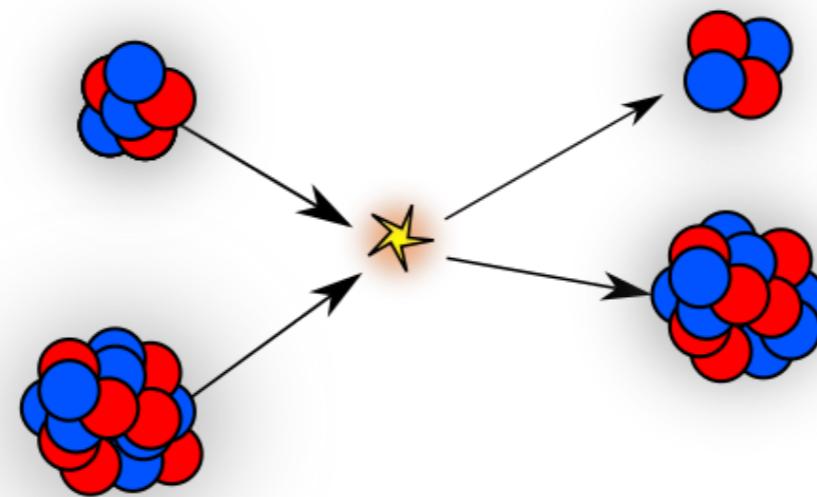


Inclusive breakup of $^{209}\text{Bi}(6\text{Li}, \alpha X)$ and related topics



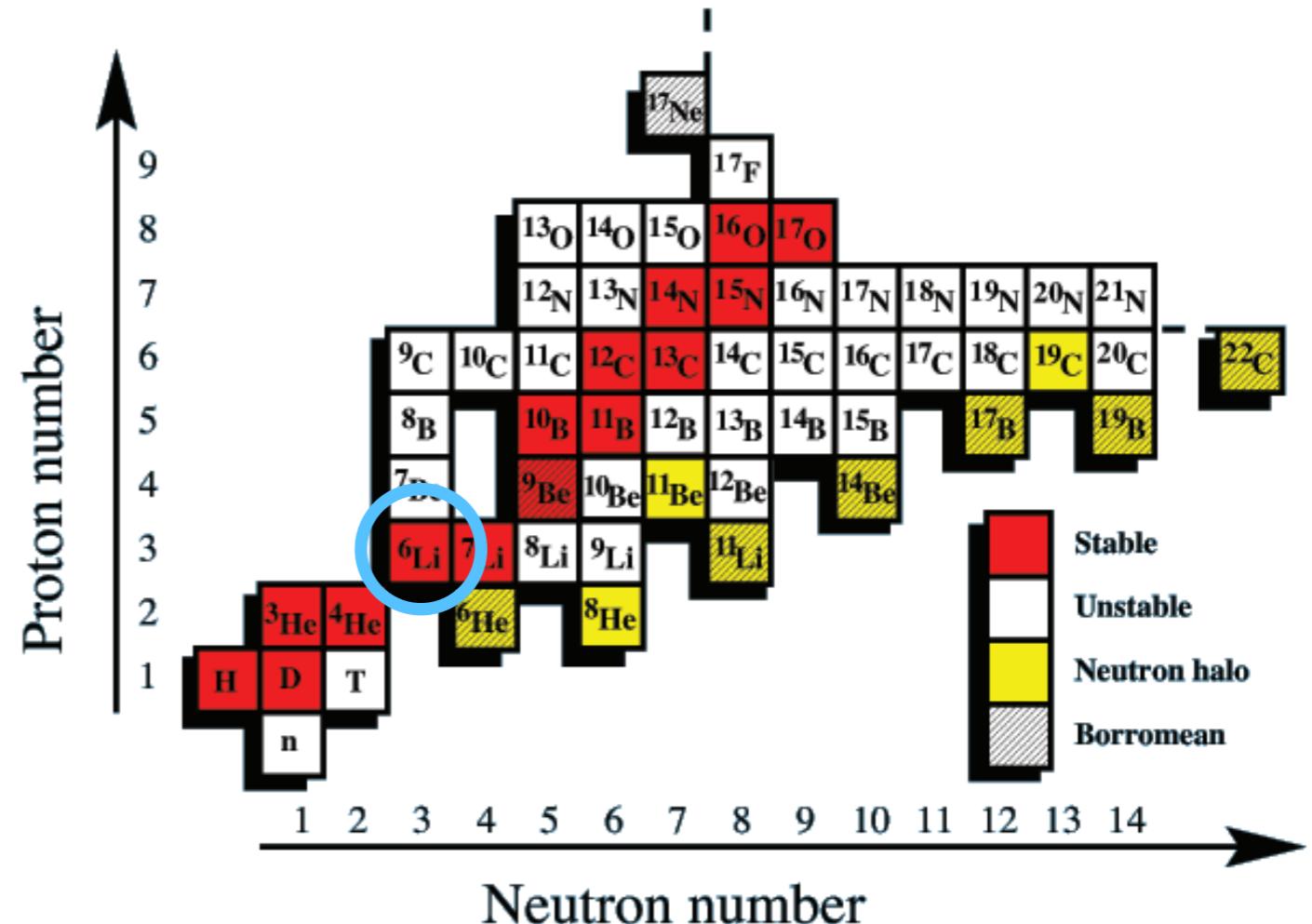
Jin Lei and Antonio M. Moro



Why 6Li

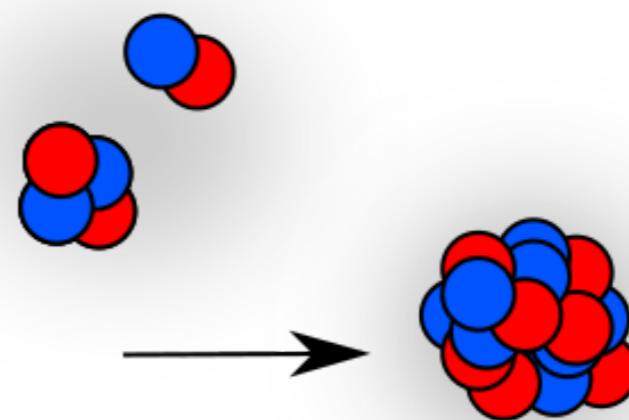
Experiment

- ${}^6\text{Li}$ is stable: high beam intensity
 - High accuracy data exist for elastic scattering, breakup, fusion, and incomplete fusion



