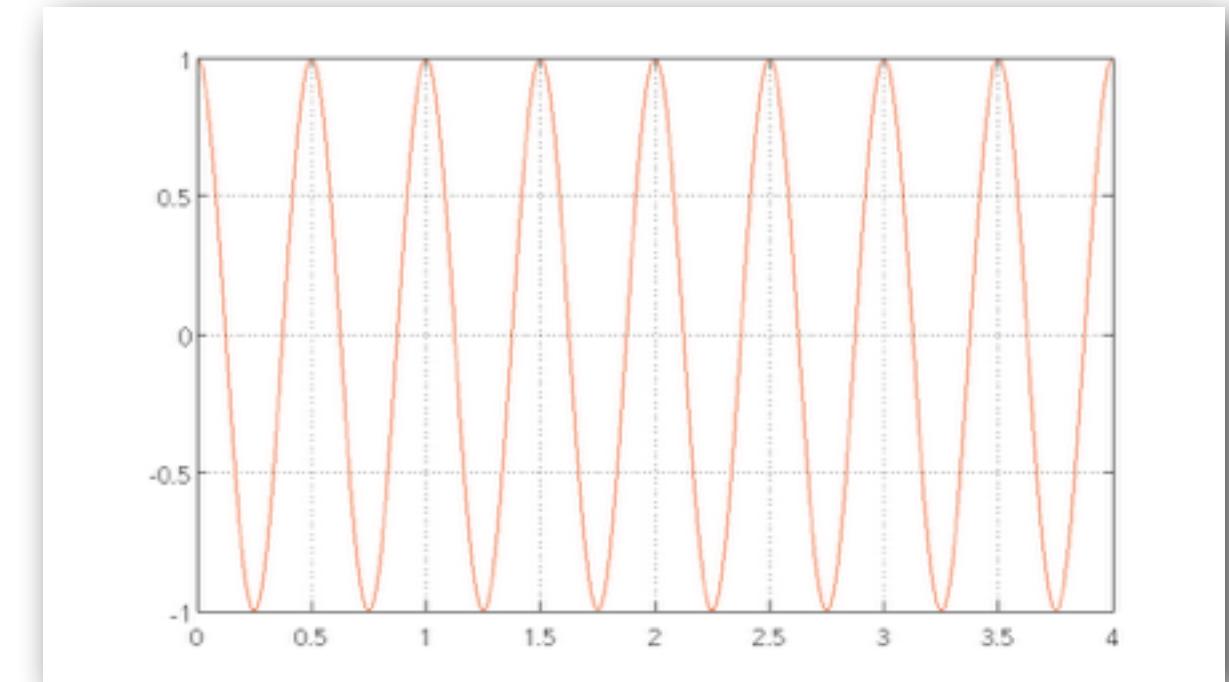
\*  $A$ : Amplitude,  $w$ : frequency

# Reconstructing a Signal

Target Signal:  $f^T(t)$



$f_2$



Repeating Impulse Function

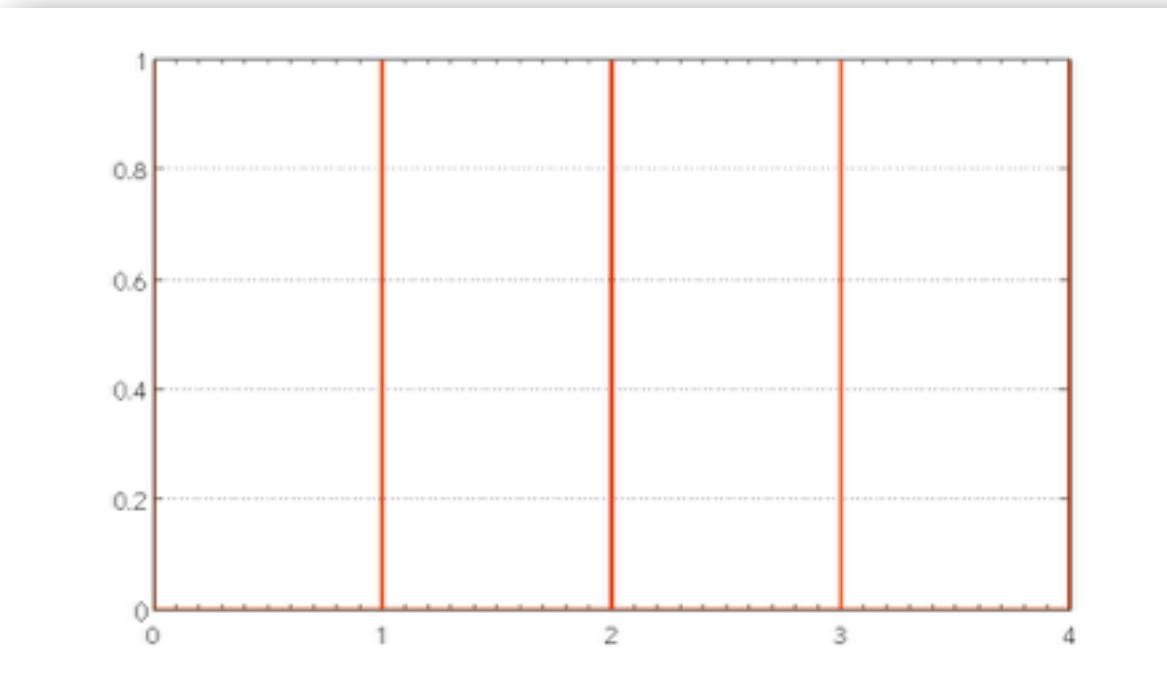
\* Basic building block

\* A: Amplitude, w: frequency

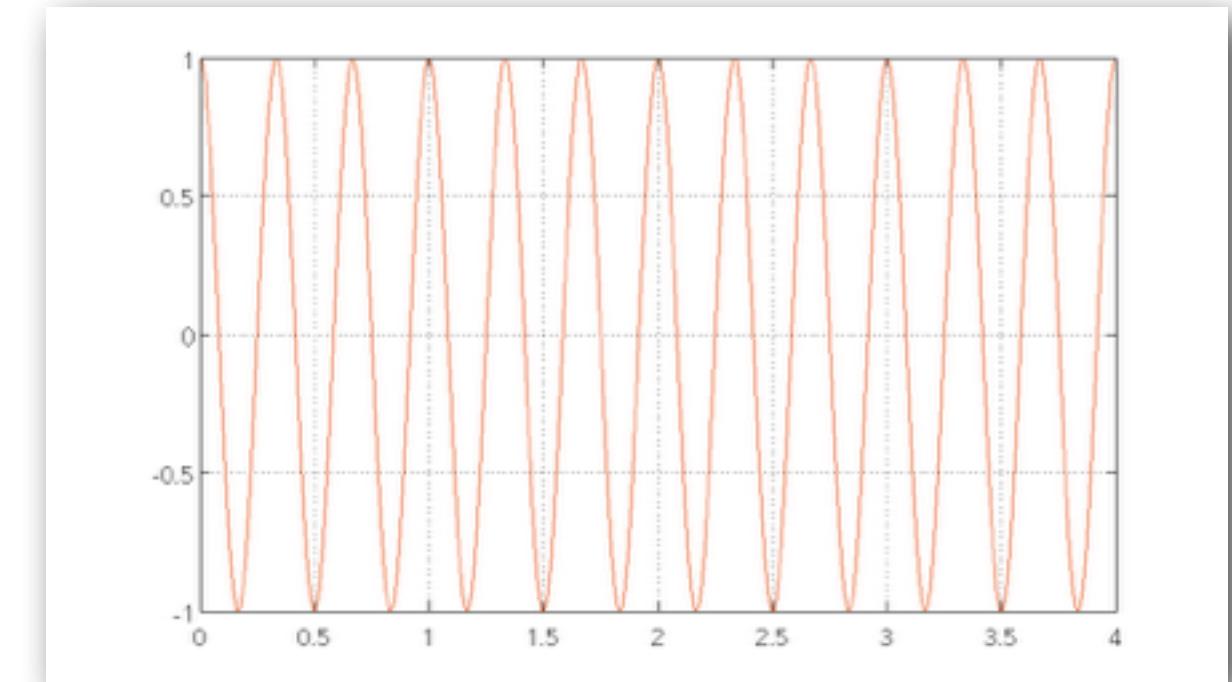
$$f(t) = A \cos(nwt)$$

# Reconstructing a Signal

Target Signal:  $f^T(t)$



$f_3$



Repeating Impulse Function

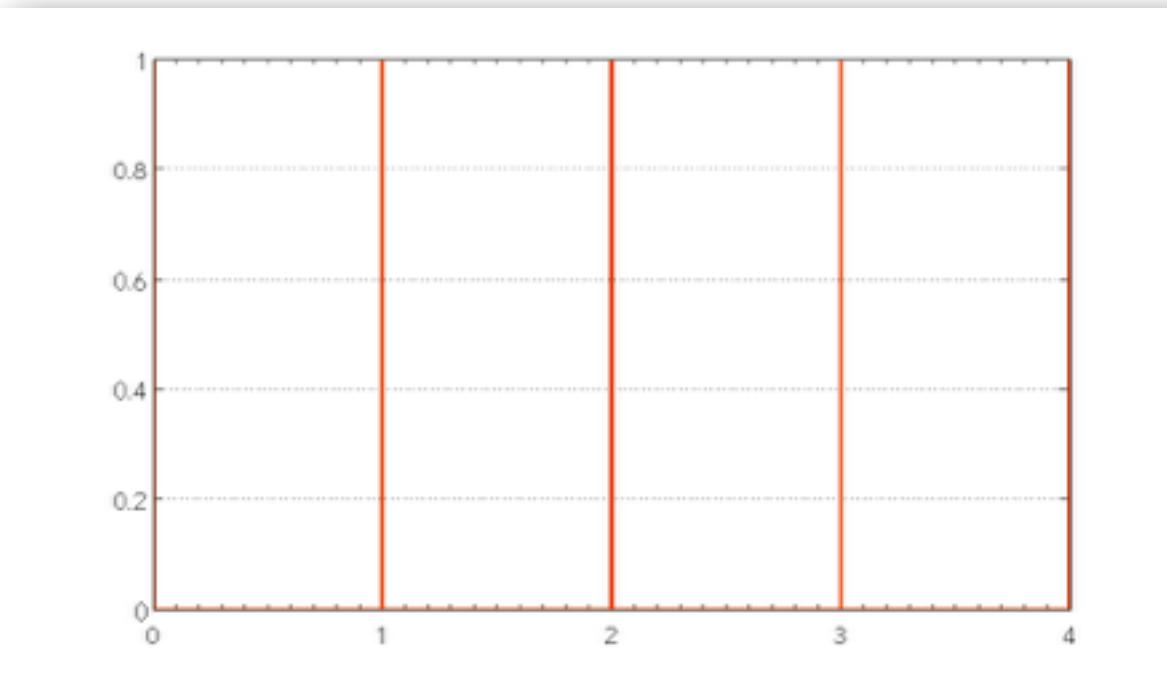
\* Basic building block

\* A: Amplitude, w: frequency

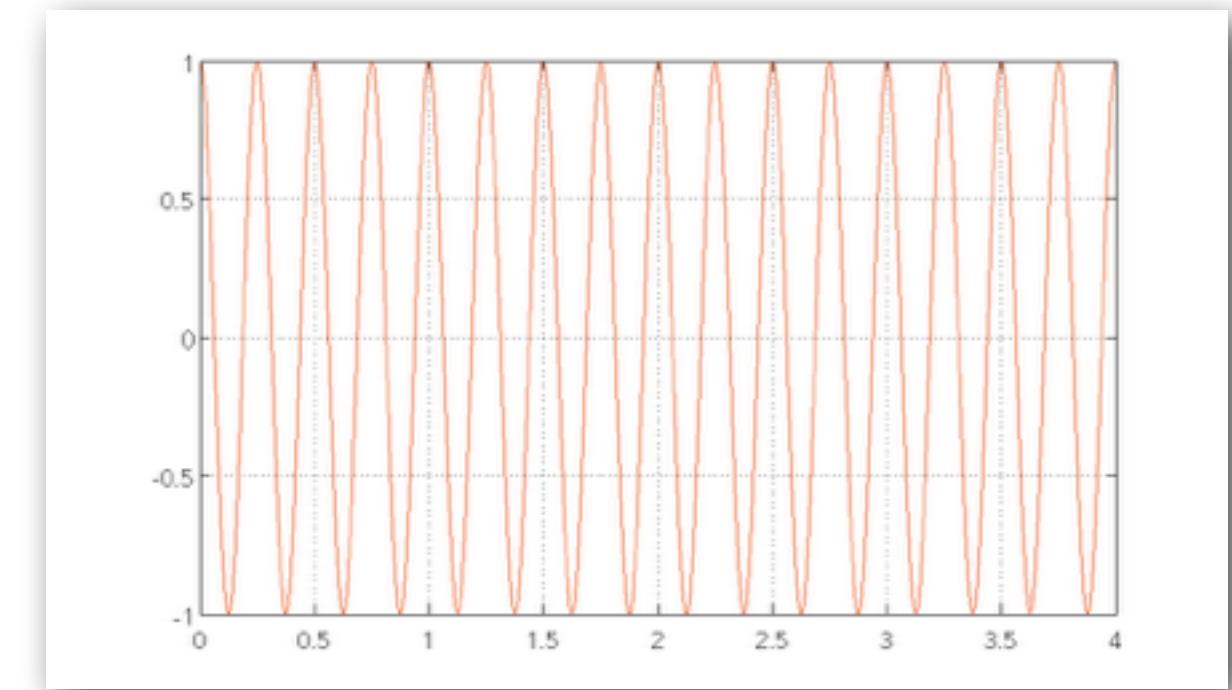
$$f(t) = A \cos(nwt)$$

# Reconstructing a Signal

Target Signal:  $f^T(t)$



$f_4$



Repeating Impulse Function

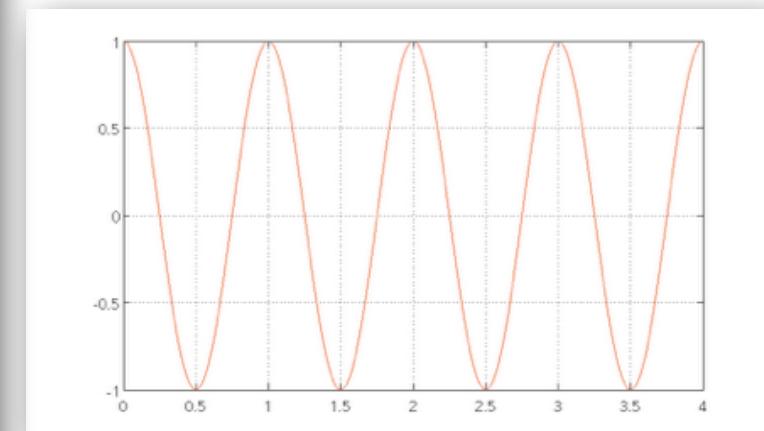
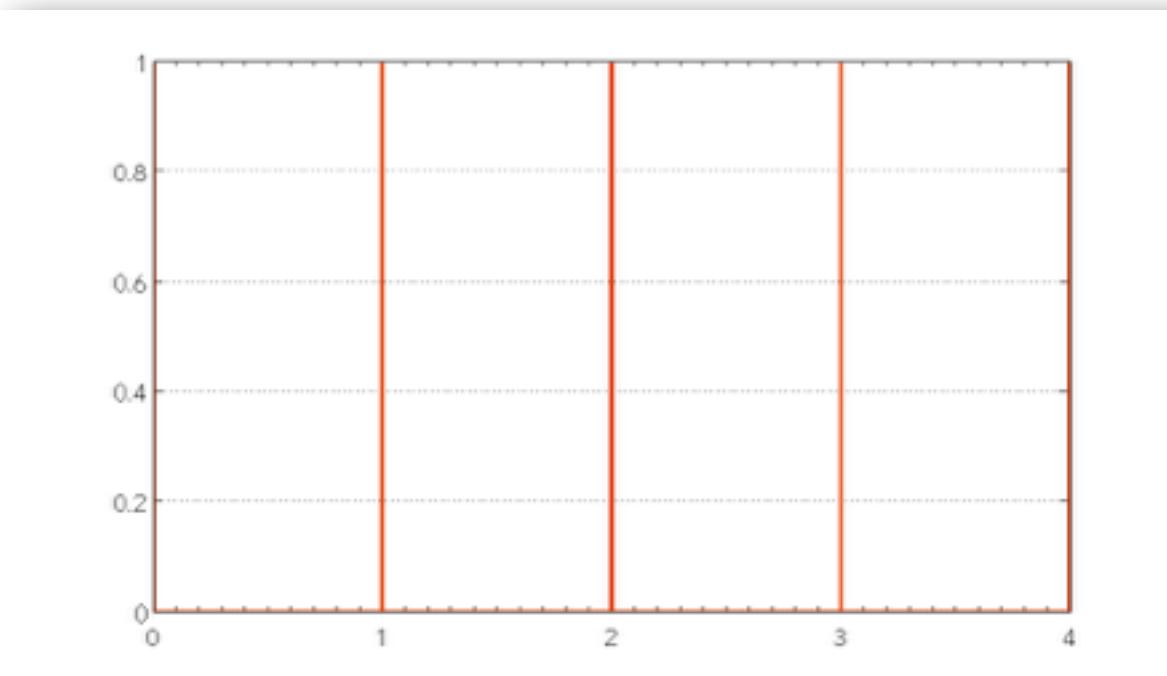
\* Basic building block

\* A: Amplitude, w: frequency

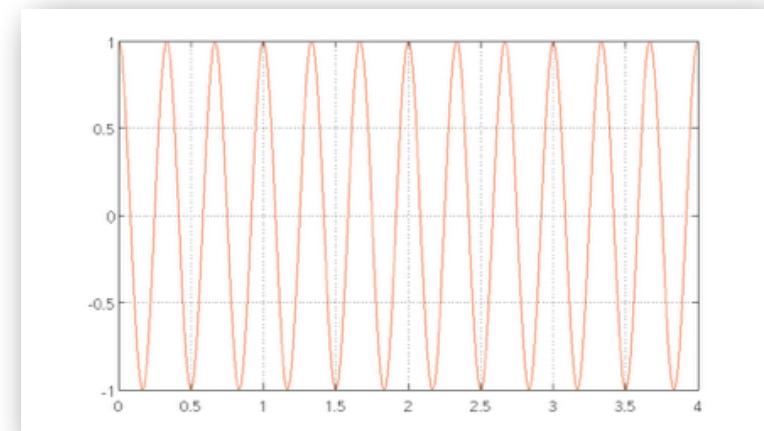
$$f(t) = A \cos(nwt)$$

# Reconstructing a Signal

Target Signal:  $f^T(t)$

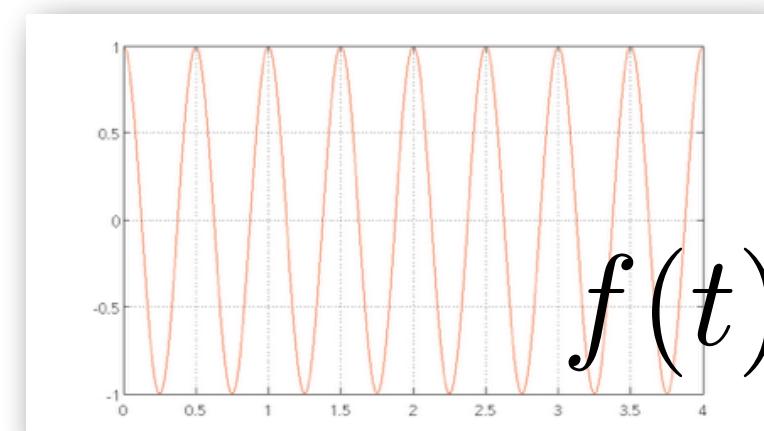


$f_1$

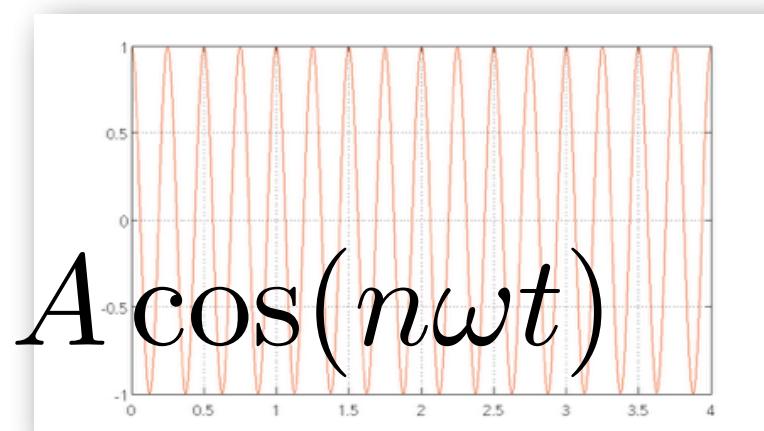


$f_3$

Repeating Impulse Function



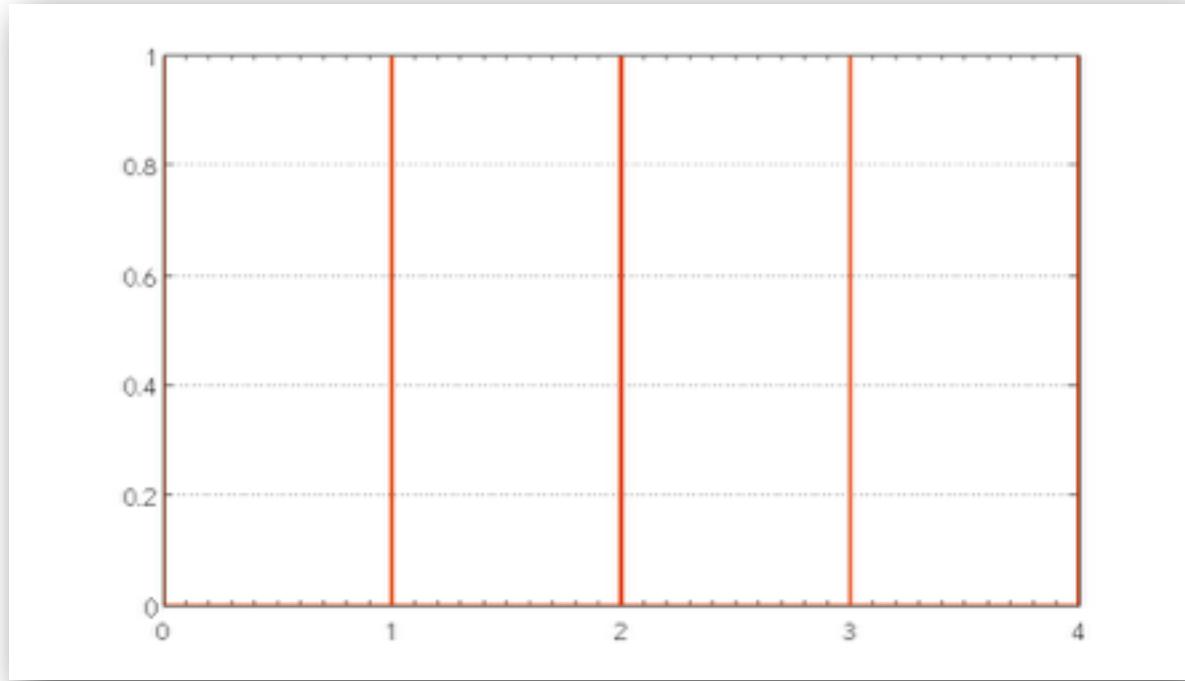
$f_2$



$f_4$

# Sum of Cosines

Target Signal:  $f^T(t)$

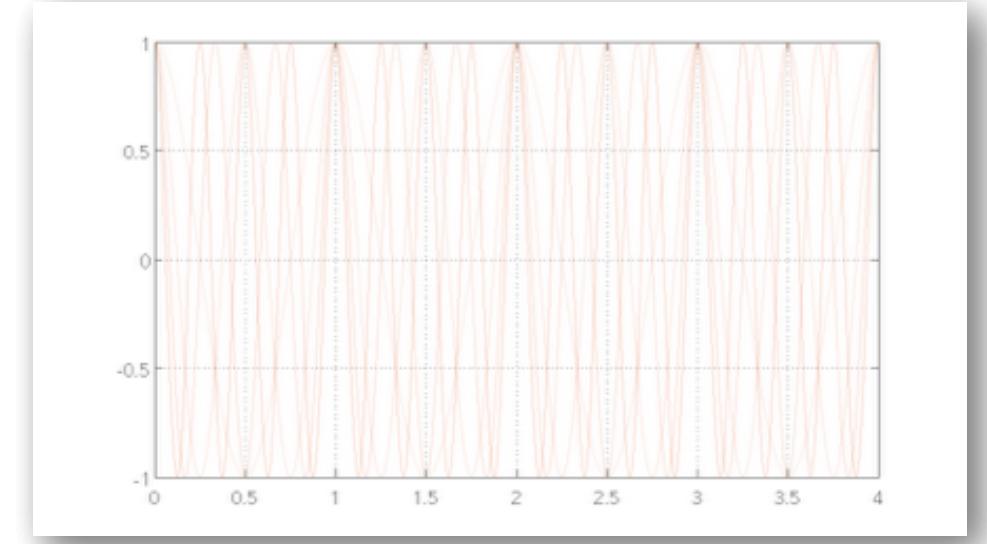


Repeating Impulse Function

\* Adding Cosines ( $N$  of them)

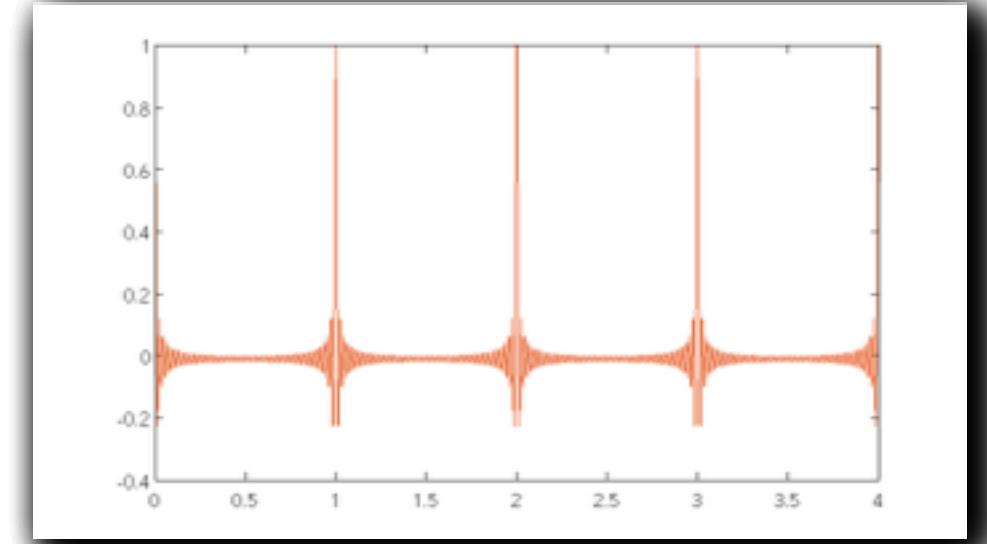
$$f^T(t) = \sum_{n=1}^N A \cos(n\omega t)$$

$f_1 \quad f_2 \quad f_3 \quad f_4$



$N=10$

$N=50$



$f_1 + f_2 + f_3 + f_4$

# A Fourier Transform

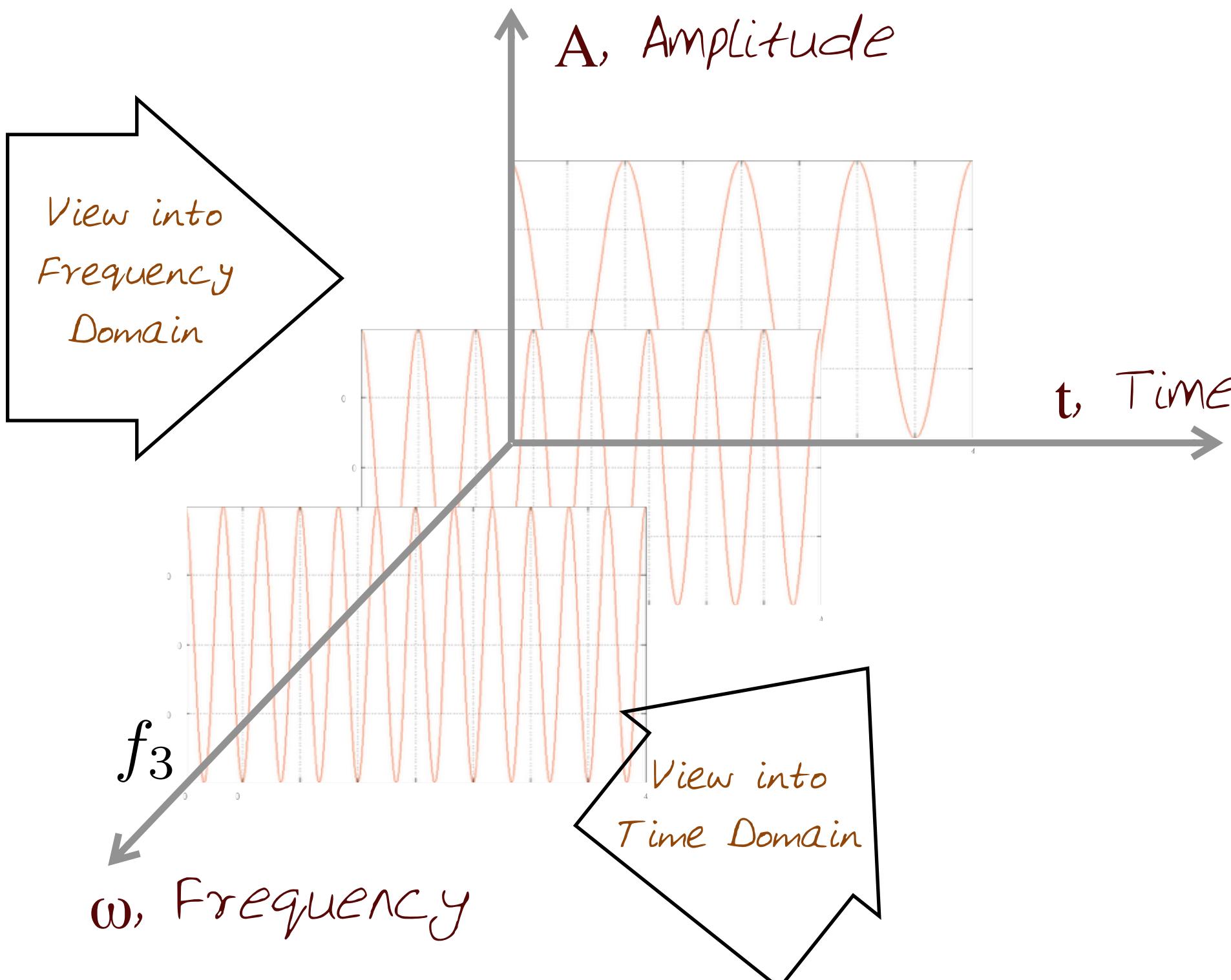


Jean Baptiste  
Joseph Fourier  
(1768-1830)

- \* Periodic function  $\Rightarrow$  a weighted sum of sines and cosines of different frequencies
- \* Transforms  $f(t)$ , into a  $F(\omega)$
- \* Frequency spectrum of the function  $f$
- \* A reversible operation
- \* For every  $\omega$  from 0 to  $\infty$  (infinity)  $F(\omega)$  holds the Amplitude  $A$  and phase  $\phi$  of a sine function

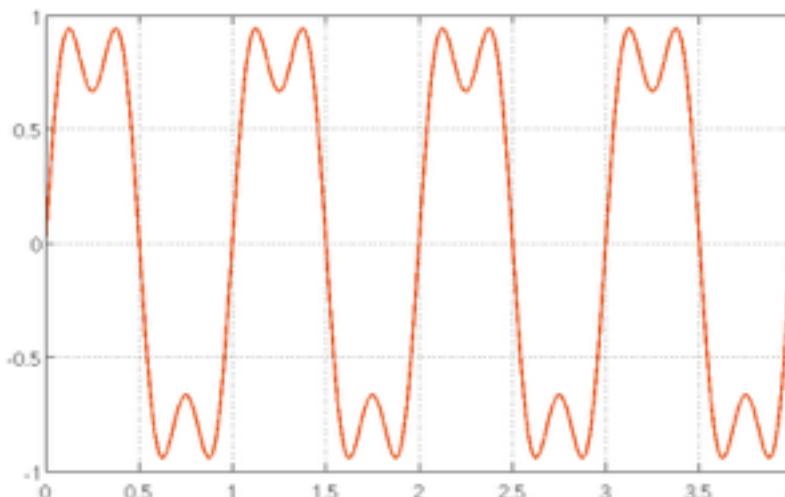
$$F(\omega) = A \cos(\omega t + \phi)$$

# Frequency Domain of a Signal

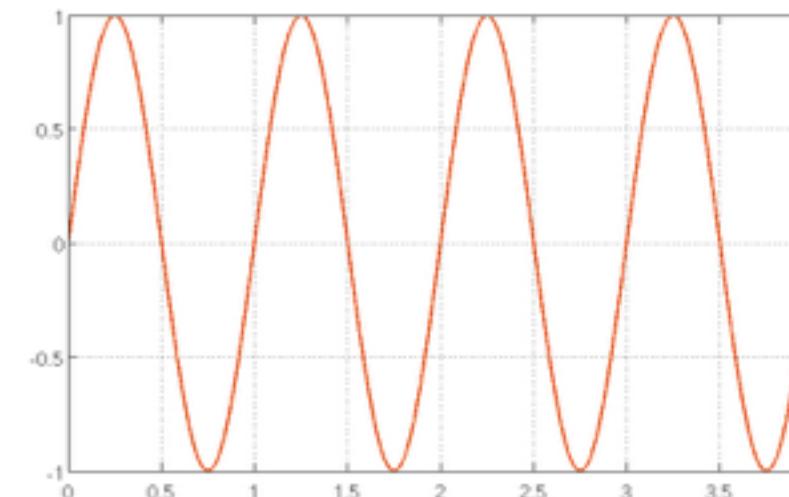


- \* How many  $N$ ?
- \* What does each control?
- \* Coarse vs. fine structure

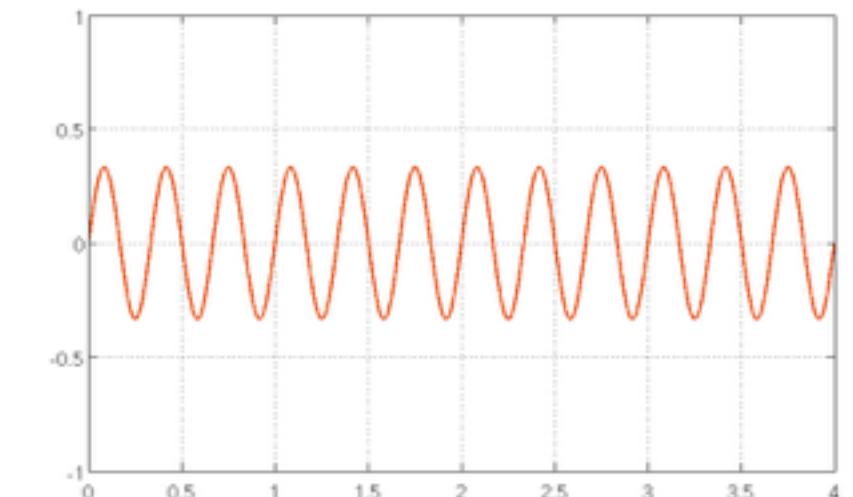
# Time, Frequency, and Frequency Spectra



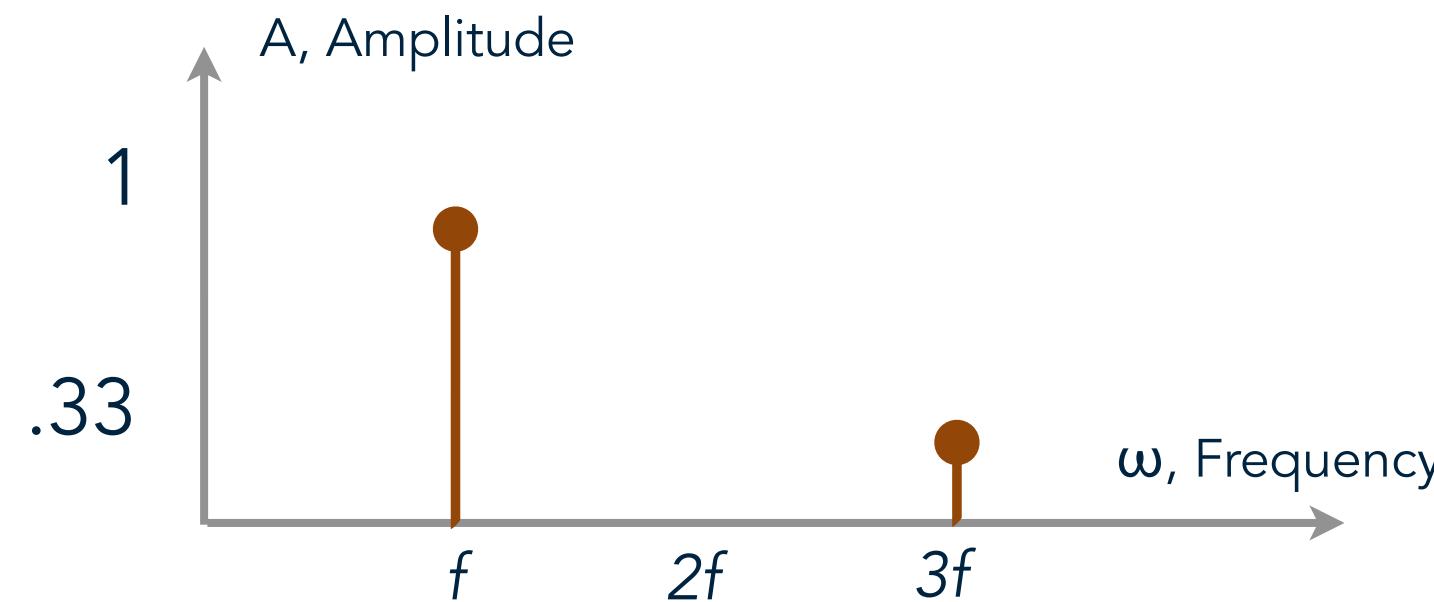
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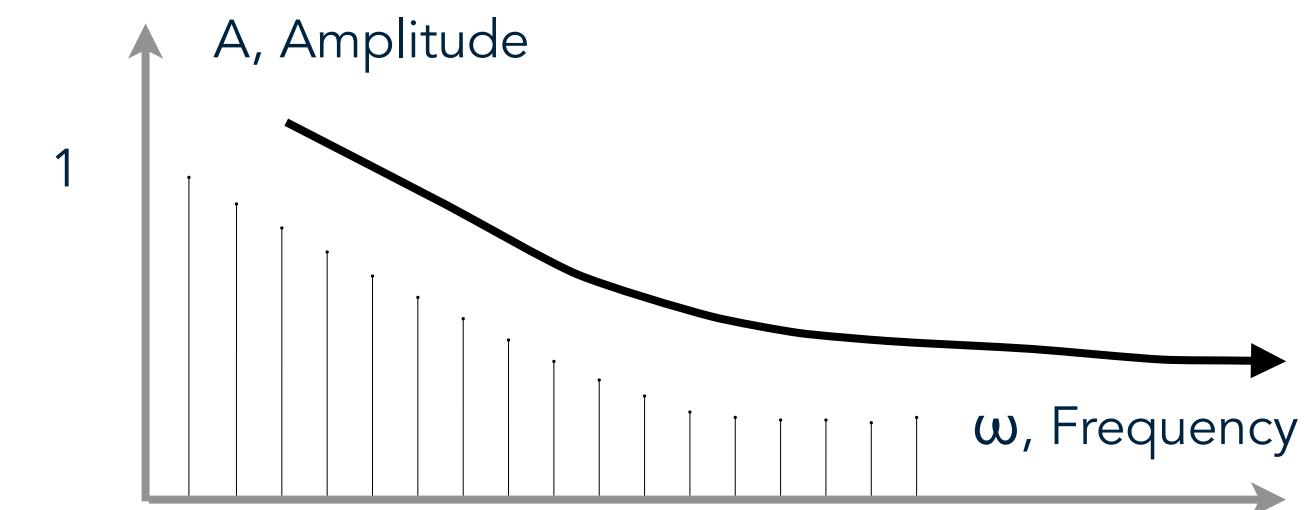
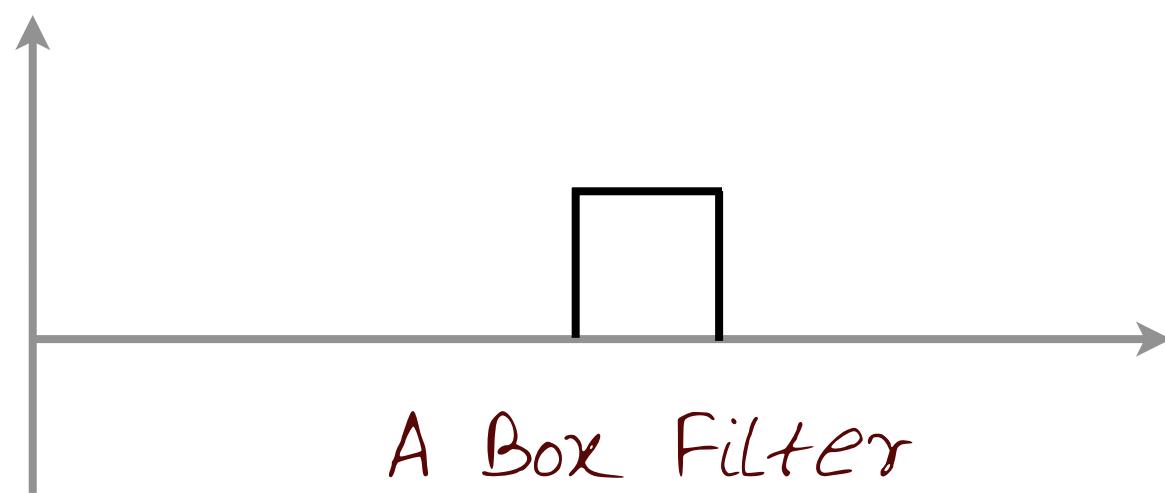
+



$$g(t) = \sin(2\pi\omega t) + \frac{1}{3} \sin(2\pi(3\omega)t)$$



# Frequency Spectra



$$A \sum_{k=1}^{\infty} \frac{1}{k} \sin(2\pi k t)$$

# Convolution Theorem and the Fourier Transform

- \* Fourier transform of a convolution of two functions = product of their Fourier transforms

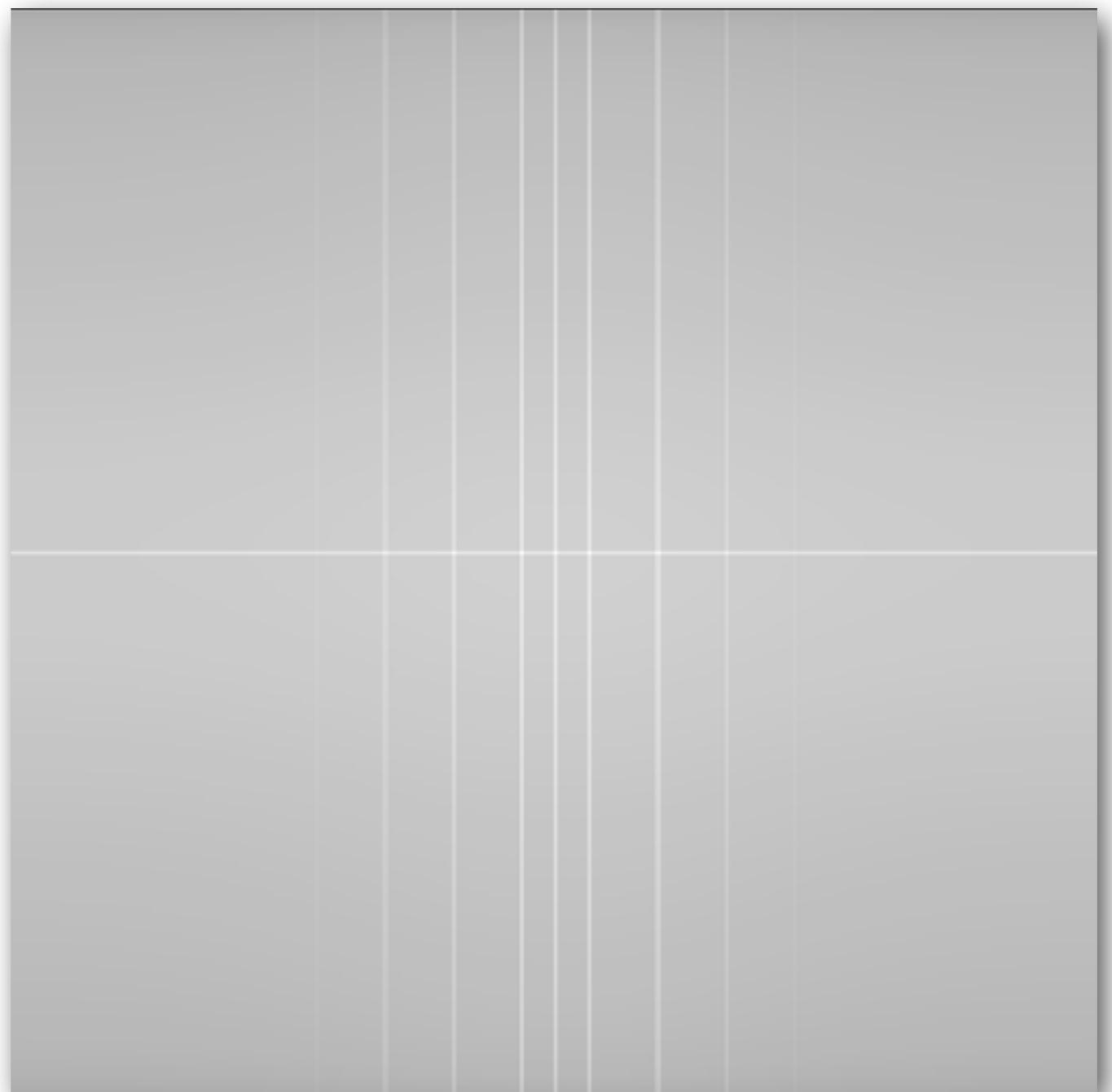
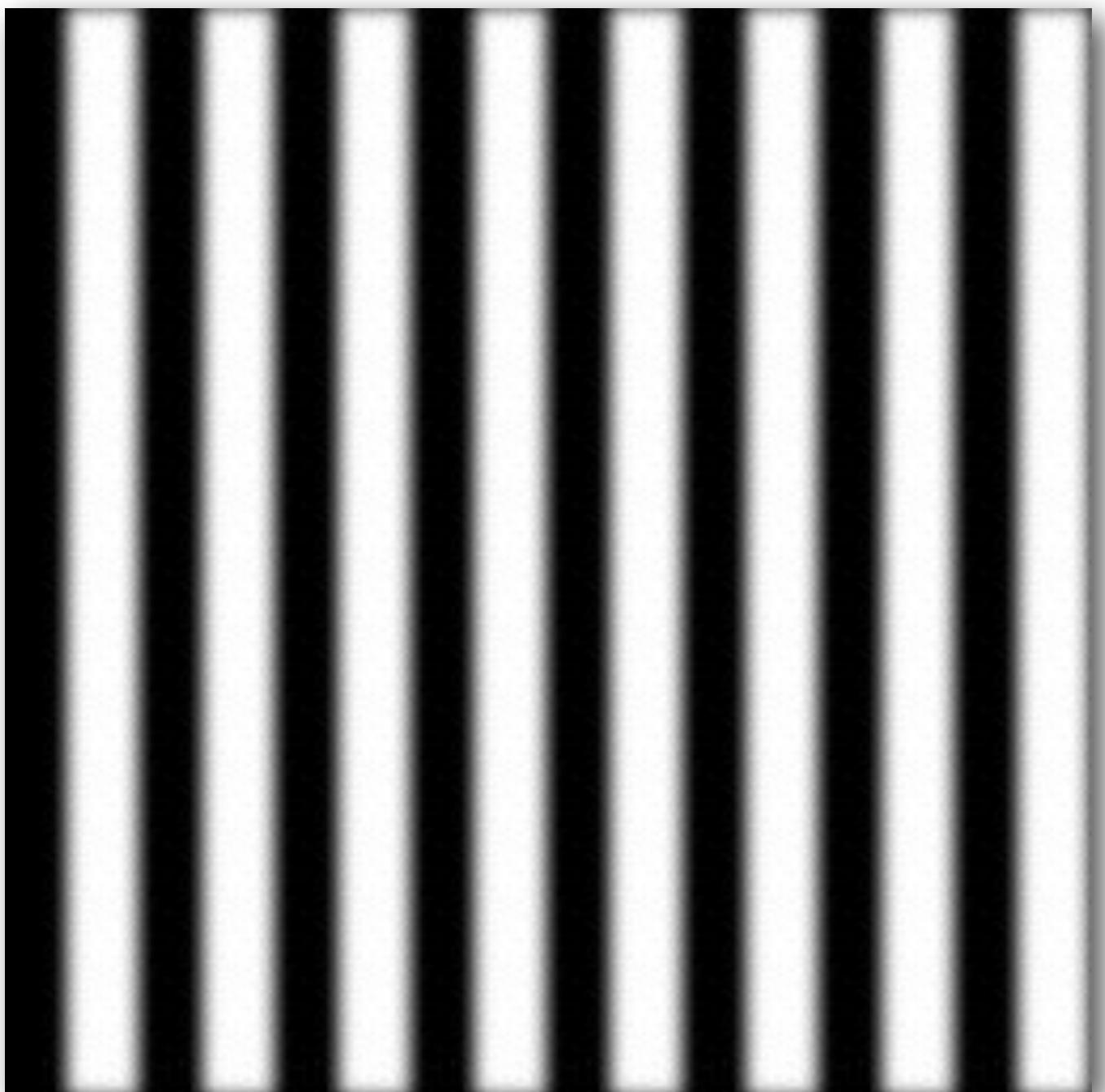
$$F[g * h] = F[g]F[h]$$

