

called **maximum-flow problems**. Several specialized algorithms exist to solve maximum-flow problems. In this section, we begin by showing how linear programming can be used to solve maximum-flow problem. Then we discuss the Ford-Fulkerson (1962) method for solving maximum-flow problems.

LP SOLUTION of Maximum-Flow Problems

EXAMPLE 3 Maximum Flow

Sunco Oil wants to ship the maximum possible amount of oil (per hour) via pipeline from node *so* to node *si* in Figure 6. On its way from node *so* to node *si*, oil must pass through some or all of stations 1, 2 and 3. The various arcs represent pipelines of different diameters. The maximum number of oil (millions of barrels per hour) that can be pumped through each arc is shown in Table 8. Each number is called an **arc capacity**. Formulate an LP that can be used to determine the maximum number of barrels of oil per hour that can be sent from *so* to node *si*.

Solution

Node *so* is called the *source* node because oil flows out of it but no oil flows into it. Analogously, node *si* is called the *sink* node because oil flows into it and no oil flows out of it. For reasons that will soon become clear, we have added an artificial arc a_0 from the

To formulate an LP that will yield the maximum flow from node *so* to *si*, we observe that Sunco must determine how much oil (per hour) should be sent through arc (i,j) . Thus, we define

x_{ij} = millions of barrels of oil per hour that will pass through arc (i,j) of pipeline

As an example of possible flow (termed a *feasible flow*), consider the flow indicated by the numbers in parentheses in Figure 6.

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$$x_{so,1} = 2, \quad x_{13} = 0, \quad x_{12} = 2, \quad x_{3,si} = 2, \quad x_{si,so} = 2, \quad x_{so,2} = 0$$

FIGURE 6

Network for Sunco Oil

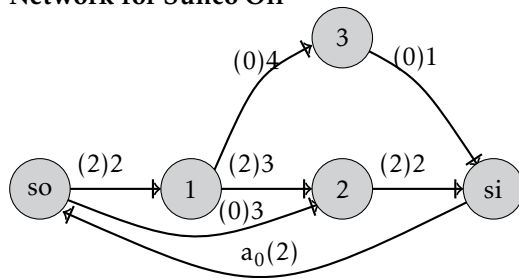


Table 8

Arc Capacities for Sunco Oil	Arc Capacity height	
	Arc	Capacity
	(so,1)	2
	(so,2)	3
	(1,2)	3
	(1,3)	4
	(3,si)	1
	(2,si)	2