Theory

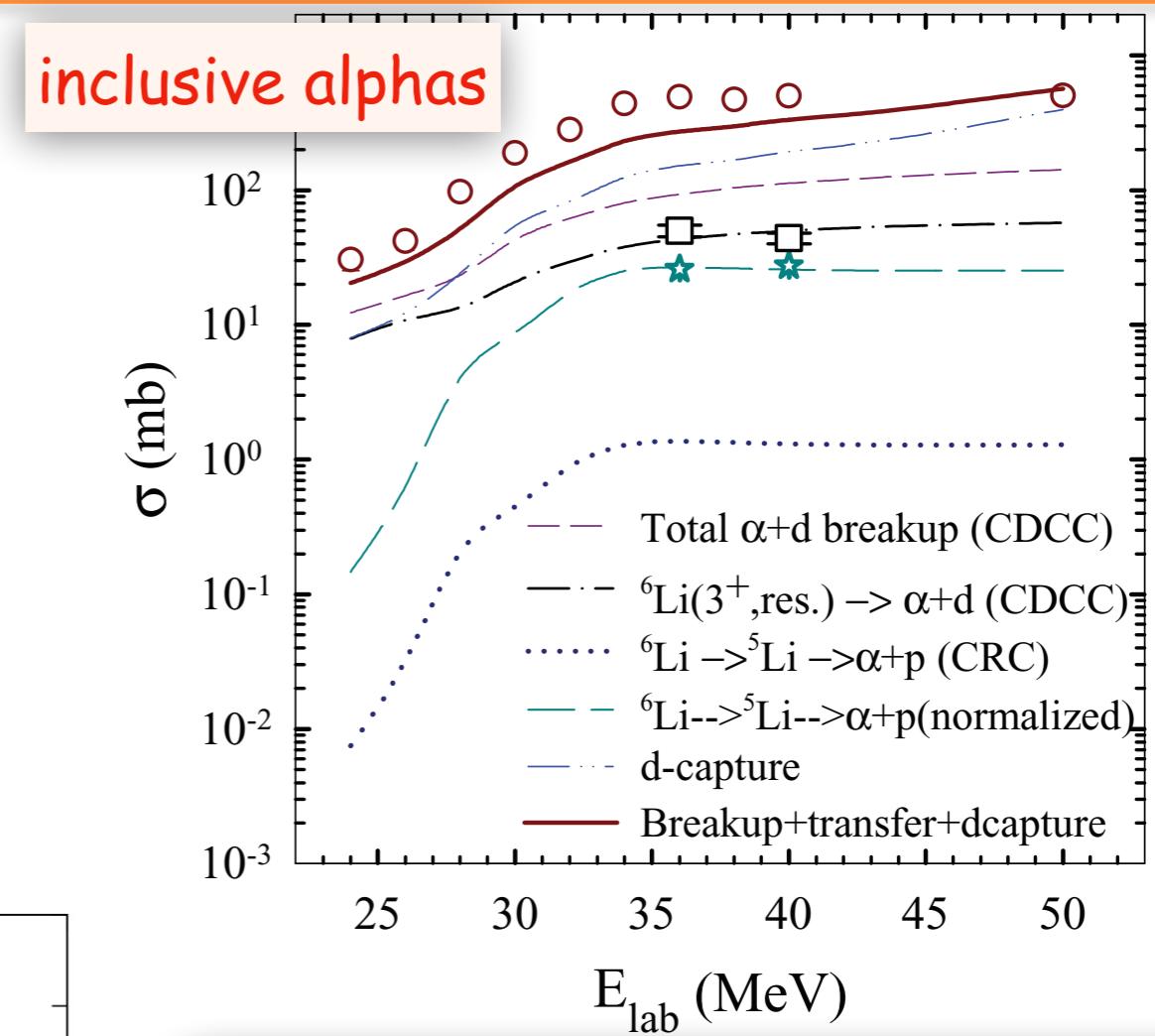
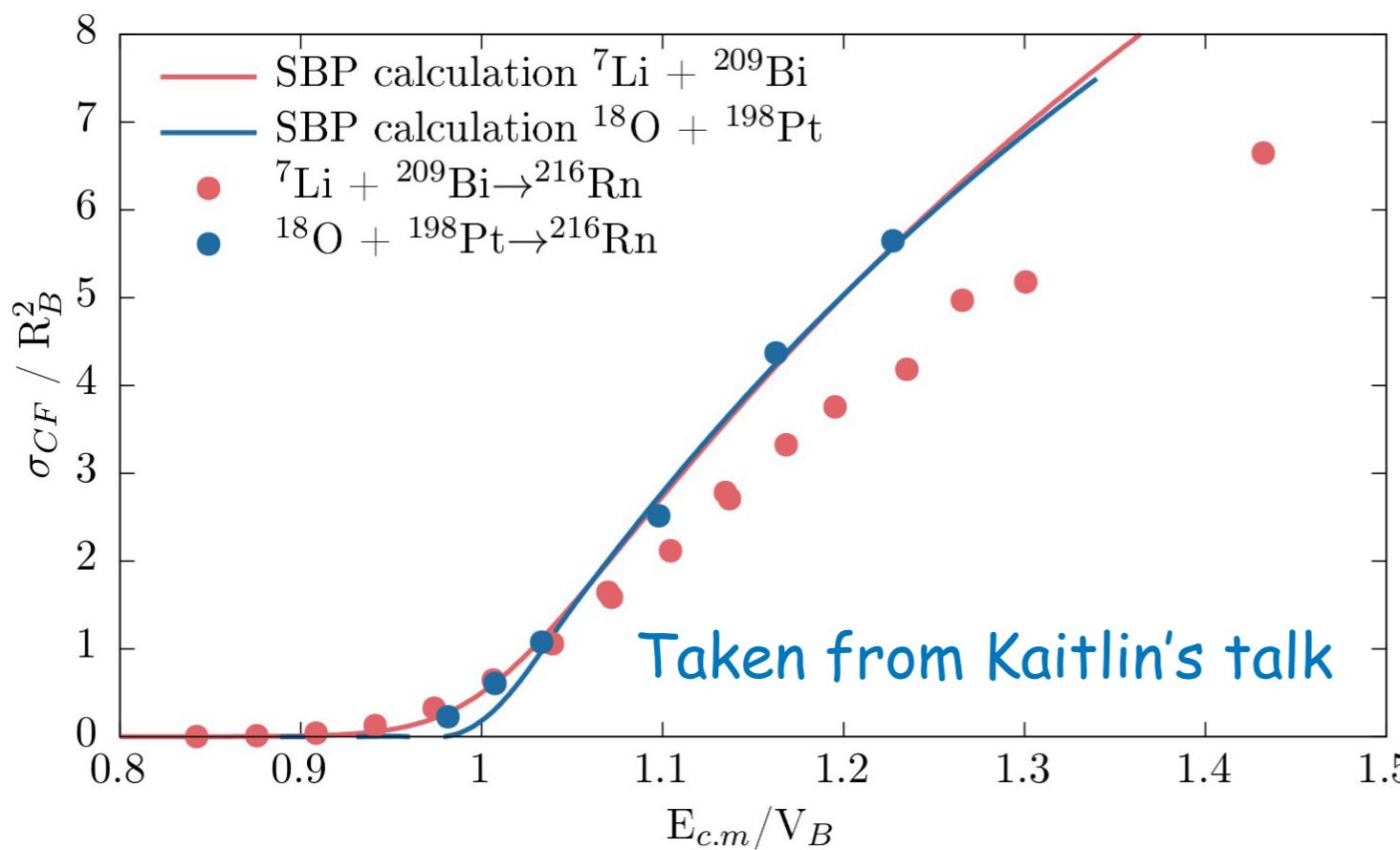
- ${}^6\text{Li}$ has two body cluster structure
 - ${}^6\text{Li}$ induced reaction can be analyzed by a three body model