- \* Inverse Fourier transform of the product of two Fourier transforms = convolution of the two inverse Fourier transforms

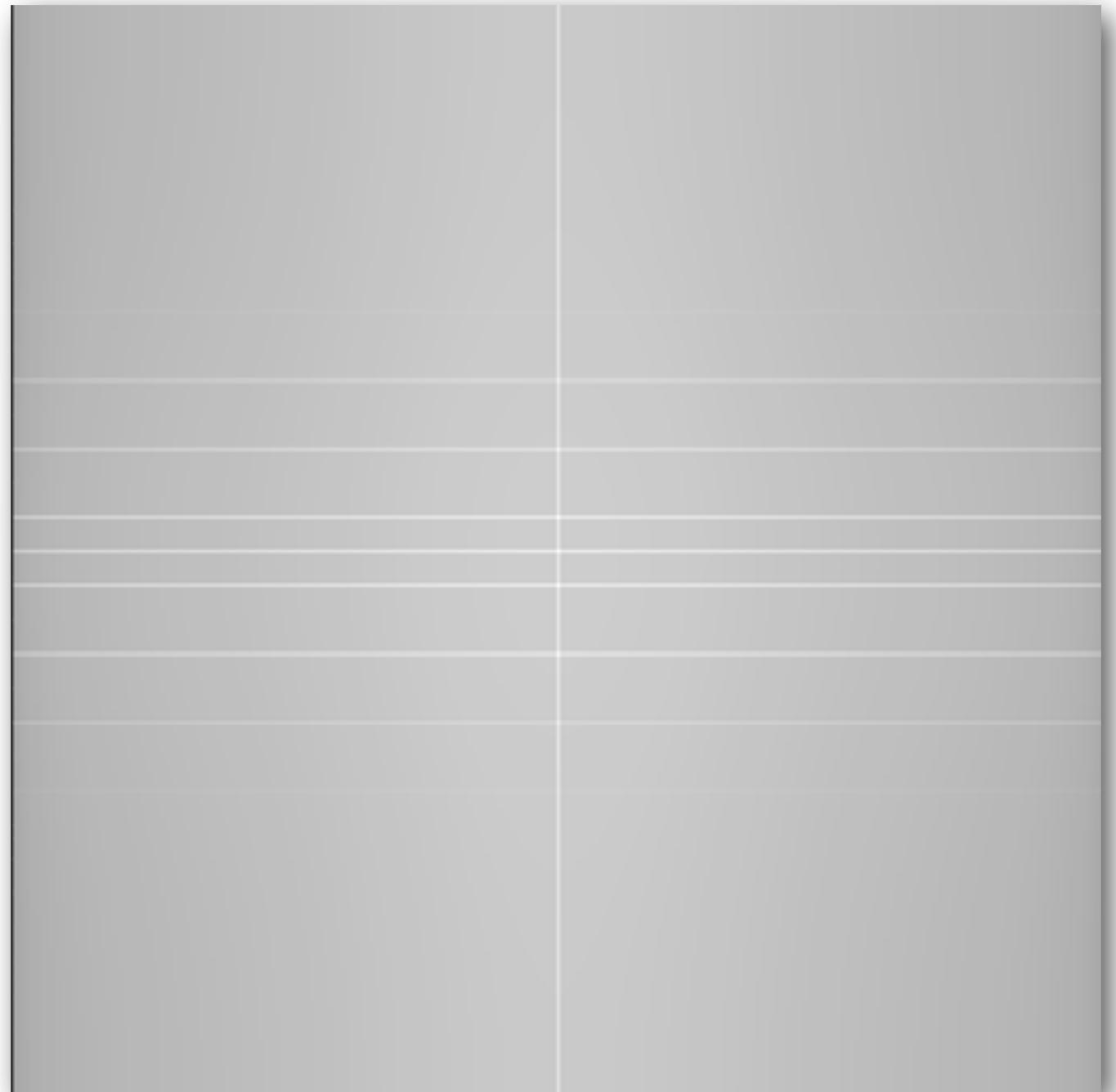
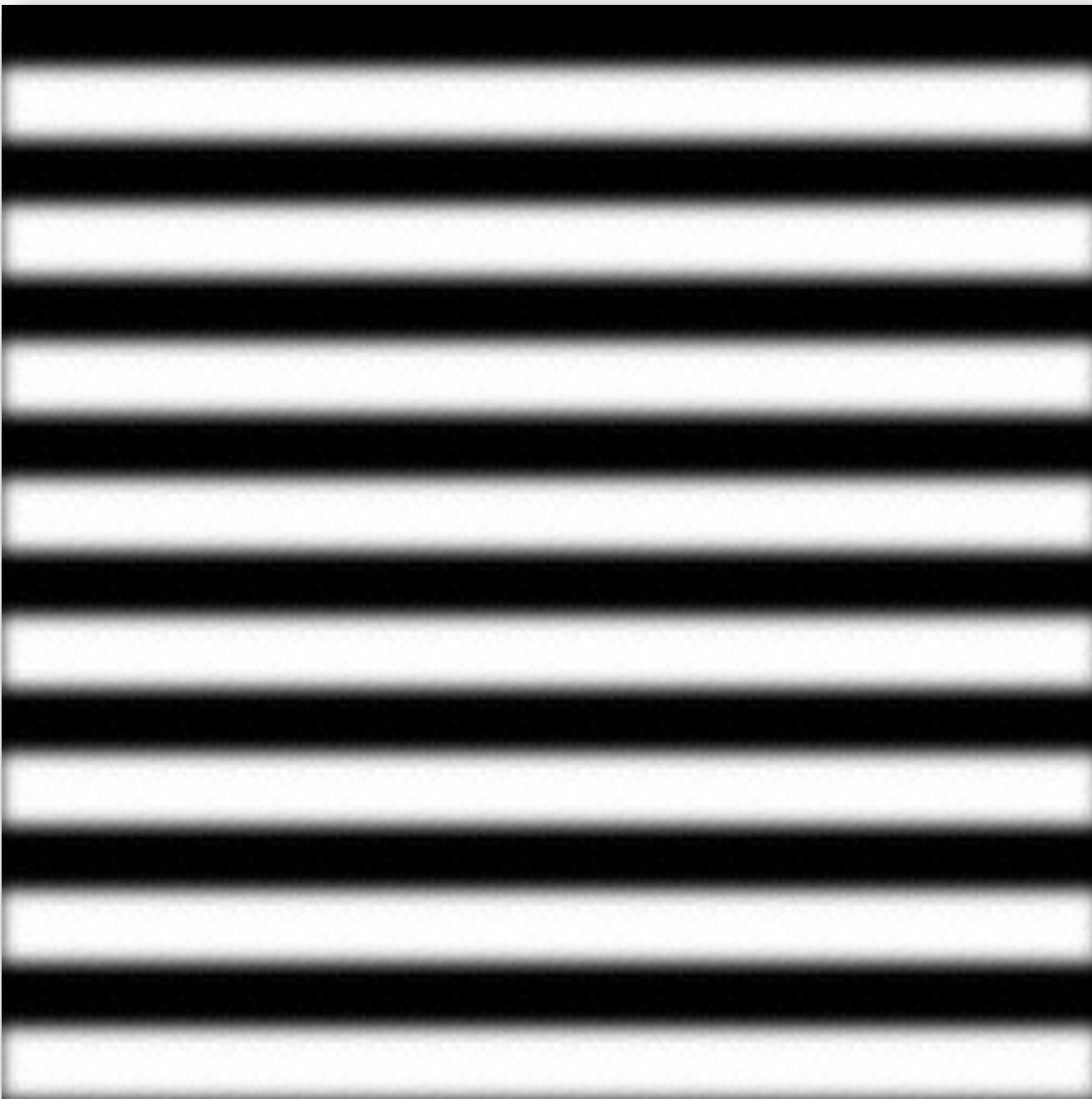
$$F^{-1}[gh] = F^{-1}[g] * F^{-1}[h]$$

- \* Convolution in spatial domain is equivalent to multiplication in frequency domain!

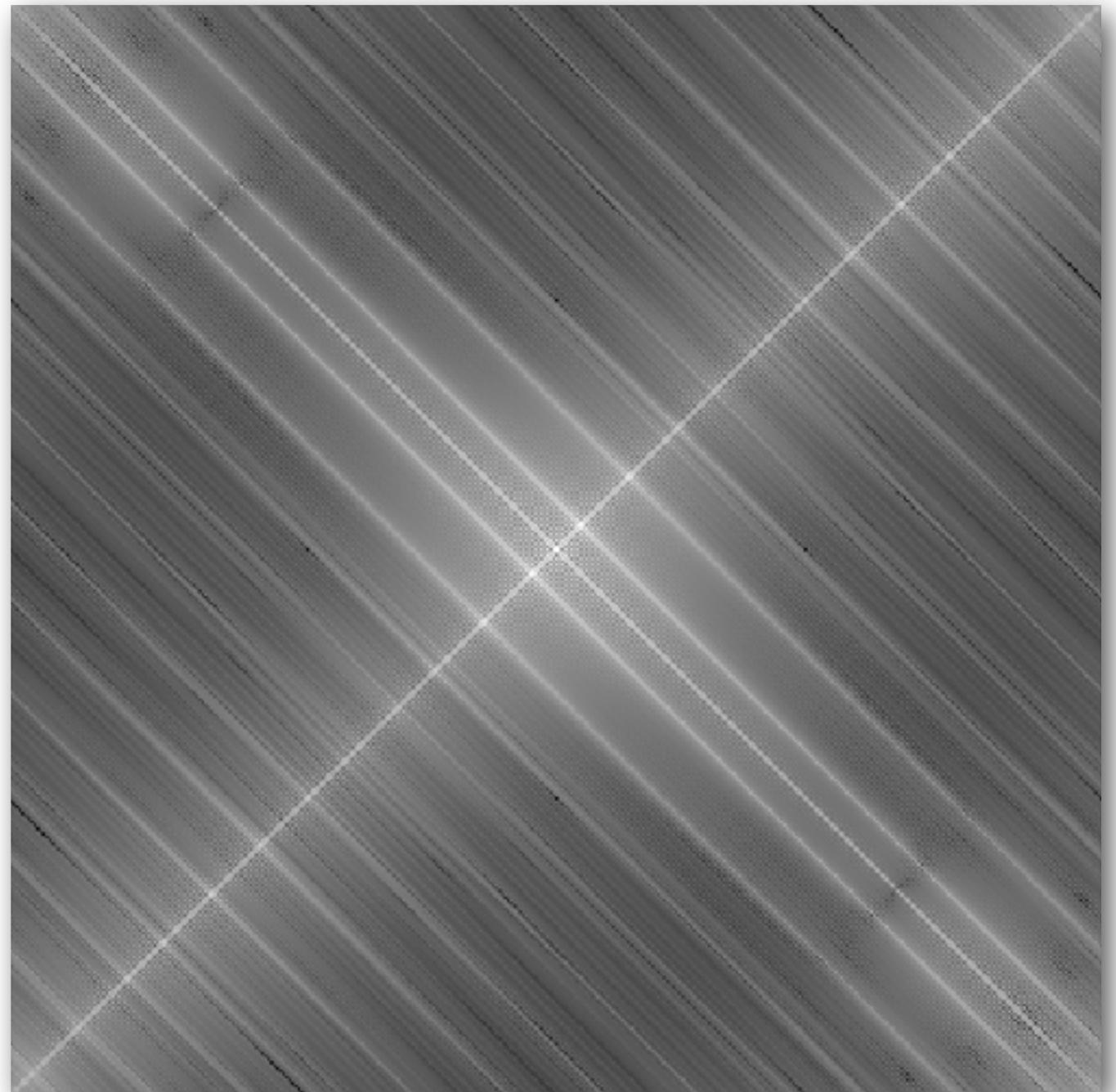
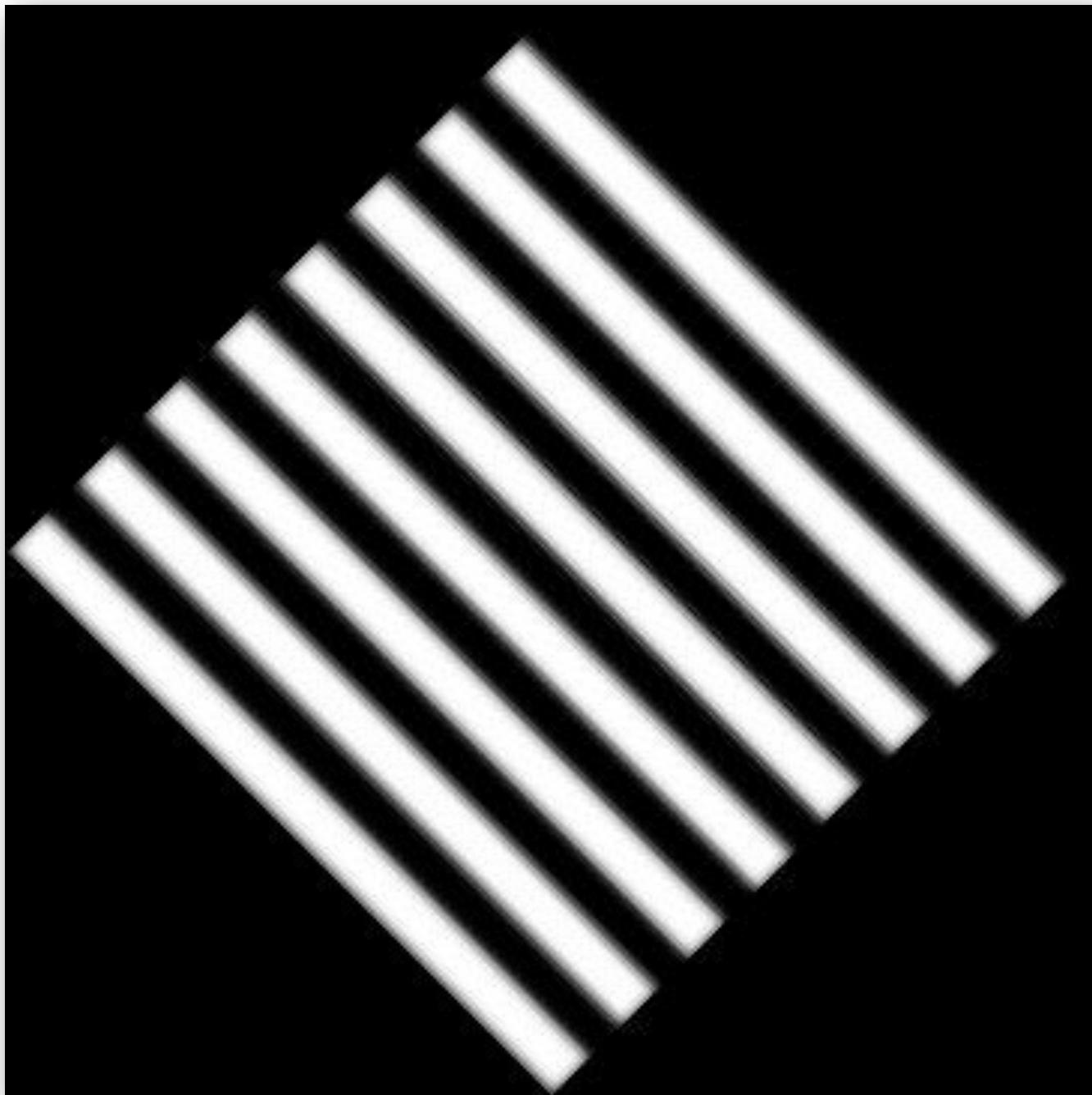
Now, Frequency Spectra for Images



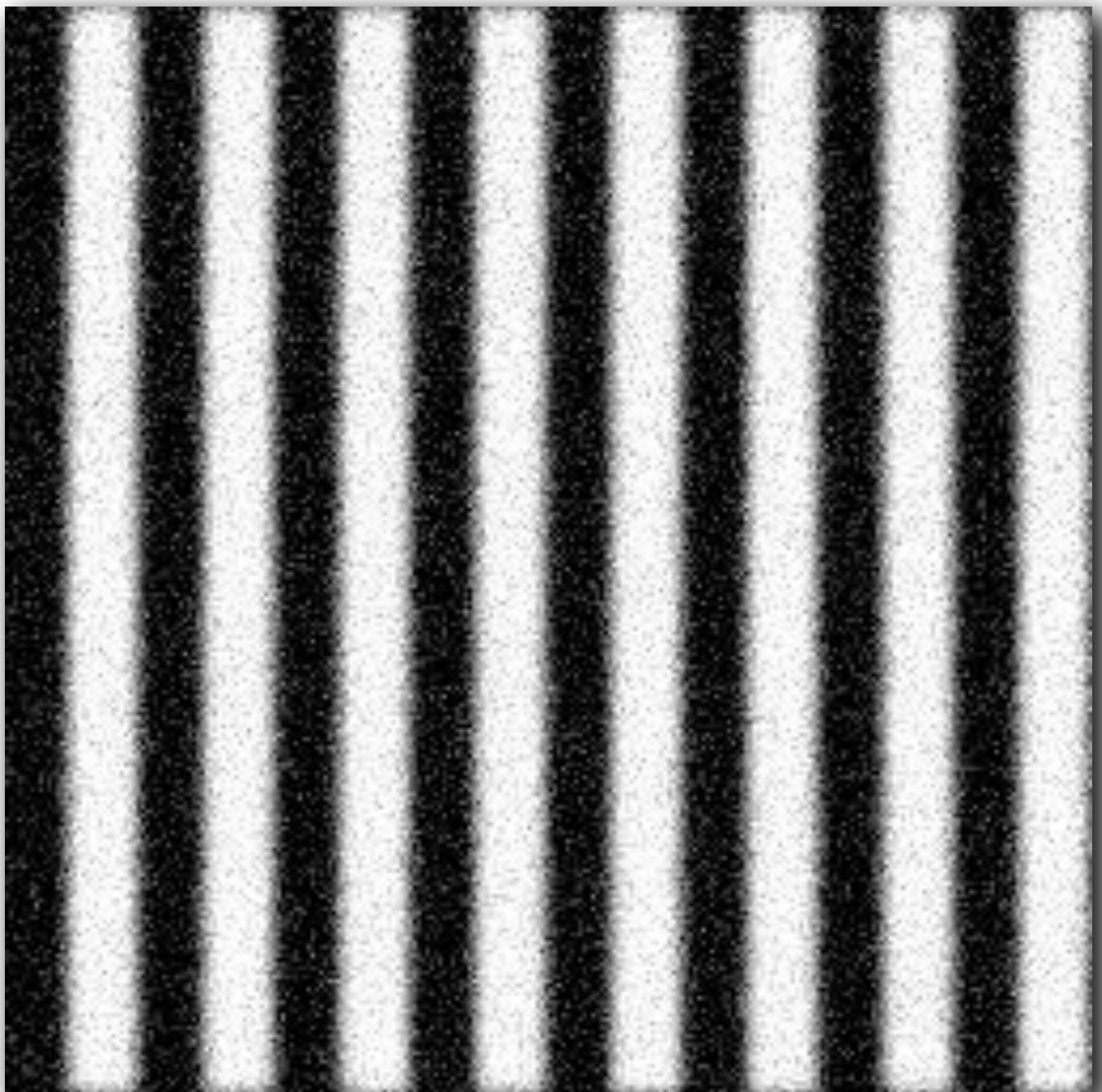
Now, Frequency Spectra for Images



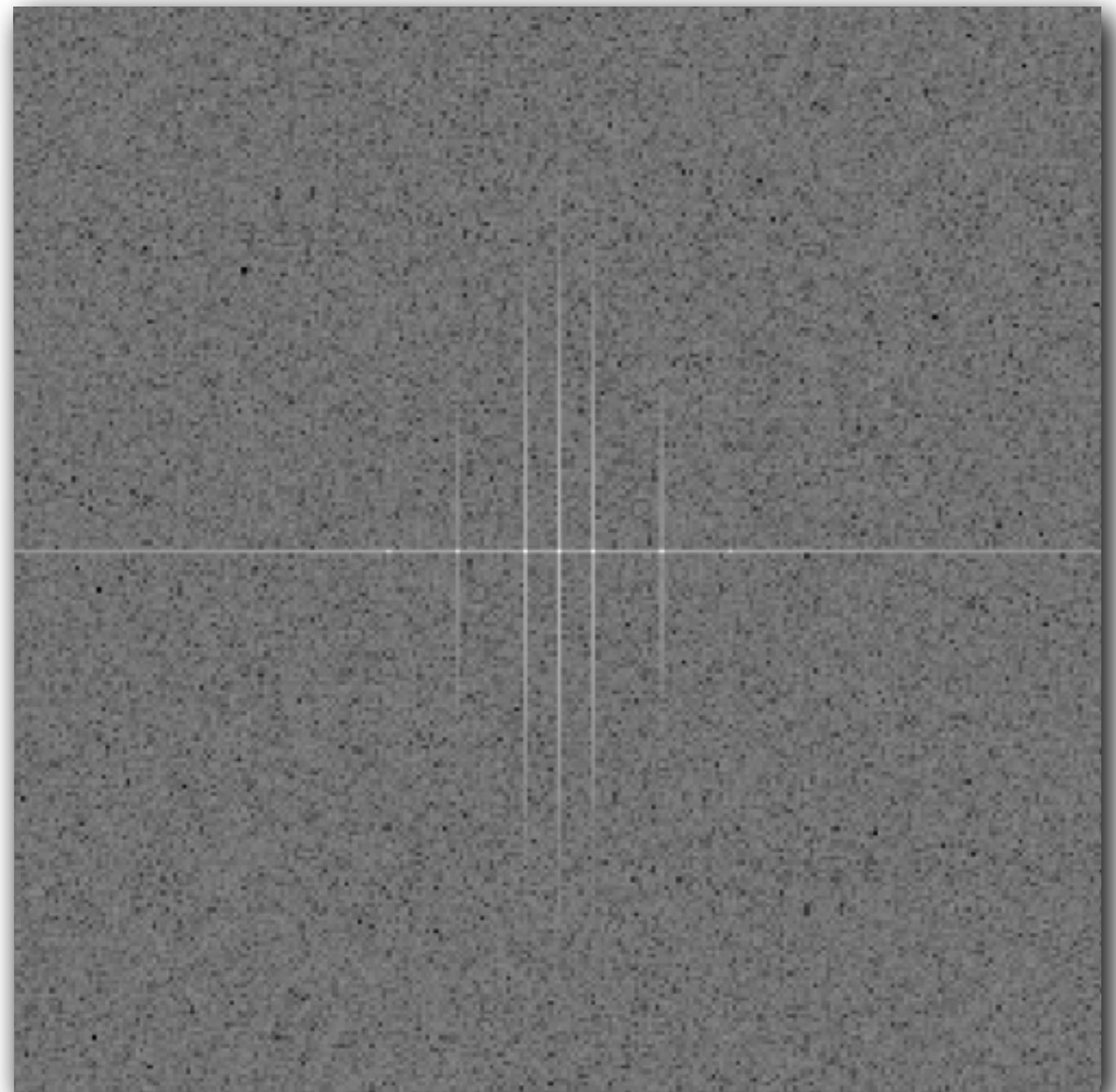
Now, Frequency Spectra for Images



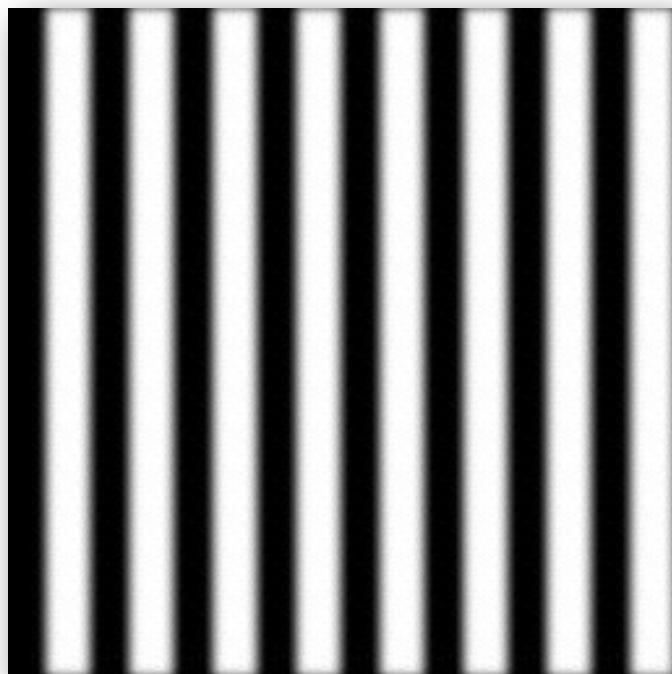
Now, Frequency Spectra for Images



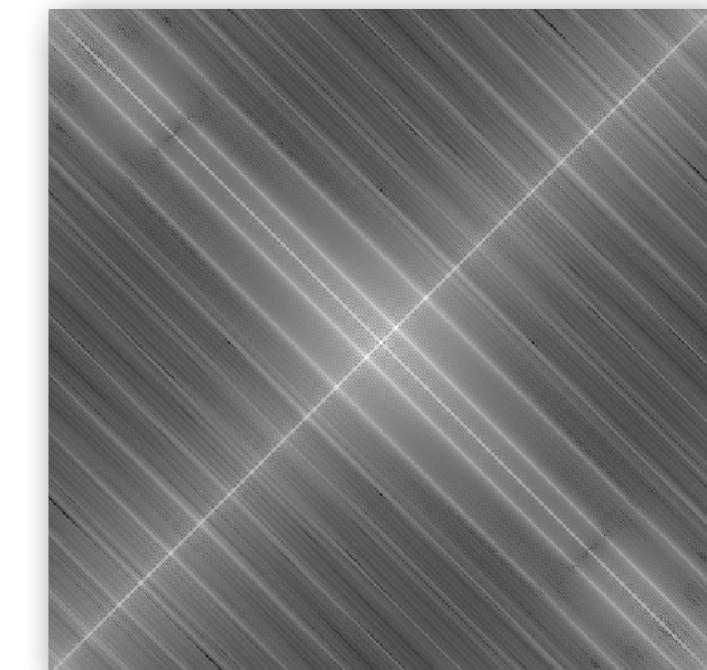
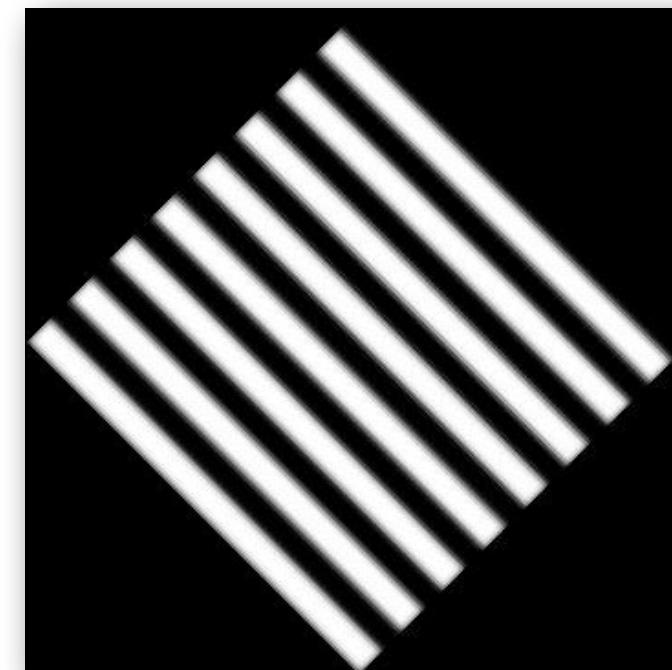
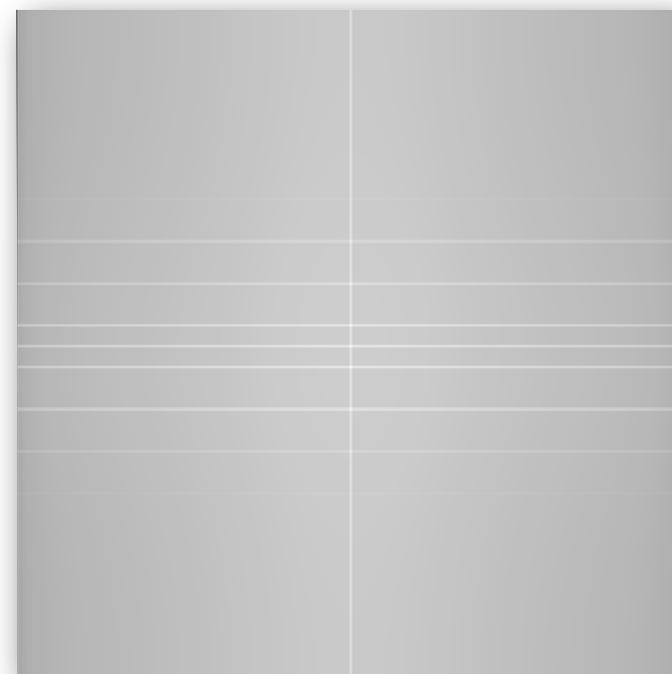
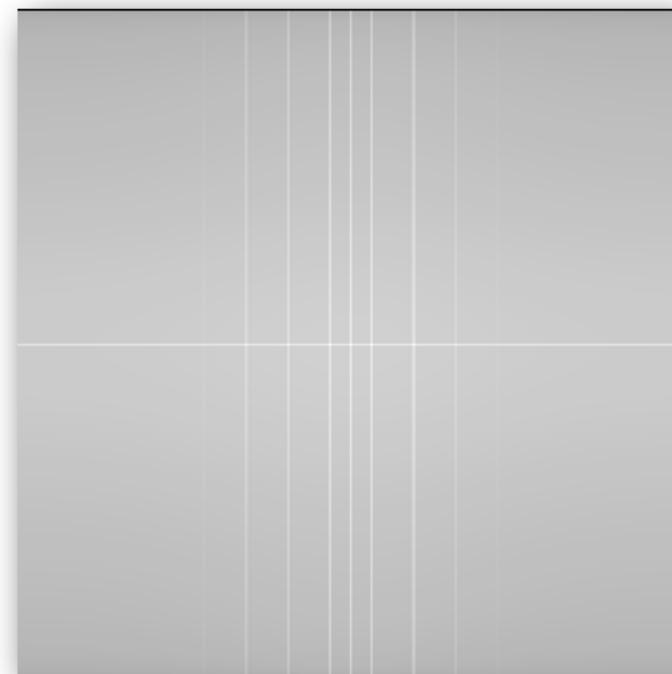
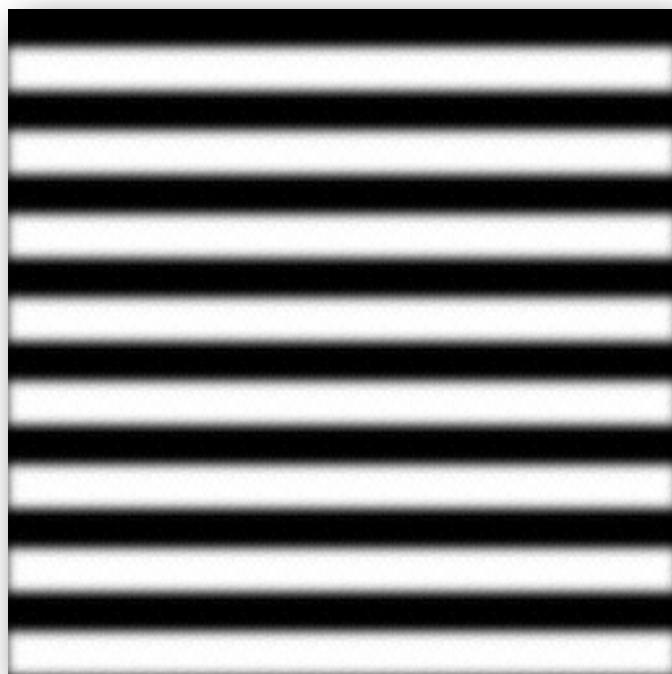
Noise Added



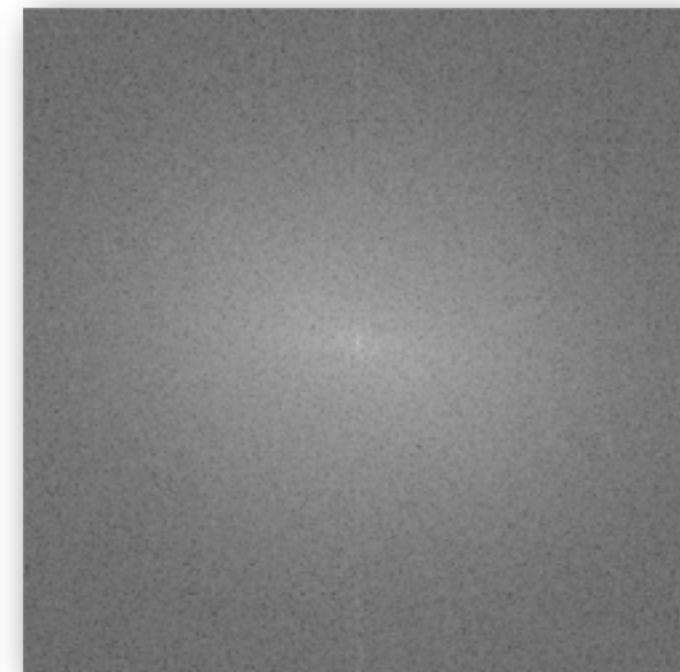
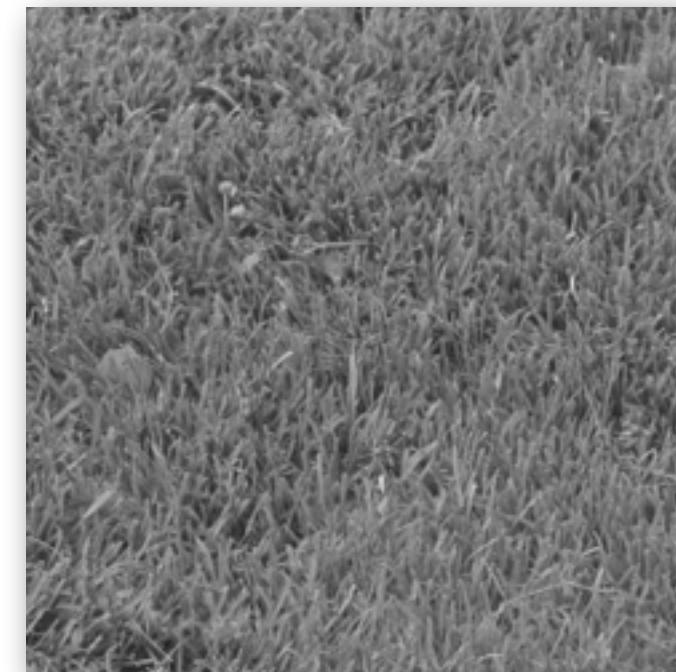
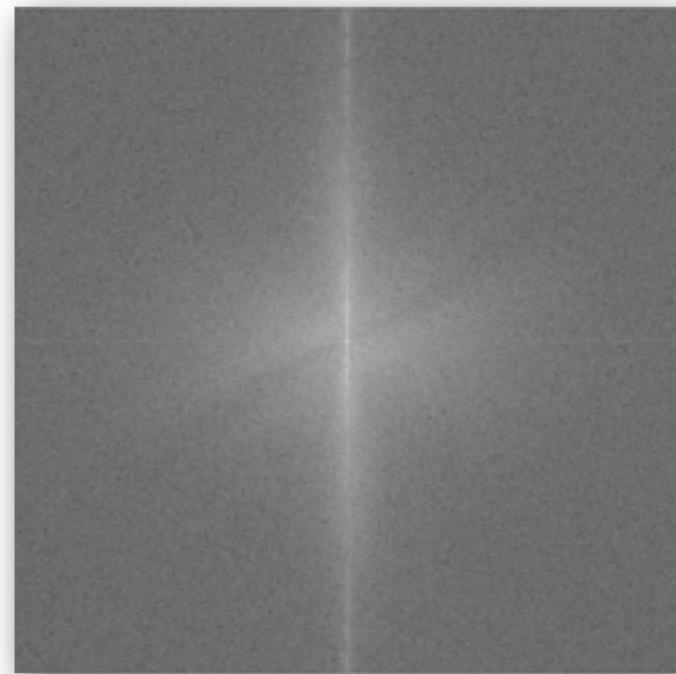
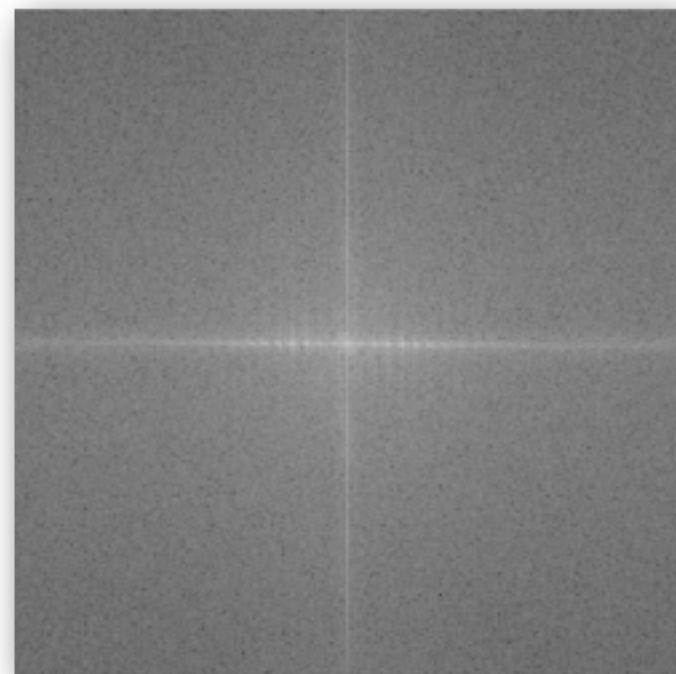
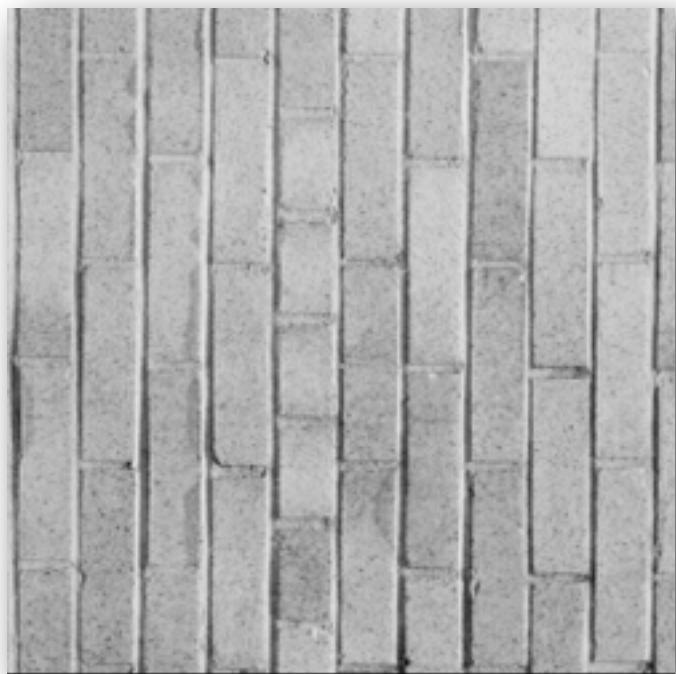
# Now, Frequency Spectra for Images



Noise Added



# Frequency Spectra for Real Images



# Fourier Transform: Some Observations

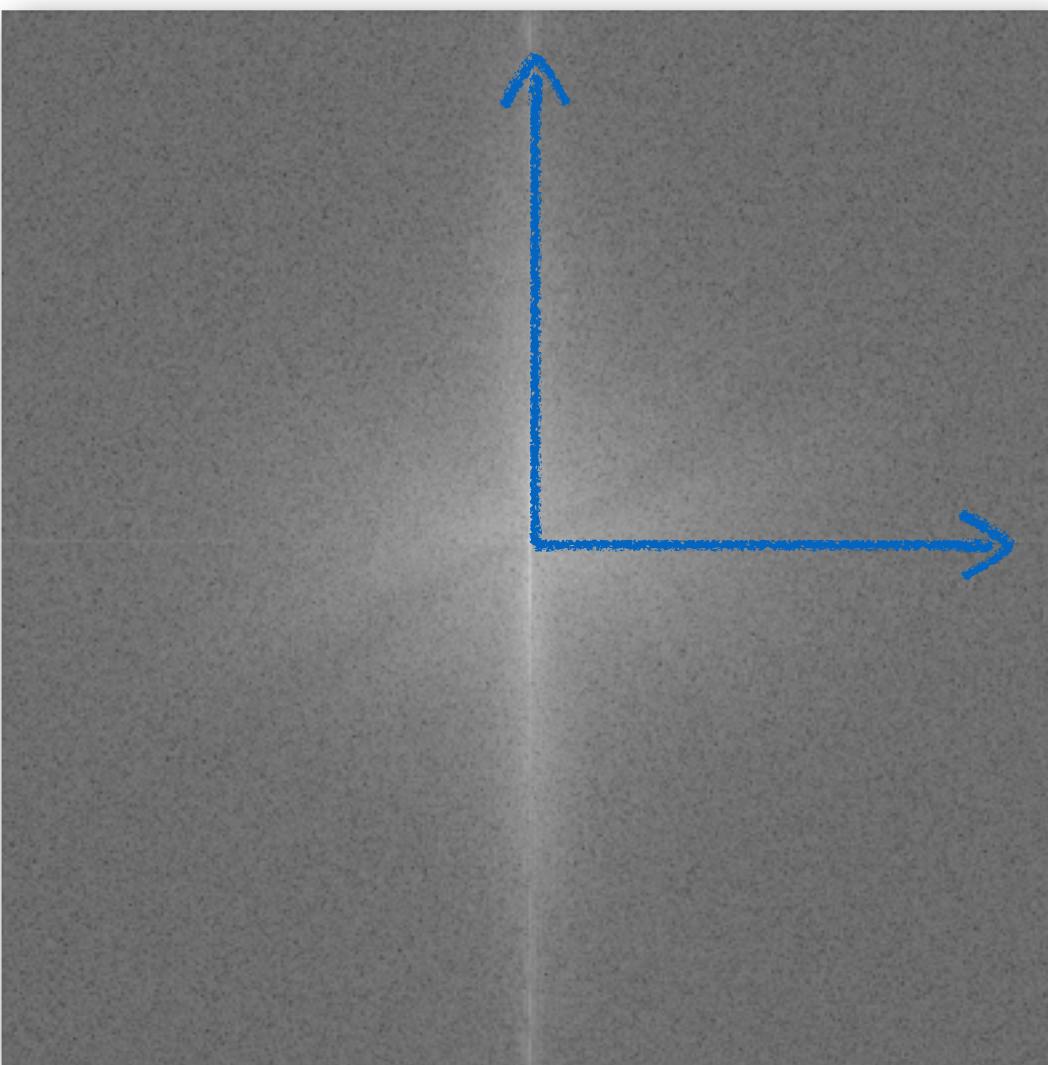
- \* For every  $\omega$  from 0 to  $\infty$  (infinity)  $F(\omega)$  holds the Amplitude  $A$  and phase  $\phi$  of a sine function.
- \* It uses Real and Complex Numbers to achieve this

$$A \sin(\omega t + \phi)$$

$$F(\omega) = R(\omega) + jI(\omega)$$

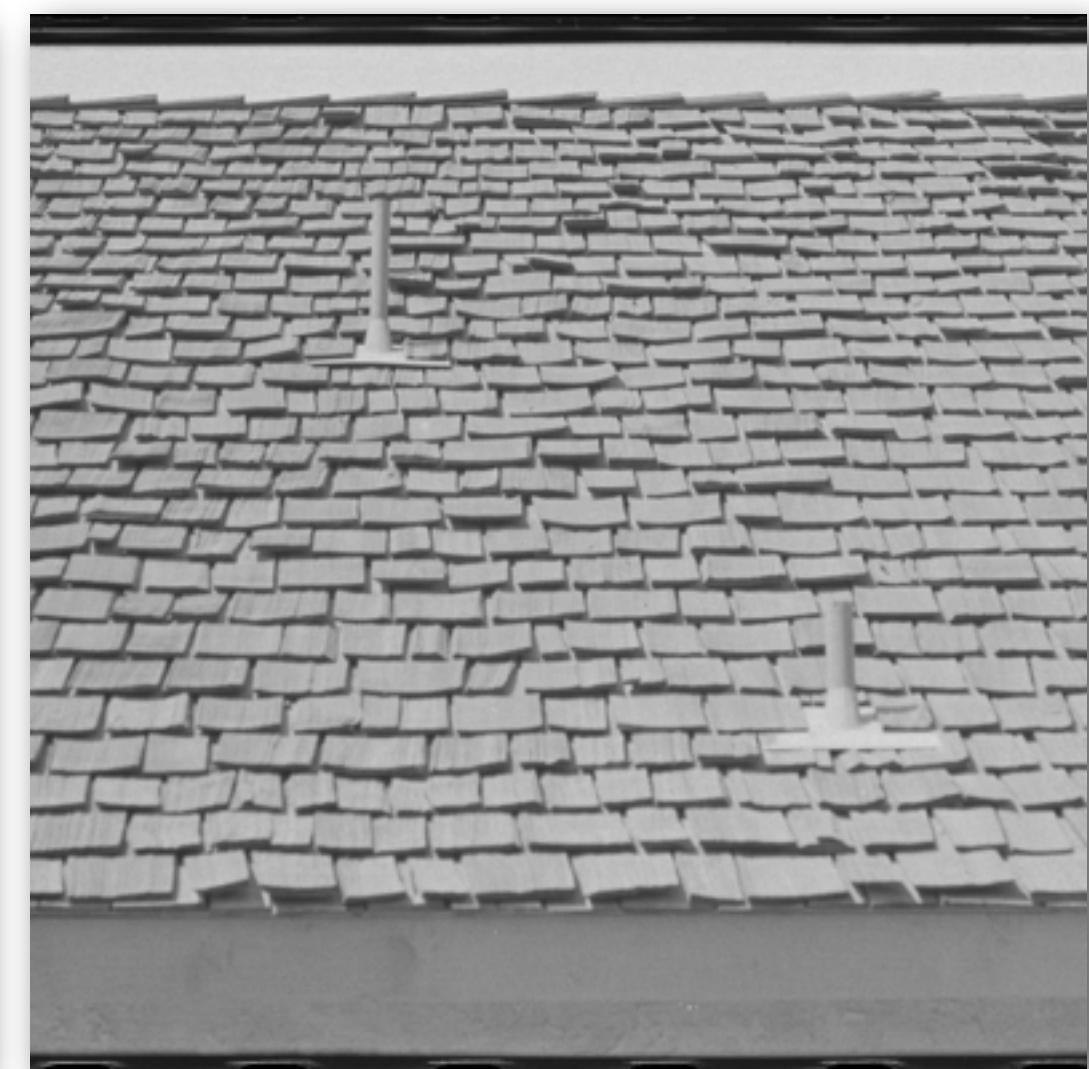
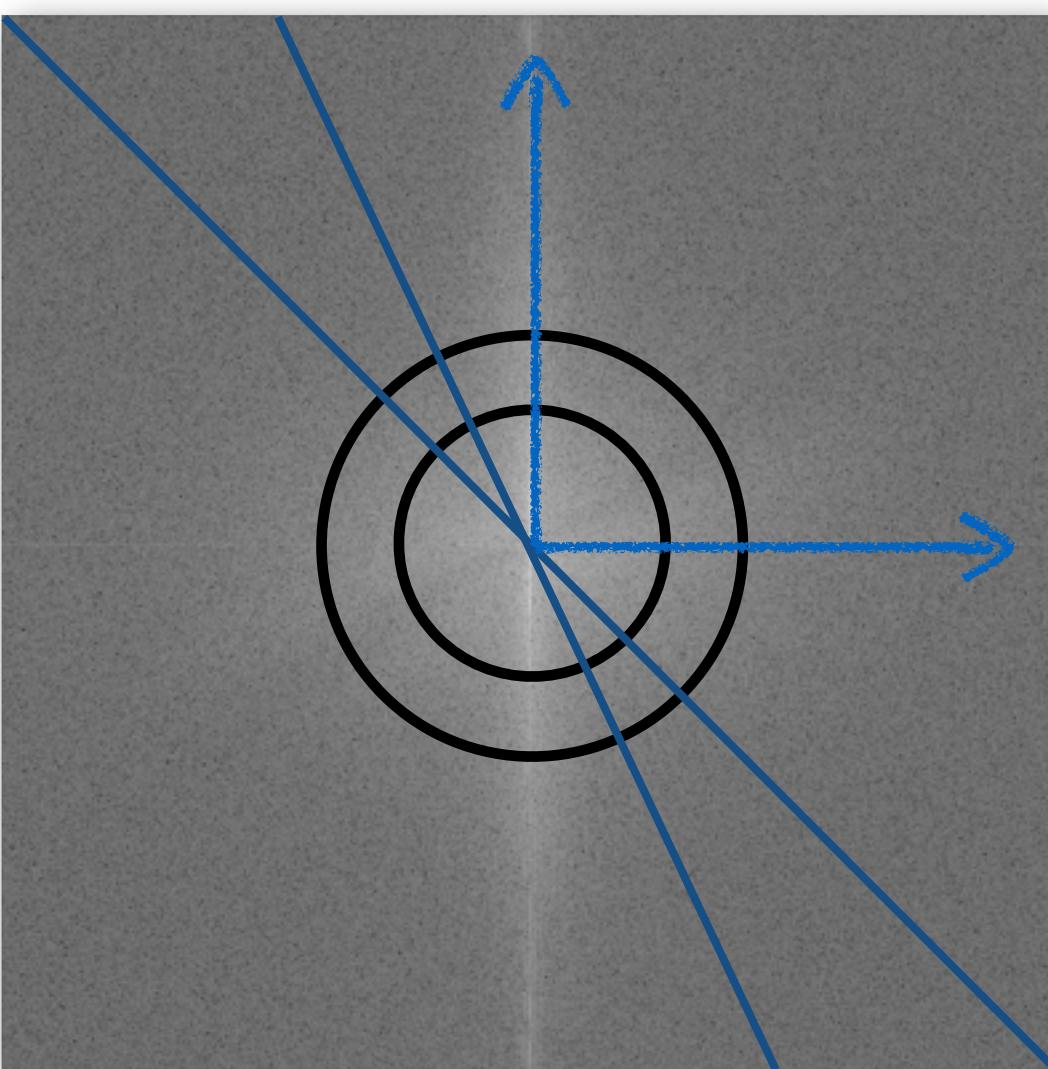
$$A = \pm \sqrt{R(\omega)^2 + I(\omega)^2}$$

$$\phi = \tan^{-1} \frac{I(\omega)}{R(\omega)}$$



# Using the Frequency Spectra

- \* Low-pass,
- \* High-Pass,
- \* Band-pass  
Filtering
- \* Change the  
spectrum and  
reconstruct



# Blurring and Frequencies



Original Image



Gaussian 5x5 Blur



Smooth - Original

# Summary



- \* Introduced how sines and cosines can be used to reconstruct a signal
- \* Characterized the Fourier Transform using Frequency, Amplitude and Phase
- \* Introduced the Frequency Domains for a Signal
- \* Identified the three properties of Convolution as it is associated with the Fourier Transform

# Neat Class

- \* merging and  
Blending of Images



# Credits



- \* For more information, see
  - \* Richard Szeliski (2010) Computer Vision: Algorithms and Applications, Springer
  - \* Forsyth & Ponce (2012), Computer Vision: A Modern Approach, Pearson
- \* Some concepts in slides motivated by similar slides by A. Efros and J. Hays
- \* Some images retrieved from
  - \* <http://commons.wikimedia.org/>
- \* List will be available on website

# Computational Photography

- \* Study the basics of computation and its impact on the entire workflow of photography, from capturing, manipulating and collaborating on, and sharing photographs.



