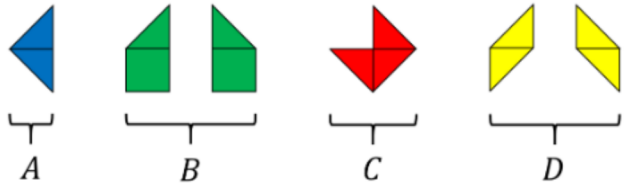
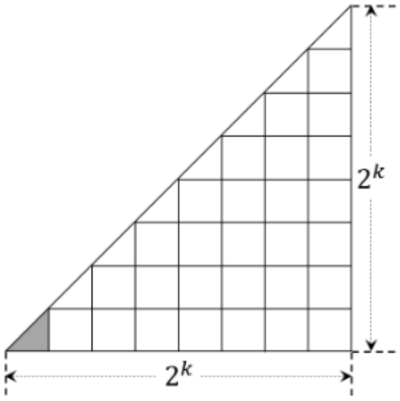
Homework #1

(Due: Mar 5)



(a) Four types of tiles



(b) A right triangular grid to tile ($k = 3$)

Figure 1: Tiling a right triangular grid.

Task 1. [70 Points] Tiling a Triangular Grid

Given an isosceles right triangular grid for some $k \geq 2$ as shown in Figure 1(b), this problem asks you to completely cover it using the tiles given in Figure 1(a). The bottom-left corner of the grid must not be covered. No two tiles can overlap and all tiles must remain completely inside the given triangular grid. You must use all four types of tiles shown in Figure 1(a), and no tile type can be used to cover more than 40% of the total grid area. You are allowed to rotate the tiles, as needed, before putting them on the grid.

(a) [**25 Points**] Design and explain a recursive divide-and-conquer algorithm for tiling the grid under the constraints given above. Include pseudocode.

(b) [**25 Points**] Write down recurrences describing the running time of your algorithm from part (a), and solve them.

(c) [**20 Points**] Write down recurrences for counting the number of tiles of each type used by your algorithm, and solve them to show that no tile type covers more than 40% of the total grid area.