Problem to solve

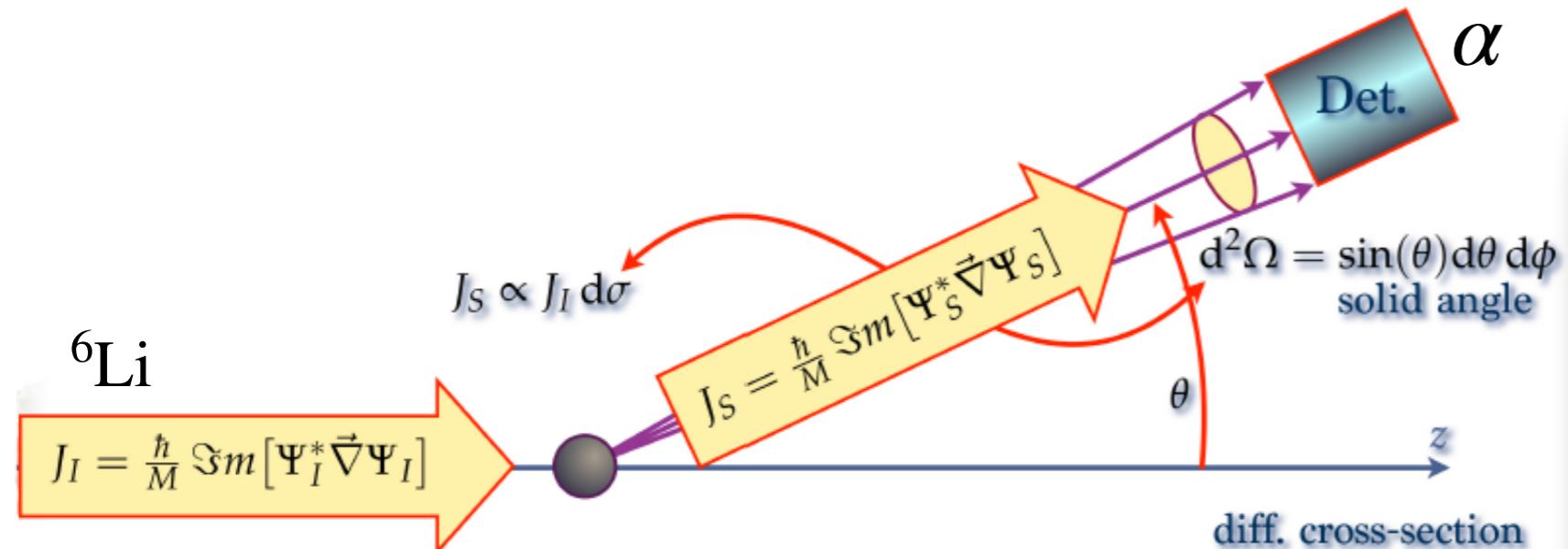
- Theoretical prediction of alphas
- Not only integrated cross section, but also angular distribution

S. Santra et al., Phys. Rev. C 85, 014612(2012).



- Find the relation between breakup and fusion
- explain the fusion suppression of weakly bound induced reaction
- Predict the fusion cross section