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# Digital Images: Merging and Blending Images

- \* Different methods for Combing multiple Images to Generate a Novel Image



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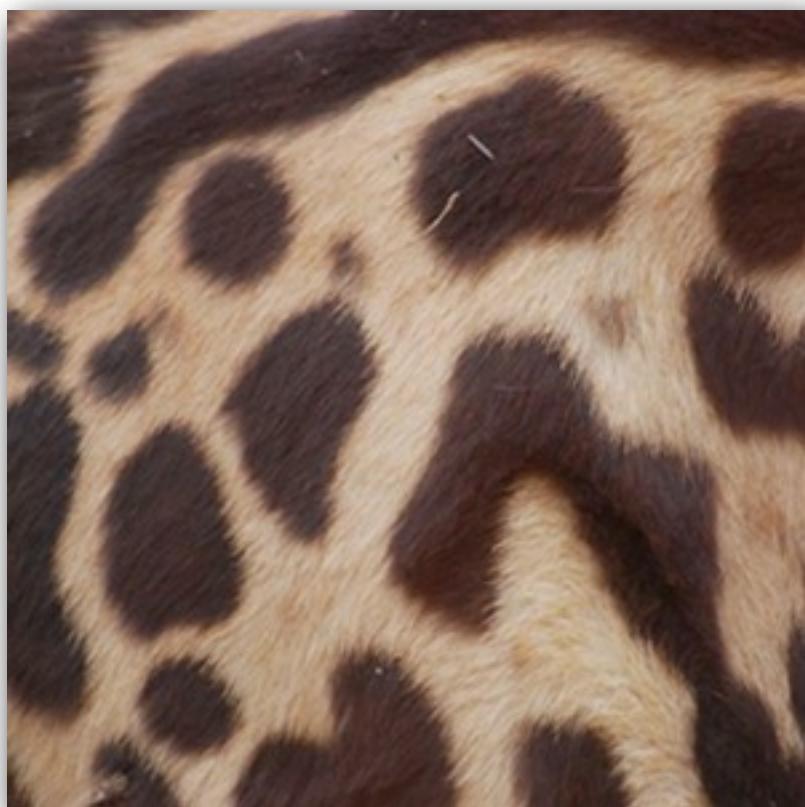
## Lesson Objectives

- \* Merging two images
- \* Window sizes used for merging images
- \* Advantages of a using the Fourier Domain

*Recall: Combine, Merge, Blend Images*



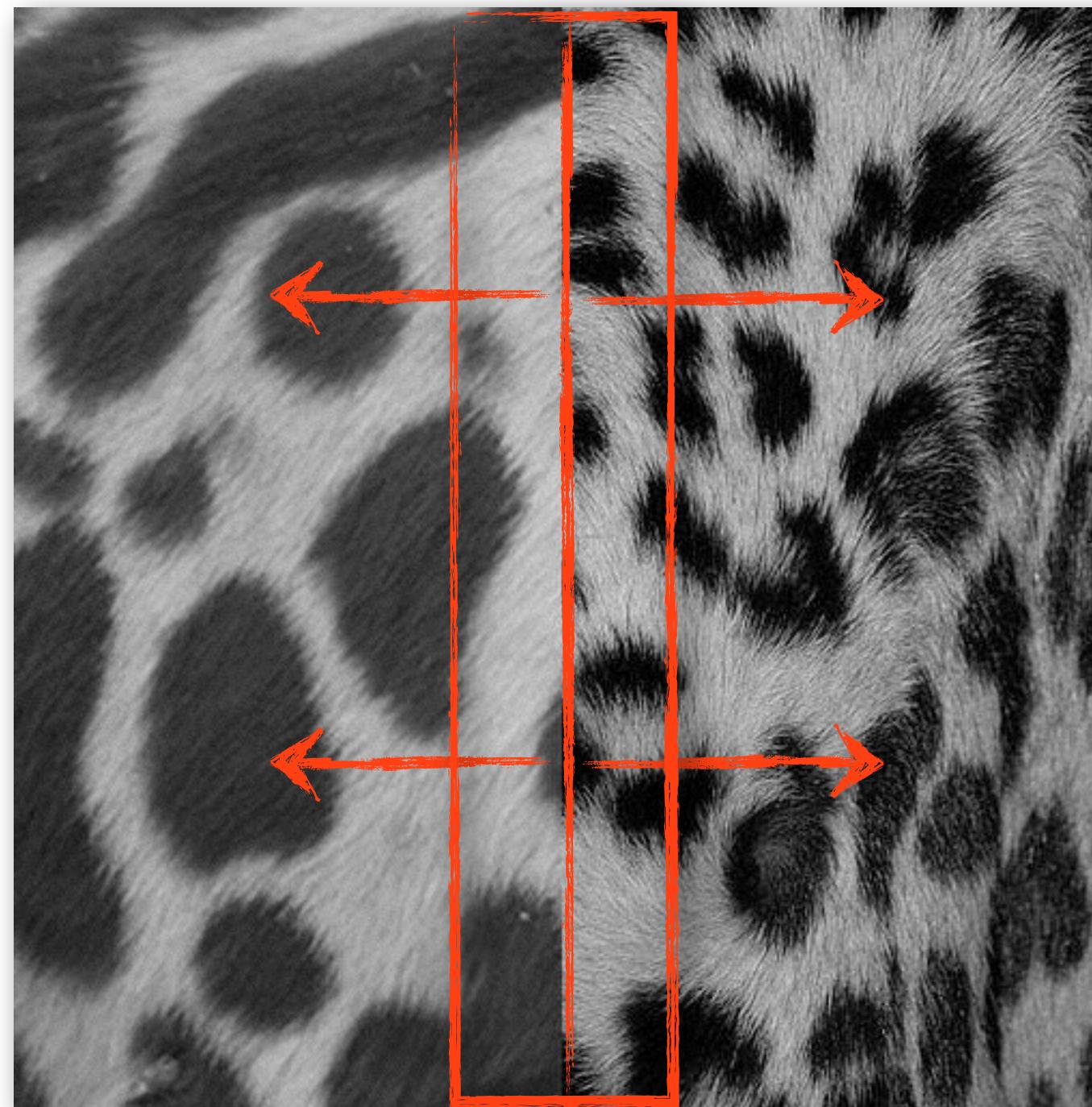
# Merging Two Images



# Merging Two Images

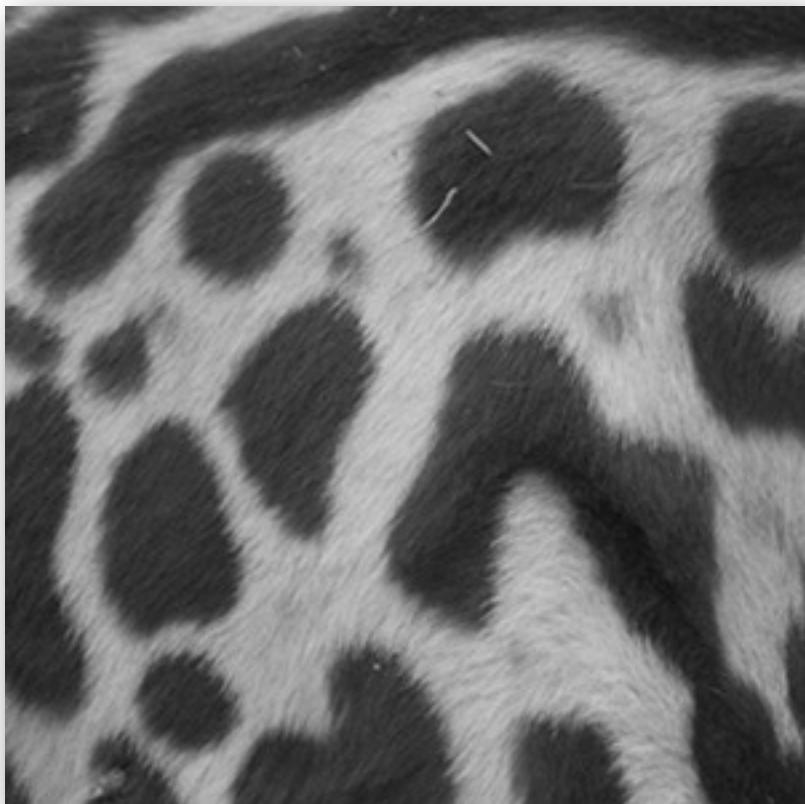


# Merging Two Images



# Cross-Fading Two Images

$I_l$



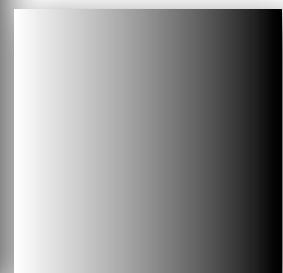
$I_r$



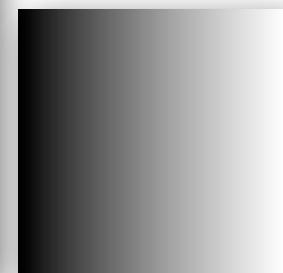
$I_t$



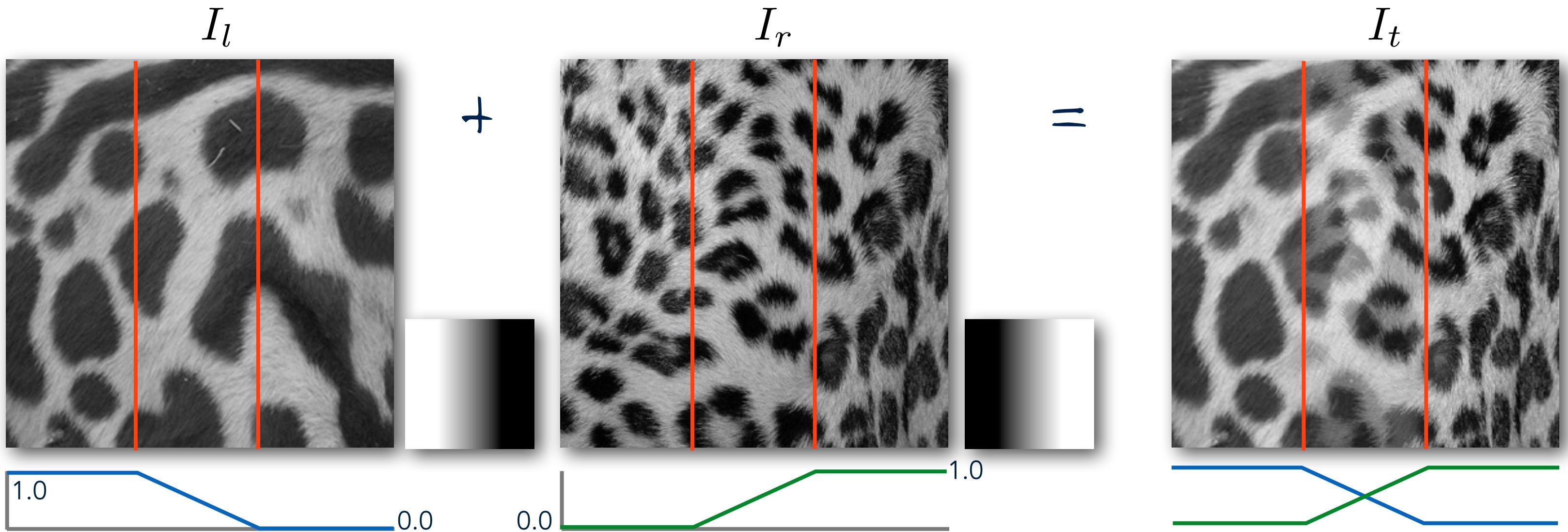
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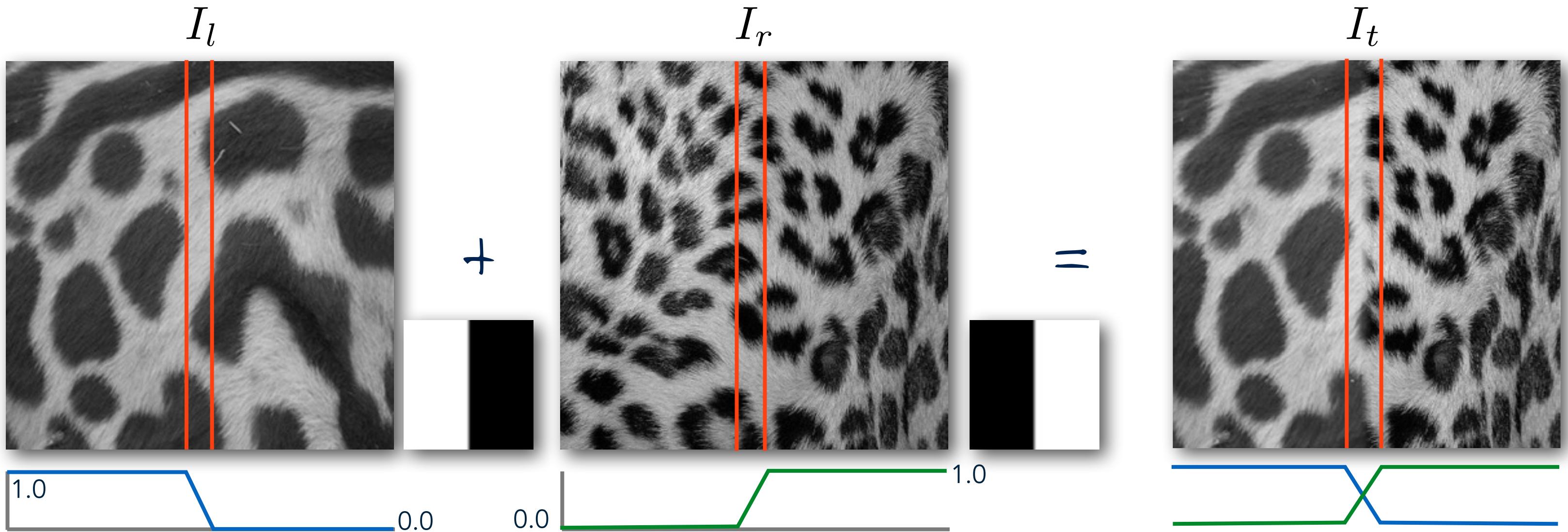
=



# Cross-Fading Two Images



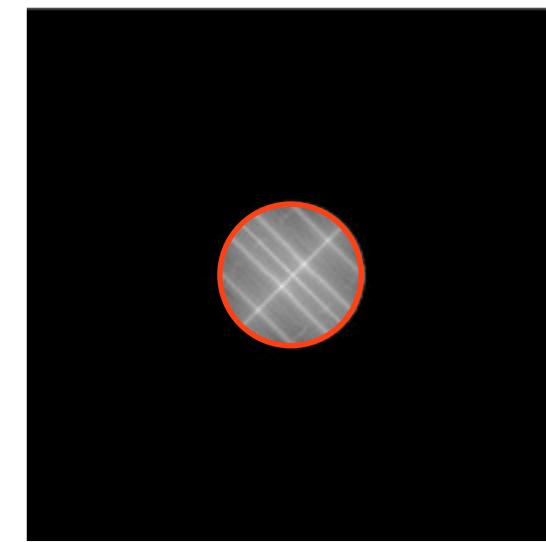
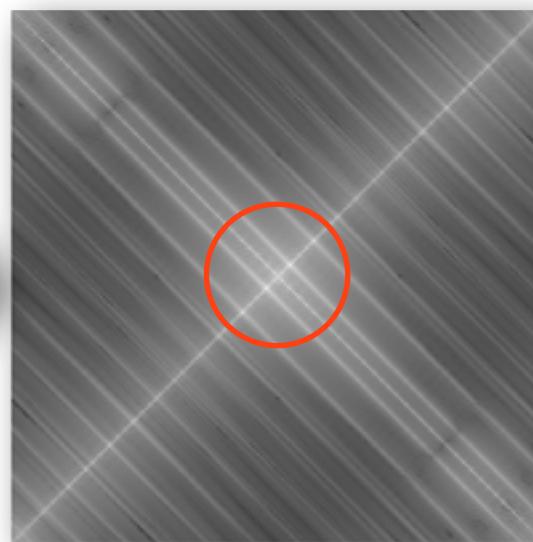
# Cross-Fading Two Images



# Cross-Fading Window Size

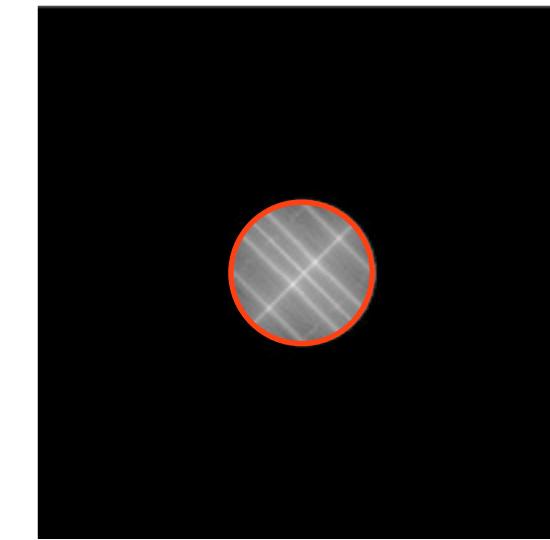
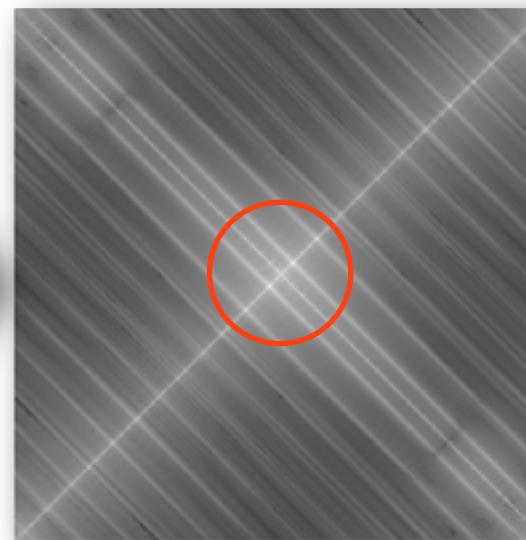
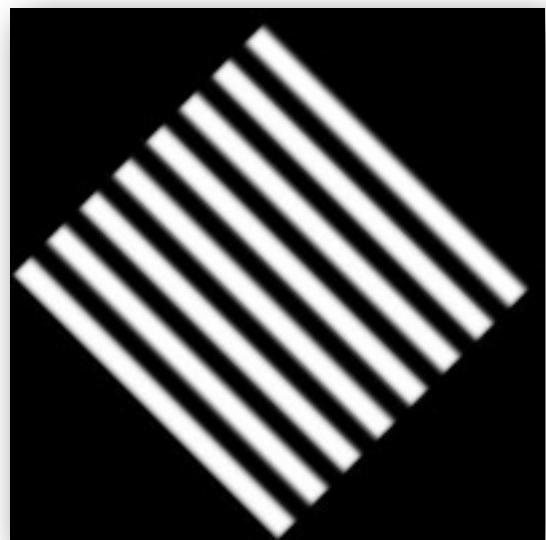


# Factors for Optimal Window Size



- \* To avoid seams: Window = size of largest prominent "feature"
- \* To avoid ghosting: Window  $\leq 2 \times$  size of smallest prominent "feature"

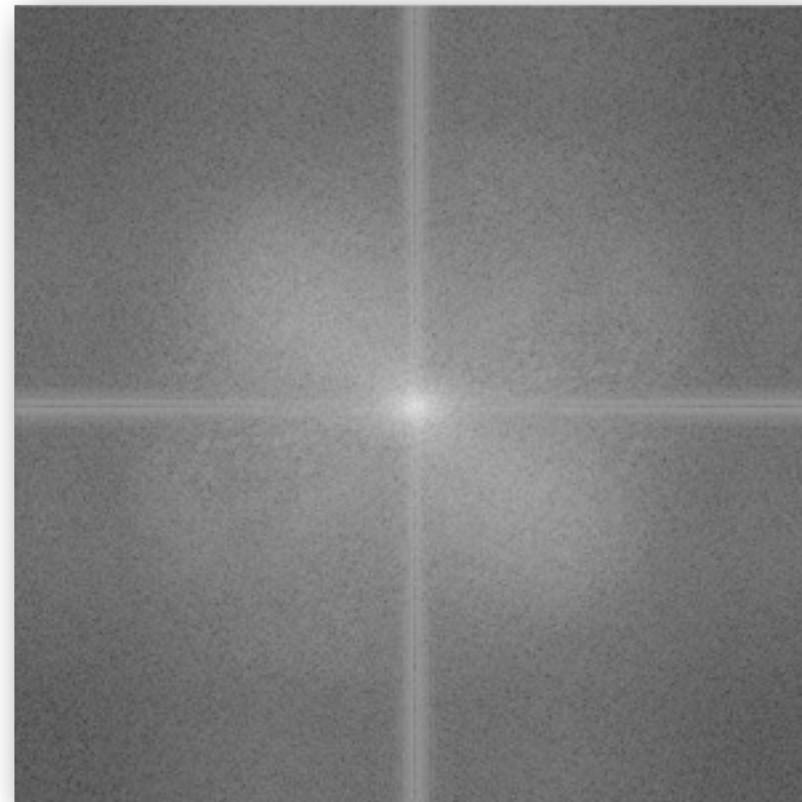
# Factors for Optimal Window Size



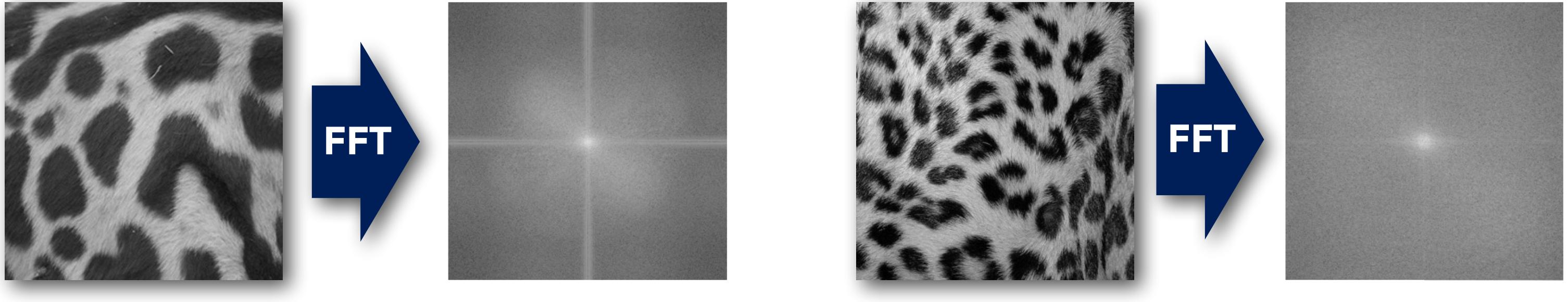
Use Fourier domain

- \* Largest frequency  $\leq 2 \times$  size of smallest frequency
- \* Image frequency content should occupy one "octave"
- \* (power of 2)

Frequency Spread needs to be Modeled



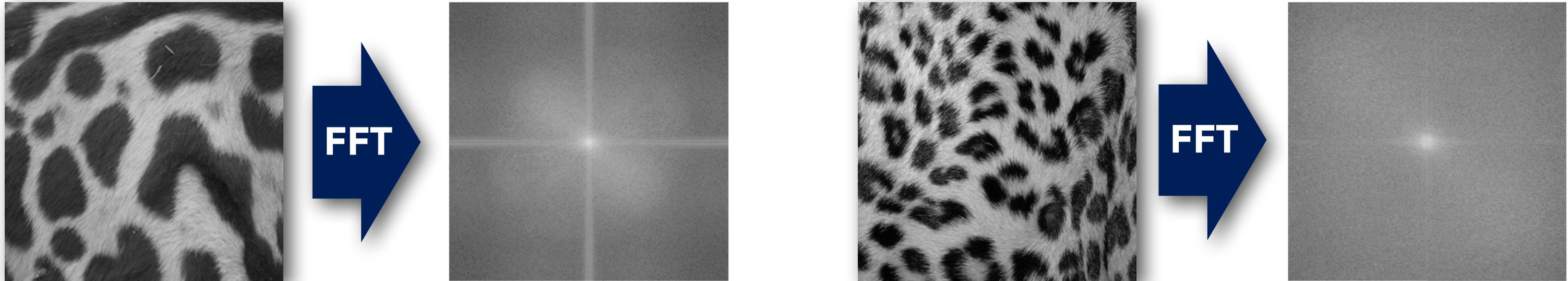
Frequency Spread needs to be Modeled



- \* Compute:  $FFT(I_l) \Rightarrow F_l, FFT(I_r) \Rightarrow F_r$
- \* Decompose Fourier image into octaves  
(bands)

$$F_l = F_l^1 + F_l^2 + F_l^3 + \dots, \quad F_r = F_r^1 + F_r^2 + F_r^3 + \dots$$

# Frequency Spread needs to be Modeled



- \* "Feather" corresponding octaves of:  $F_l$      $F_r$
- \* Compute inverse FFT and feather in spatial domain
- \* Sum feathered octave images in frequency domain

Burt and Adelson (1983)

# Feathering



- \* “Blur” the edges before applying the blend operations.
- \* Makes the blend, smoother.

# Summary



- \* Merging two images
- \* Presented the two issues caused by not being able to determine the window used for merging images
- \* Presented the two advantages of using the Fourier Domain

# Neat Class

- \* merging and  
Blending of Images.  
Use of Pyramids



# Credits



- \* For more information, see
  - \* Szeliski (2010) Computer Vision: Algorithms and Applications, Springer.
  - \* Burt and Adelson (1983) "The Laplacian Pyramid as a Compact Image Code", In IEEE Transactions on Communications, 31 (4). p 532-540. 1983 (DOI)
  - \* Burt and Adelson (1983) "A multiresolution spline with application to image mosaics". In ACM Transactions on Graphics, 2 (4). 1983 (DOI)
- \* Some concepts in slides motivated by similar slides by A. Efros and J. Hays.
- \* Some images retrieved from
  - \* <http://commons.wikimedia.org/>.
  - \* List will be available on website
  - \* by Irfan Essa

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# Digital Images: Merging and Blending Images using Image Pyramids

- \* Combining multiple Images to generate a Novel Image.' Image Pyramids



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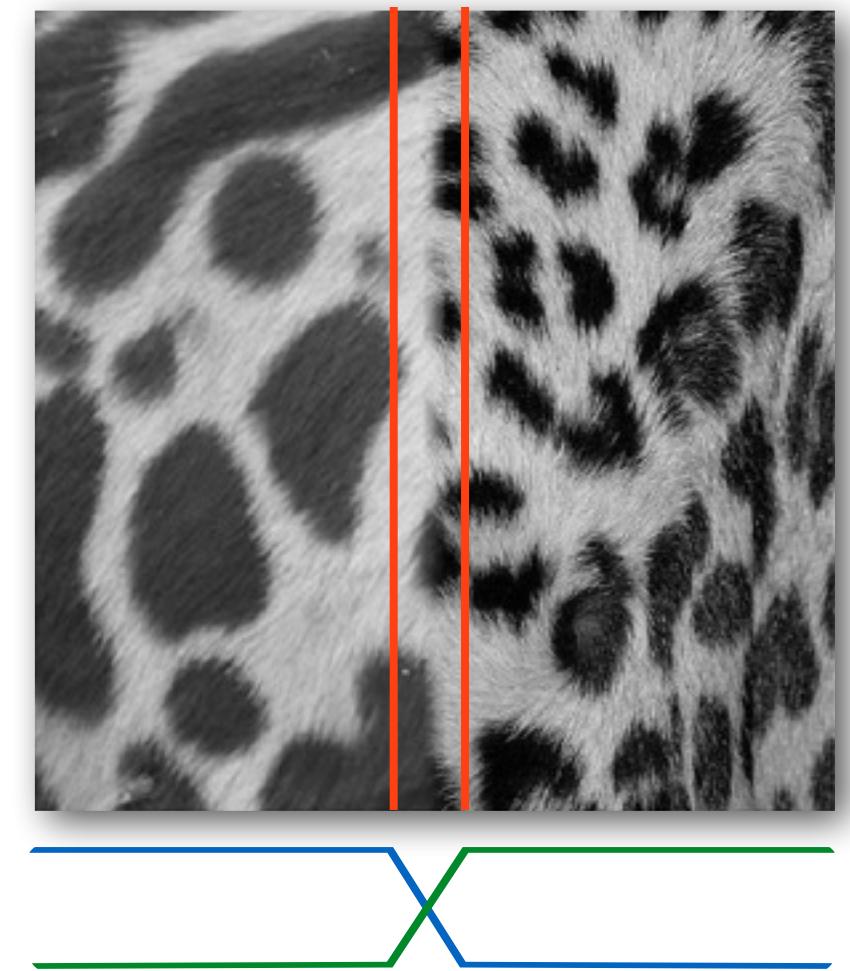


## Lesson Objectives

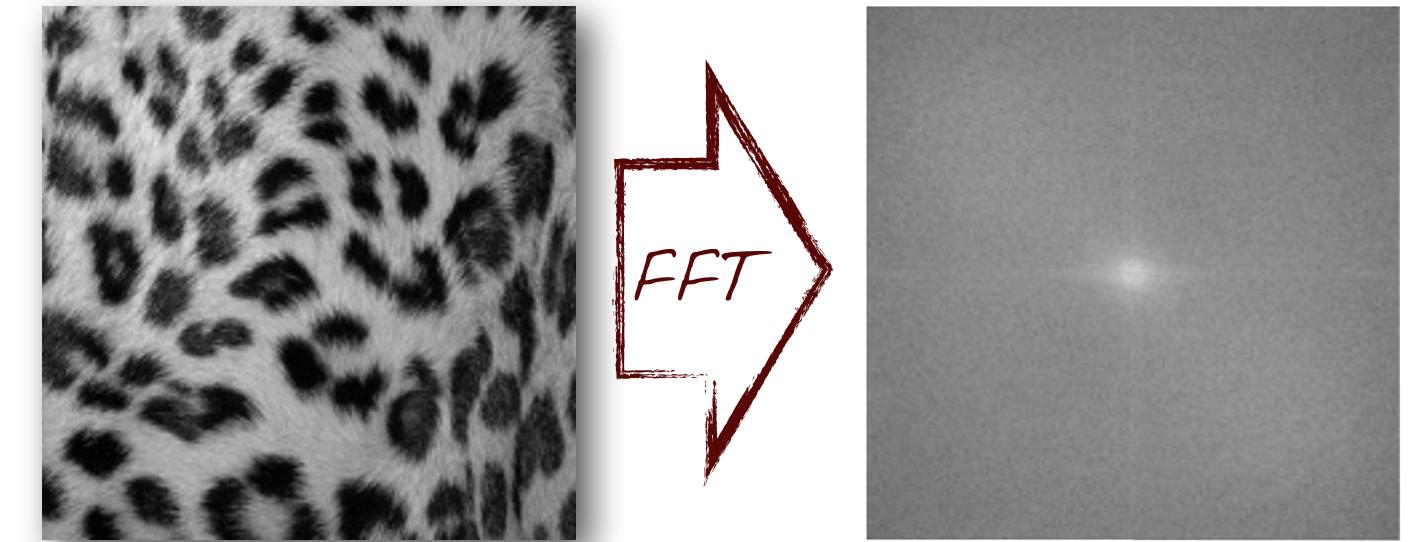
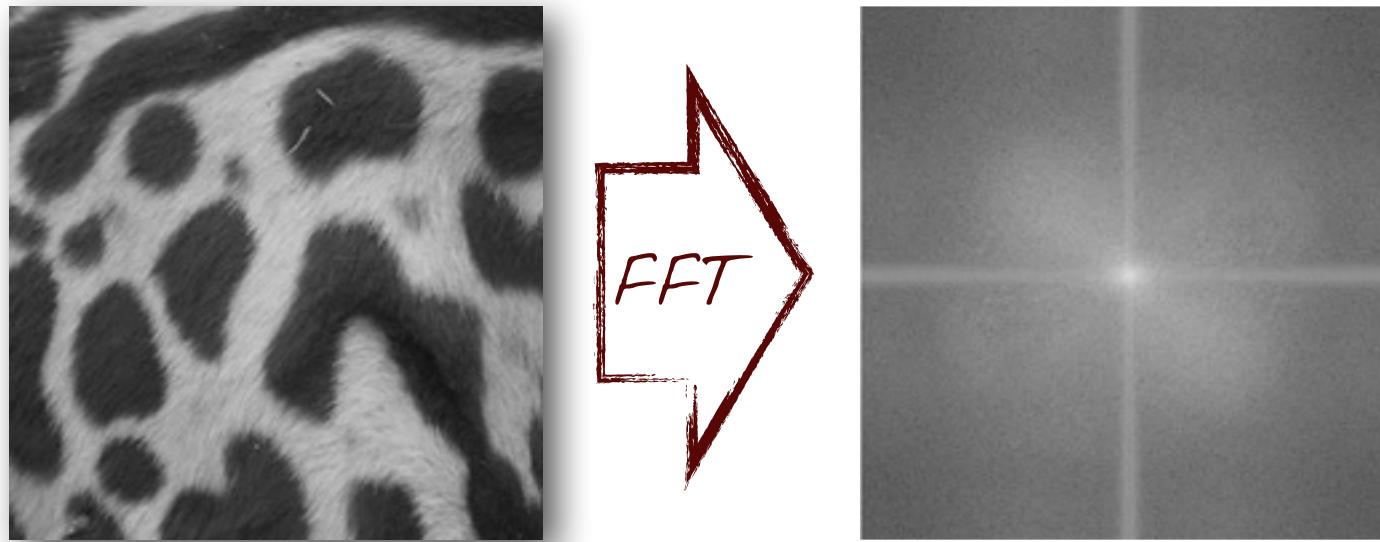
1. Gaussian and the Laplacian Pyramids
2. Use of Pyramids to encode the Frequency domain
3. Compute a Laplacian Pyramid from a Gaussian Pyramid
4. Blend two images using Pyramids

## Recall: Optimal Window Size

- \* Avoid seams: Window = size of largest prominent "feature"
- \* Avoid ghosting: Window  $\leq 2 \times$  size of smallest prominent "feature"
- \* Use Fourier domain
  - \* Largest frequency  $\leq 2 \times$  size of smallest frequency
  - \* Image frequency content should occupy one "octave" (power of two)



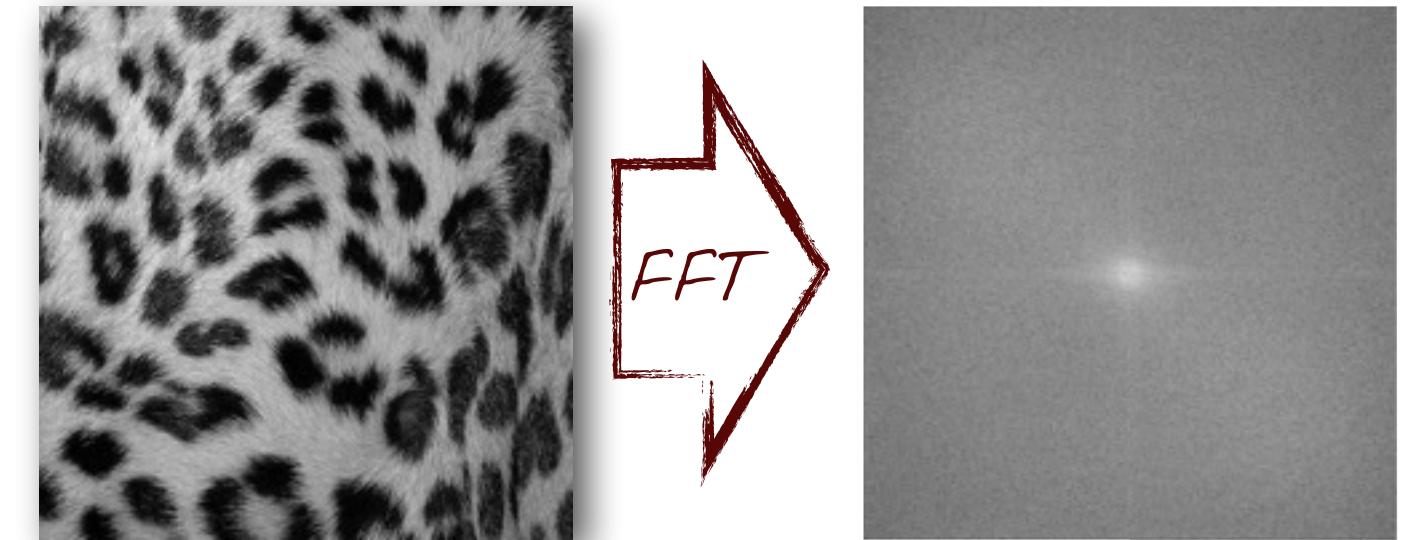
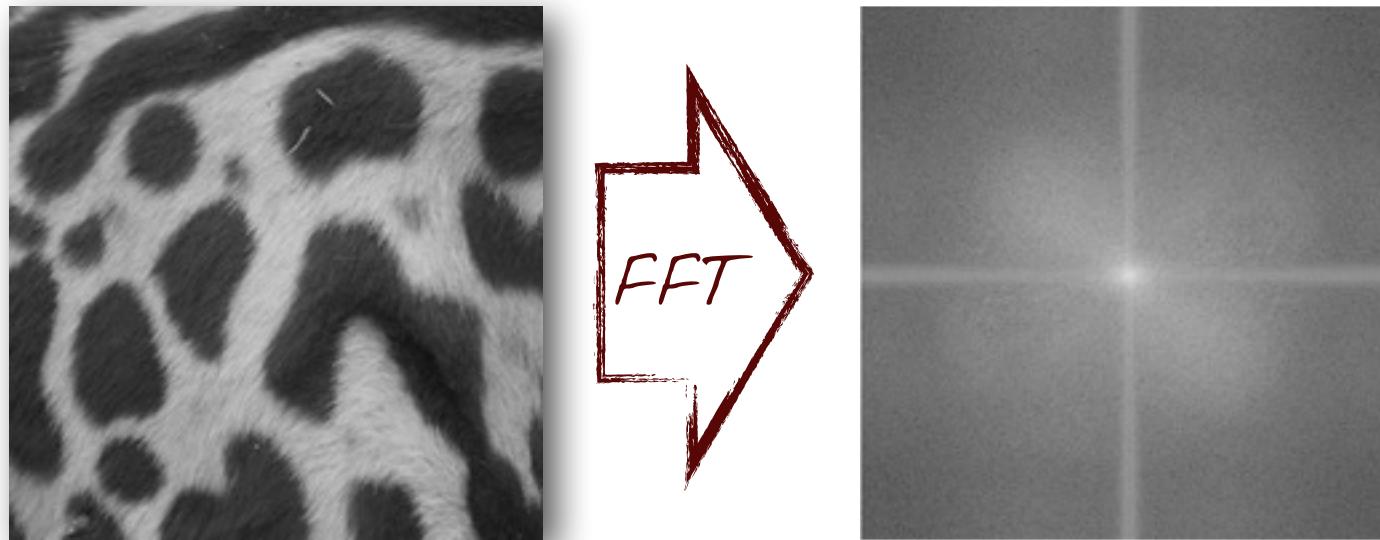
Recall: Frequency Spread needs to be Modeled



- \* Compute:  $\text{FFT}(I_l) \Rightarrow F_l, \quad \text{FFT}(I_r) \Rightarrow F_r$
- \* Decompose Fourier image into octaves  
(bands)

$$F_l = F_l^1 + F_l^2 + F_l^3 + \dots, \quad F_r = F_r^1 + F_r^2 + F_r^3 + \dots$$

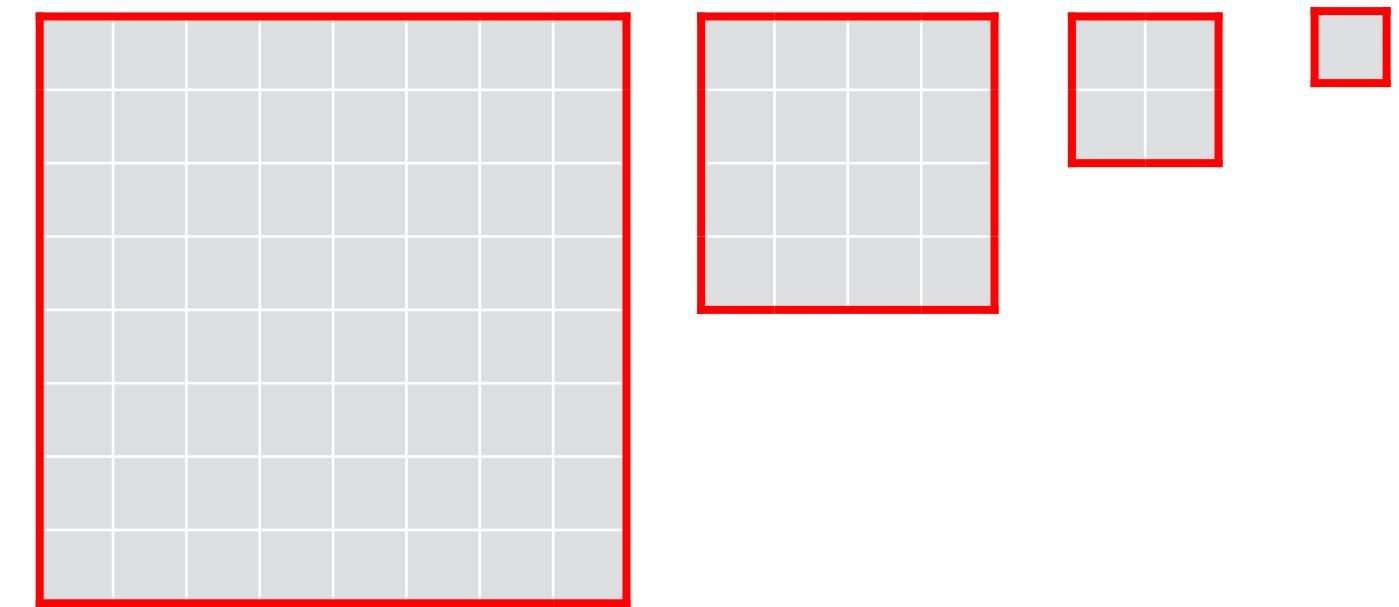
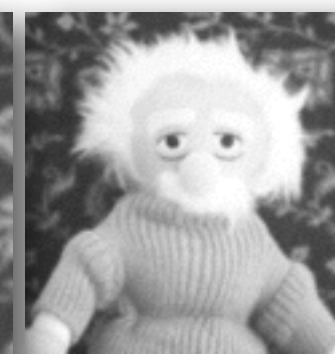
## Recall: Frequency Spread needs to be Modeled



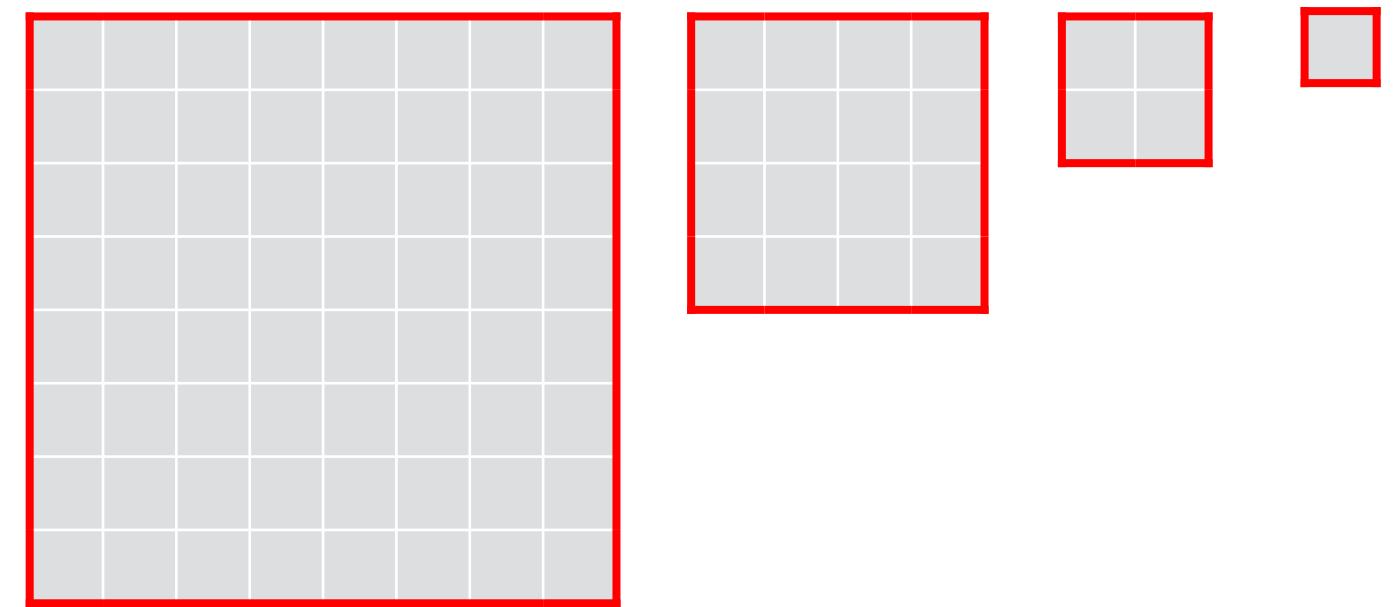
- \* "Feather" > corresponding octaves of:  $F_l$      $F_r$
- \* Compute inverse FFT and feather in spatial domain
- \* Sum feathered octave images in frequency domain

Burt and Adelson (1983)

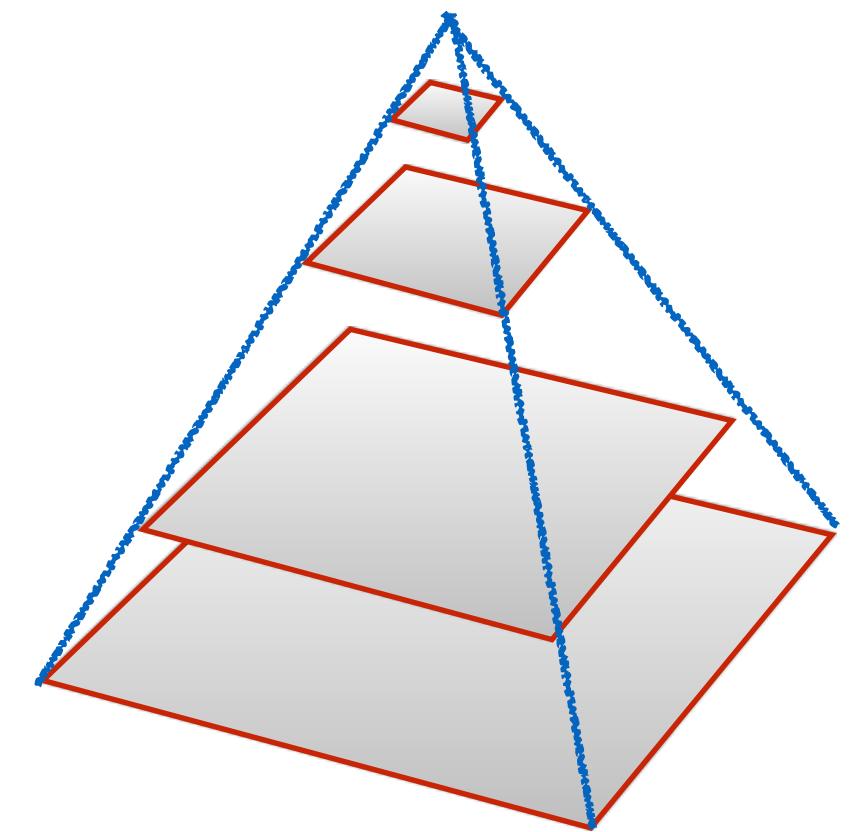
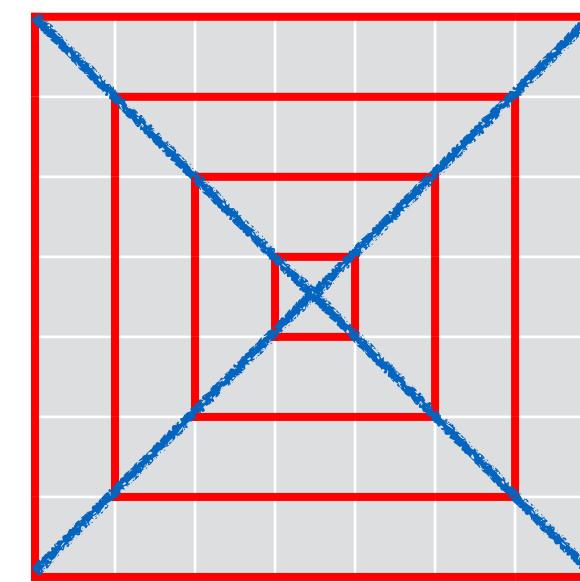
# Pyramid Representation: A Gaussian Pyramid



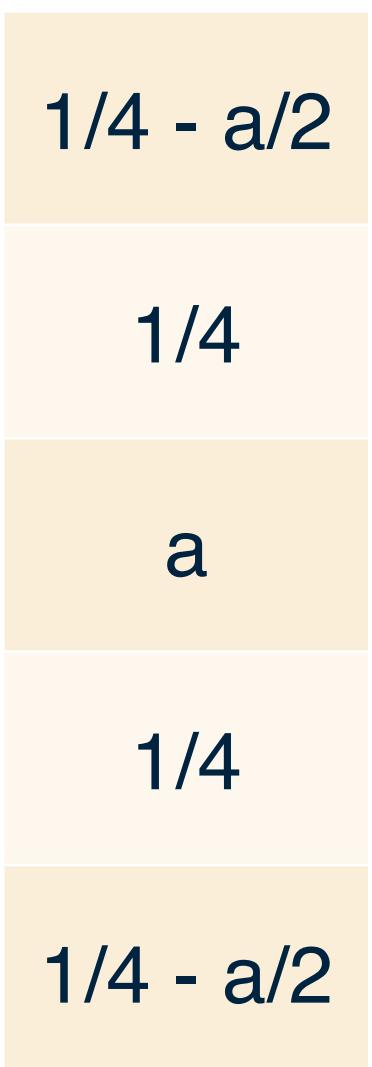
# Pyramid Representation: A Gaussian Pyramid



# Pyramid Representation: A Gaussian Pyramid



# Pyramid Representation of Images (A Gaussian Pyramid)

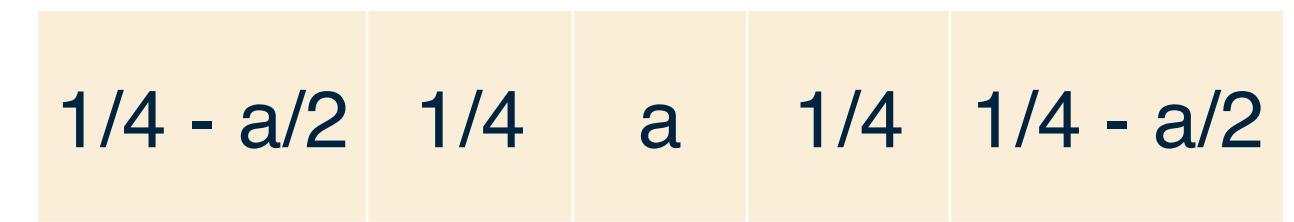


$$a = 0.3 - 0.6 (.38)$$

$$h = \omega_h \star \omega_v$$

$g_0$

$\omega_h =$



Burt and Adelson (1983)

# *Pyramid Representation of Images (A Gaussian Pyramid)*



$g_1$

$g_2$

$g_3$

$g_4$

$g_5$

$$g_k = h \star g_{(k-1)}$$

$$g_k = \text{REDUCE}(g_{(k - 1)})$$

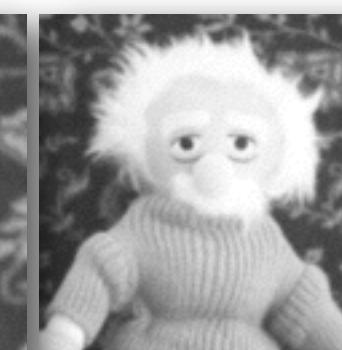
# Pyramid Representation of Images (A Gaussian Pyramid)



$g_0$



$g_1$



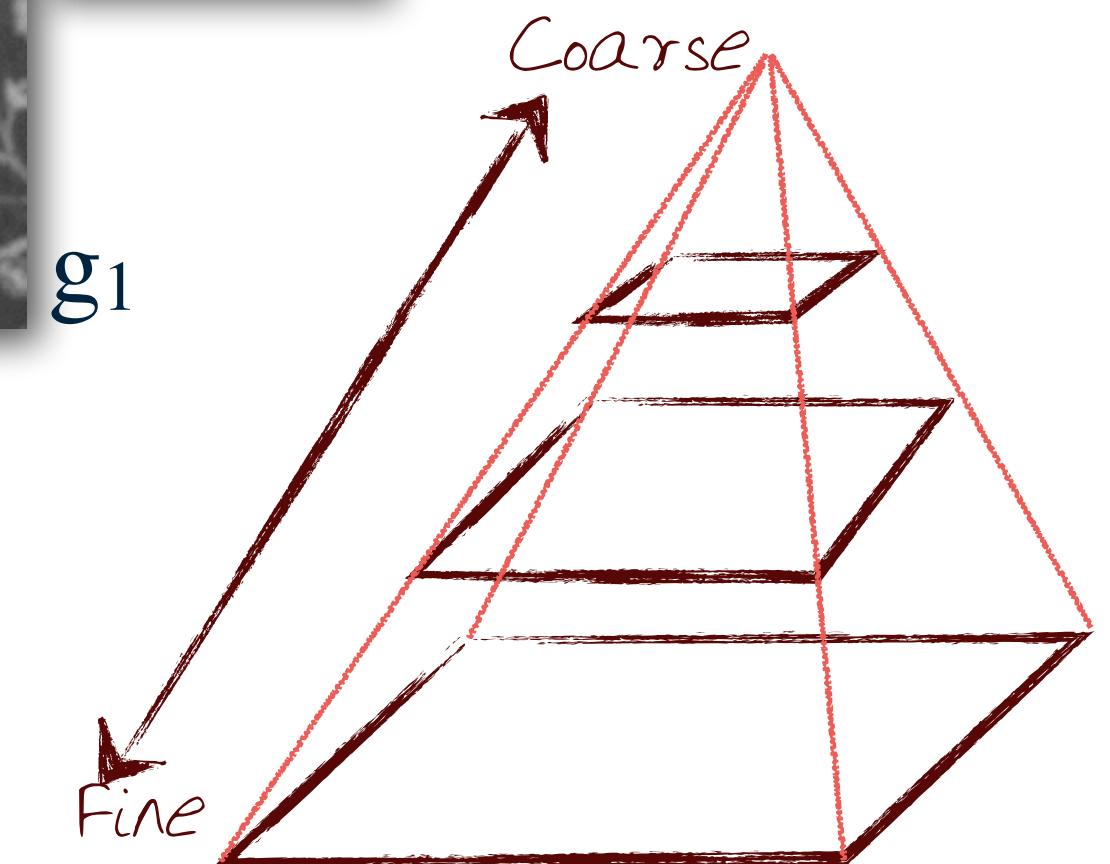
$g_2$



$g_3$

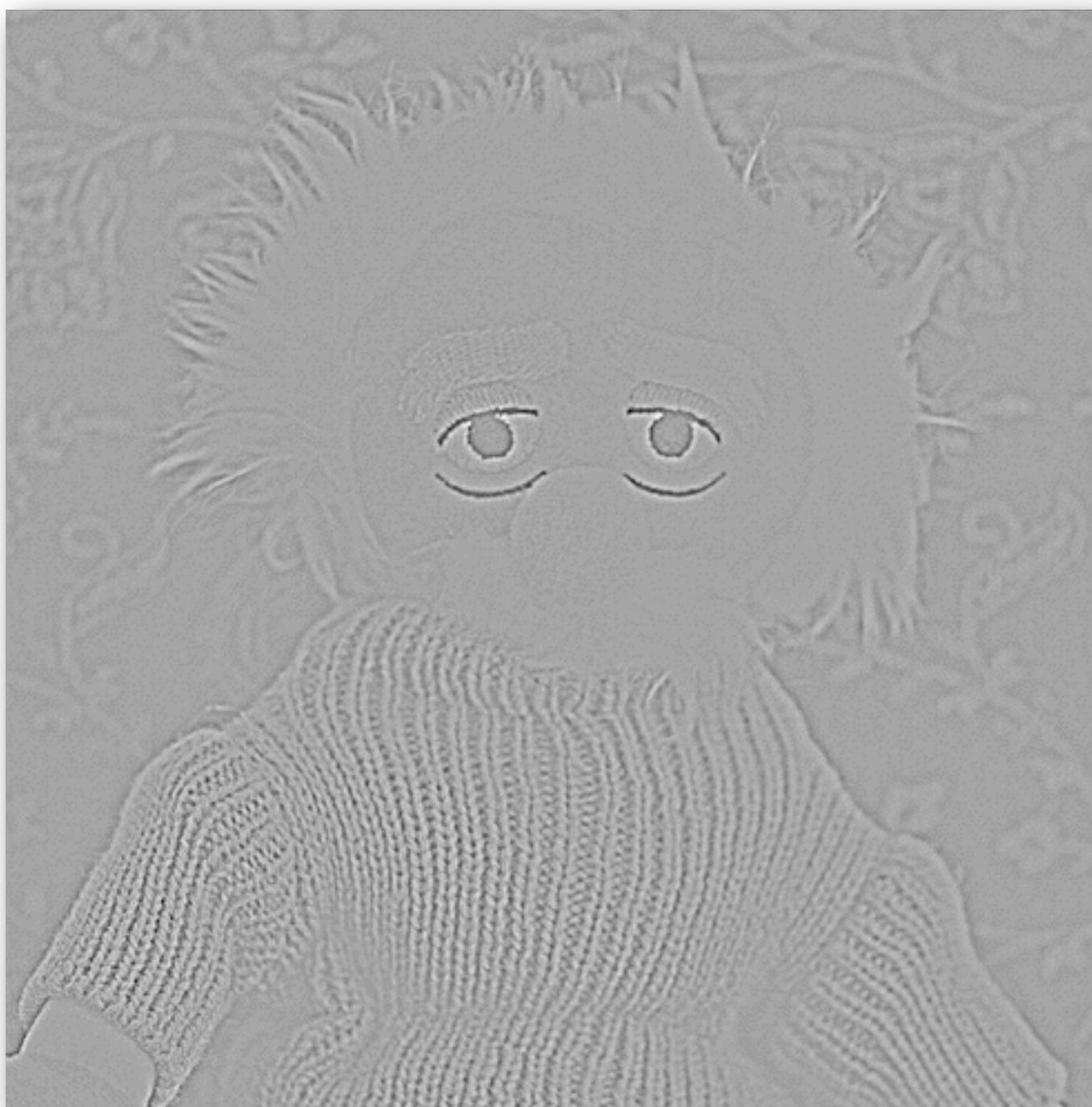


$g_4$



$$g_k = \text{REDUCE}(g_{(k - 1)})$$

# Pyramid Representation of Images (A Gaussian Pyramid)



$g_0$

$g_{0,1}$

$g_0 - g_{0,1}$

$g_1$

$g_k = \text{REDUCE}(g_{k-1})$

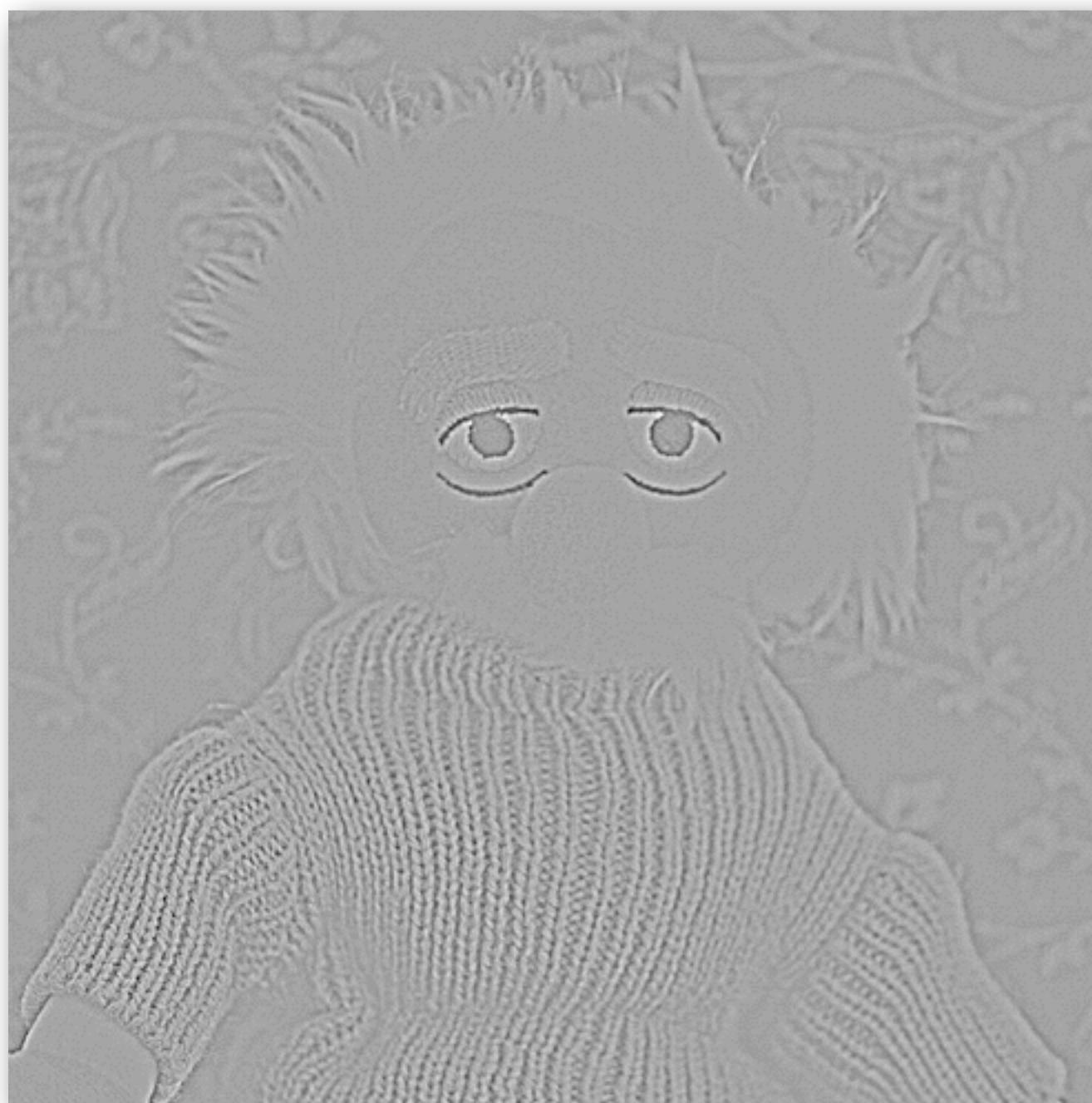
$g_{j,n} = \text{EXPAND}(g_{j,n-1})$

EXPAND is inverse of  
REDUCE, as it seeks to

add new values in  
between knowns ones.

$g_{j,n}$  is  $g_j$  expanded  $n$   
times

# Pyramid Representation of Images (A Laplacian Pyramid)



$L_3$

$L_1$

$L_2$

$$L_l = g_l - \text{EXPAND}(g_{l+1})$$

- \* A series of "error" images,
- \* A difference between two levels of a Gaussian Pyramid

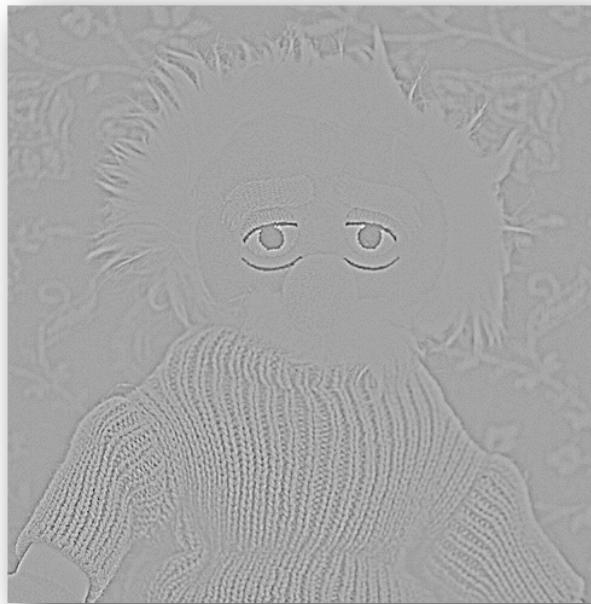
# Computing Gaussian and Laplacian Pyramids

Blur



$g_1$

$g_{1,1}$



$L_1$

Sub sample



$L_4$

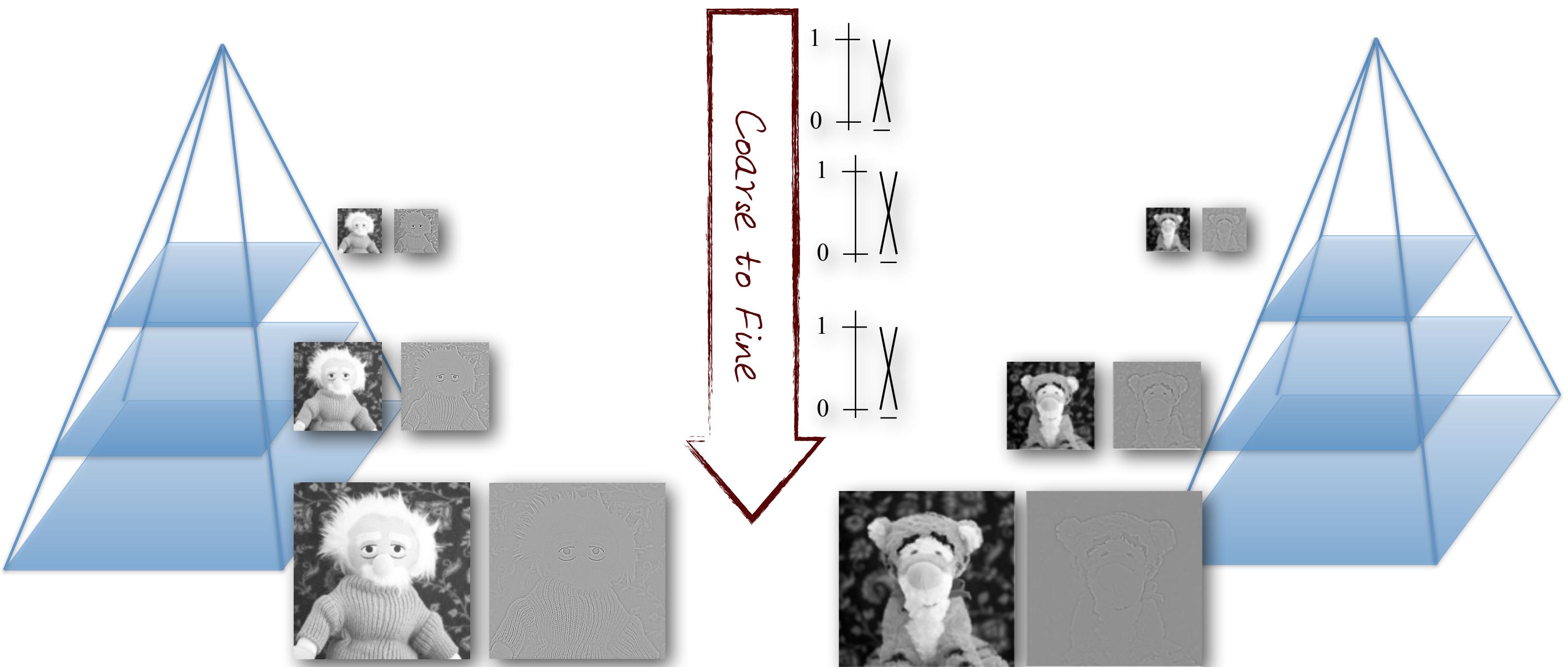


$L_2$



$L_3$

# Pyramid Blending



# *Blend 1*



# Pyramid Blending Process



A



B



R

- \* Build Laplacian pyramids
- \* Build a Gaussian pyramid from selected region R
- \* Form a combined pyramid using  $G_R$  as weights:
  - \* 
$$L_O(i,j) = G_R(i,j) * L_A(i,j) + (1-G_R(i,j)) * L_B(i,j)$$
- \* Collapse the  $L_O$  pyramid to get the final blended image

*Blend 2*



# Summary



- \* merge two images leveraging the Frequency Domain
- \* Gaussian and the Laplacian Pyramids
- \* mathematical formulation to compute a Laplacian
- \* Blend two images using Pyramids

# Neat Class

- \* merging and  
Blending of Images.  
Cutting Images



# Further Reading



- \* Richard Szeliski (2010) Computer Vision: Algorithms and Applications, Springer.
- \* Burt and Adelson (1983) "The Laplacian Pyramid as a Compact Image Code" , IN IEEE Transactions on Communications, 31 (4). p 532-540 . 1983 (DOI)
- \* Burt and Adelson (1983) "A multiresolution spline with application to image mosaics" . IN ACM Transactions on Graphics, 2 (4). 1983 (DOI)
- \* Look for "pyramids" on OpenCV and Matlab sites

# Credits



- \* For more information, see
  - \* Richard Szeliski (2010) Computer Vision: Algorithms and Applications, Springer.
  - \* Some concepts in slides motivated by similar slides by A. Efros and J. Hays.
  - \* Some images retrieved from
    - \* List will be available on website.
    - \* by Irfan Essa

# Computational Photography

- \* Study the basics of computation and its impact on the entire workflow of photography, from capturing, manipulating and collaborating on, and sharing photographs.



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# Digital Images: Cutting Images for Merging

- \* methods for Combing multiple Images by Cutting to Generate a Novel Image



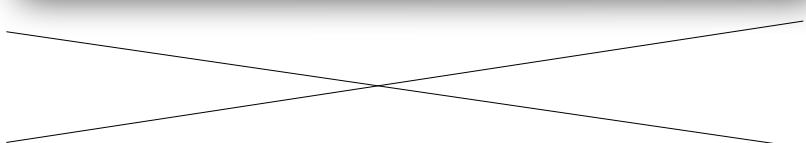
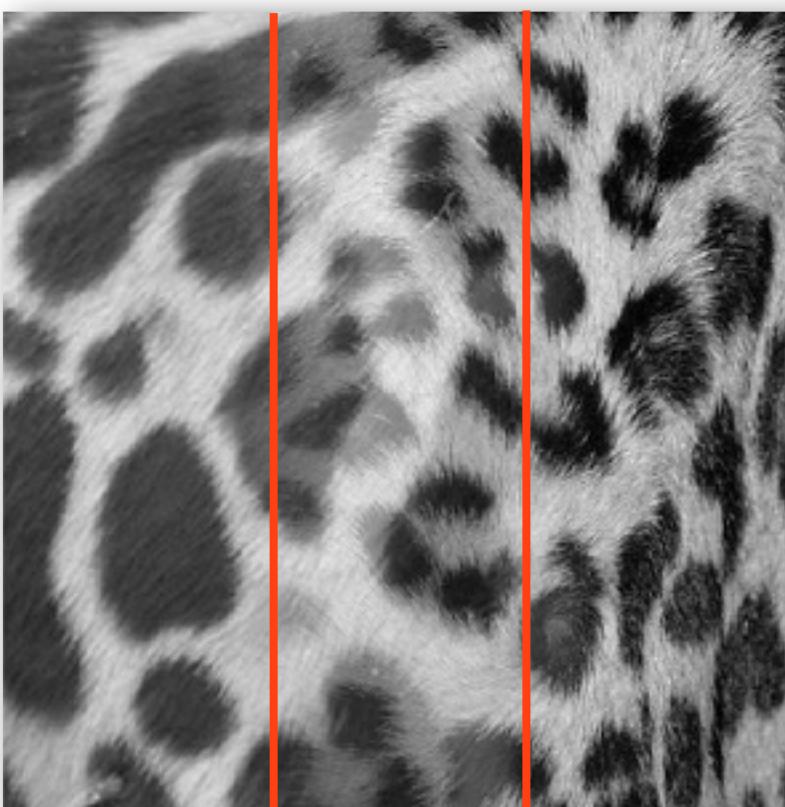
## Lesson Objectives

1. An additional method for merging images besides blending
2. Finding seams in images
3. Benefits of cutting images over blending images

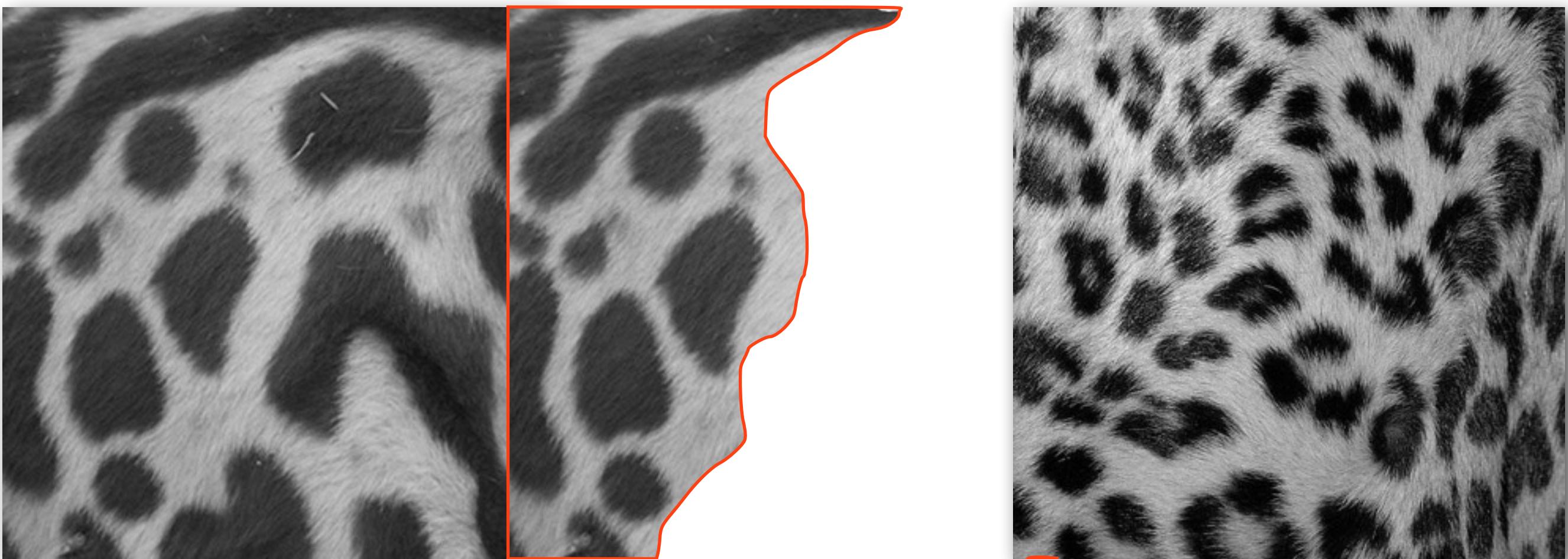
*Recall: Combine, Merge, Blend Images*



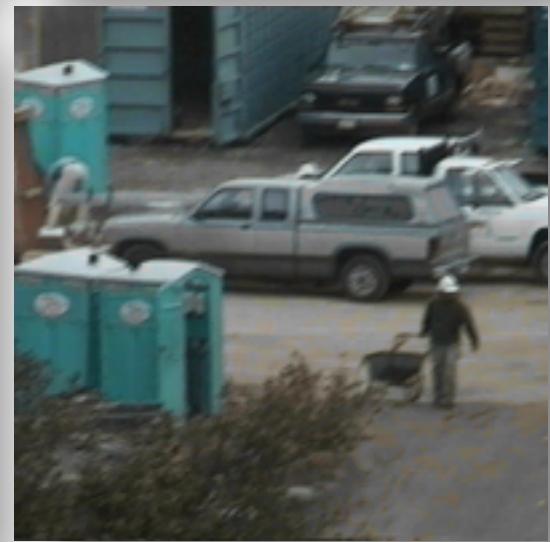
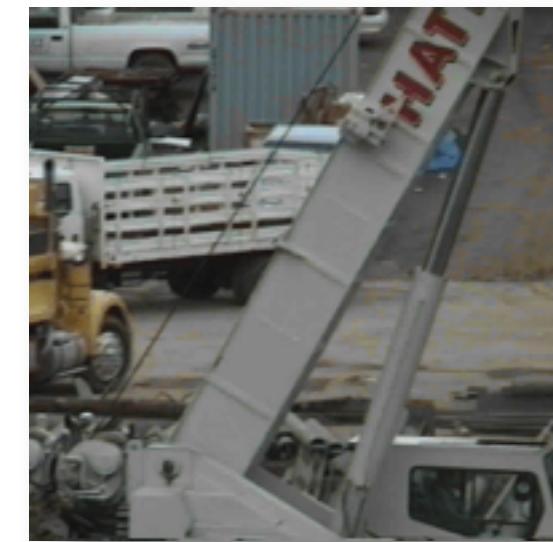
# Recall: Cross-Fading Window Size



*Cut, Don't Blend!*

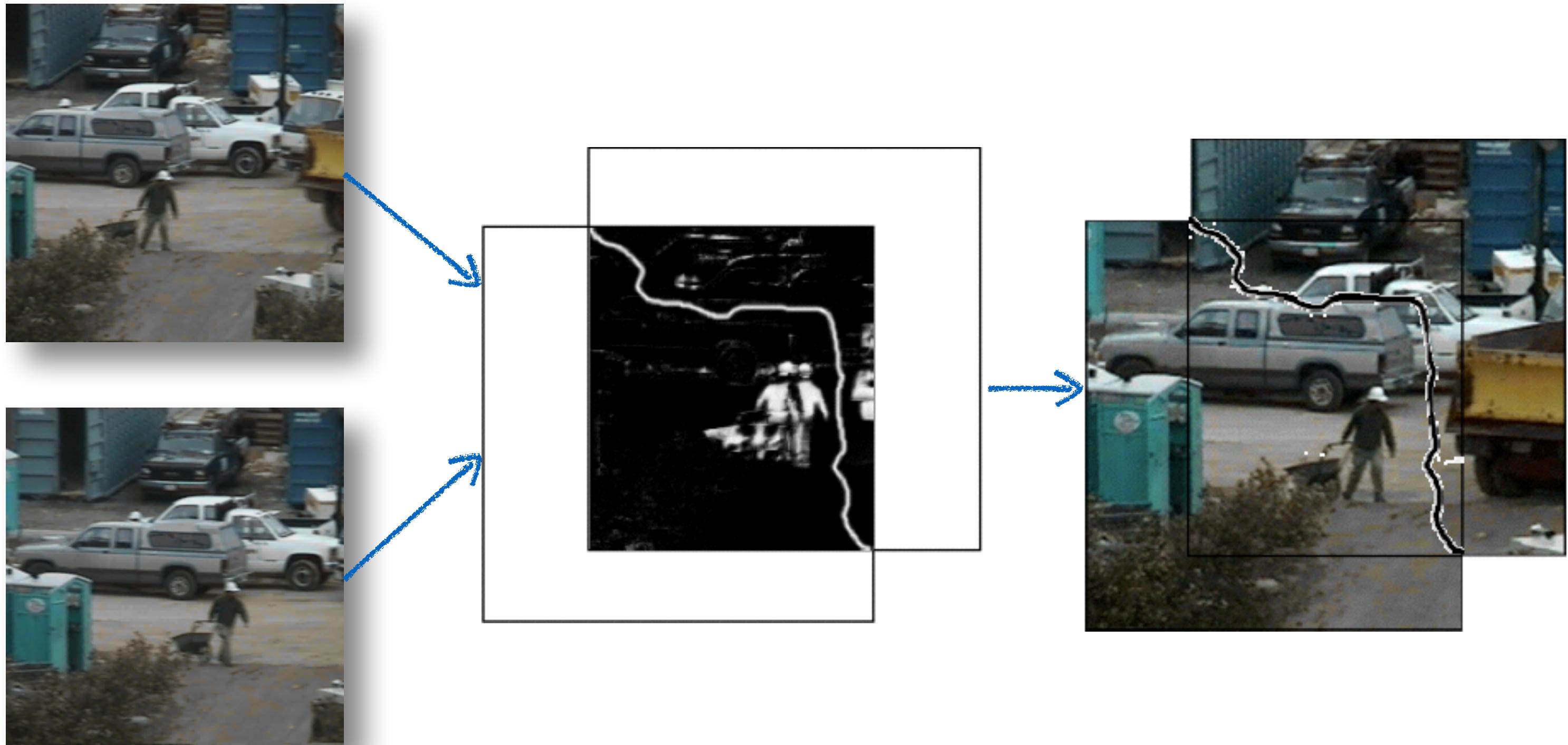


# Cut, Don't Blend



Davis (1998)

# Cut, Don't Blend



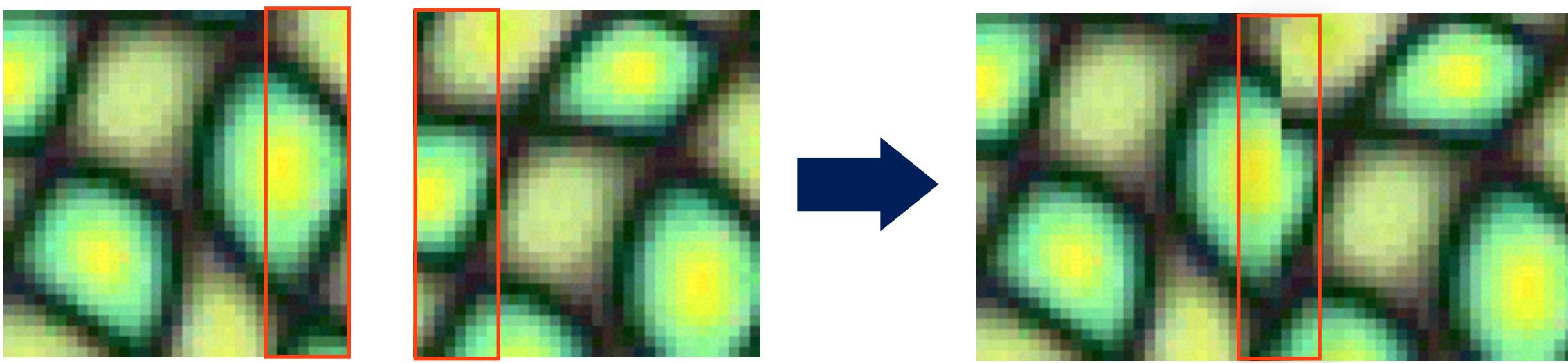
Davis (1998)



- \* Moving objects cause "ghosting"
- \* Find an optimal seam as opposed to blend between images
- \* Final has exact pixels from an image

Davis (1998)

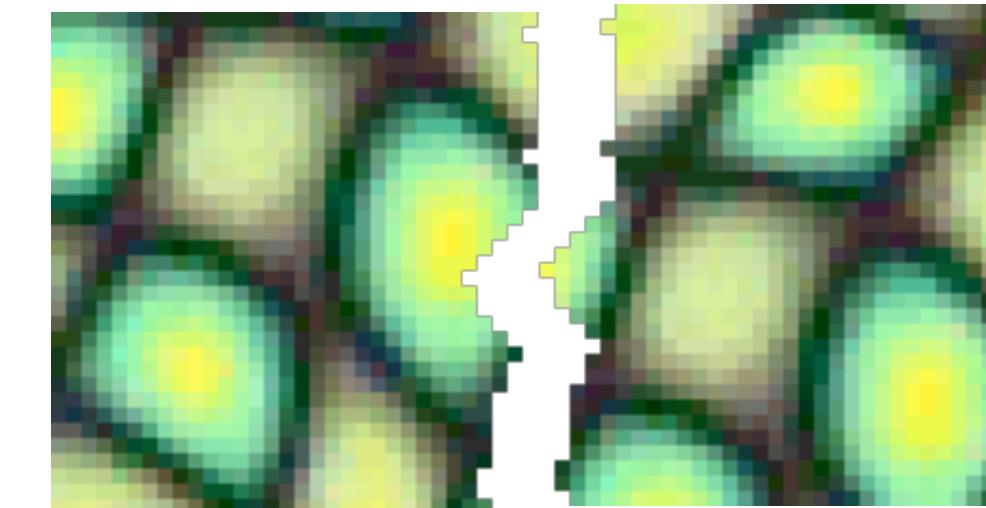
# Finding the Seams



overlapping blocks

vertical boundary

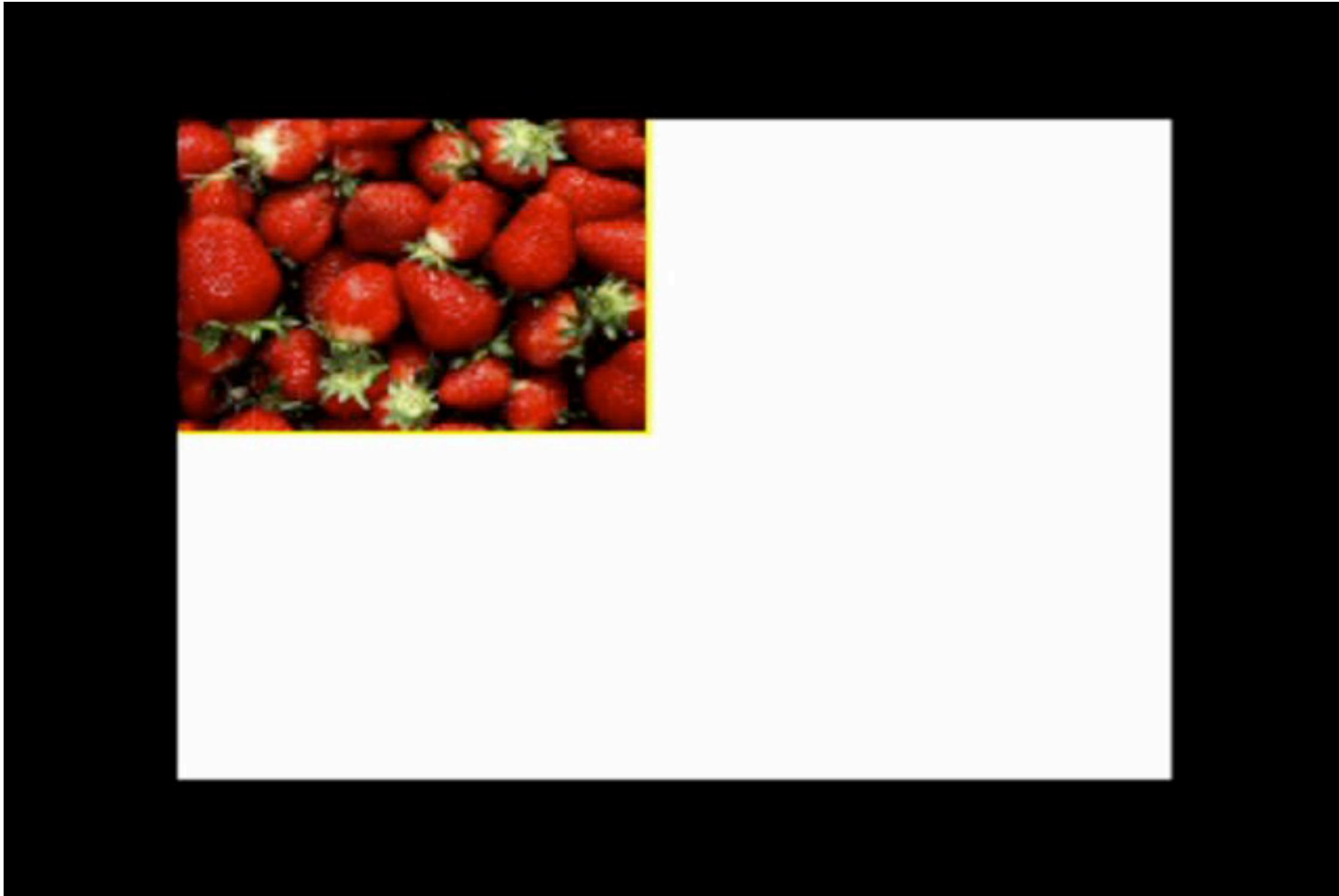
$$\left( \begin{array}{c} \text{block 1} \\ - \\ \text{block 2} \end{array} \right)^2 = \text{overlap error}$$



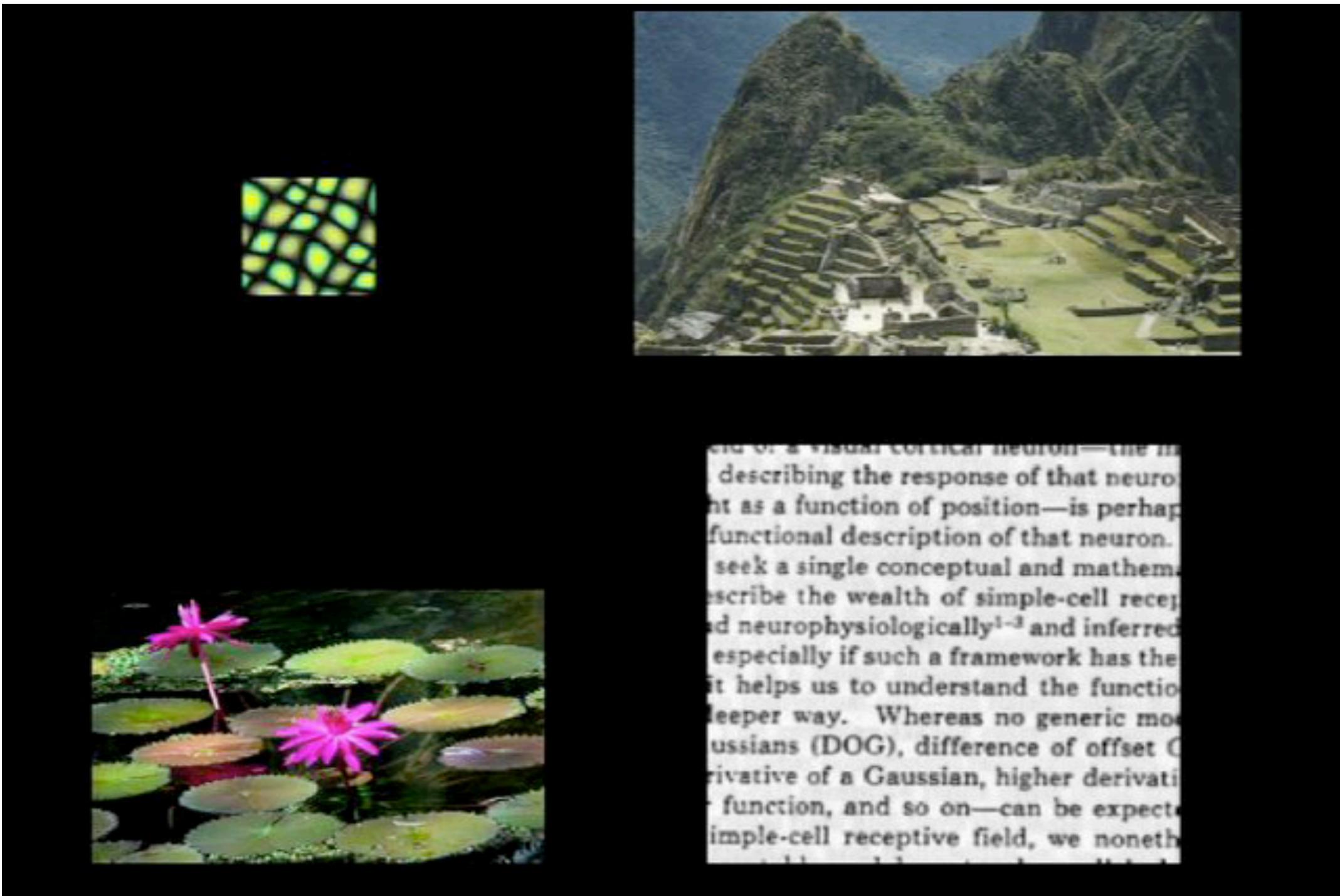
min. error boundary

Efros and Freeman (2001)