Inclusive breakup of ${}^6\text{Li}$ induced reaction



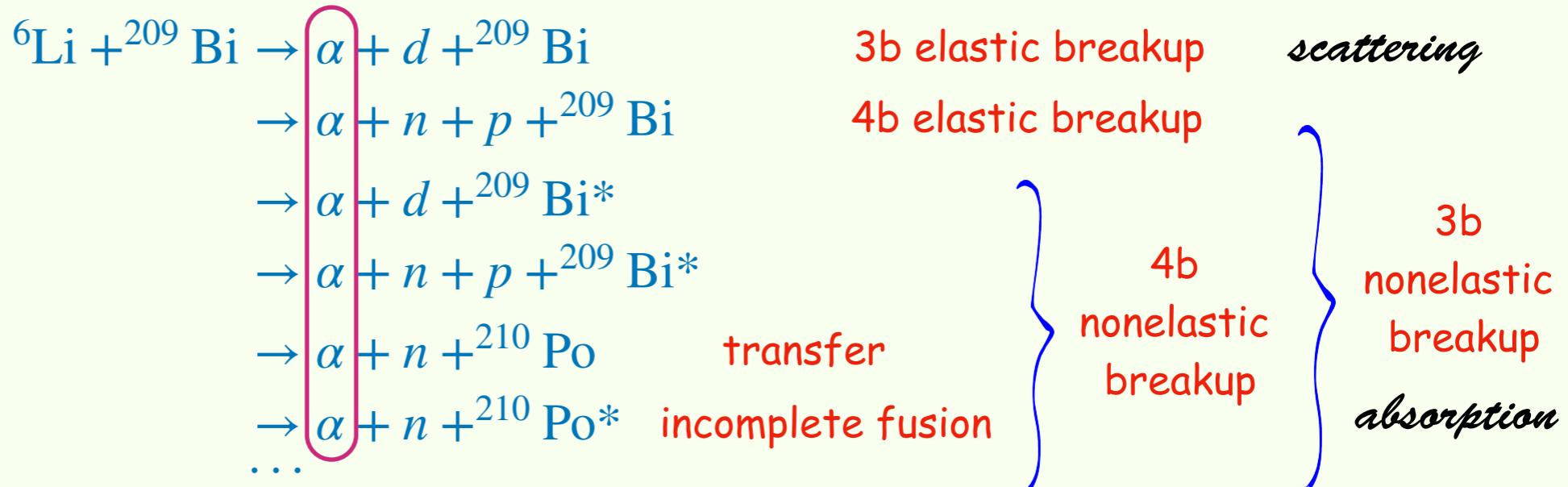
Determine J_S/J_I as a function of θ : $\frac{J_S}{J_I} = \frac{d\sigma}{r^2 d^2\Omega} =: \frac{1}{r^2} \underbrace{\left[\frac{d\sigma}{d\Omega} := \frac{d\sigma}{d^2\Omega} \right]}_{:= \sigma(\theta, \phi)}$

Experimental point of view

- ${}^6\text{Li}$ induced reaction
- Only alpha is detected

Theoretical point of view

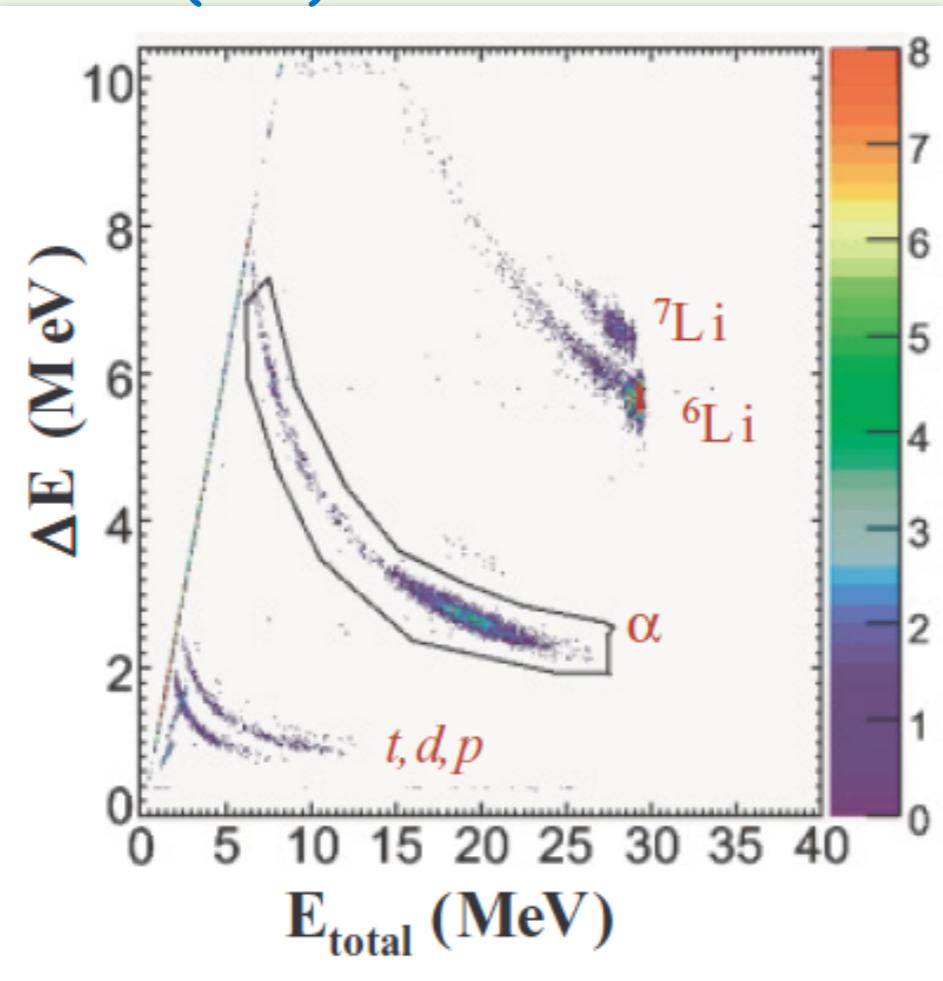
- Contributions of alphas



- solve the problem by using a three body model (alpha+d+209Bi)

Experimental examples of inclusive breakup

${}^6\text{Li}(\text{d}+\alpha)$ induced reaction



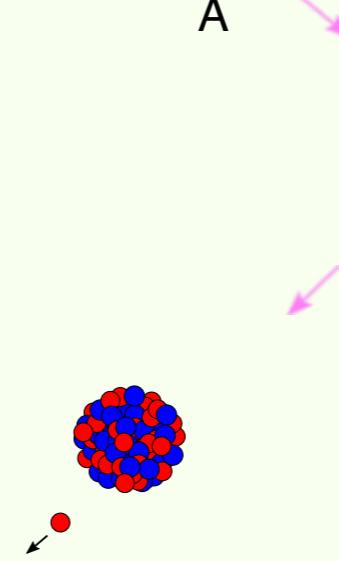
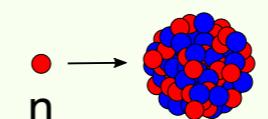
S. Santra et al, Phys. Rev. C 85, 014612 (2012)