# Finding Seams



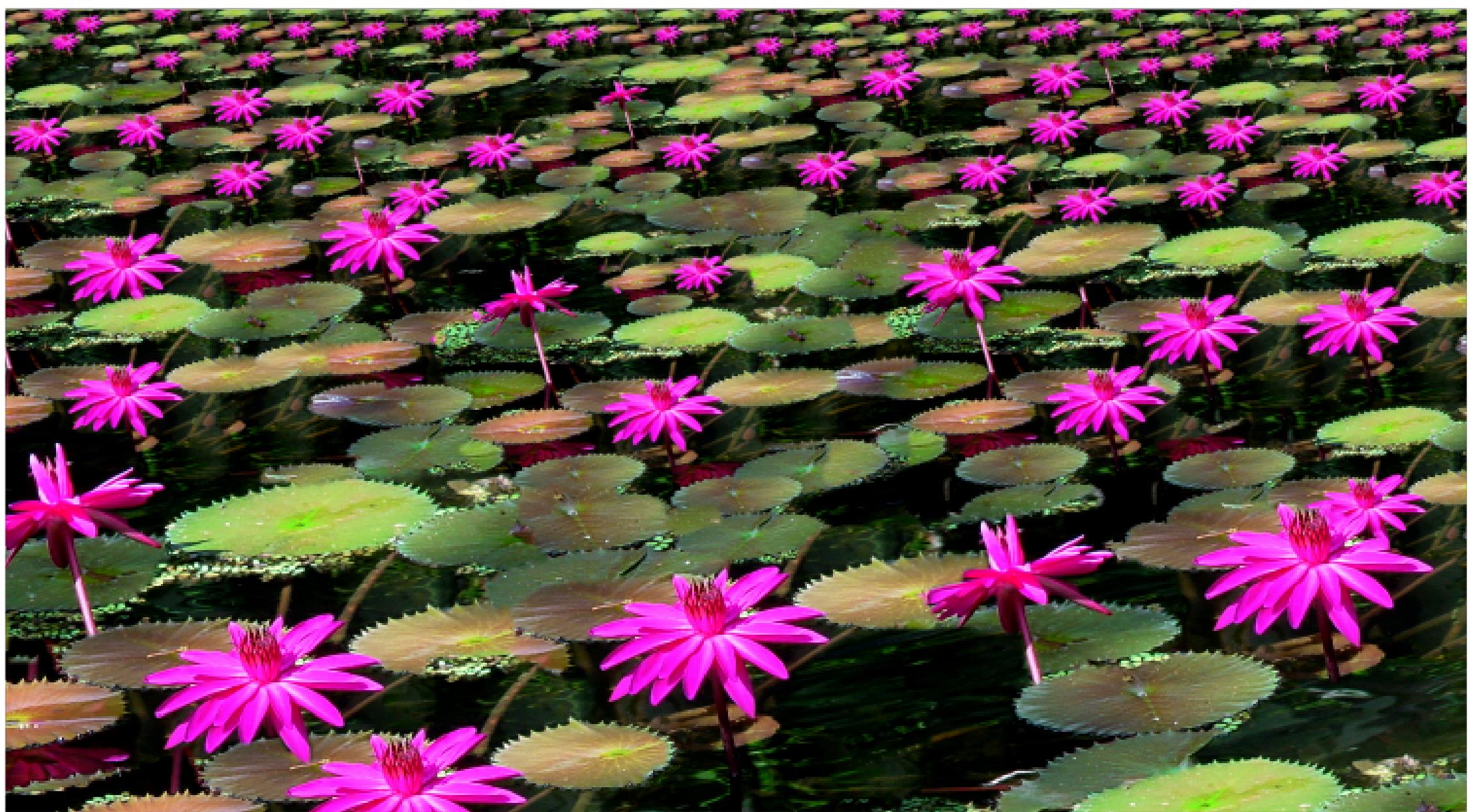
# Generating Novel Images



Kwatra et al. (2003)

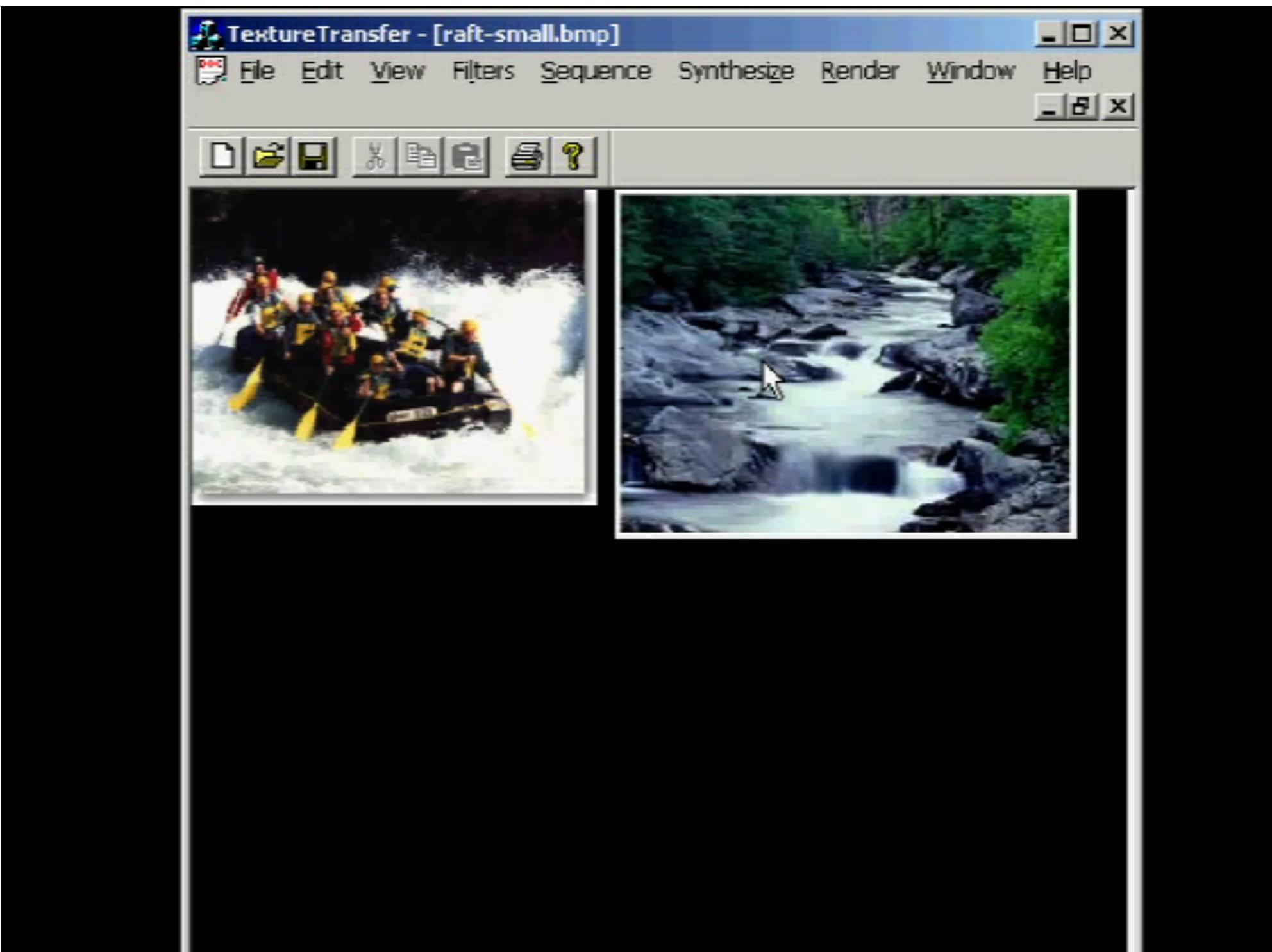
© 2015 Irfan Essa, Georgia Tech, All Rights Reserved

# Extending Images

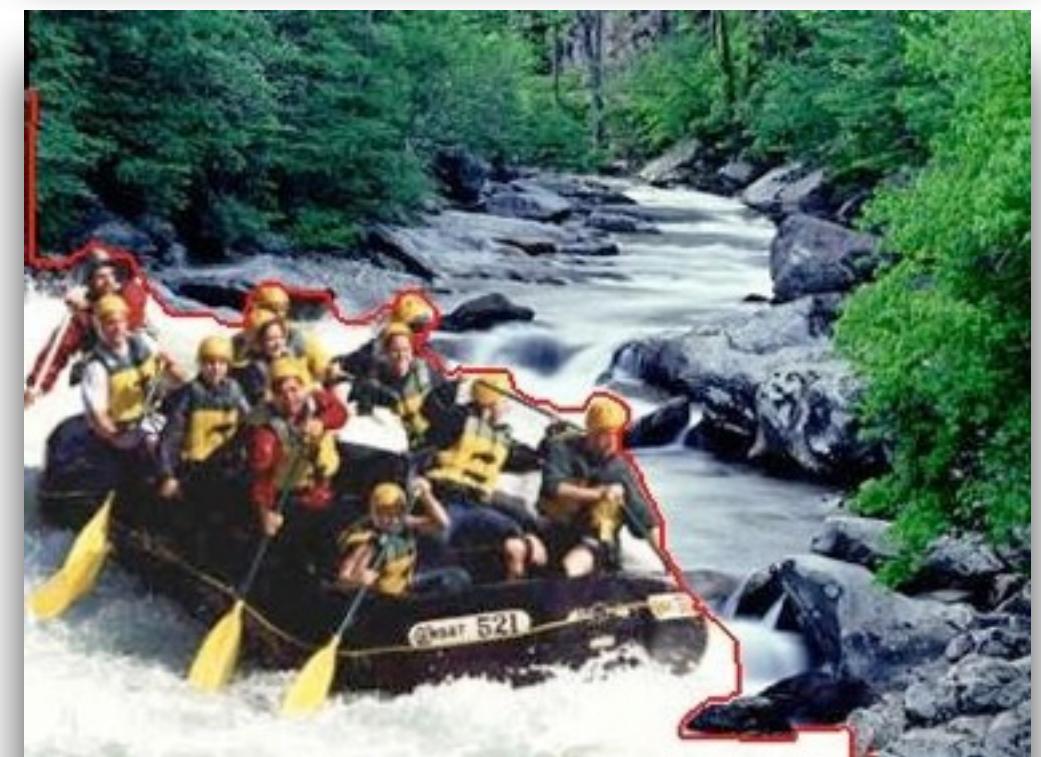


Kwatra et al. (2003)

# Editing Images



# Editing Images



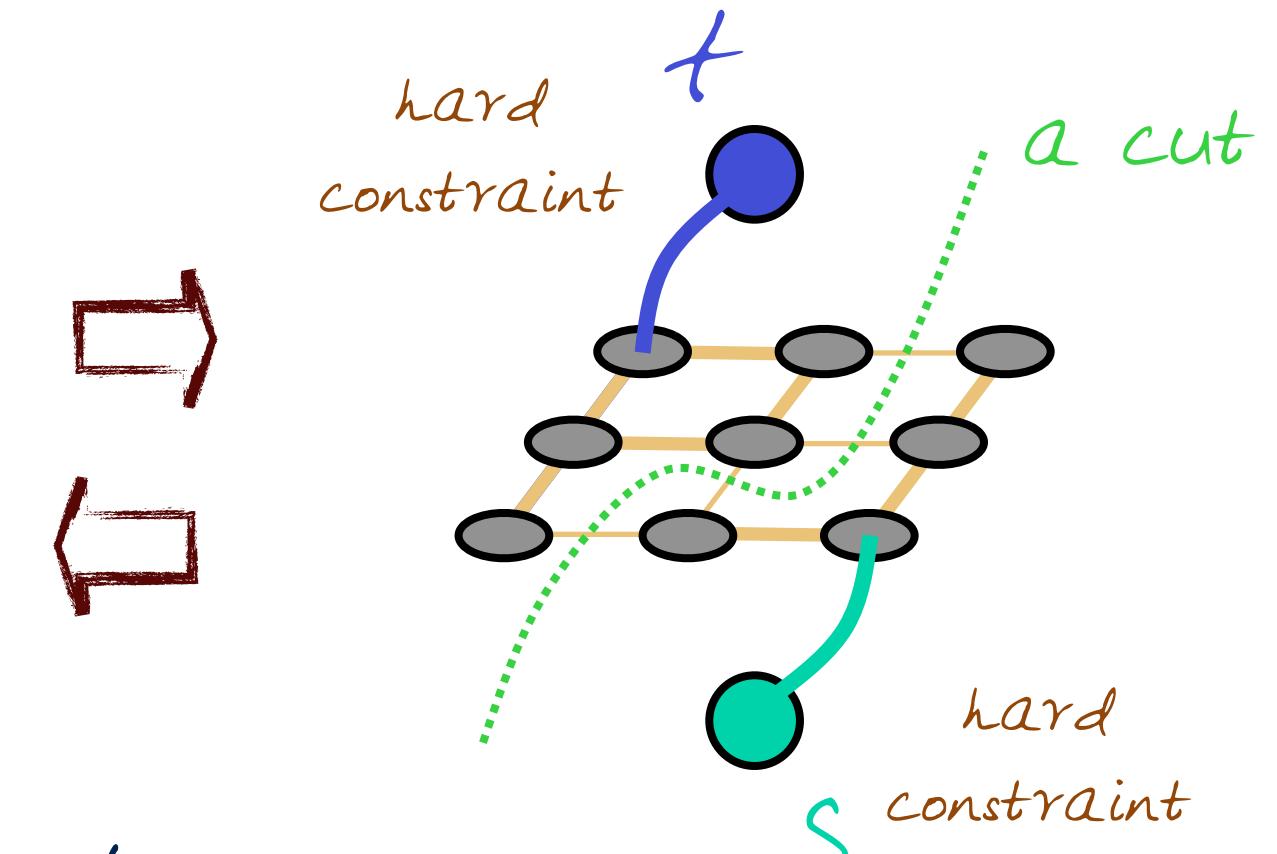
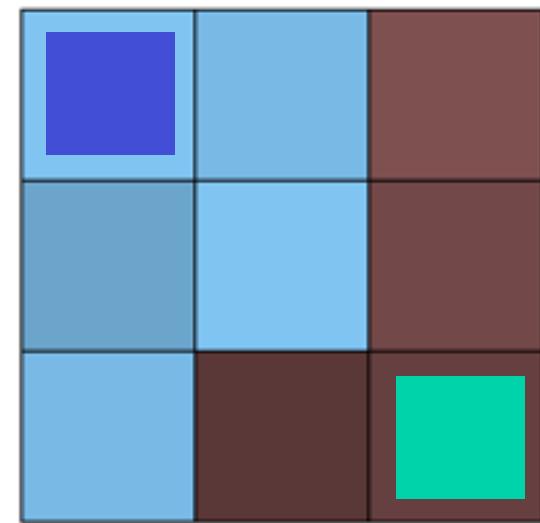
Kwatra et al. (2003)

# Seam Finding using Graph-cuts

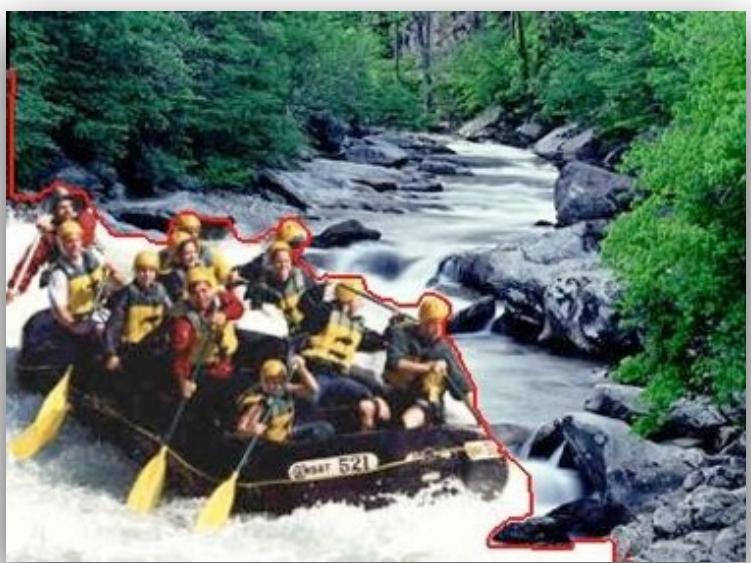


$t$

$s$



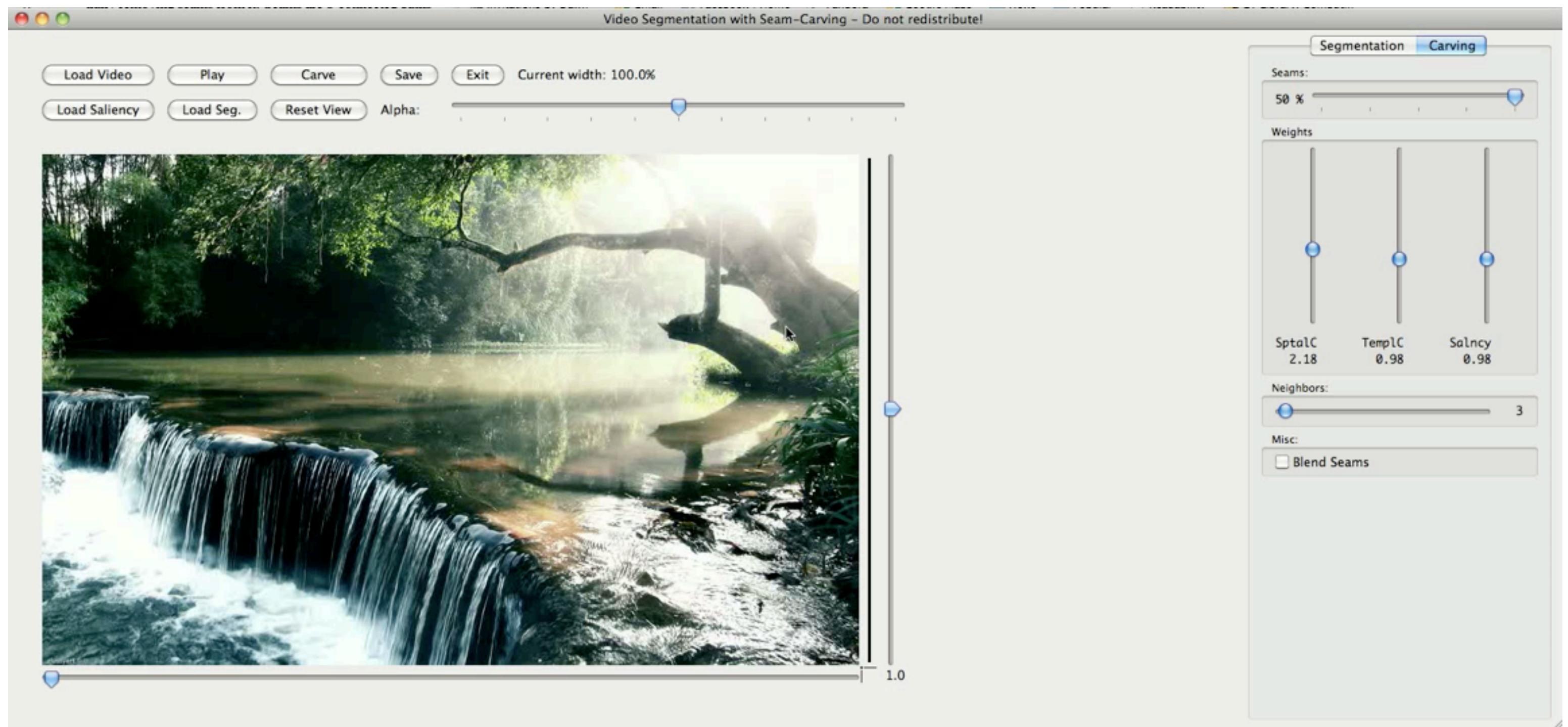
Minimum cost cut can be computed in polynomial time (max-flow/min-cut algorithms)



Another approach is to use Dynamic Programming to find seams

Boykov and Jolly (2001) & Kwatra et al. (2003)

# Seam Carving



Grundmann et al. (2010), Avidan and Shamir (2007)

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# Summary



- \* Introduced an additional method for merging images besides blending
- \* Described how seams are found in images
- \* Discussed the benefits of cutting images over blending images

# Further Reading



- \* Davis (1998), "Mosaics of Scenes with Moving Objects" » IEEE Computer Vision and Pattern Recognition (CVPR), 1998 .
- \* Efros and Freeman (2001), "Image Quilting for Texture Synthesis and Transfer" » SIGGRAPH 2001
- \* Kwatra, Schödl, Essa, Turk, Bobick (2003), "Graphcut textures: image and video synthesis using graph cuts" » SIGGRAPH 2003
- \* Boykov and Jolly (2001), "Interactive Graph Cuts for Optimal Boundary & Region Segmentation of Objects in N-D images, ICCV 2001 .
- \* Avidan and Shamir (2007), "Seam carving for content-aware image resizing" , SIGGRAPH 2007 .
- \* Agarwala, Dontcheva, Agrawala, Drucker, Colburn, Curless, Salesin, Cohen (2004), "Interactive Digital Photomontage" » SIGGRAPH 2004 .

# Neat Class

- \* Feature Detection  
and matching



# Credits



- \* For more information, see
  - \* Richard Szeliski (2010) Computer Vision: Algorithms and Applications, Springer.
  - \* Some concepts in slides motivated by similar slides by A. Efros and J. Hays.
  - \* Some images retrieved from
    - \* <http://commons.wikimedia.org/>.
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# Computational Photography

- \* Study the basics of computation and its impact on the entire workflow of photography, from capturing, manipulating and collaborating on, and sharing photographs.



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# Feature Detection and Matching

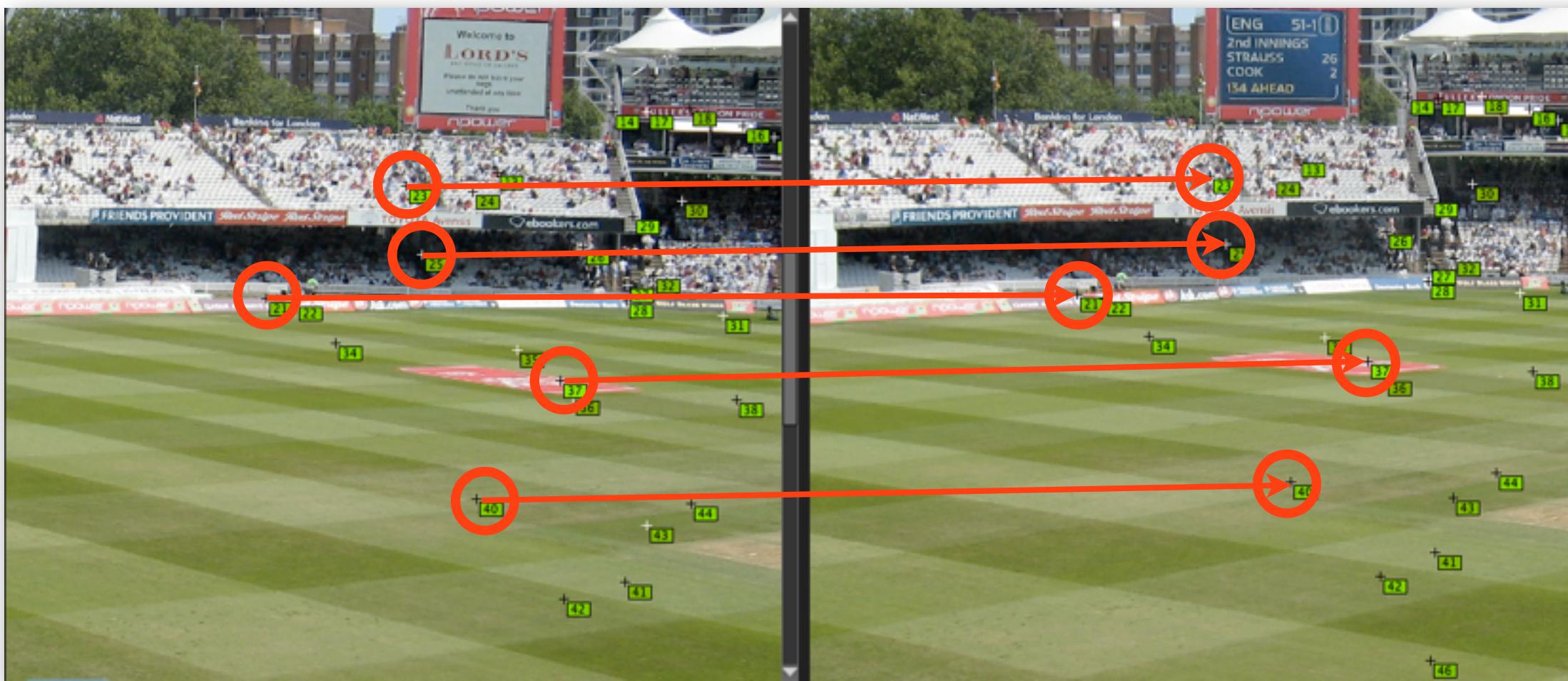
\* Methods for Detecting Features  
in Images that can be matched  
for Registration and Alignment.



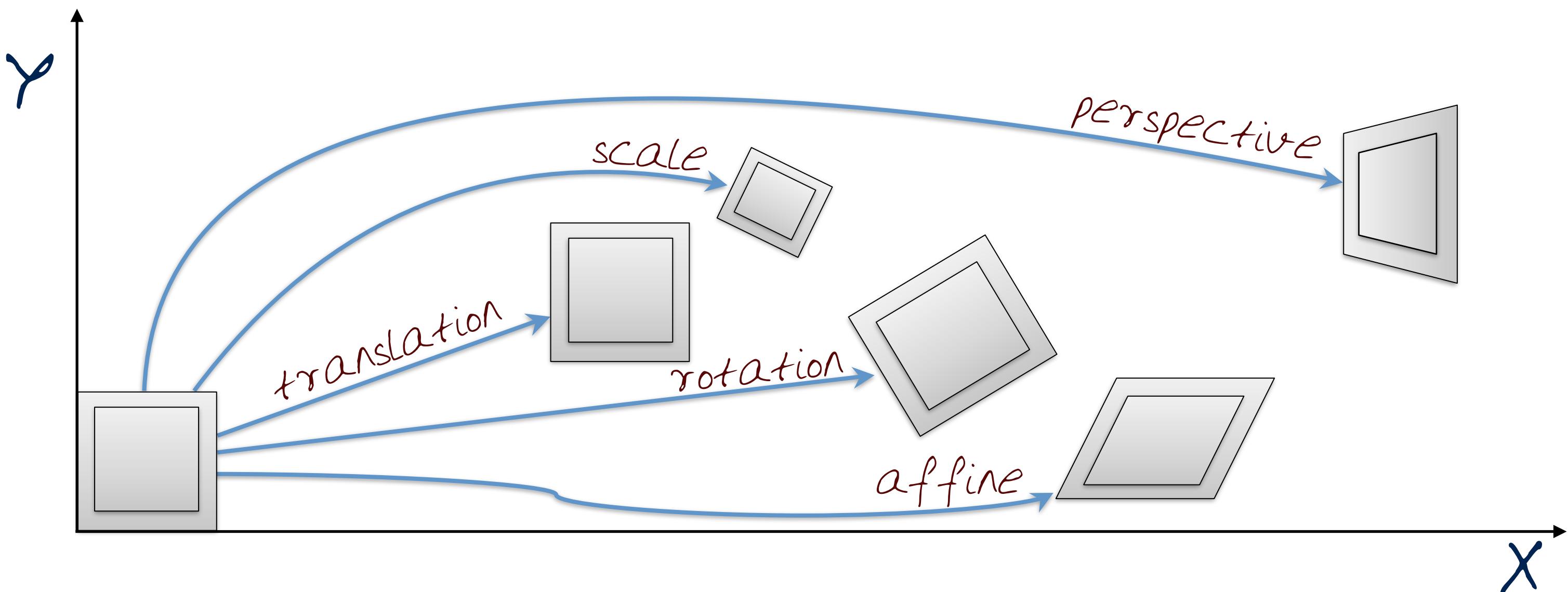
## Lesson Objectives

1. Benefits of Feature Detection and matching in images
2. Characteristics of Good Features
3. Corners are Good Features
4. Harris Corner Detector Algorithm
5. Stages of a SIFT detector

# Recall: Detection and Matching

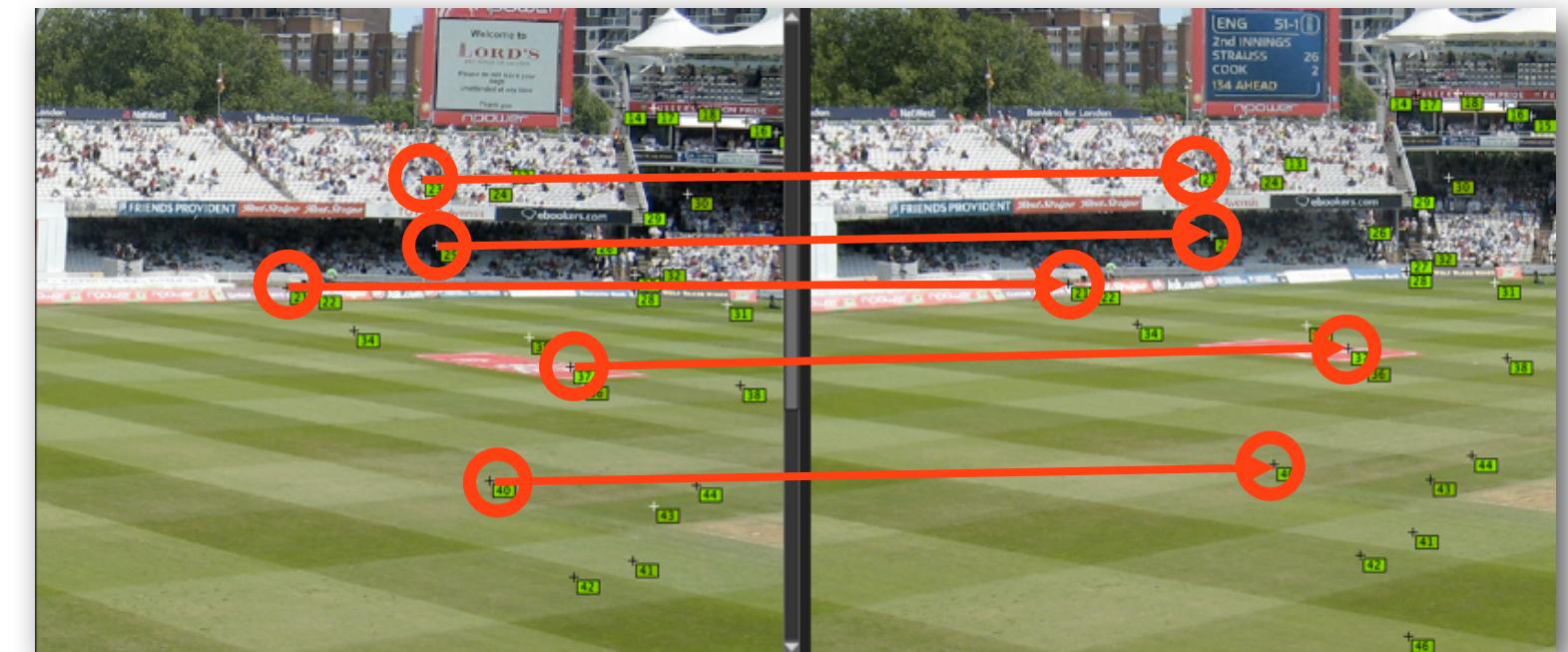


# Image Matching



# Finding Features

- \* Goal - Find points in an image that can be:
  - \* Found in other images
  - \* Found precisely - well localized
  - \* Found reliably - well matched



# Characteristics of Good Features

- \* Repeatability/Precision
- \* Saliency/matchability
- \* Compactness and efficiency
- \* Locality



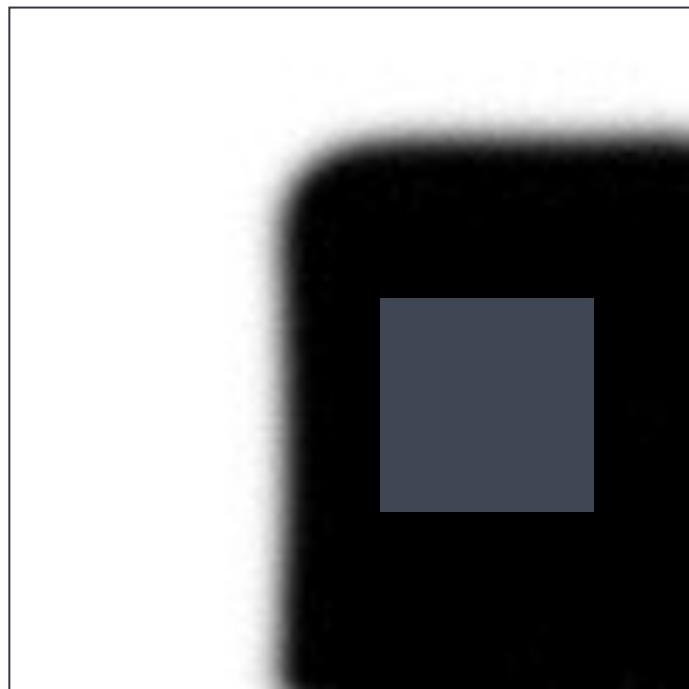
# Find Corners



- \* Key property: In the region around a corner, image gradient has two or more dominant directions
- \* Corners are repeatable and distinctive

Harris and Stephens (1988)

# Corner Detection: The Basics



“flat”  
No change in  
any directions



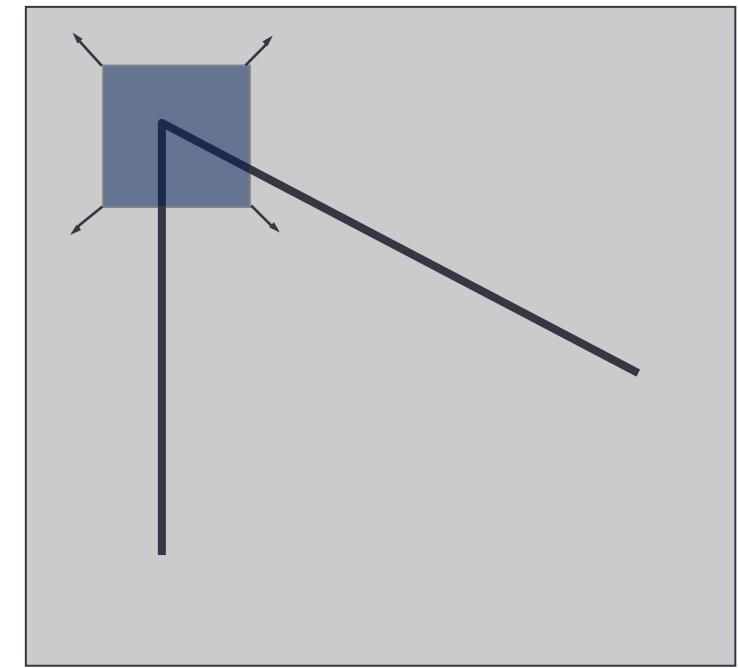
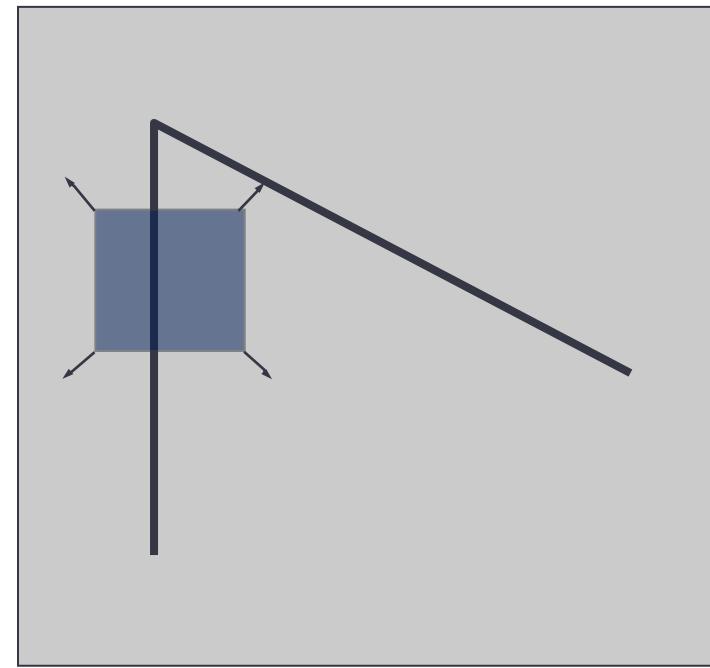
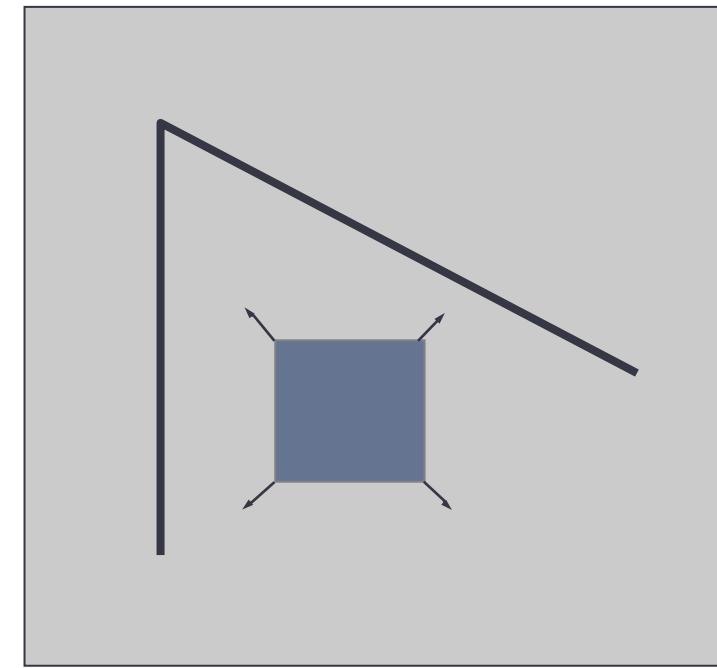
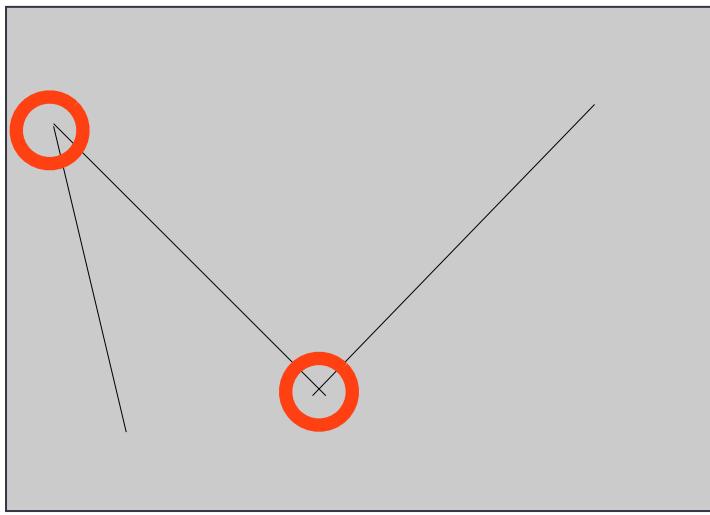
“edge”  
No change  
along the  
edge direction



“corner”  
Change in all  
directions

- \* Recognize a point by looking through a small window
- \* Shifting a window in any direction causes a large change in intensity

# Corner Detection: The Basics



“flat”

No change in  
any directions

“edge”

No change  
along the  
edge direction

“corner”

Change in all  
directions

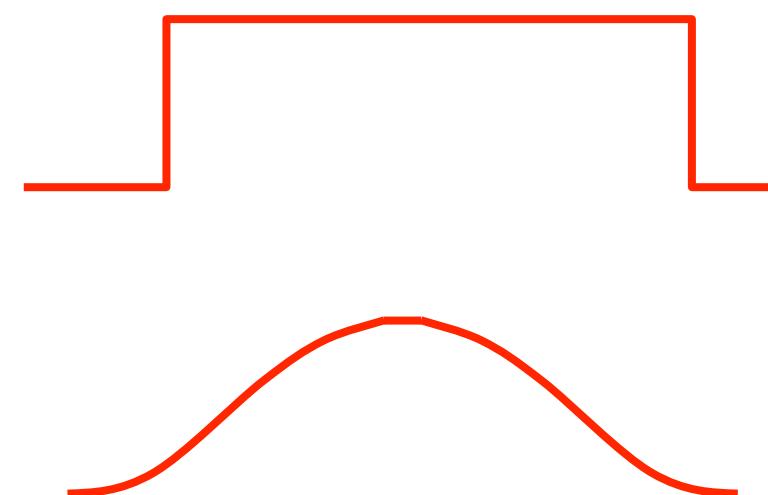
# Corner Detection: Mathematics

Compute the change in appearance by shifting the window by  $u, v$ :

$$E(u, v) = \sum_{x, y} w(x, y) [I(x + u, y + v) - I(x, y)]^2$$

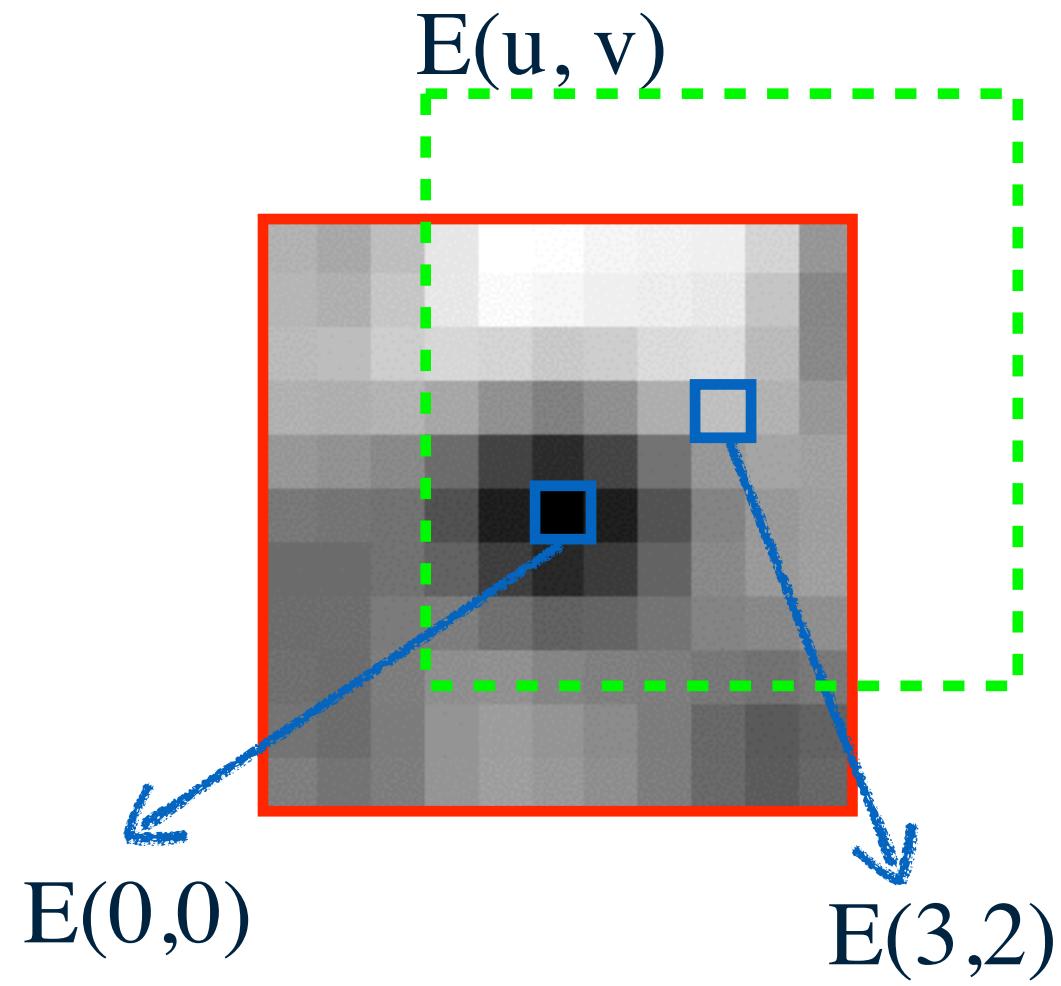
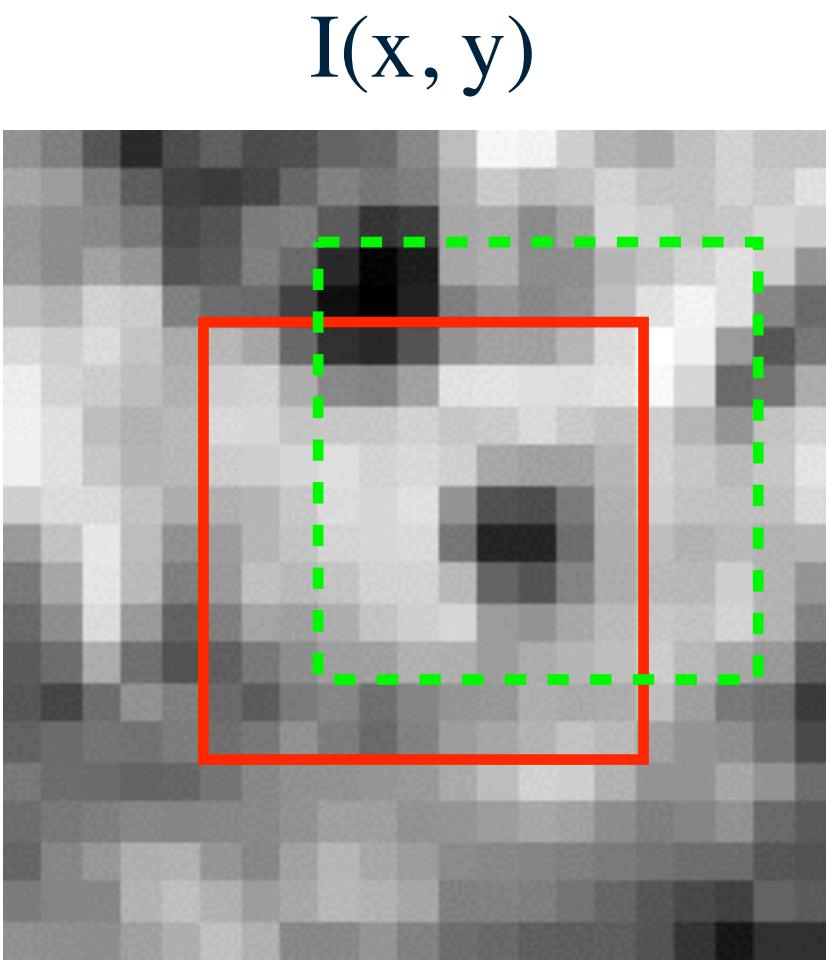
change in appearance      shifted intensity  
window function      intensity

$w(x, y)$  { box function  
a Gaussian



# Corner Detection: Mathematics

$$E(u, v) = \sum_{x,y} w(x, y) [I(x + u, y + v) - I(x, y)]^2$$



Computation  
of the  
change in  
appearance  
by shifting  
the window  
by  $u, v$ :

# Corner Detection: Mathematics

$$E(u, v) = \sum_{x, y} w(x, y) [I(x + u, y + v) - I(x, y)]^2$$

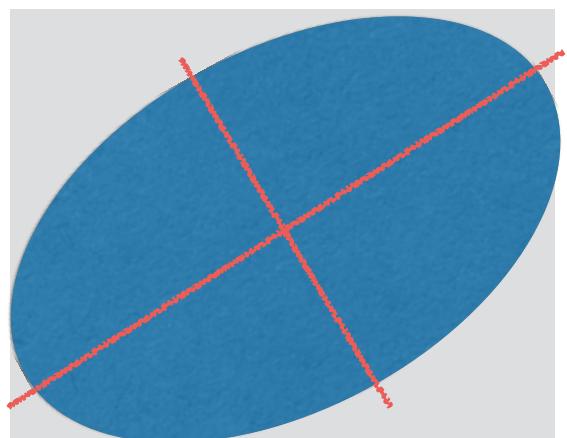
The quadratic approximation, following Taylor Expansion, simplifies to:

$$E(u, v) \approx \begin{bmatrix} u & v \end{bmatrix} M \begin{bmatrix} u \\ v \end{bmatrix}$$

where  $M$  is a second moment matrix computed from image

derivatives  $I_x$  and  $I_y$ .

$$M = \sum_{x, y} w(x, y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$



# Corner Detection: Mathematics

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

$$E(u,v) = \sum_{x,y} \omega(x,y) (u^2 I_x^2 + 2uv I_x I_y + v^2 I_y^2)$$

The surface  $E(u,v)$  is locally approximated by a quadratic form

# Corner Detection: Mathematics

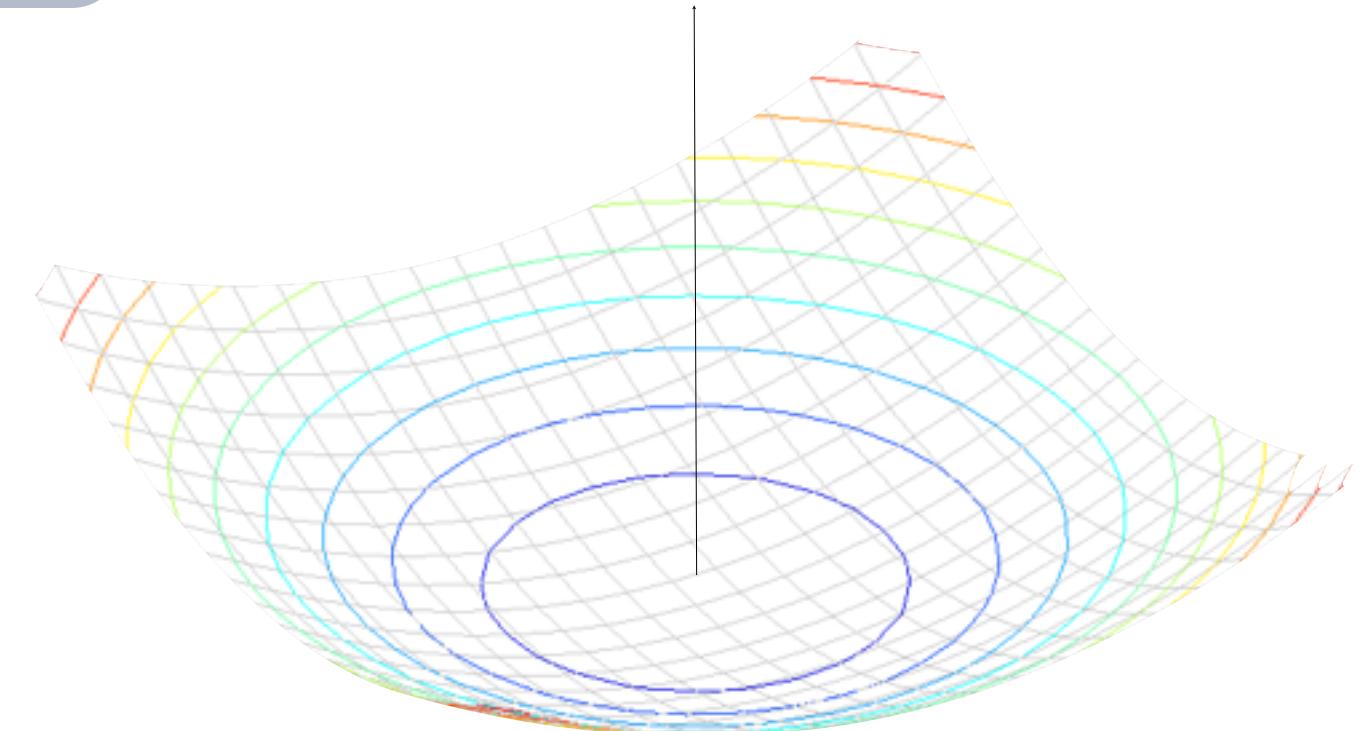
$$E(u, v) = \sum_{x,y} \omega(x, y) (u^2 I_x^2 + 2uv I_x I_y + v^2 I_y^2)$$

The surface  $E(u,v)$  is locally approximated by a quadratic form.

$$E(u, v) = \sum_{x,y} \omega(x, y) (u^2 I_x^2 + 2uv I_x I_y + v^2 I_y^2)$$

consider a "slice" :

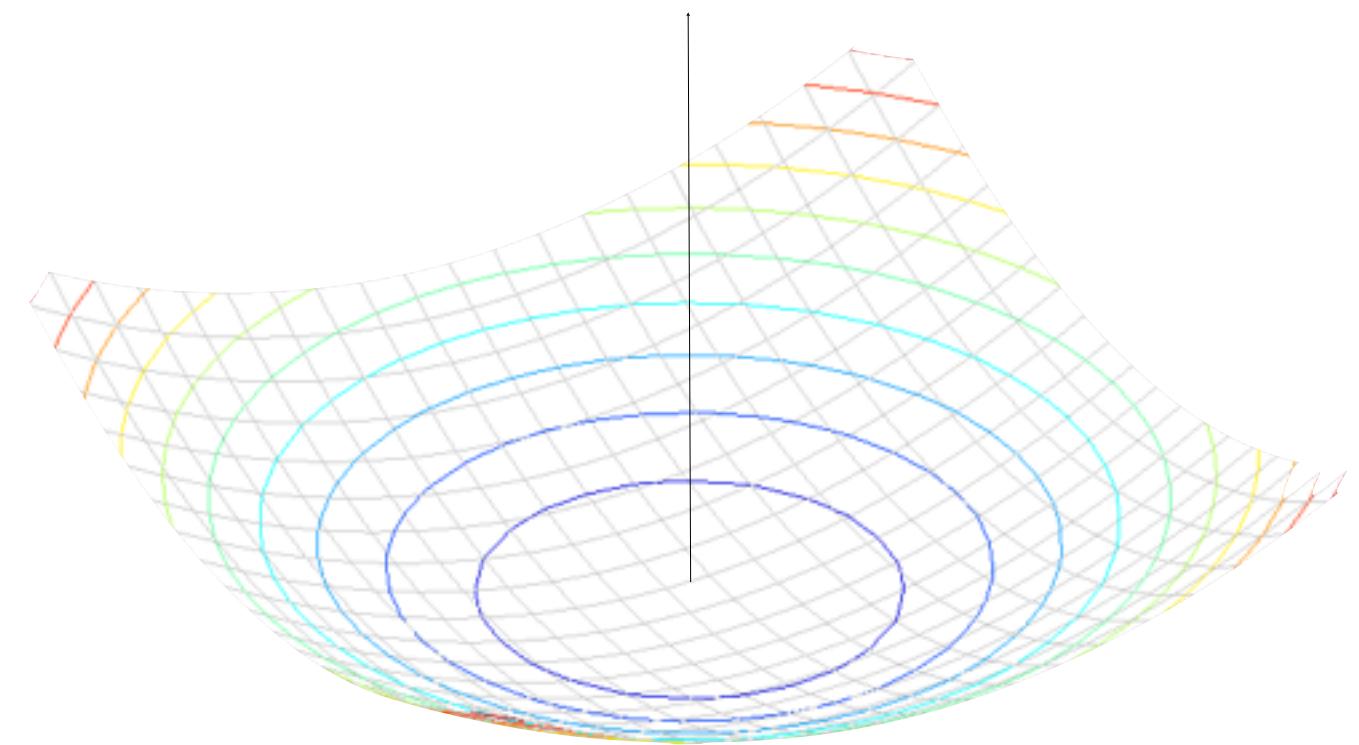
$$u^2 I_x^2 + 2uv I_x I_y + v^2 I_y^2 = k$$



which is an equation of an ellipse

The surface  $E(u,v)$  is locally approximated by a quadratic form.