Surrogate reaction

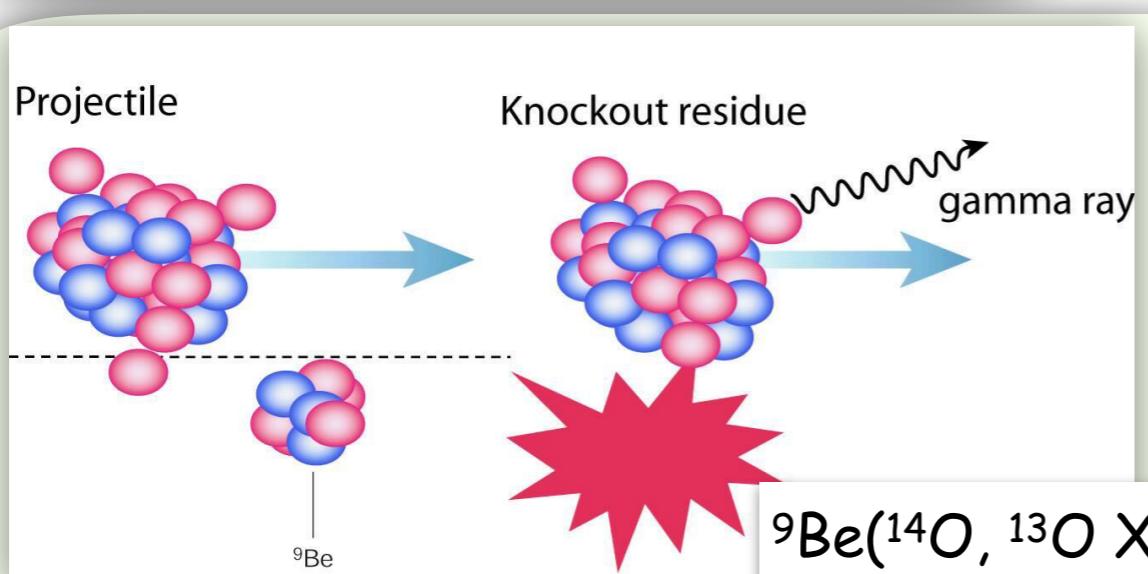
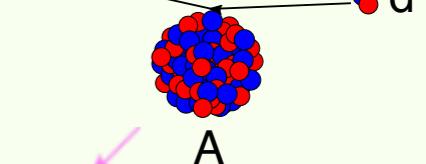
Andrew Ratkiewicz's talk on April 7th

Jutta Escher's talk on June 11th

Desired Reaction



Surrogate Reaction



Knockout reaction

Angela Bonaccorso's talk on April 30th

Study the Spectroscopic factor

- Current theory based on semi-classical (eikonal approximation)
- Fully quantum model is needed

Theoretical models for inclusive (nonelastic) breakup

- Requires inclusion of all possible processes through which the breakup fragment can interact with the target. Impractical in most cases.

In 1980s

- Ichimura, Austern, and Vincent developed a spectator-participant model (post-form)
Phys. Rev. C 23, 1847 (1981)
Phys. Rev. C 32, 431 (1985)
- Udagawa and Tamura suggested a breakup-fusion model (prior-form)
Phys. Rev. C 24, 1348 (1981)
Phys. Lett. B 135, 333(1984)
- Hussein and McVoy adopted a spectator model with the Feshbach projection method
Nucl. Phys. A 445, 124 (1985)
- Three different approaches with different predictions

Goals

- Find a suitable model for inclusive breakup
- Explore relations between these models

Challenges

- Numerically difficult
- No numerical implementation in 1980s-2000s even for Finite Range DWBA

The Ichimura, Austern, Vincent (IAV) model

- A three body model:



Any possible states between
 x and A (including all nucleons
degree of freedom)

Two body scattering with an optical potential

$(x+A)^*$ \rightarrow $x+A$

Key point
relative wave function
between x and A

elastic scattering

cross section: related to asymptotic part

$$\sigma_{\text{el}} = \frac{\pi}{k^2} \sum_{L=0}^{\infty} (2L + 1) \left| 1 - \mathbf{S}_L \right|^2$$

nonelastic scattering
(absorption/reaction)

inelastic scattering
breakup reaction
transfer reaction
fusion reaction
...

cross section: related to interior part

$$\sigma_A = \frac{2}{\hbar v} \frac{4\pi}{k^2} \sum_L (2L + 1) \int_0^\infty [-W(R)] \left| \chi_L(R) \right|^2 dR$$

The Ichimura, Austern, Vincent (IAV) model

- A three body model:



Any possible states between x and A (including all nucleons degree of freedom)

- Relative wave function between x and A in three body reaction with optical potentials

$$(E_x - K_x - U_x) \varphi_x(\mathbf{k}_b, \mathbf{r}_x) = \langle \mathbf{r}_x \chi_b(\mathbf{k}_b) | V_{\text{post}} | \Psi^{3b} \rangle$$

$$\varphi_x^\ell(\mathbf{k}_b, r_x) \xrightarrow{r \rightarrow \infty} -S_\ell \mathcal{H}_l^+$$

- Elastic breakup: equivalent to CDCC (three body scattering)

$$\frac{d^3\sigma}{dE_b d\Omega_b d\Omega_x} = \frac{(2\pi\hbar)^3}{\mu_x^2 v_a} \rho_b(E_b) \rho_x(E_x) \left| f\left(\hat{\mathbf{k}}_b, \hat{\mathbf{k}}_x\right) \right|^2$$

$$f\left(\hat{\mathbf{k}}_b, \hat{\mathbf{k}}_x\right) = - \sum \mathcal{C}\mathcal{G} Y_{l_b}^{m_b}(\hat{\mathbf{k}}_b) Y_{l_x}^{m_x}(\hat{\mathbf{k}}_x) i^{-l_x} S_\ell$$

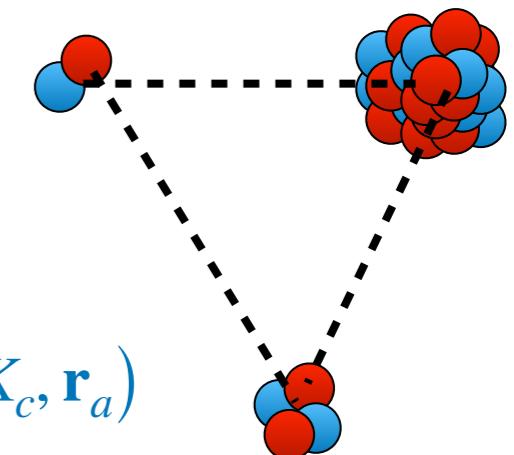
- Nonelastic breakup: (absorption)

$$\frac{d^2\sigma}{dE_b d\Omega_b} = -\frac{2}{\hbar v_a} \rho_b(E_b) \langle \varphi_x(\vec{\mathbf{k}}_b) | W_x | \varphi_x(\vec{\mathbf{k}}_b) \rangle$$

Effective two body interactions

- Effective two body interaction in three body model
- Testing the effective interactions in CDCC model

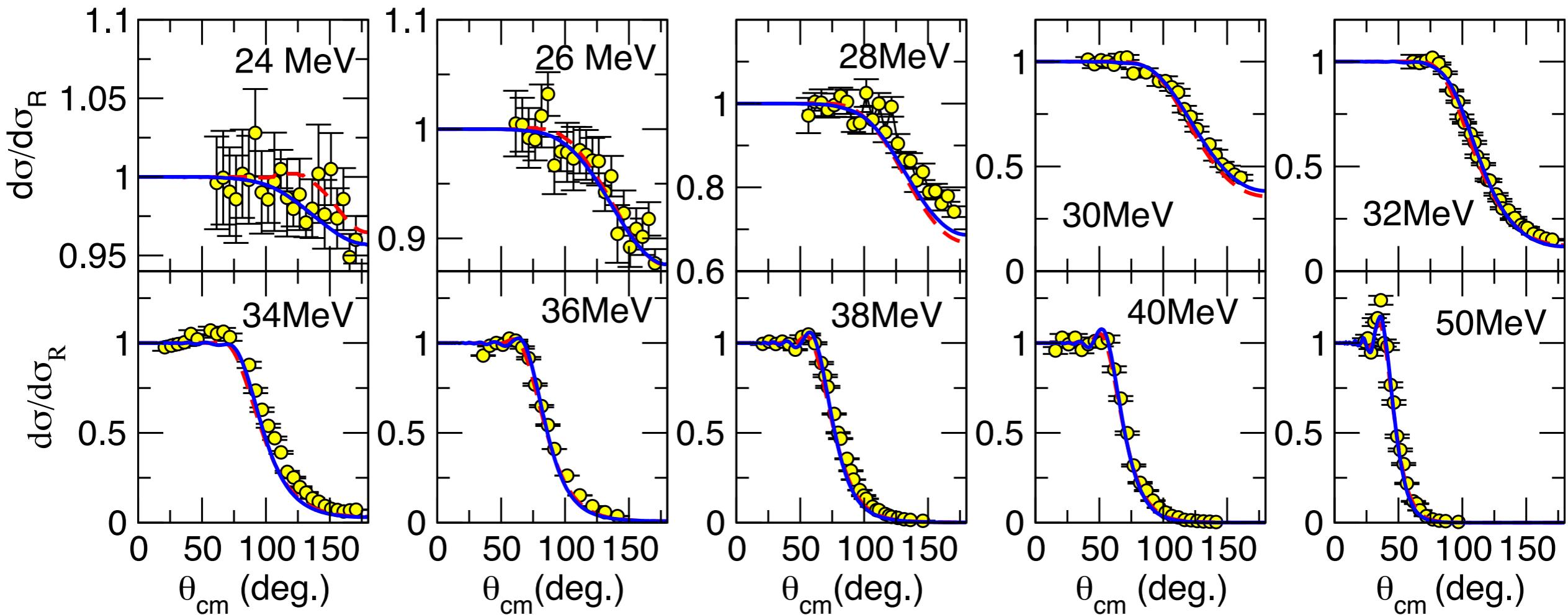
$$\Psi^{3b(+)} \simeq \Psi^{\text{CDCC}(+)}(\mathbf{r}_a, \mathbf{r}_{bx}) = \sum_i \phi_a^i(\mathbf{r}_{bx}) \chi_a^{i(+)}(\mathbf{r}_a) + \sum_c \phi_a^c(k_c, \mathbf{r}_{bx}) \chi_a^{c(+)}(K_c, \mathbf{r}_a)$$



- Compare the elastic scattering cross section with the data

data: S. Santra et al., Phys. Rev. C 83, 034616 (2011).