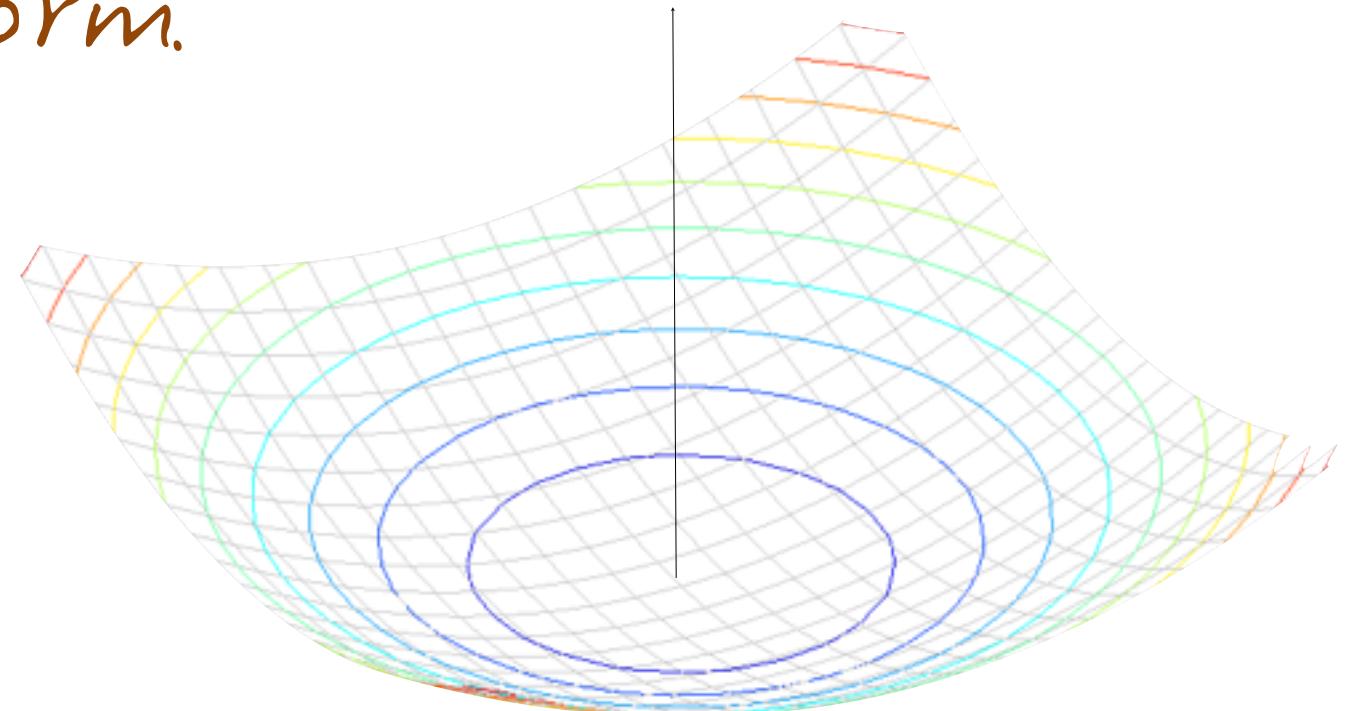
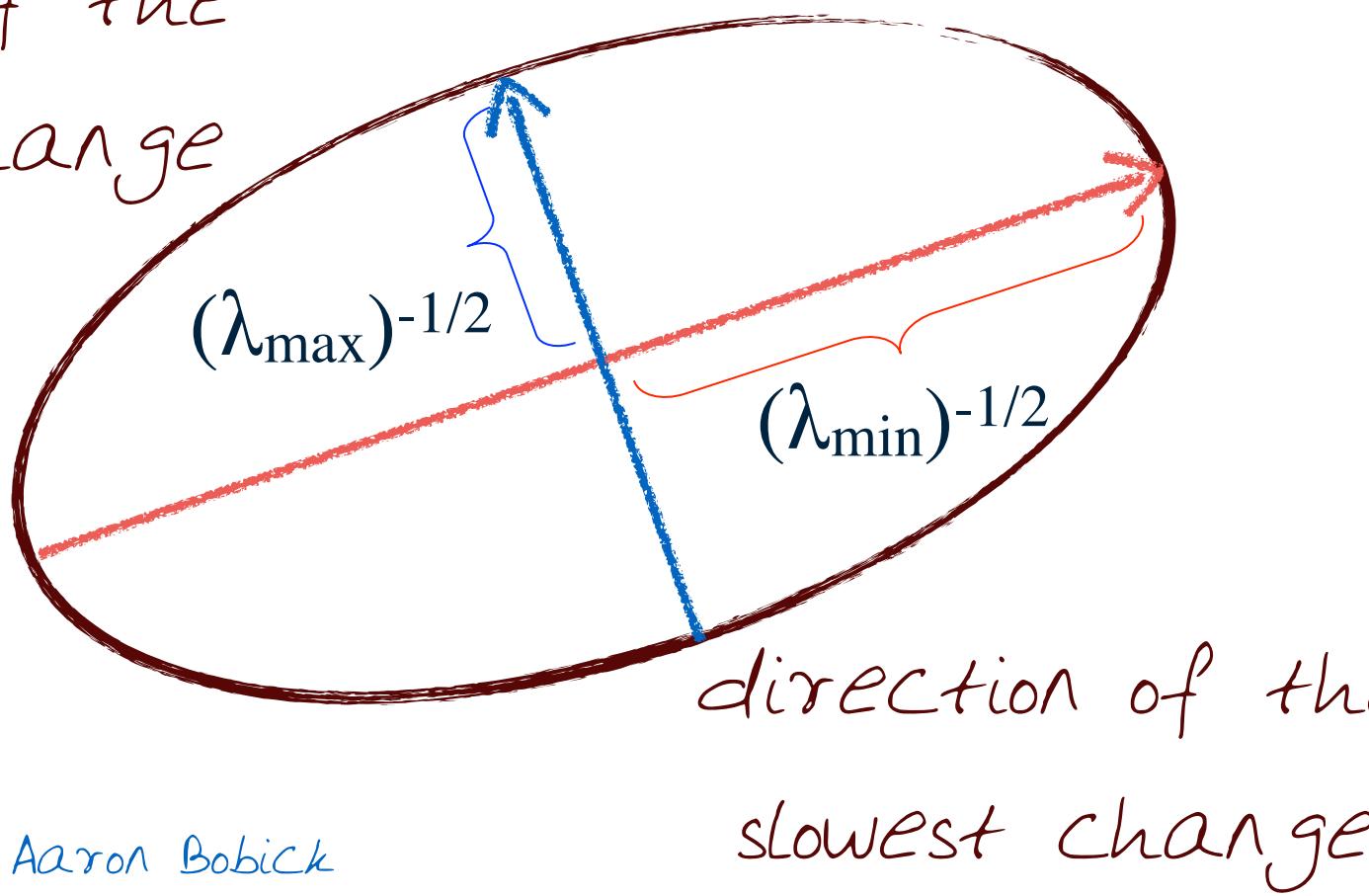
$$u^2 I_x^2 + 2uv I_x I_y + v^2 I_y^2 = k$$



The surface  $E(u,v)$  is locally approximated by a quadratic form.

$$u^2 I_x^2 + 2uv I_x I_y + v^2 I_y^2 = k$$

direction of the  
fastest change



Eigenvalue Analysis:

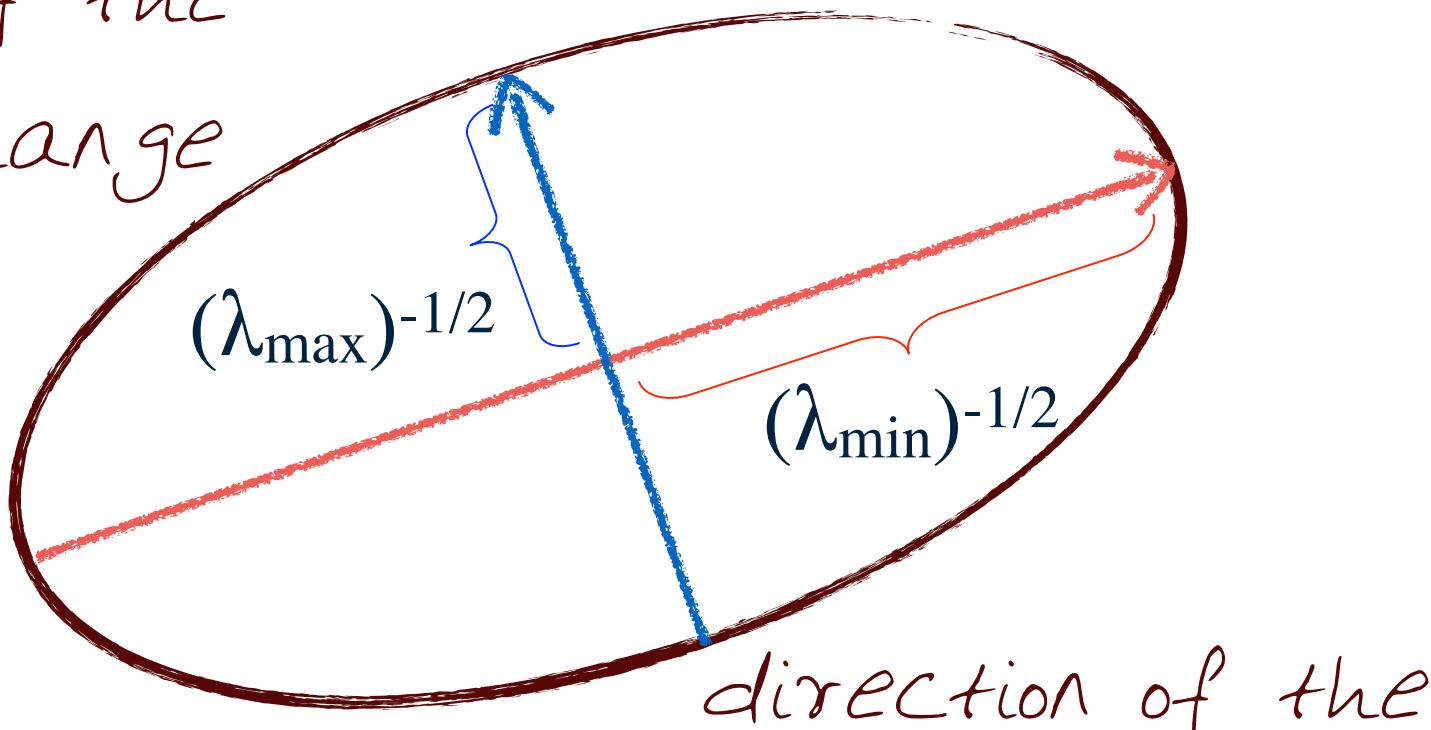
- ◆ Axis lengths  $\Rightarrow$  Eigenvalues
- ◆ Orientation  $\Rightarrow$  Eigenvectors

The surface  $E(u,v)$  is locally approximated by a quadratic form.

$$u^2 I_x^2 + 2uv I_x I_y + v^2 I_y^2 = k$$

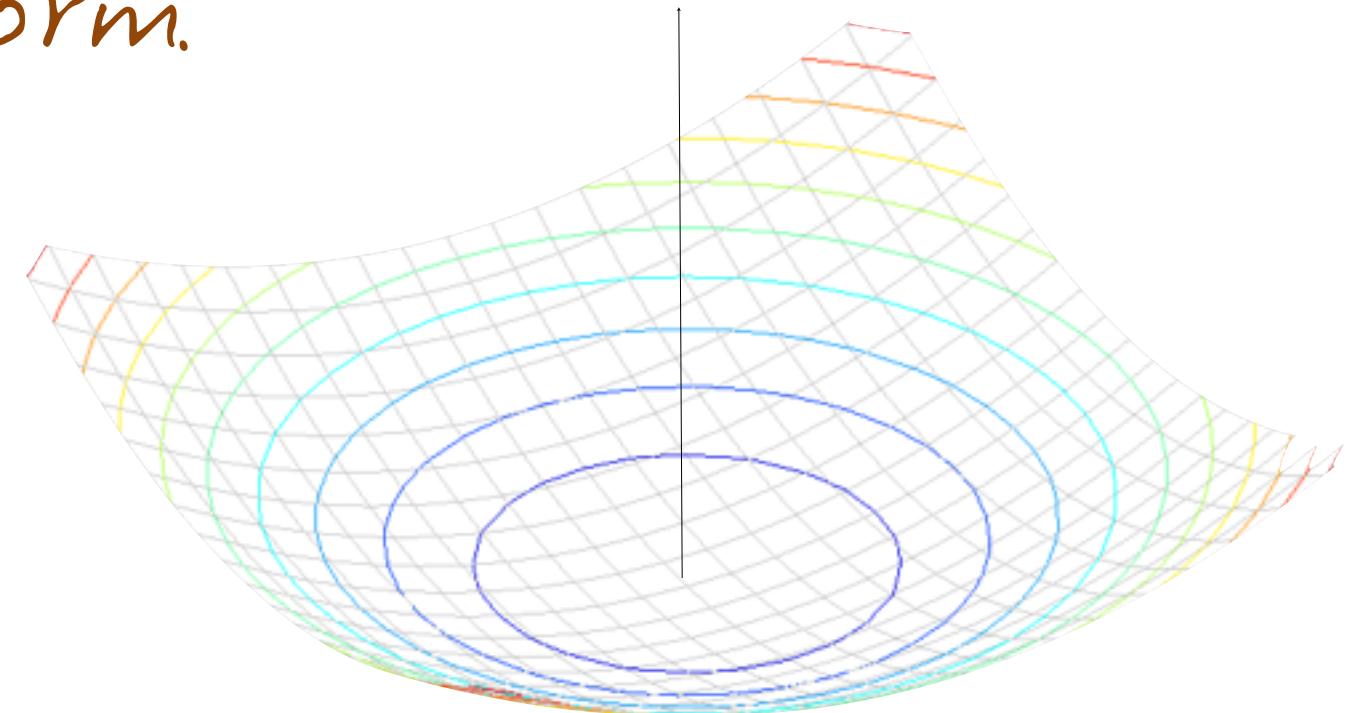
$$\begin{bmatrix} u & v \end{bmatrix} M \begin{bmatrix} u \\ v \end{bmatrix} = k$$

direction of the  
fastest change



$M$  is a diagonal matrix

$$M = R^{-1} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} R$$

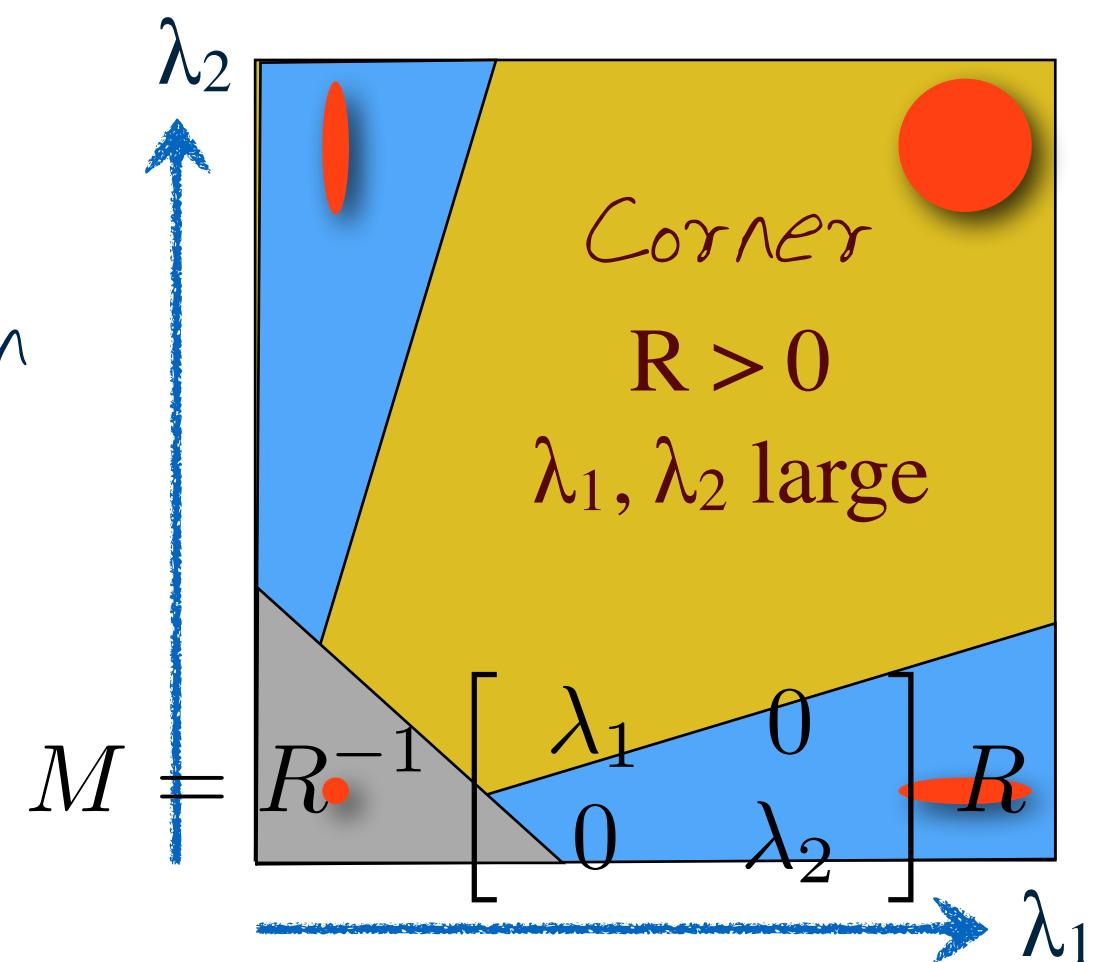


# Eigenvalues

$$R = \det(M) - \alpha \operatorname{trace}(M)^2 = \lambda_1 \lambda_2 - \alpha (\lambda_1 + \lambda_2)^2$$

$\alpha$ : constant (0.04 to 0.06)

- \*  $R$  depends only on eigenvalues of  $M$
- \*  $R \Rightarrow$  large for a corner
- \*  $R \Rightarrow$  negative with large magnitude for an edge
- \*  $|R| \Rightarrow$  small for a flat region
- \* Note: No explicit computation of eigenvalues required



# Harris Detector Algorithm (Preview)

- \* Compute Gaussian derivatives at each pixel
- \* Compute second moment matrix  $M$  in a Gaussian window around each pixel
- \* Compute corner response function  $R$
- \* Threshold  $R$
- \* Find local maxima of response function (non-maximum suppression)



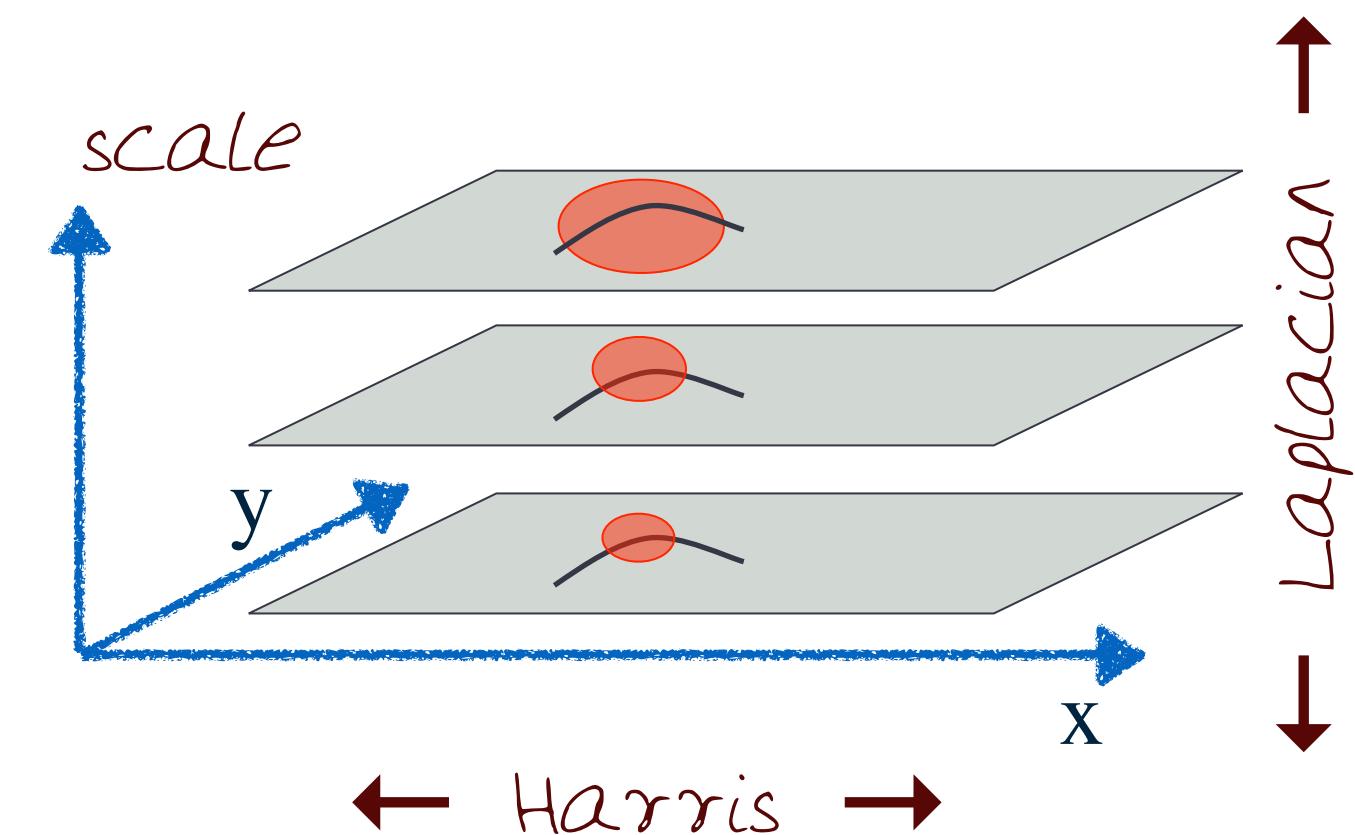
# Properties of the Harris Detector

- \* Rotation Invariant?
  - \* Ellipses rotate, but shape (eigenvalues) remain same
  - \* Corner Response R is invariant
- \* Intensity Invariant?
  - \* Partial invariance to additive and multiplicative intensity changes (threshold issue for multiplicative)
  - \* ONLY Image derivatives are used
- \* Scale Invariant
  - \* NO! Dependent on Window Size!
  - \* USE Pyramids (or Frequency Domain!)



# Scale Invariant Detectors

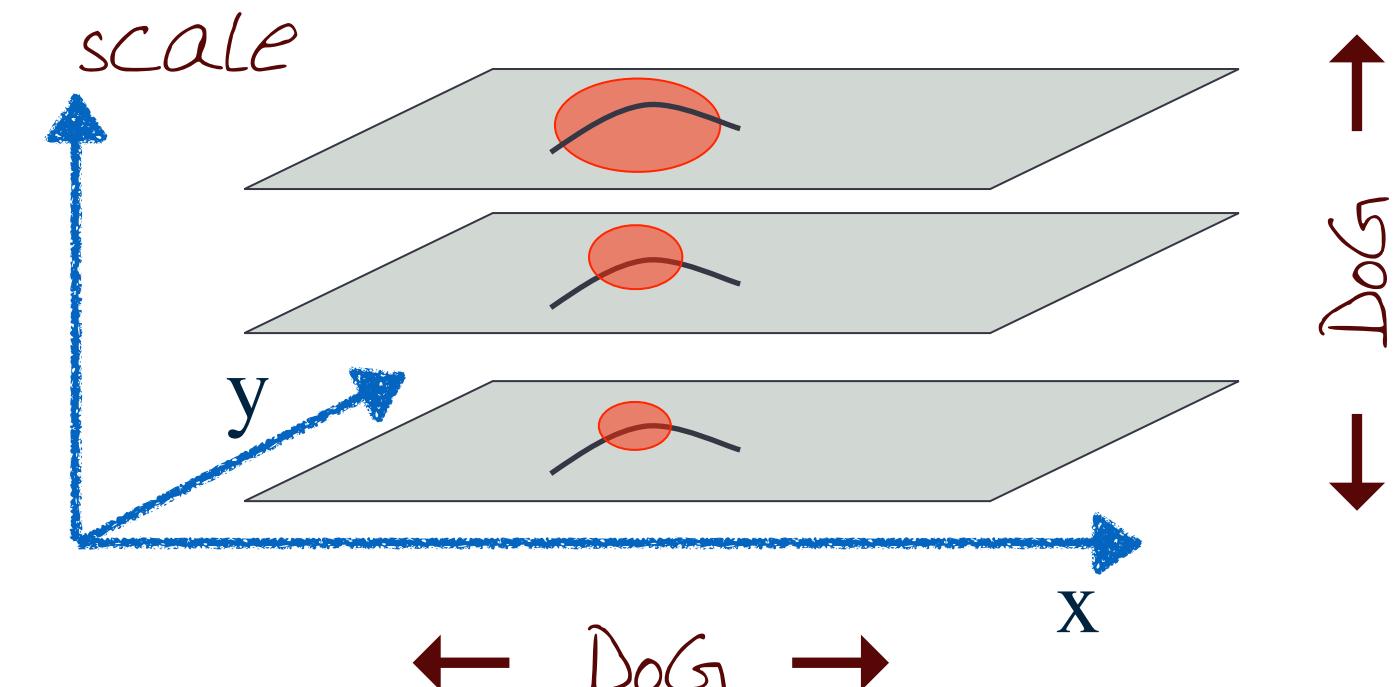
- \* Harris-Laplacian
- \* Find local maximum of:
  - \* Harris corner detector in space (image coordinates)
  - \* Laplacian in scale



(mikolajczyk and schmid, 2001)

# Scale Invariant Detectors

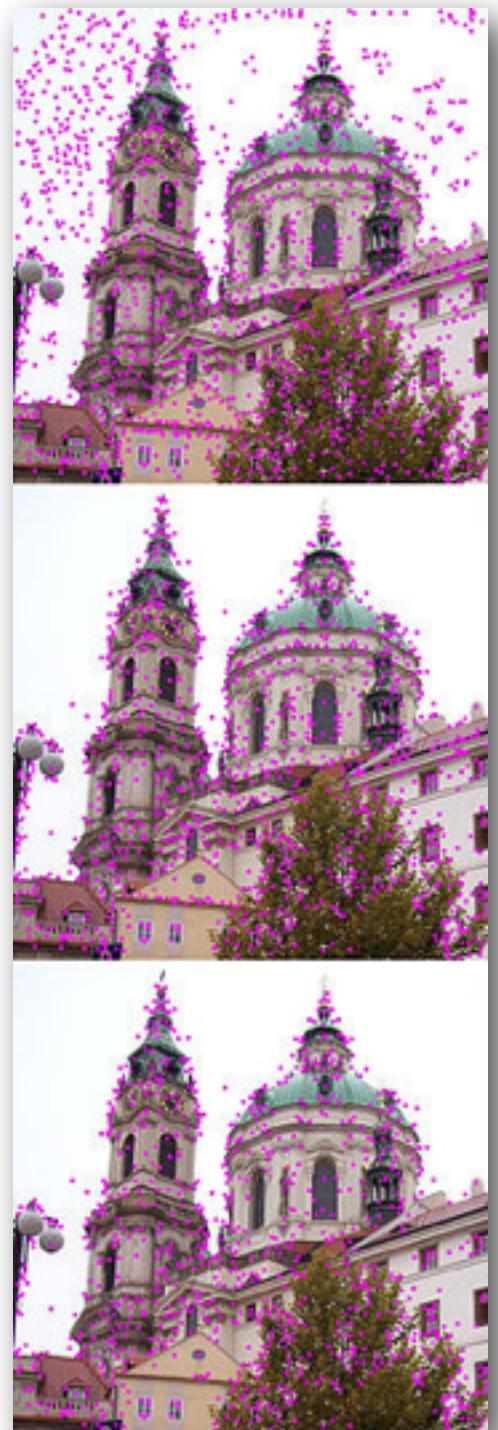
- \* SIFT (Lowe, 2004)
- \* Find local maximum of:
  - \* Difference of Gaussians (DoG) in space and scale
  - \* DoG is simply a pyramid of the difference of Gaussians within each octave



Lowe 2004

# SIFT (Scale-Invariant Feature Transform)

- \* Orientation assignment
- \* Compute best orientation(s) for each keypoint region.
- \* Keypoint description
- \* Use local image gradients at selected scale and rotation to describe each keypoint region.



# Invariant Local Features

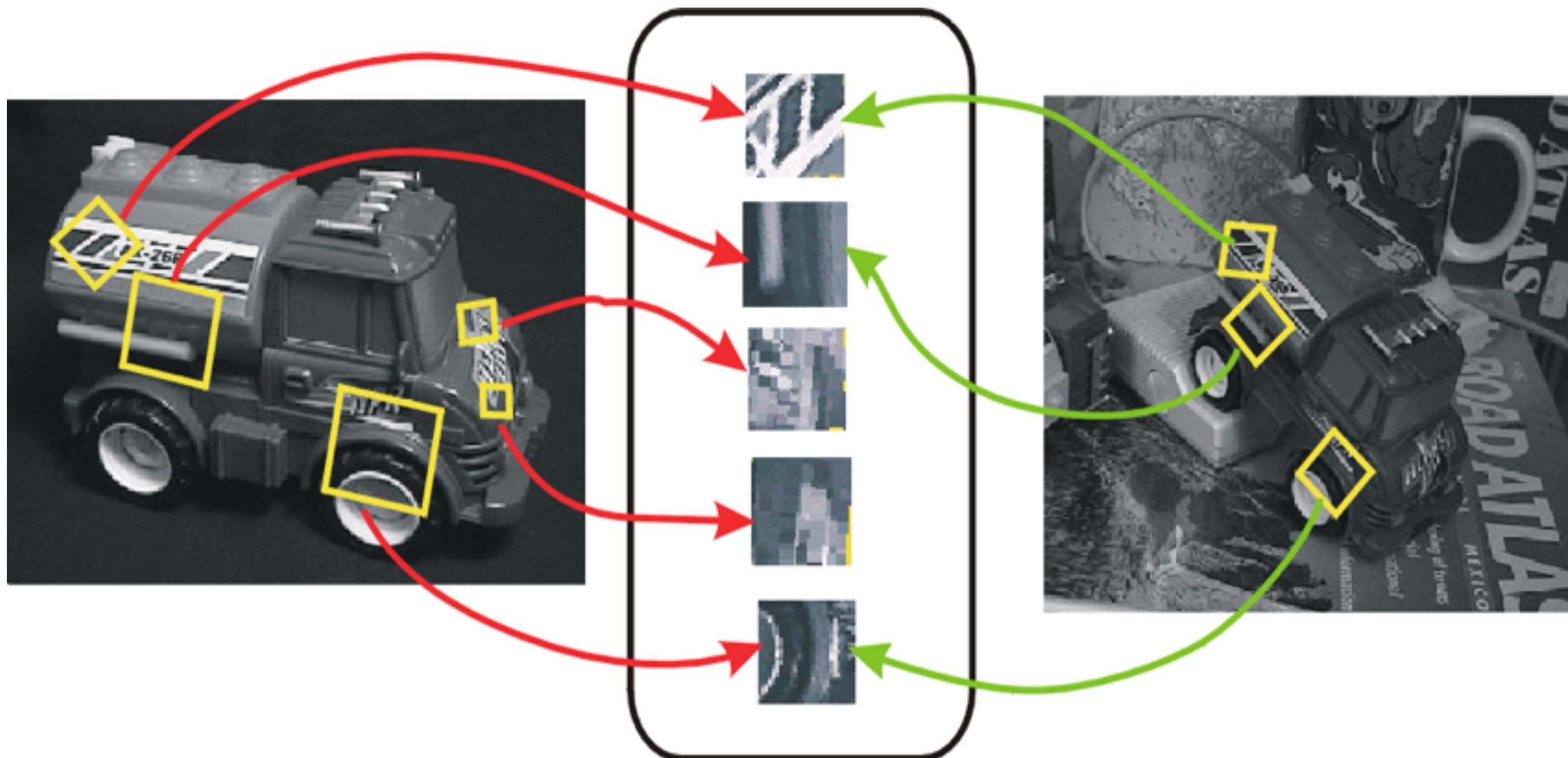
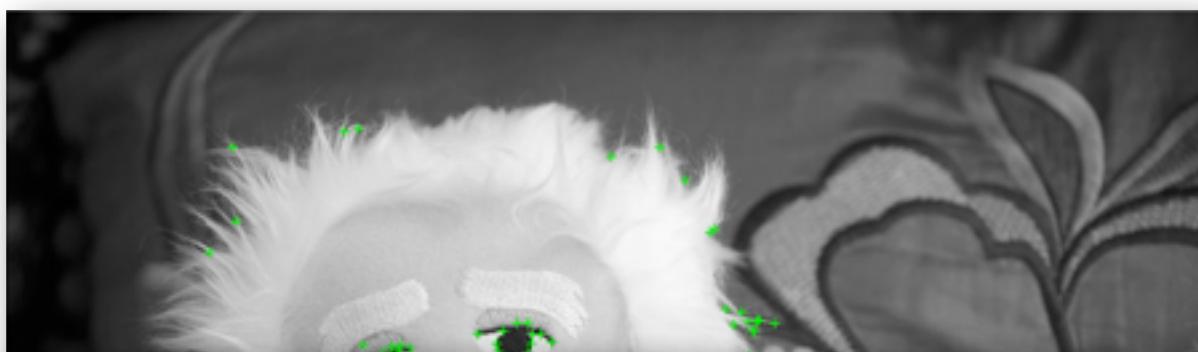


Image content is transformed into local feature coordinates that are invariant to translation, rotation, scale, and other imaging parameters

Lowe 2004

# Results

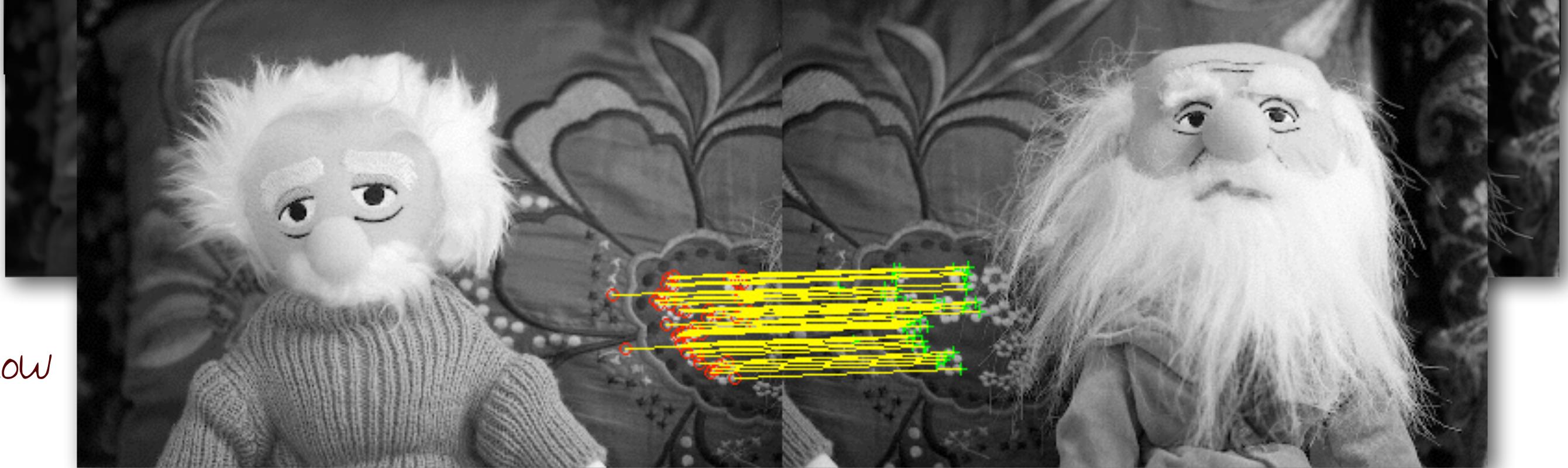
Detect



match



Show



# Summary



- \* Introduced Feature Detection and matching for images
- \* Discussed the four (4) Characteristics of Good Features
- \* Introduced the Harris Corner Detector Framework
- \* Introduced the SIFT detector

# Further Reading



- \* Harris and Stephens (1988) "A Combined Corner and Edge Detector." Proceedings of the 4th Alvey Vision Conference, 1988, [PDF][DOI]
- \* Mikolajczyk and Schmid (2001). "Indexing Based on Scale Invariant Interest Points". ICCV 2001
- \* Lowe (2004) "Distinctive Image Features from Scale-Invariant Keypoints". IJCV 2004
- \* Search for "Features" on OpenCV site

# Neat Class

- \* More details on SIFT  
and Harris Corner  
Detectors!



# Credits



- \* For more information, see:
  - \* Richard Szeliski (2010) Computer Vision: Algorithms and Applications, Springer
- \* Some concepts in slides motivated by similar slides by A. Efros and J. Hays
- \* Some images retrieved from
  - \* <http://commons.wikimedia.org/>
  - \* List will be available on website

# Computational Photography

- \* Study the basics of computation and its impact on the entire workflow of photography, from capturing, manipulating and collaborating on, and sharing photographs.

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# Feature Detection and Matching

- \* More details on Harris and SIFT Features



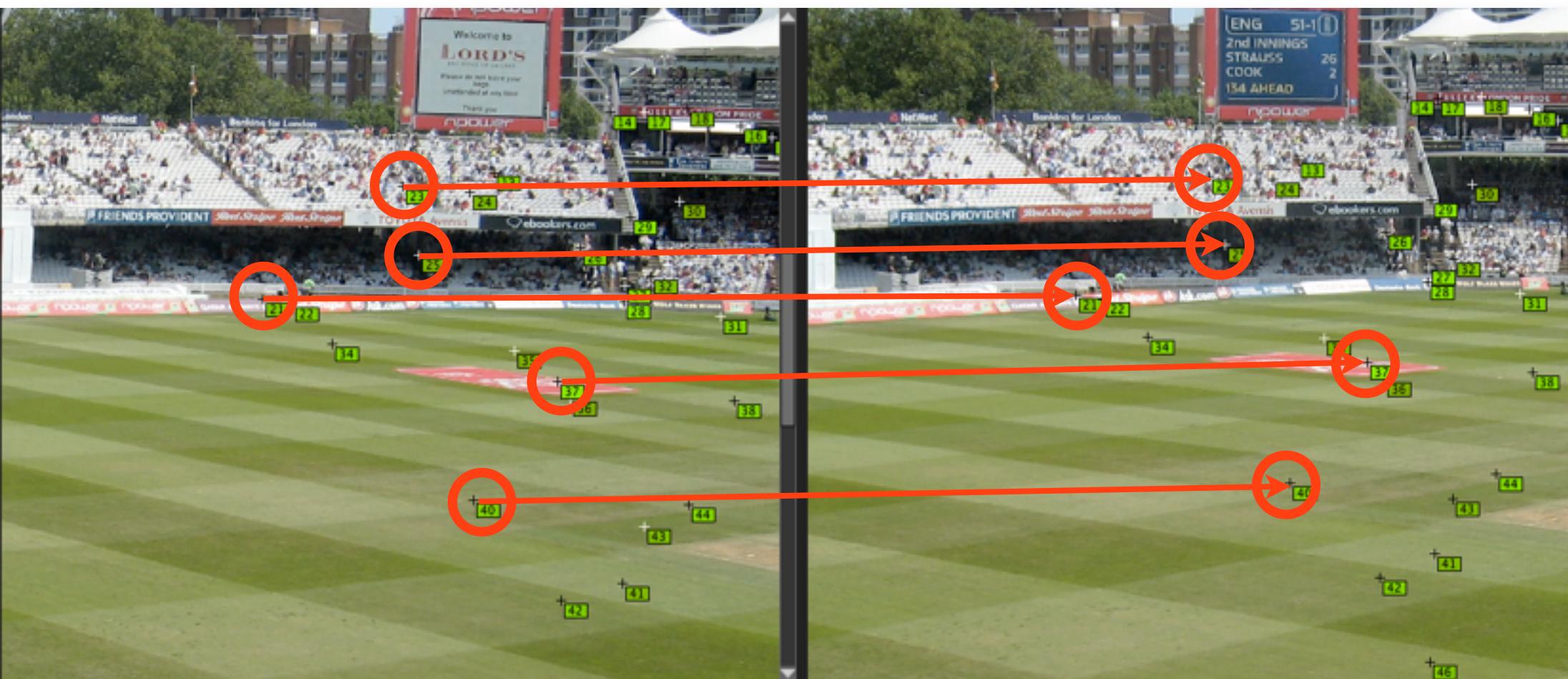
© 2014 Irfan Essa, Georgia Tech, All Rights Reserved



## Lesson Objectives

1. Harris Corner Detector Algorithm
2. SIFT

# Recall: Detection and Matching



## Recall: Corner Detection: Mathematics

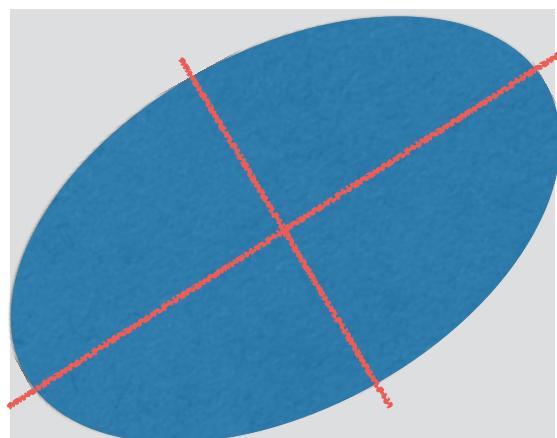
$$E(u, v) = \sum_{x, y} w(x, y) [I(x + u, y + v) - I(x, y)]^2$$

The quadratic approximation, following Taylor Expansion, simplifies to:

$$E(u, v) \approx \begin{bmatrix} u & v \end{bmatrix} M \begin{bmatrix} u \\ v \end{bmatrix}$$

where  $M$  is a second moment matrix computed from image derivatives  $I_x$  and  $I_y$ :

$$M = \sum_{x, y} w(x, y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

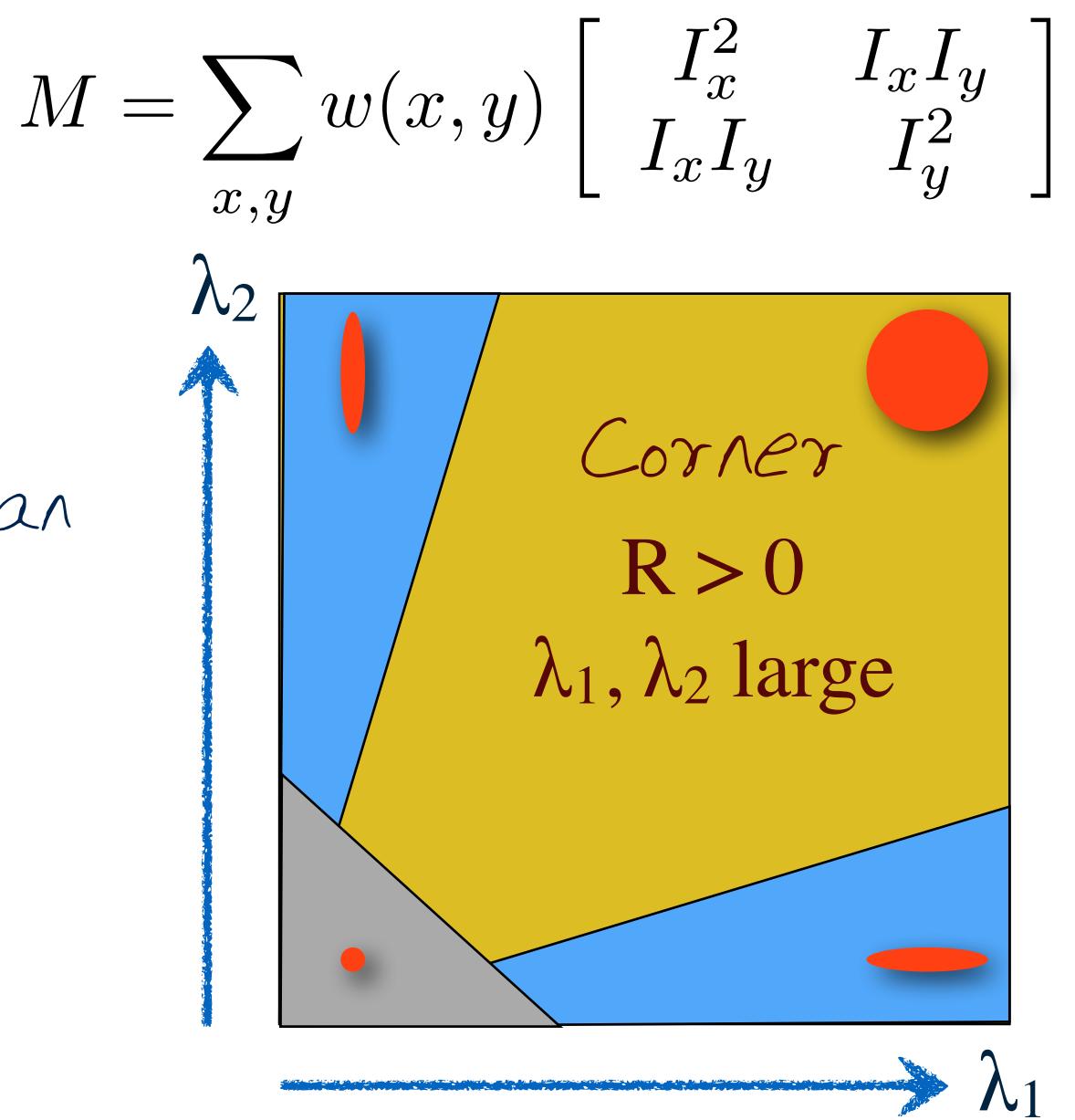


# Recall: Harris Corner Response Function

$$R = \det(M) - \alpha \operatorname{trace}(M)^2 = \lambda_1 \lambda_2 - \alpha (\lambda_1 + \lambda_2)^2$$

$\alpha$ : constant (0.04 to 0.06)

- \*  $R$  depends only on eigenvalues of  $M$
- \*  $R \Rightarrow$  large for a corner
- \*  $R \Rightarrow$  negative with large magnitude for an edge
- \*  $|R| \Rightarrow$  small for a flat region
- \* Note: No explicit computation of eigenvalues required



# Harris Detector: Step by Step

1. Compute horizontal and vertical derivatives of the image (convolve with derivative of Gaussians)
2. Computer outer products of gradients  $M$
3. Convolve with larger Gaussian
4. Compute scalar interest measure  $R$
5. Find local maxima above some threshold, detect features!