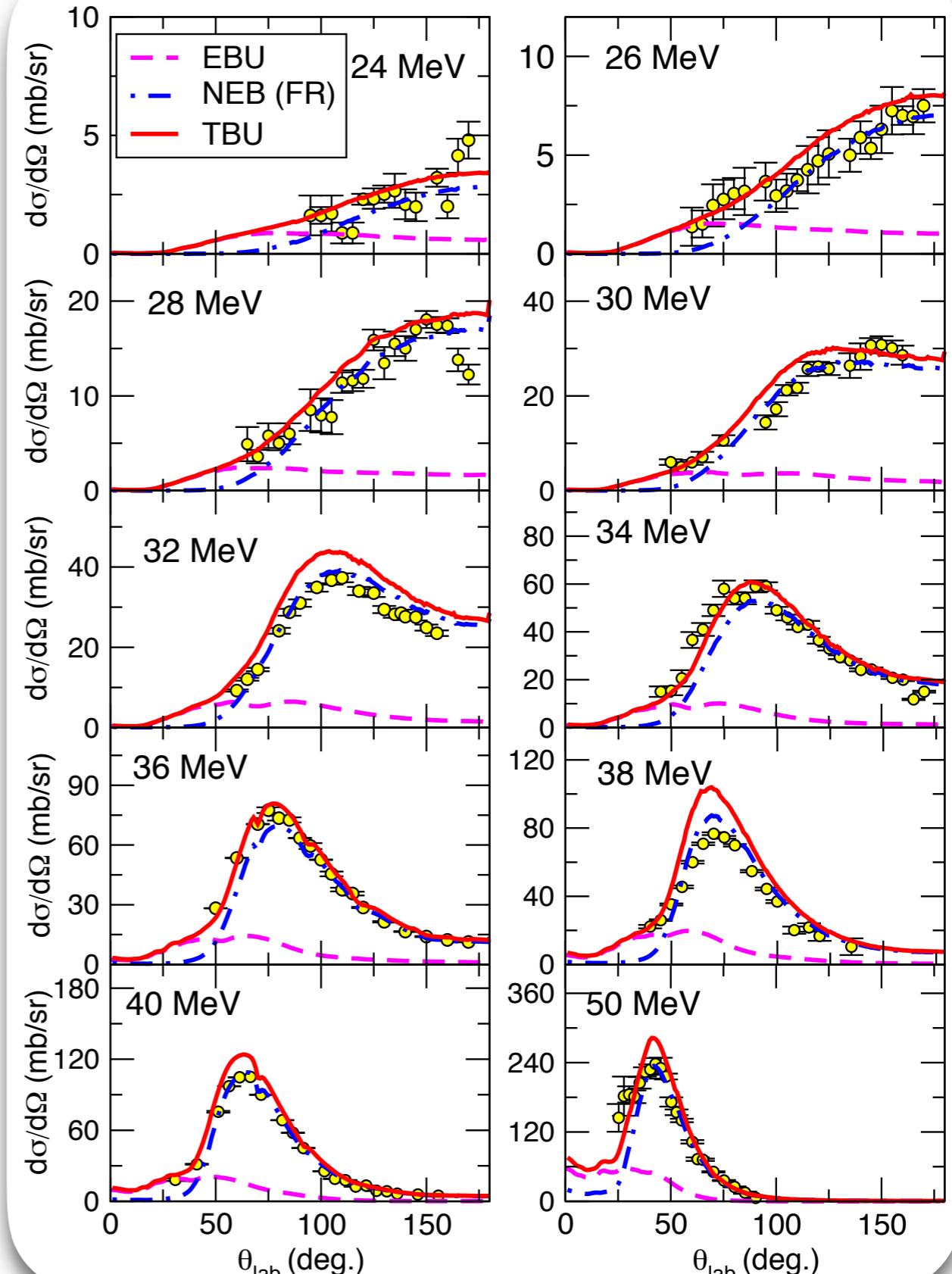
— OMP
— CDCC



Inclusive breakup of $^{209}\text{Bi}(^6\text{Li}, \alpha X)$

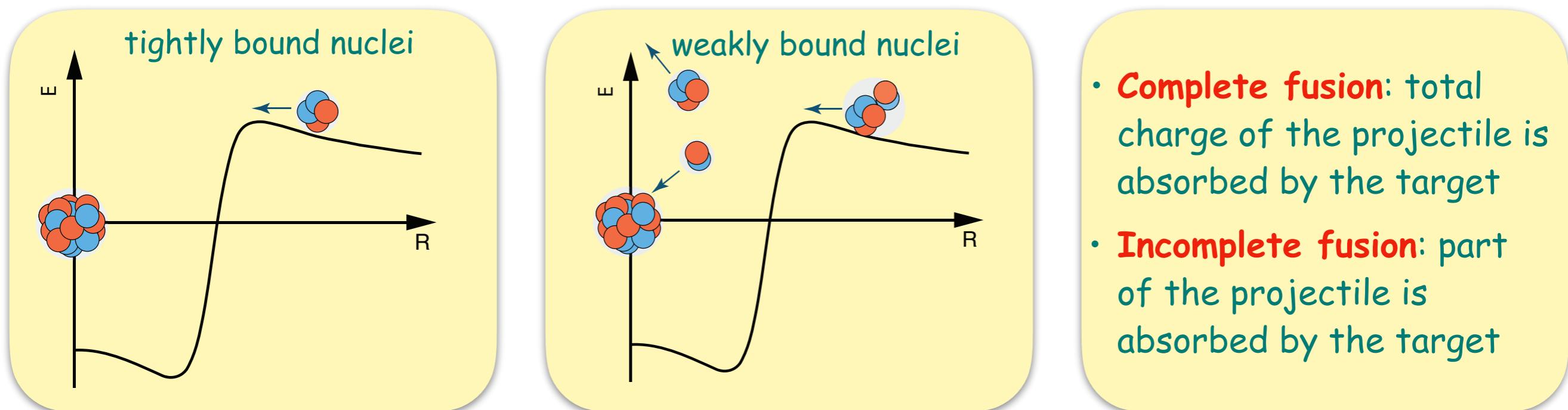
- ${}^6\text{Li} \Rightarrow (\text{alpha} + \text{d})$, $S(\text{d})=1.474 \text{ MeV}$
- Only alpha is detected
 - data: S. Santra et al.,
Phys. Rev. C 85, 014612(2012).
- EBU : CDCC (FRESCO)
- NEB : IAV model
 - DWBA $\Psi^{3b(+)} \simeq \Psi^{\text{DWBA}(+)} = \chi_a^{(+)} \phi_a$
- Total Breakup (TBU)=EBU+NEB
- Dominated by NEB
- EBU has large contributions at small angles
- Supports IAV model

JL and A. M. Moro,
Phys. Rev. C 92, 044616 (2015)