# Harris Detector: Workflow

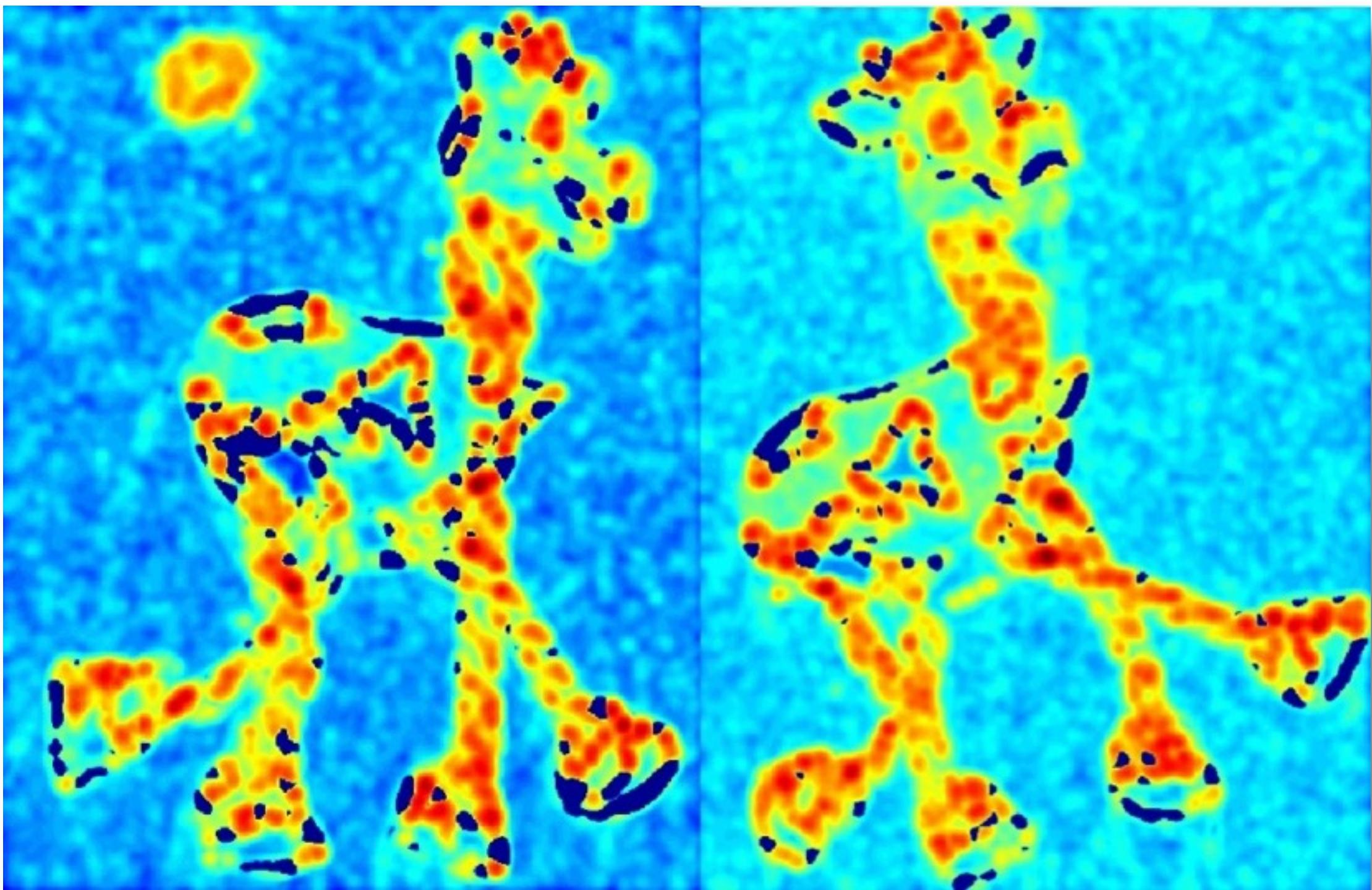
Image Pair



Slides adapted from Aaron Bobick, Alyosha Efros, etc.

# Harris Detector: Workflow

Compute corner  
response R

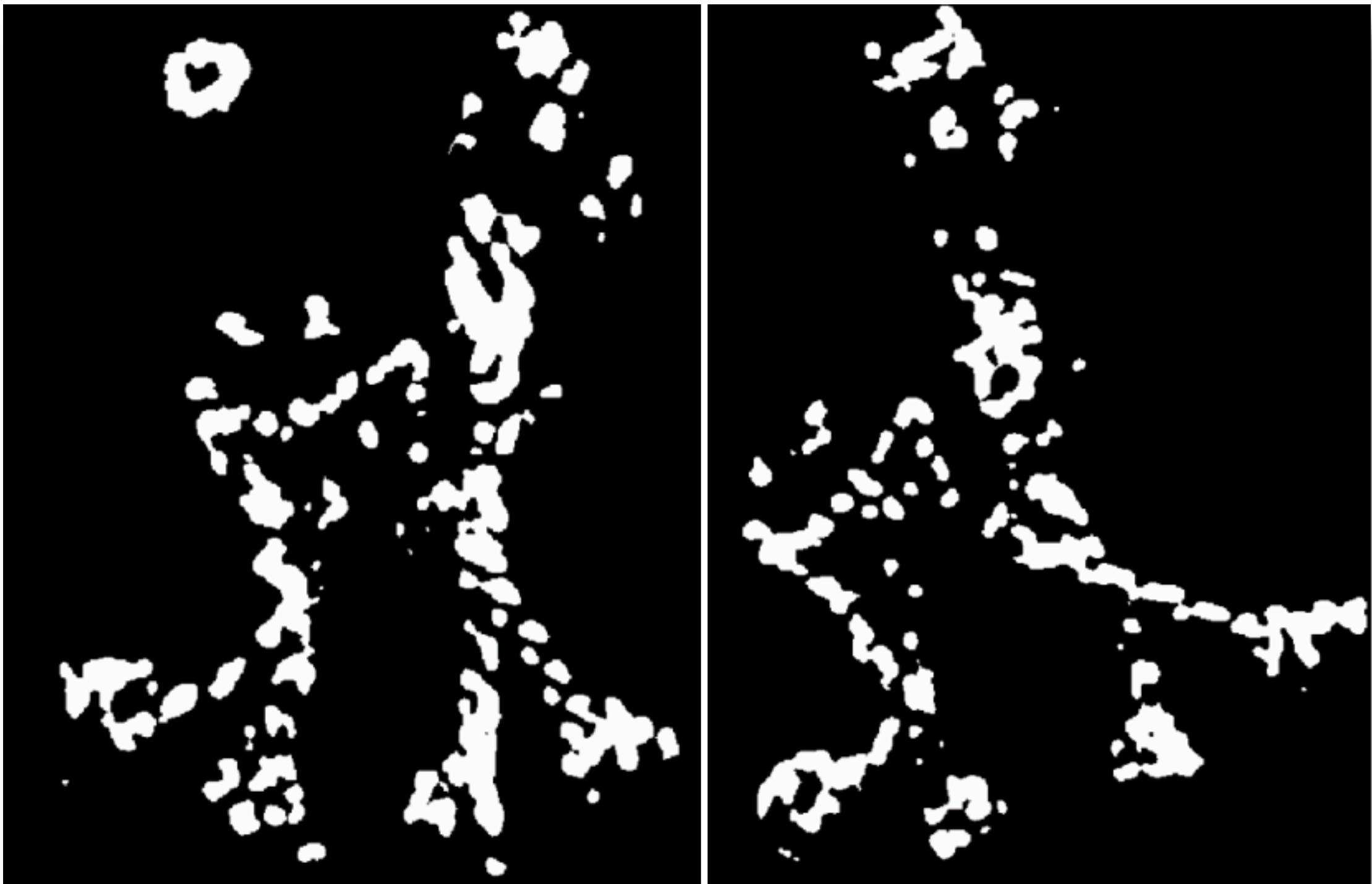


Slides adapted from Aaron Bobick, Alyosha Efros, etc.

# Harris Detector: Workflow

Find points  
with large  
corner response.

$R > \text{threshold}$



Slides adapted from Aaron Bobick, Alyosha Efros, etc.

# Harris Detector: Workflow

Take only the  
points of local  
maxima of  $R$



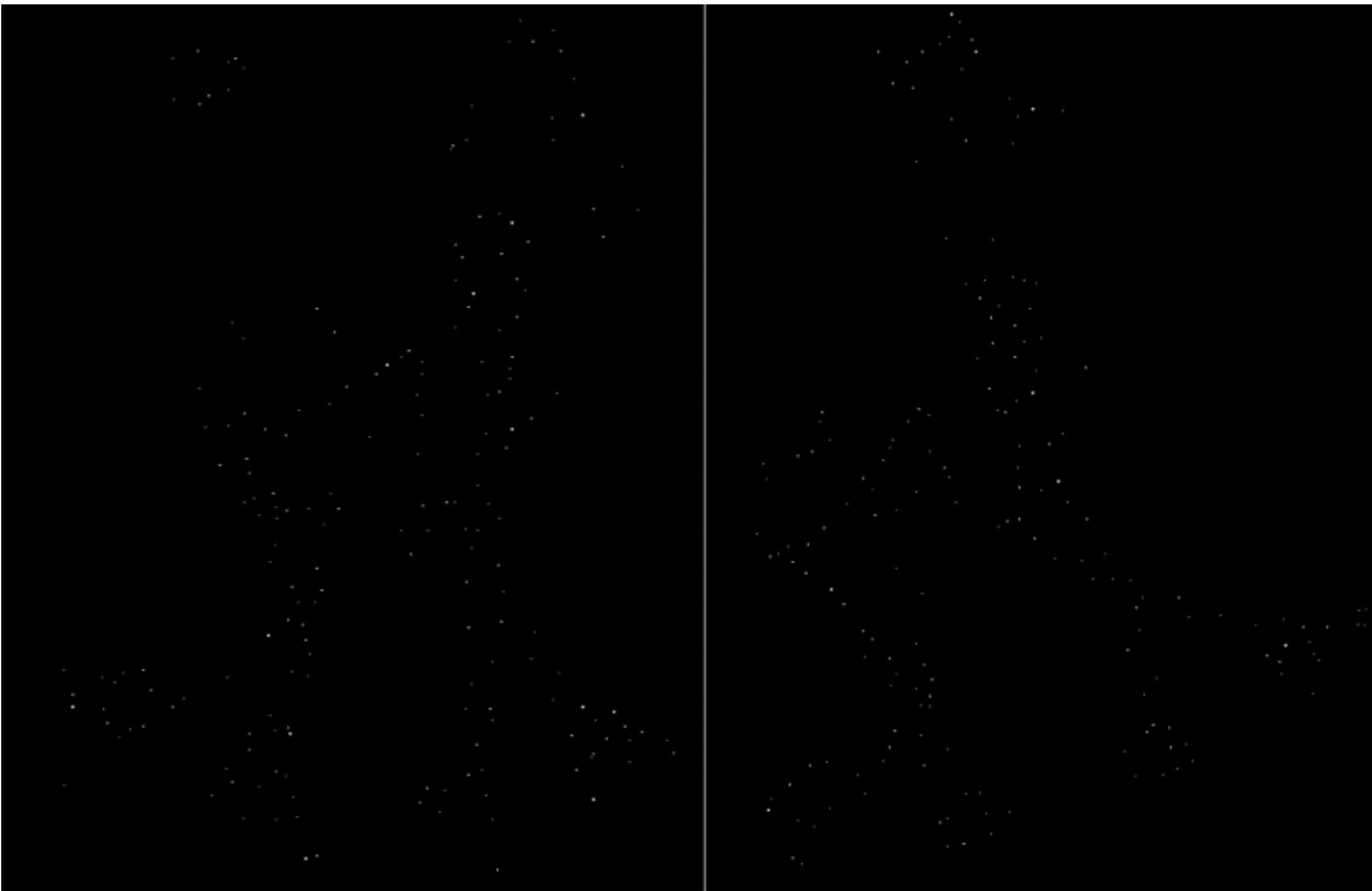
Slides adapted from Aaron Bobick, Alyosha Efros, etc.

# Harris Detector: Workflow

Take only the  
points of local  
maxima of  $R$



Slides adapted from Aaron Bobick, Alyosha Efros, etc.





# Harris Detector: Workflow

Output



# Harris Detector Algorithm (Preview)

- \* Compute Gaussian derivatives at each pixel
- \* Compute second moment matrix  $M$  in a Gaussian window around each pixel
- \* Compute corner response function  $R$
- \* Threshold  $R$
- \* Find local maxima of response function (non-maximum suppression)

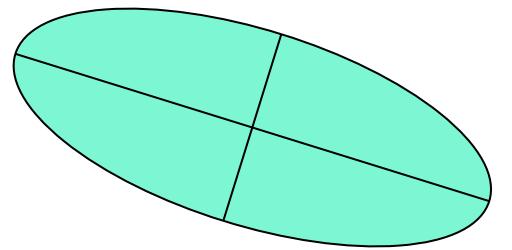
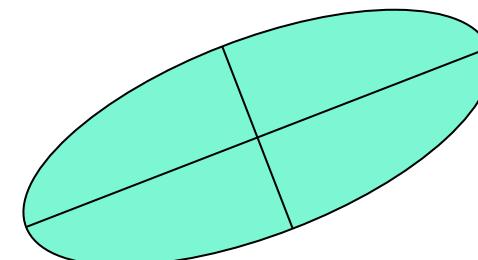
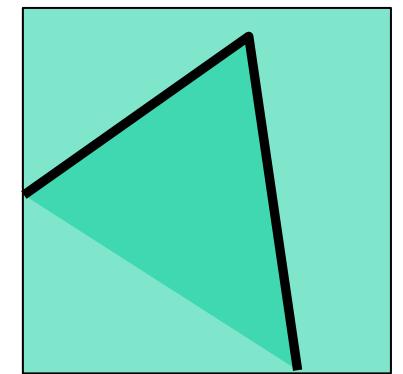
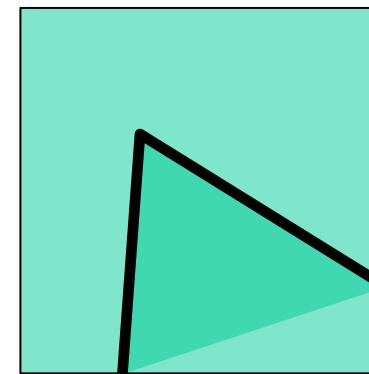


# Harris Detector: Some Properties

- \* Invariant to Rotation?
- \* Invariance to image intensity?
- \* Invariant to image scale?

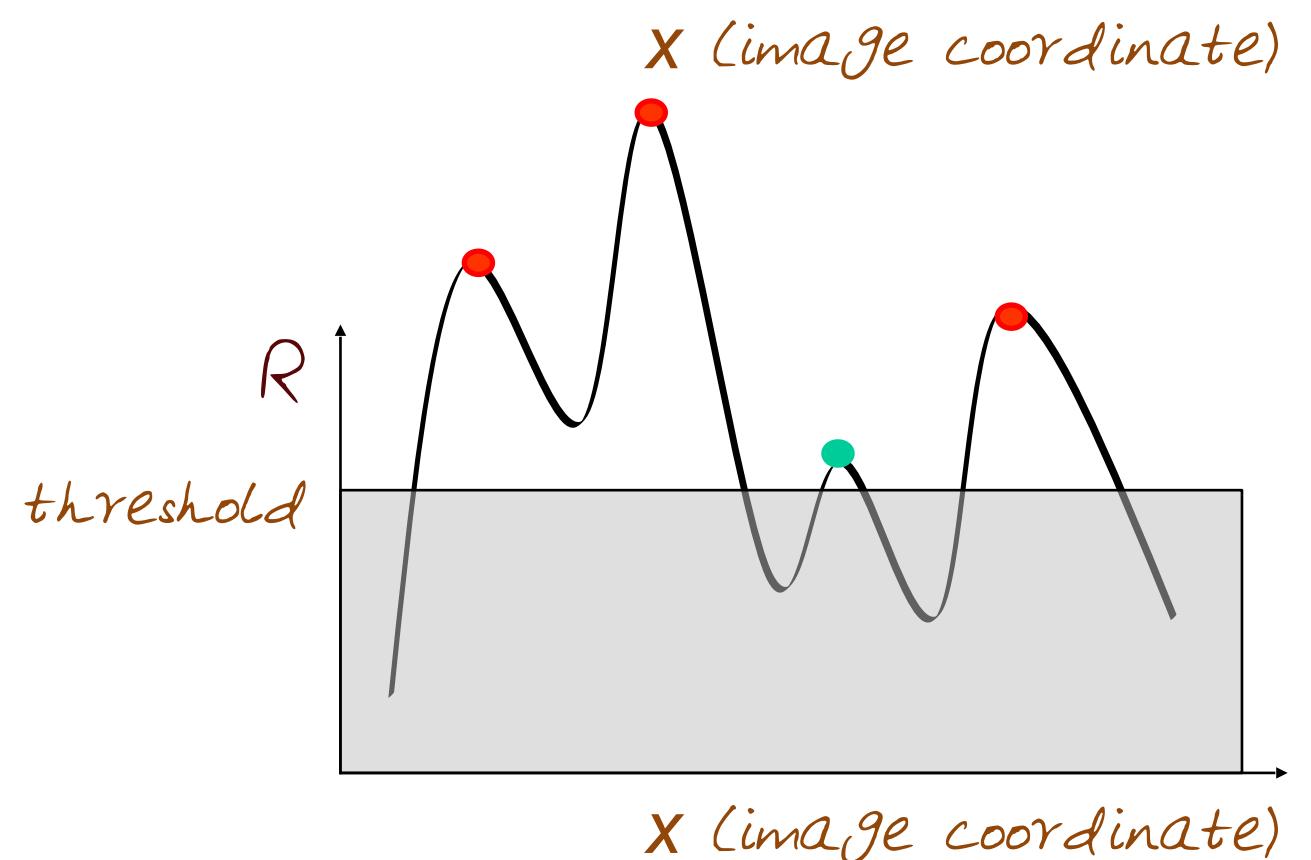
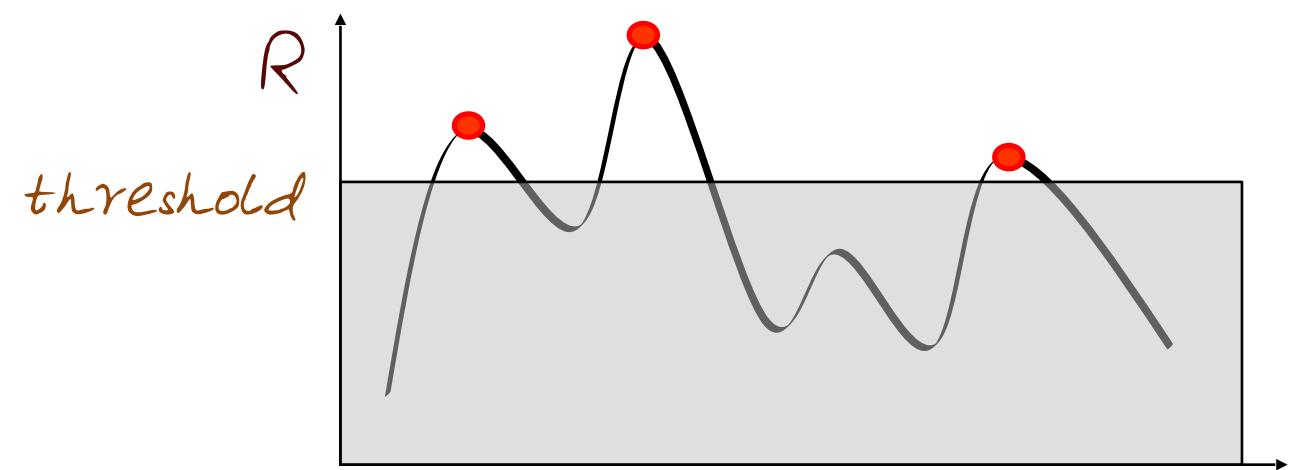
# Harris Detector: Some Properties

- \* Invariant to Rotation?
- \* Ellipse rotates but its shape (i.e. eigenvalues) remains the same
- \* Corner response  $R$  is invariant to image rotation



# Harris Detector: Some Properties

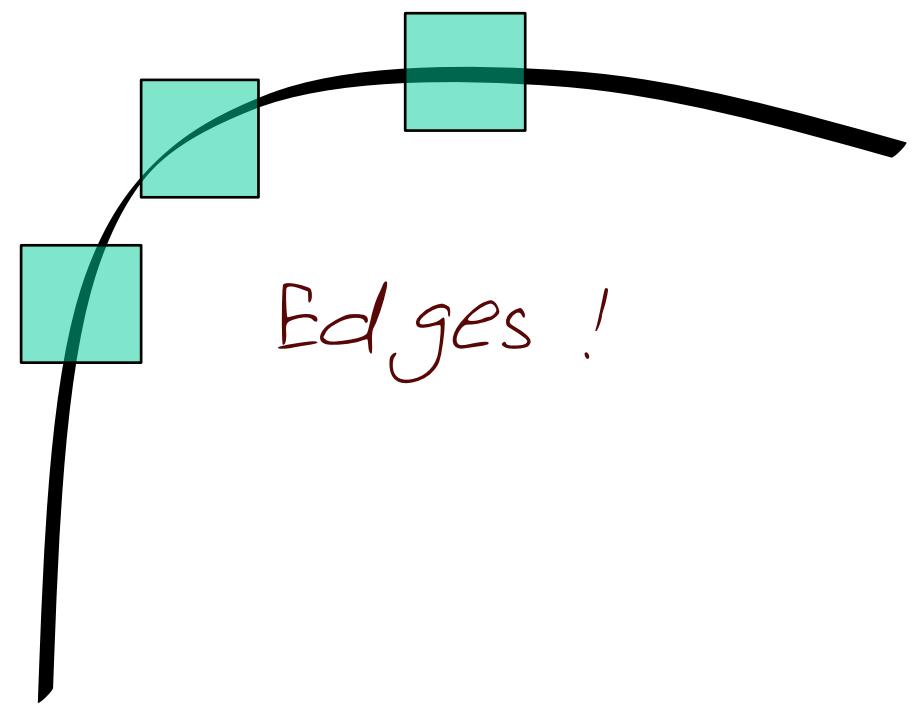
- \* Invariance to image intensity?
- \* Mostly invariant to additive and multiplicative intensity changes
- \* Only derivatives are used
- \* Invariance to intensity shift:
  - \*  $I \rightarrow I + b$
  - \* Intensity scale:
    - \*  $I \rightarrow a I$



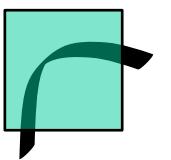
Slides adapted from Aaron Bobick

# Harris Detector: Some Properties

- \* Invariant to image scale?
- \* Not Invariant to image scale,
- \* But can we do something about this?



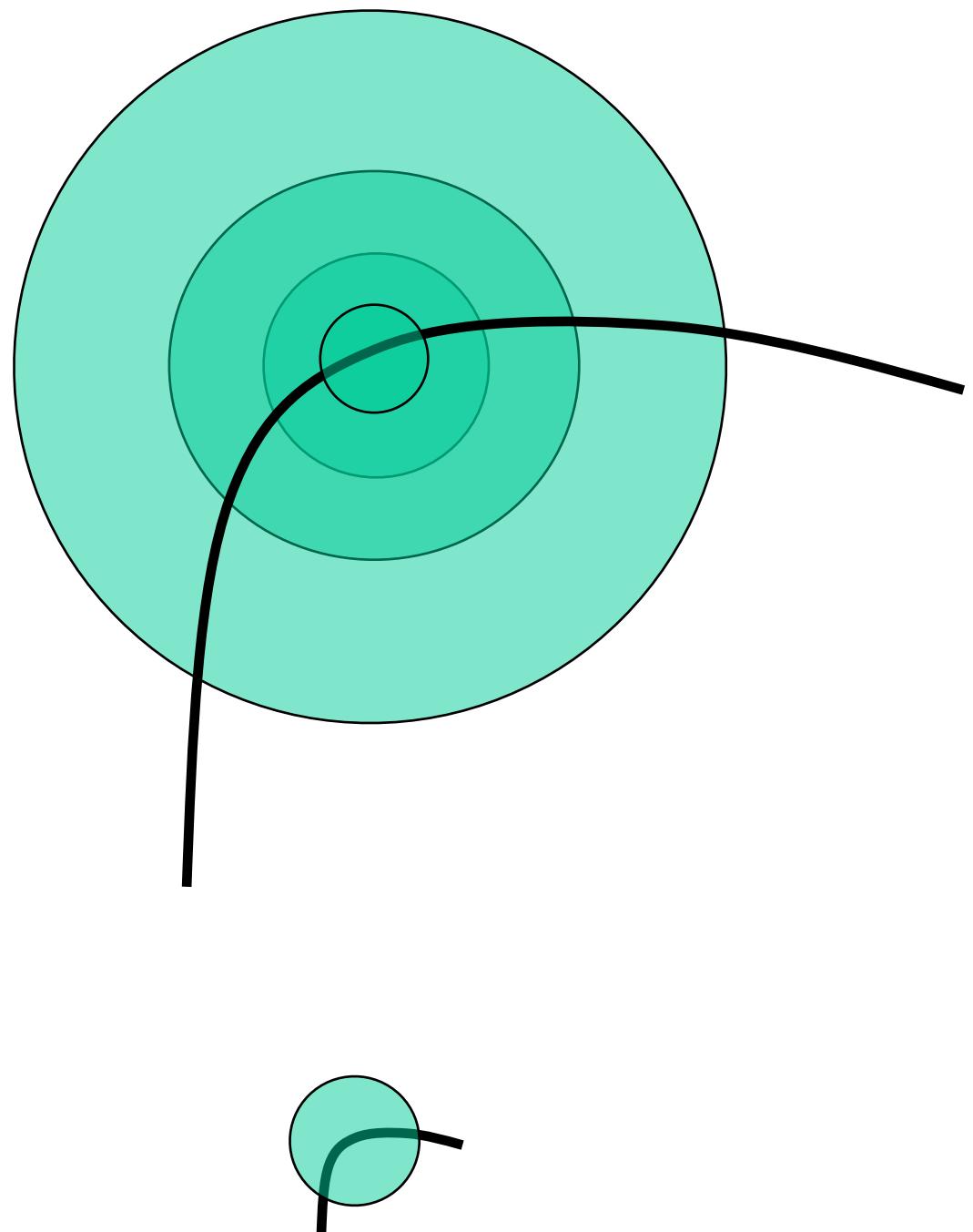
Edges !



Corner !

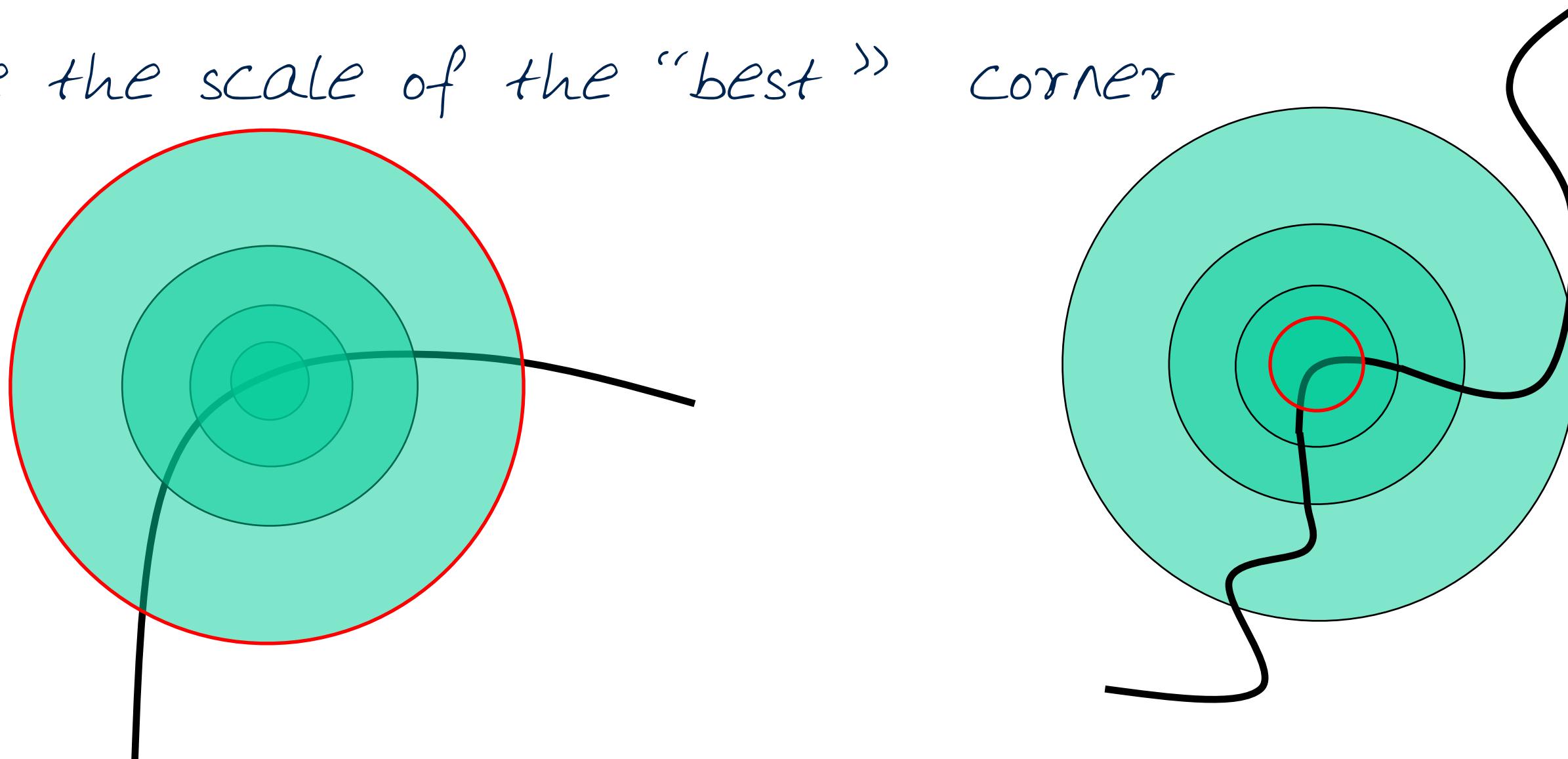
# Scale Invariant Detection

- \* Consider regions (e.g. circles) of different sizes around a point
- \* Regions of corresponding sizes will look the same in both images



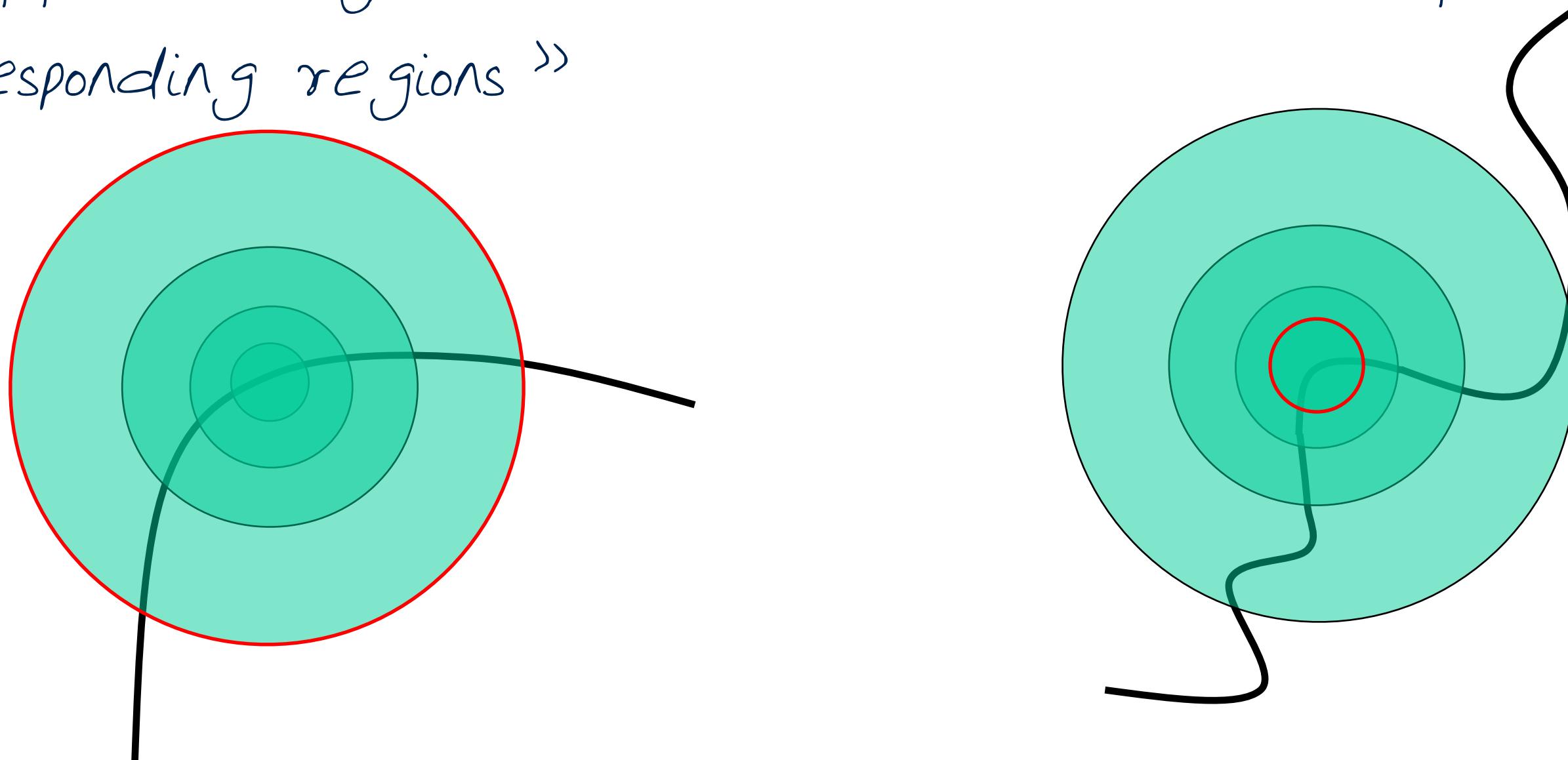
# Scale Invariant Detection

- \* The problem: how do we choose corresponding circles independently in each image?
- \* Choose the scale of the “best” corner



# Scale Invariant Detection

- \* A region (circle), which is "scale invariant"
- \* Not affected by the size but will be the same for "corresponding regions"

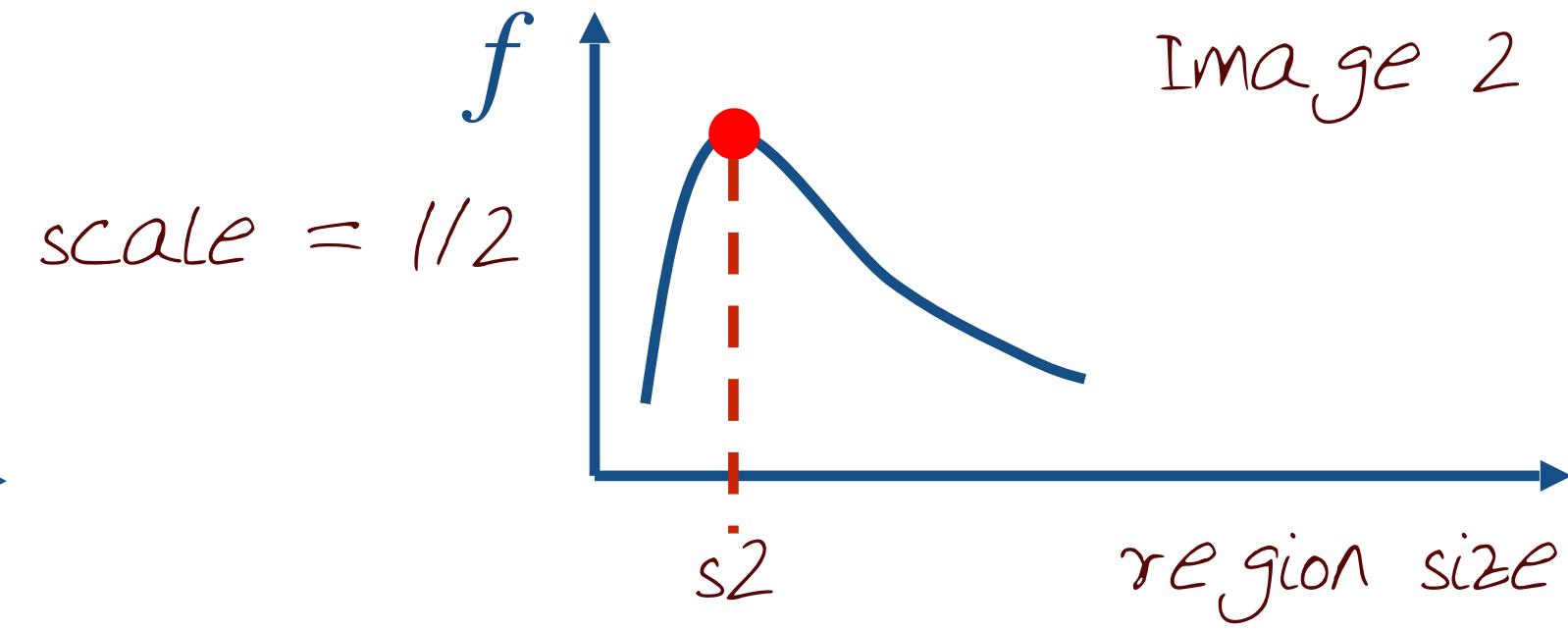
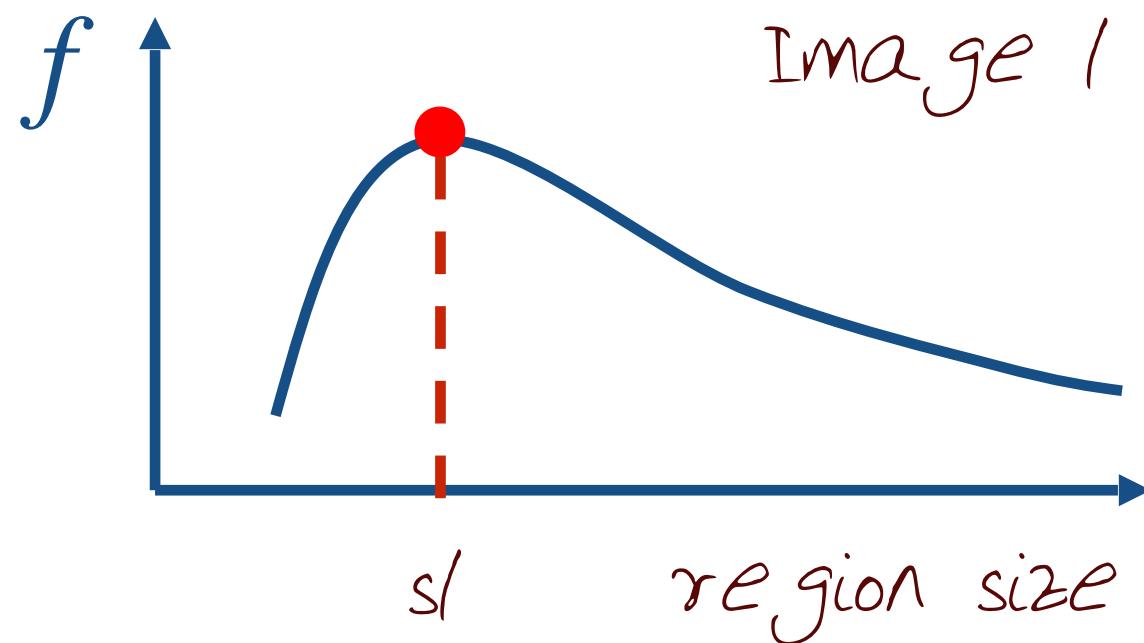


# Scale Invariant Detection

- \* A region (circle), which is "scale invariant"
- \* Not affected by the size but will be the same for "corresponding regions"
- \* Example: Average intensity. For corresponding regions (even of different sizes) it will be the same

# Scale Invariant Detection

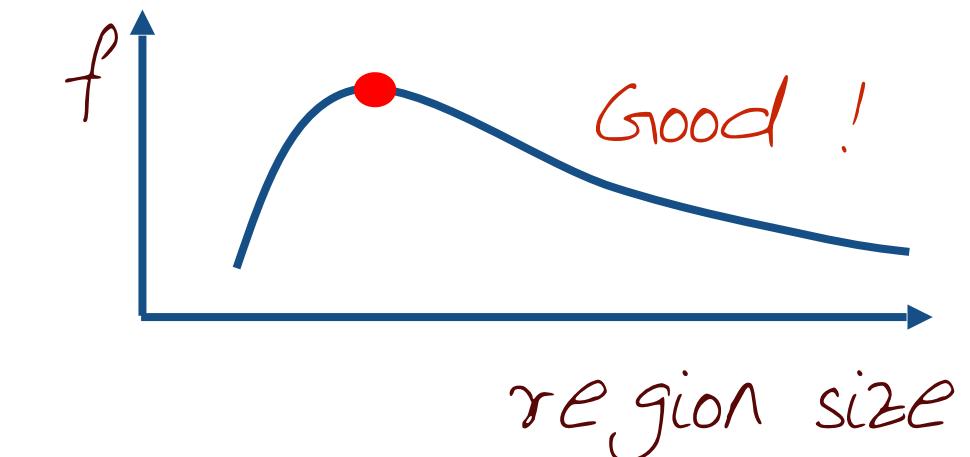
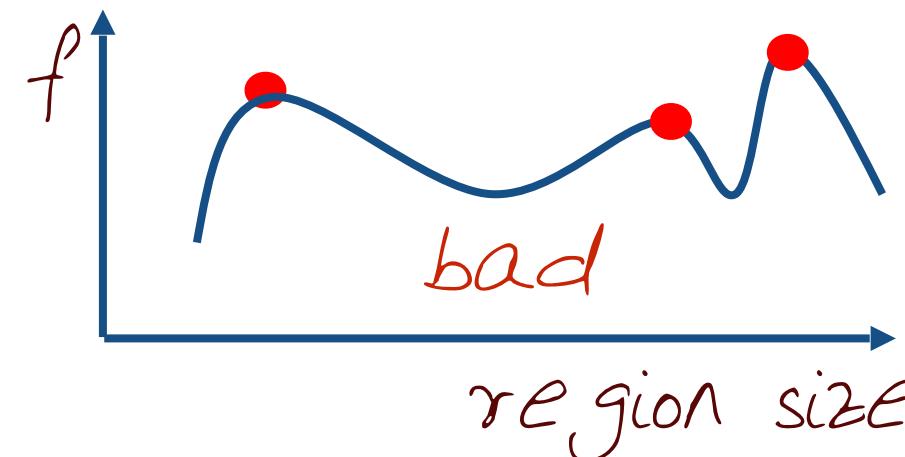
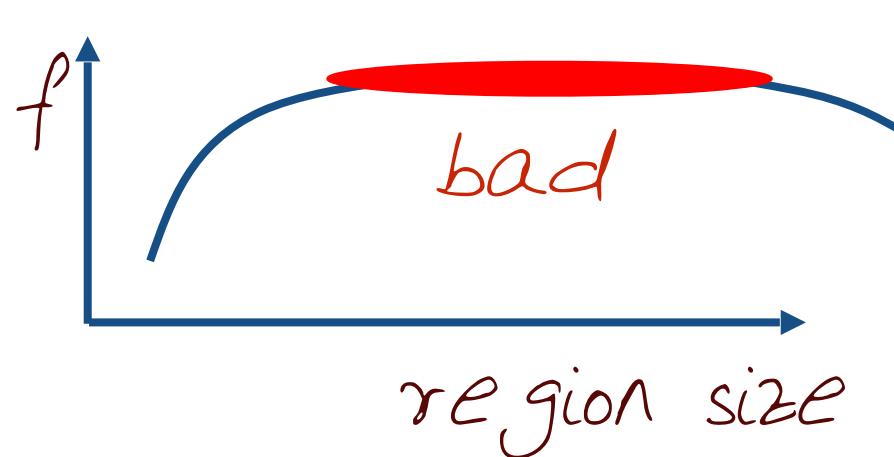
- \* At a point, compute the scale invariant function over different size neighborhoods (different scales)
- \* Choose the scale for each image at which the function is a maximum



Slides adapted from Aaron Bobick

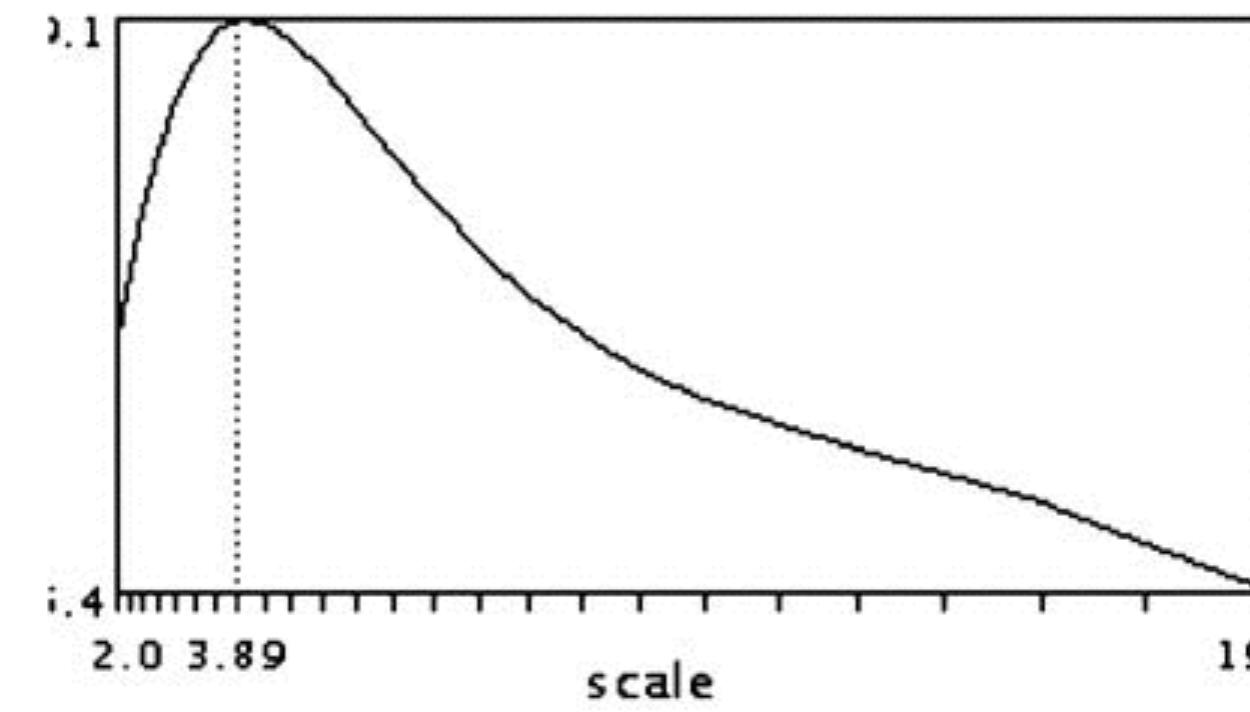
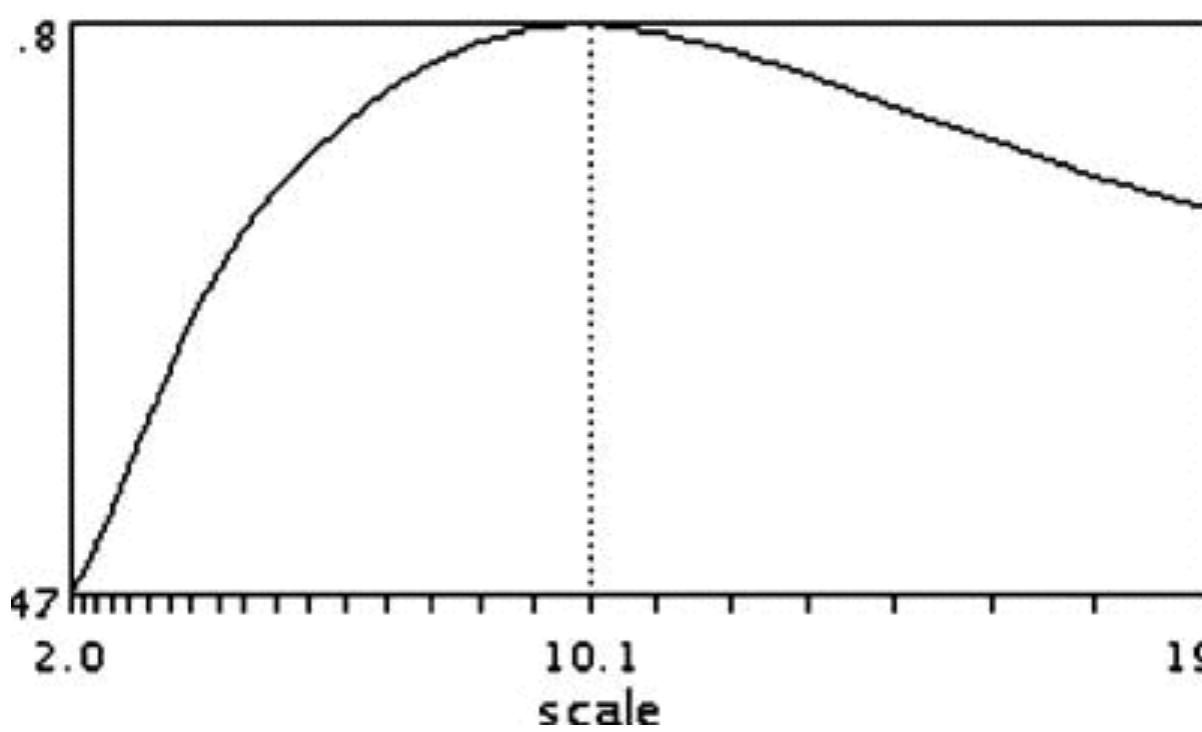
# Scale Invariant Detection

- \* A “good” function for scale detection has one stable sharp peak



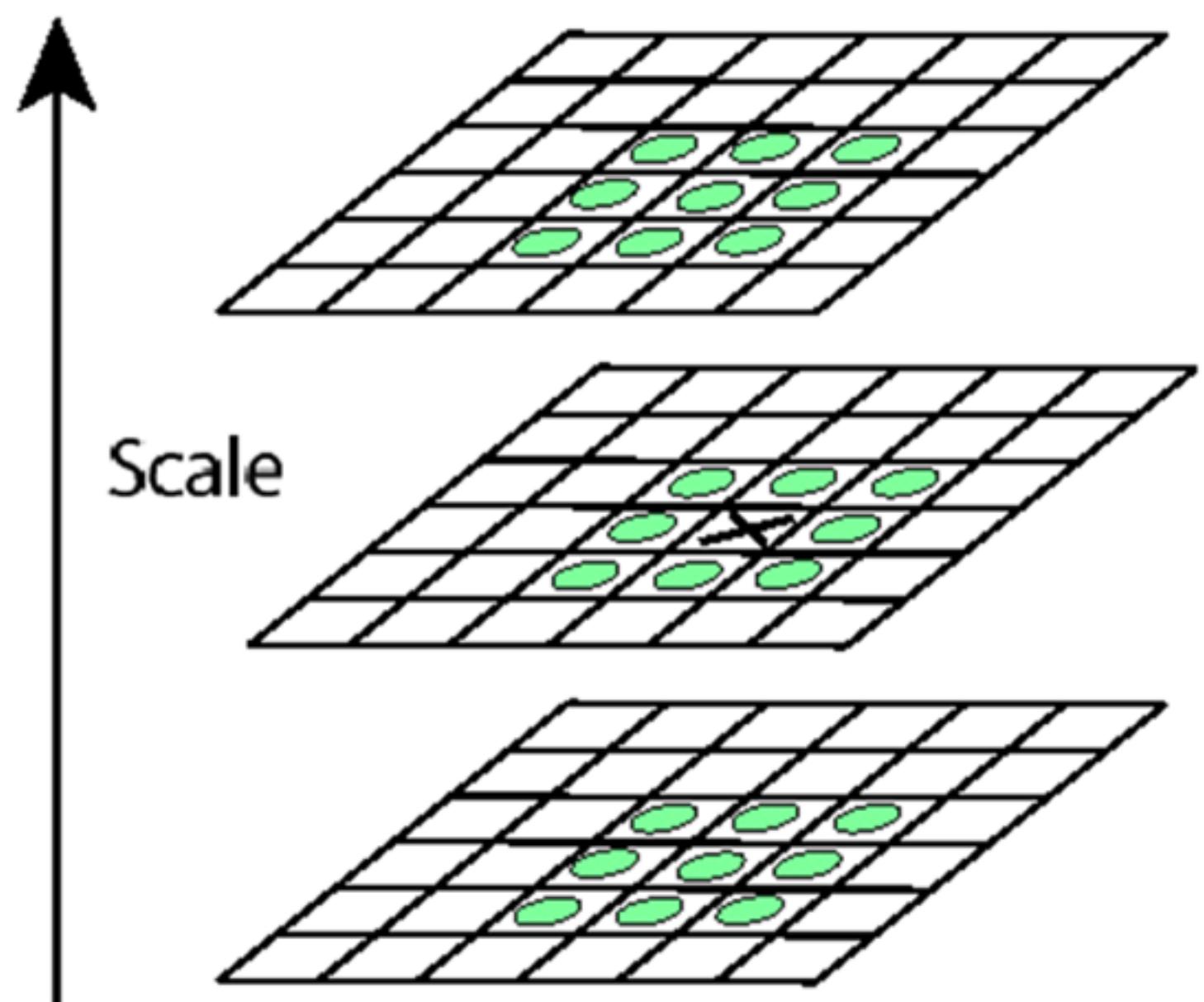
- \* For usual images, a good function would be one which responds to contrast (sharp local intensity change)

# Scale Sensitive Response



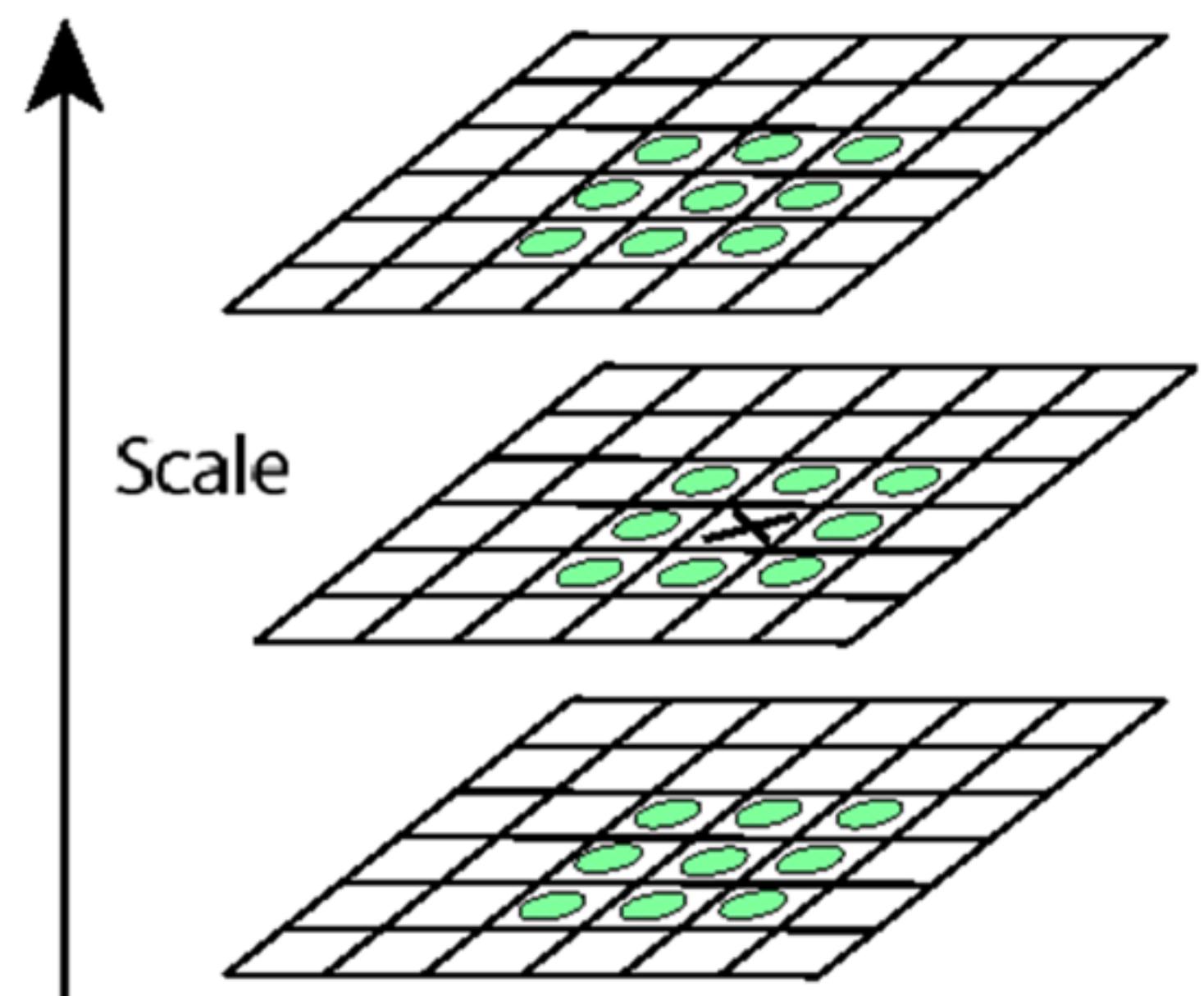
# Key Point Localization in SPACE

- \* Find robust extremum  
(maximum or minimum) both  
in space and in scale



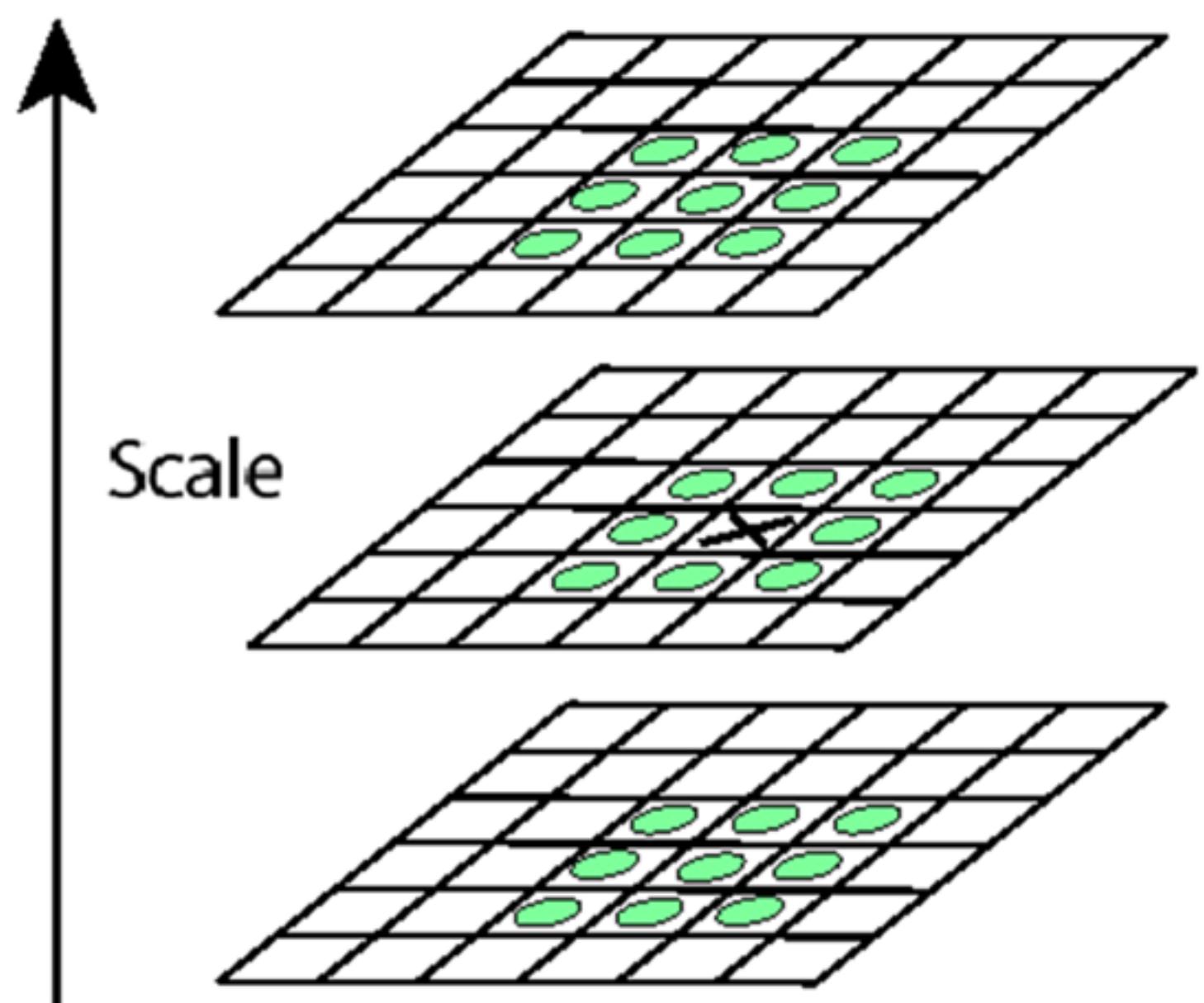
# Key Point Localization

- \* SIFT: Scale Invariant Feature Transform
- \* Specific suggestion: use pyramid to find maximum values (remember edge detection?) – then eliminate “edges” and pick only corners

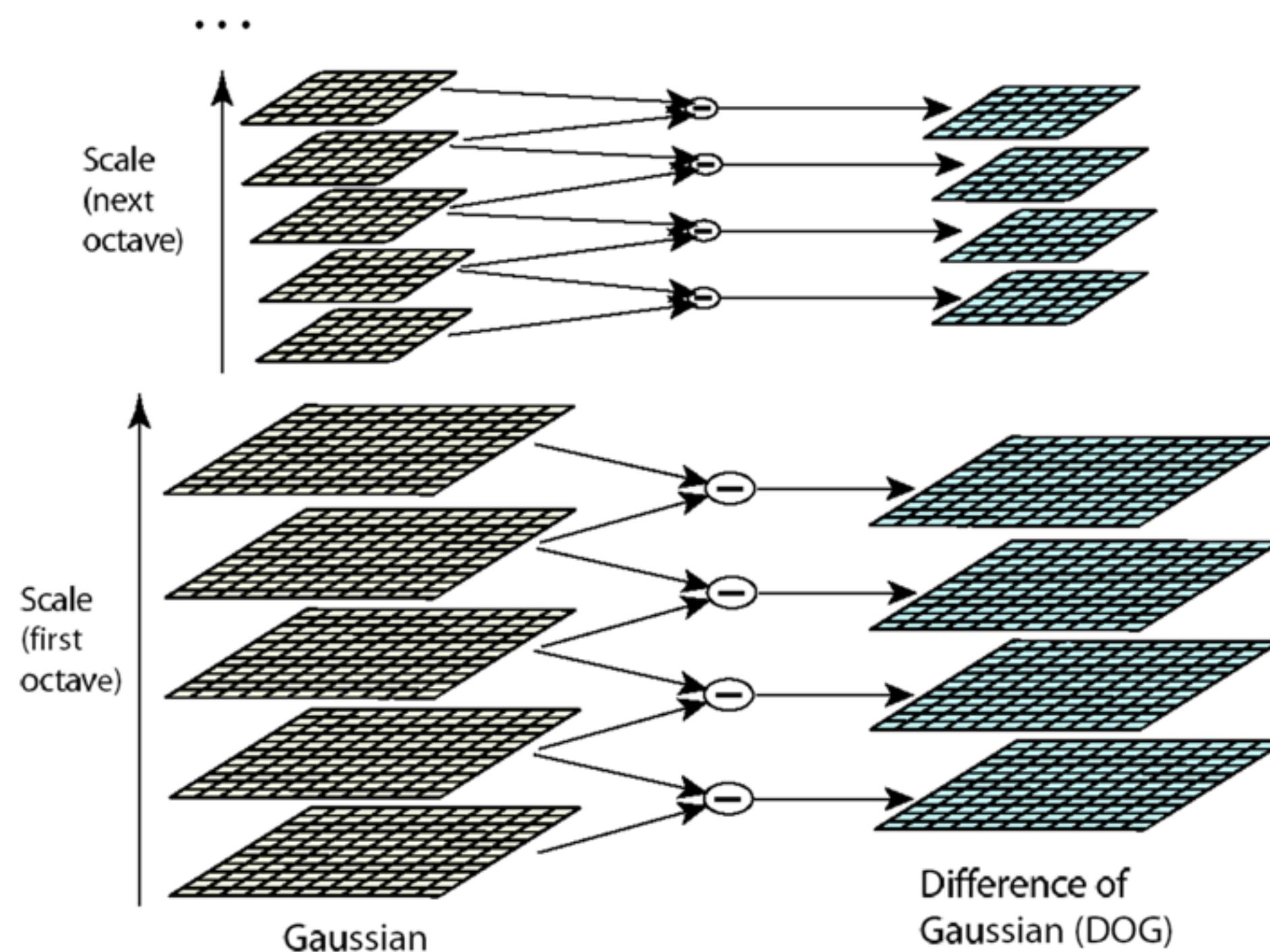


# Key Point Localization

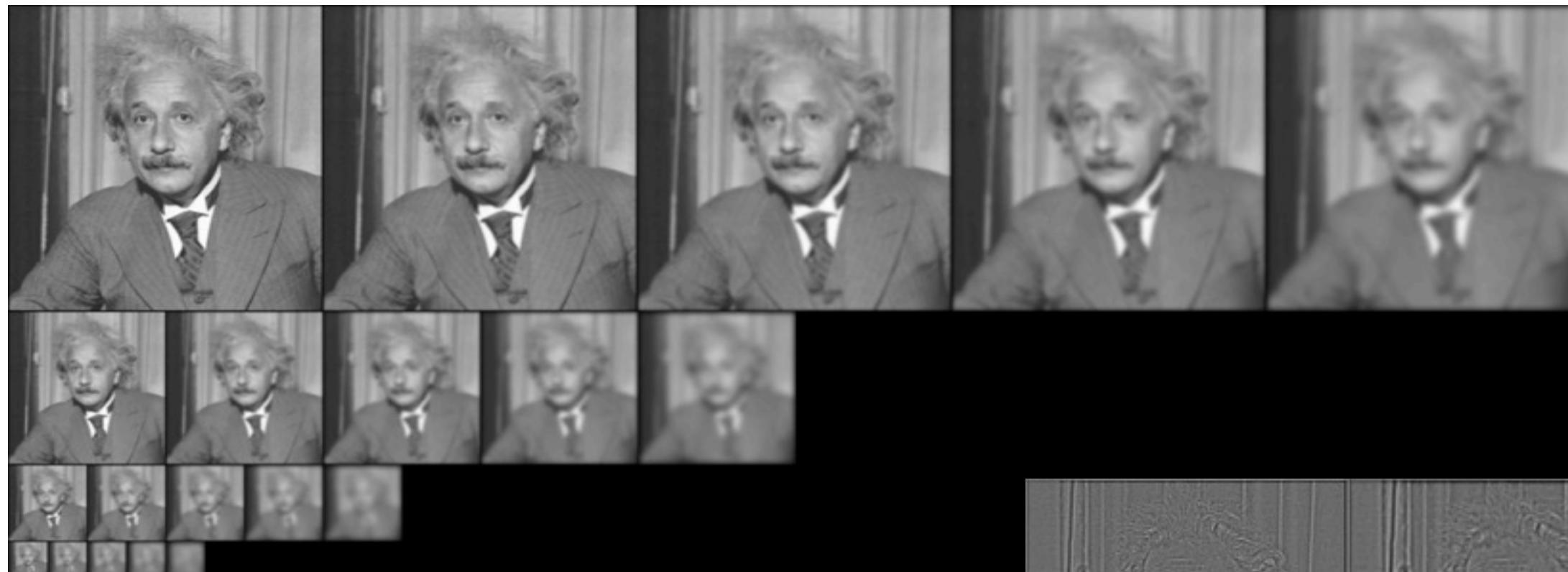
- \* Each point is compared to its 8 neighbors in the current image and 9 neighbors each in the scales above and below)



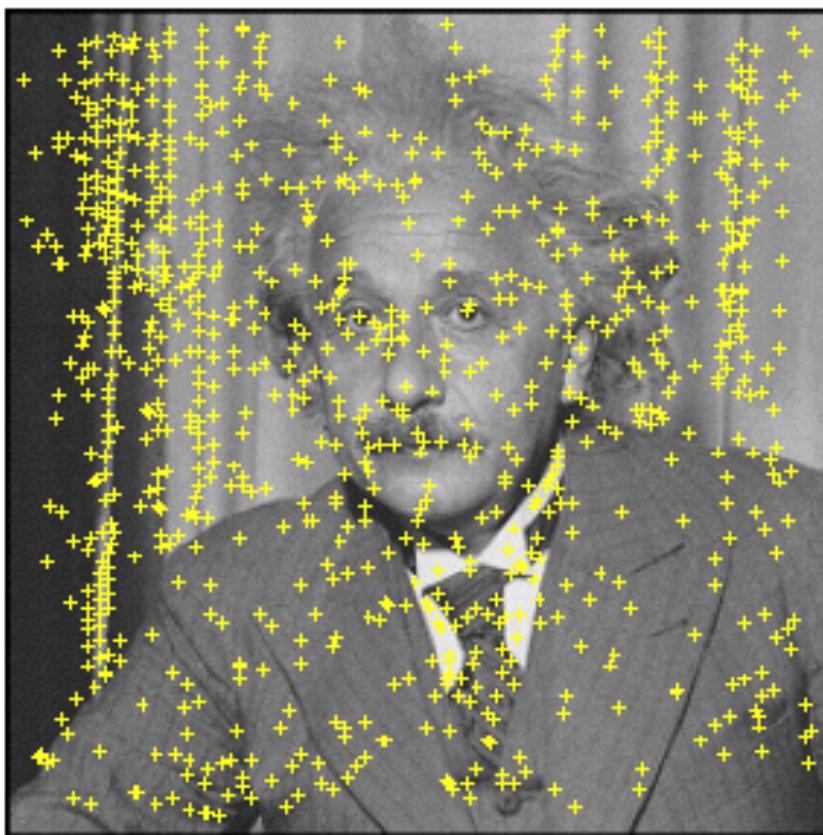
# Scale Space Processed One Octave at a Time



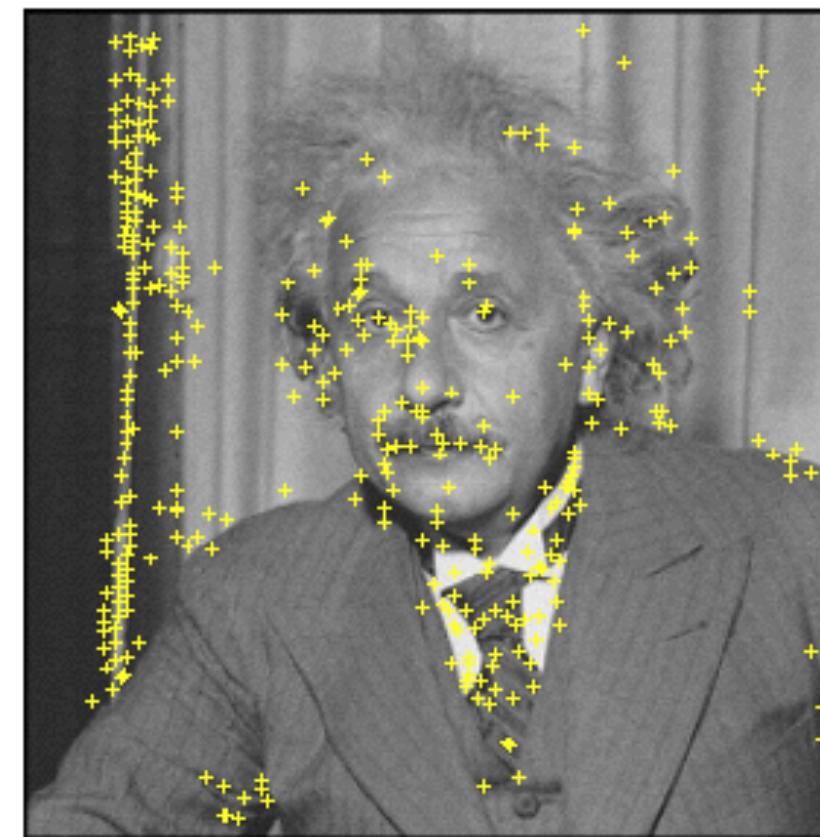
# Extrema at Different Scales



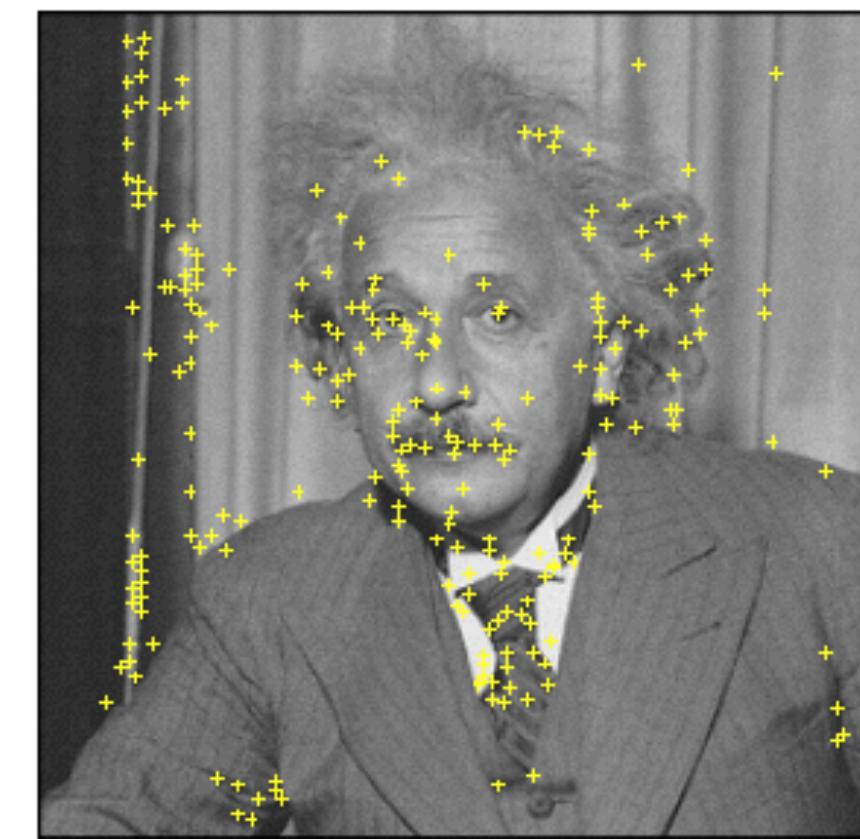
# Remove Low Contrast, Edge Bound



Extrema points



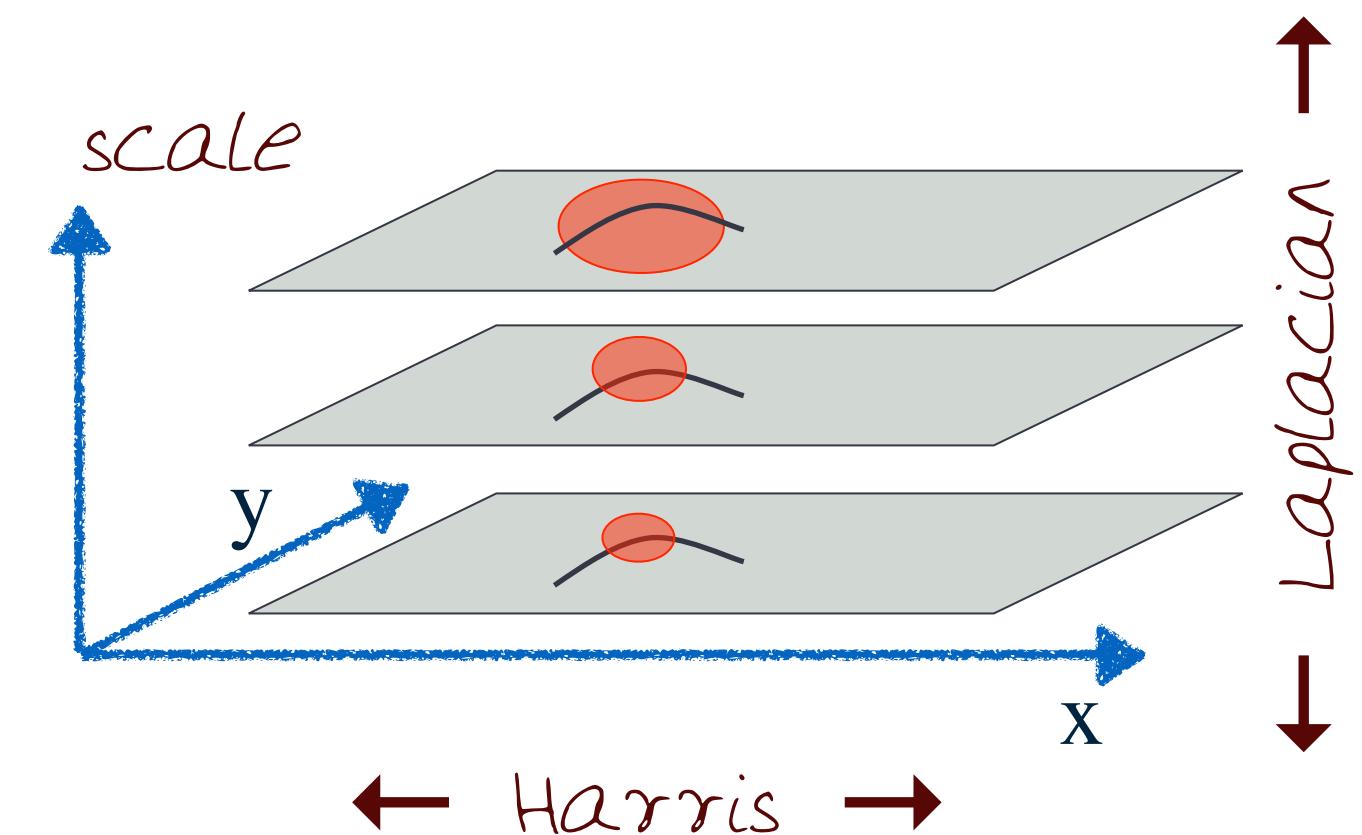
Contrast > C



Not on edge

# Scale Invariant Detectors

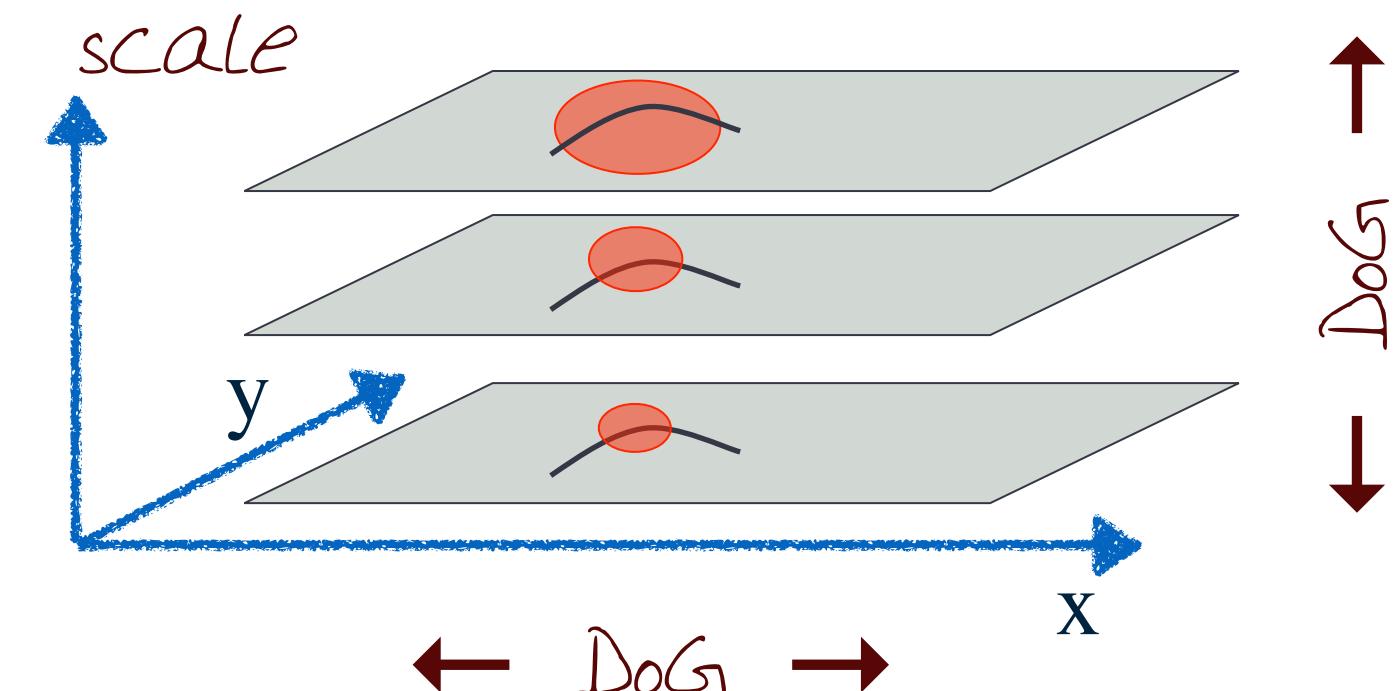
- \* Harris-Laplacian
- \* Find local maximum of:
  - \* Harris corner detector in space (image coordinates)
  - \* Laplacian in scale



(mikolajczyk and schmid, 2001)

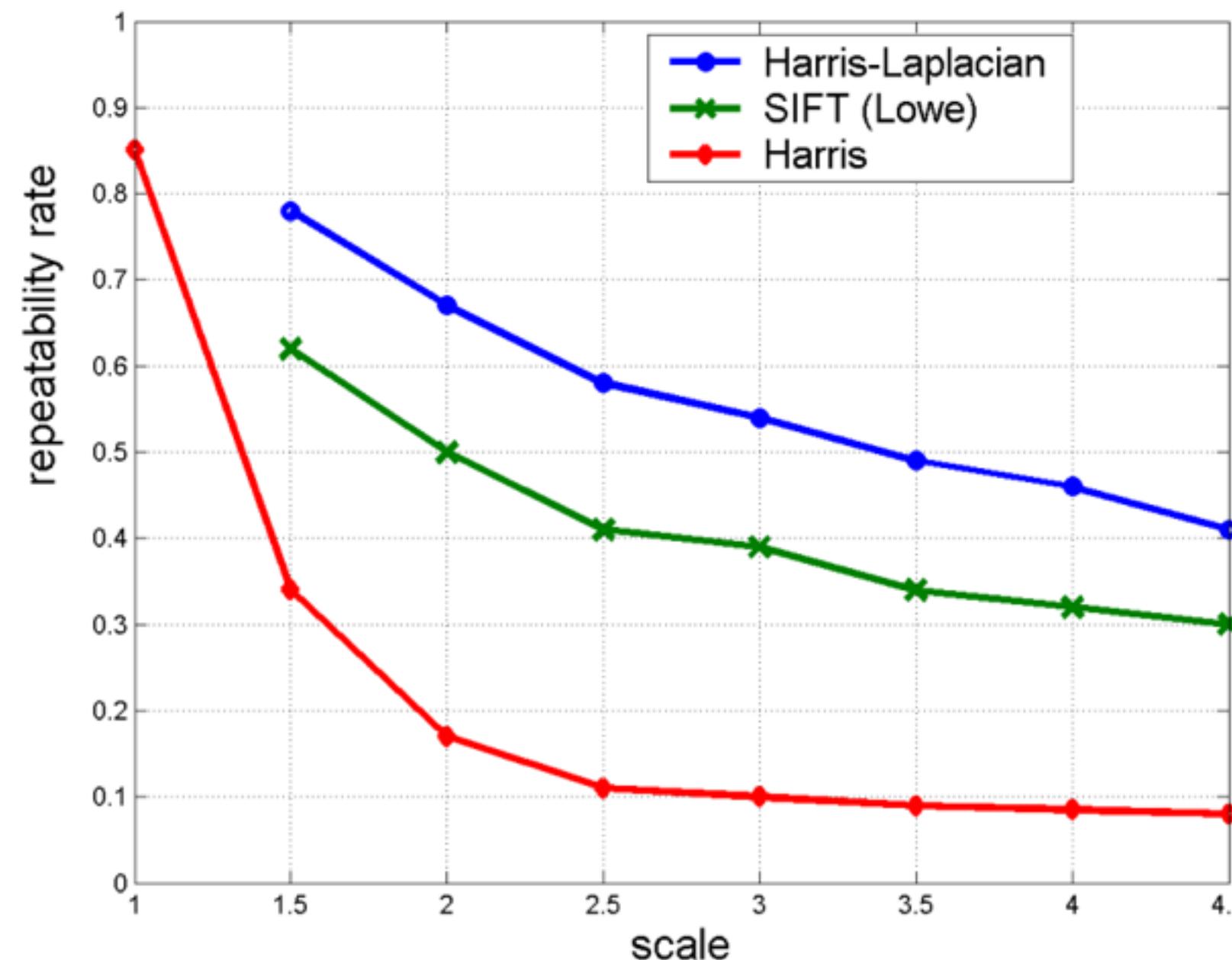
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Lowe 2004

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K. Mikolajczyk, C. Schmid (2001) "Indexing Based on Scale Invariant Interest Points" . ICCV 2001

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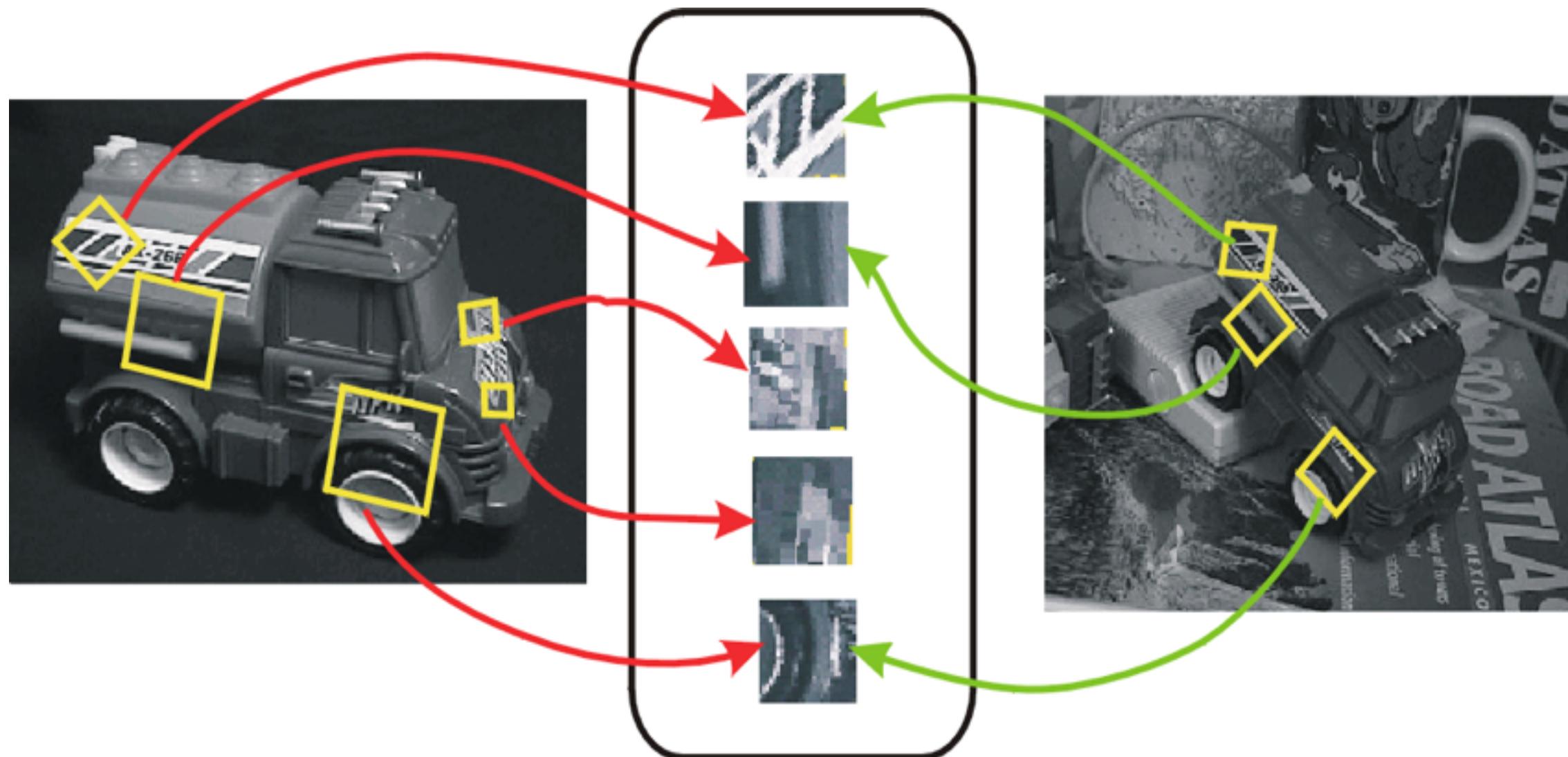
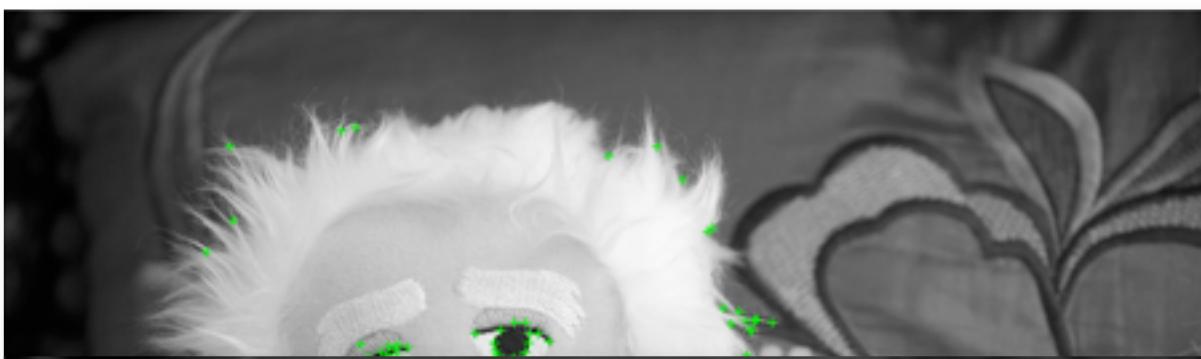


Image content is transformed into local feature coordinates that are invariant to translation, rotation, scale, and other imaging parameters

Lowe 2004

# Results

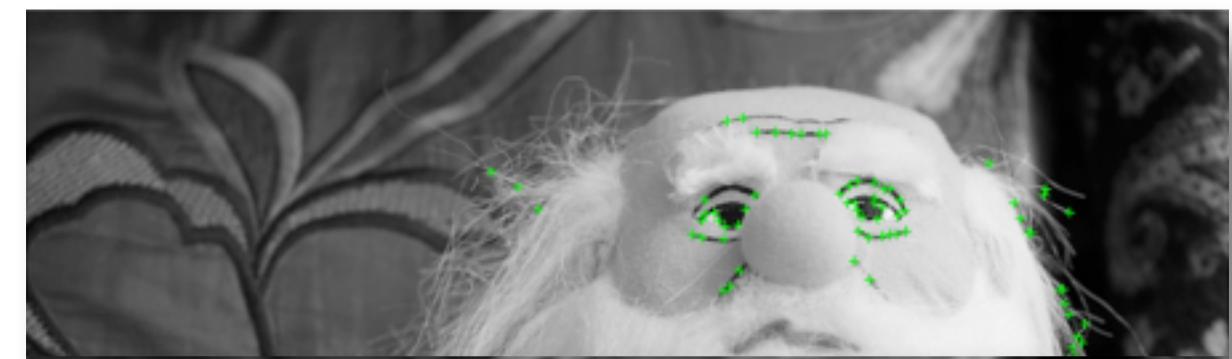
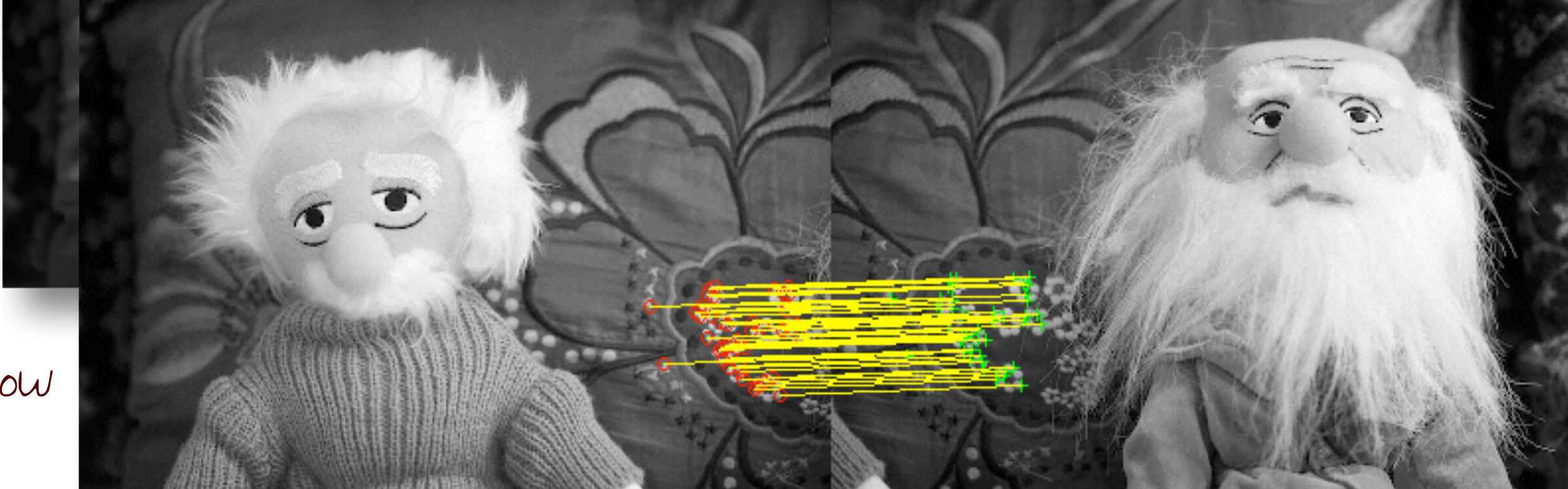
Detect



match



Show



# Summary



- \* Discussed invariants  
for feature  
detection
- \* Harris Corner  
Detector Framework
- \* SIFT detector

# Further Reading



- \* Harris and Stephens (1988) "A Combined Corner and Edge Detector." Proceedings of the 4th Alvey Vision Conference, 1988,
- \* Mikolajczyk and Schmid (2001). "Indexing Based on Scale Invariant Interest Points". ICCV 2001
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# Neat Class

- \* Moving on to more advanced topics



# Credits



- \* For more information, see:
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