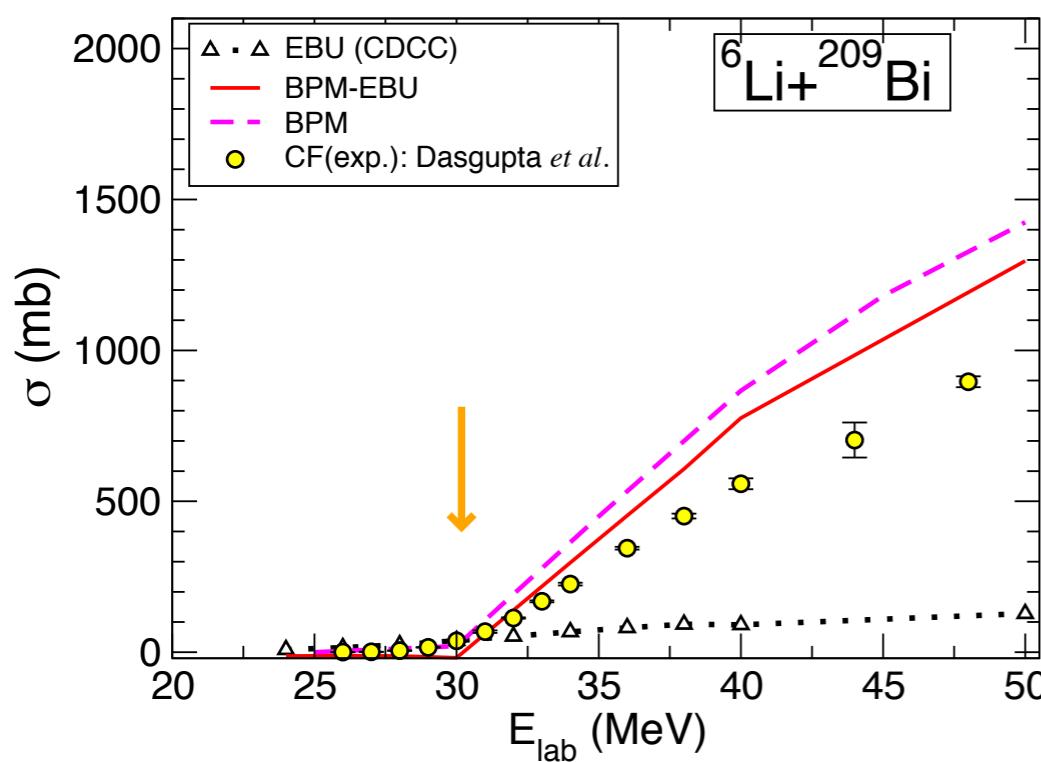


Breakup and fusion

- From the barrier penetration picture



- Complete Fusion is suppressed due to weak binding of the projectile

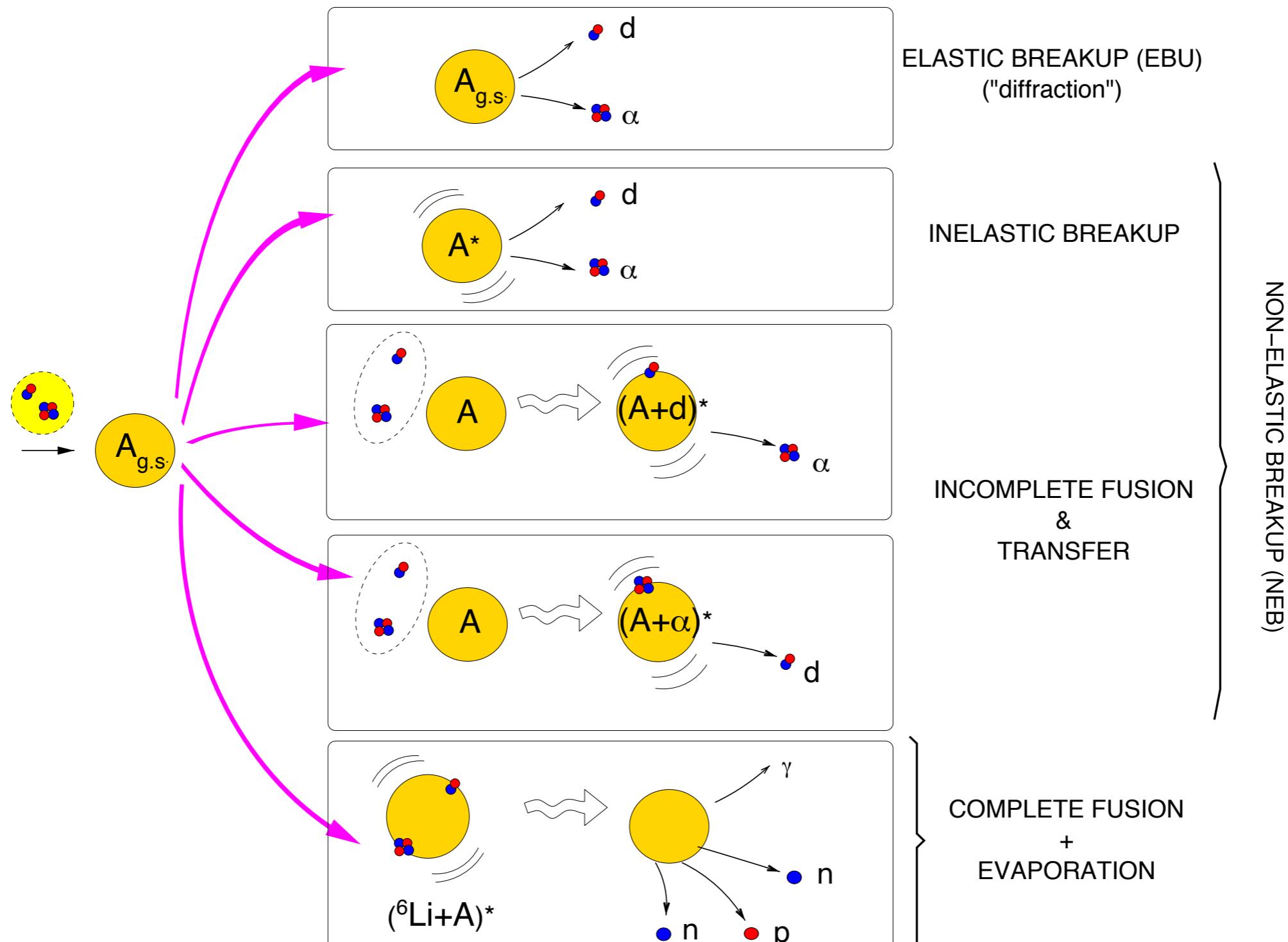


Challenges

- To correctly understand fusion suppression (not only from semi-classical picture) and simultaneously predict the complete fusion cross section
- To study incomplete fusion is breakup-fusion (two-step) or transfer to continuum (one-step)

Study the fusion cross section through a three body model

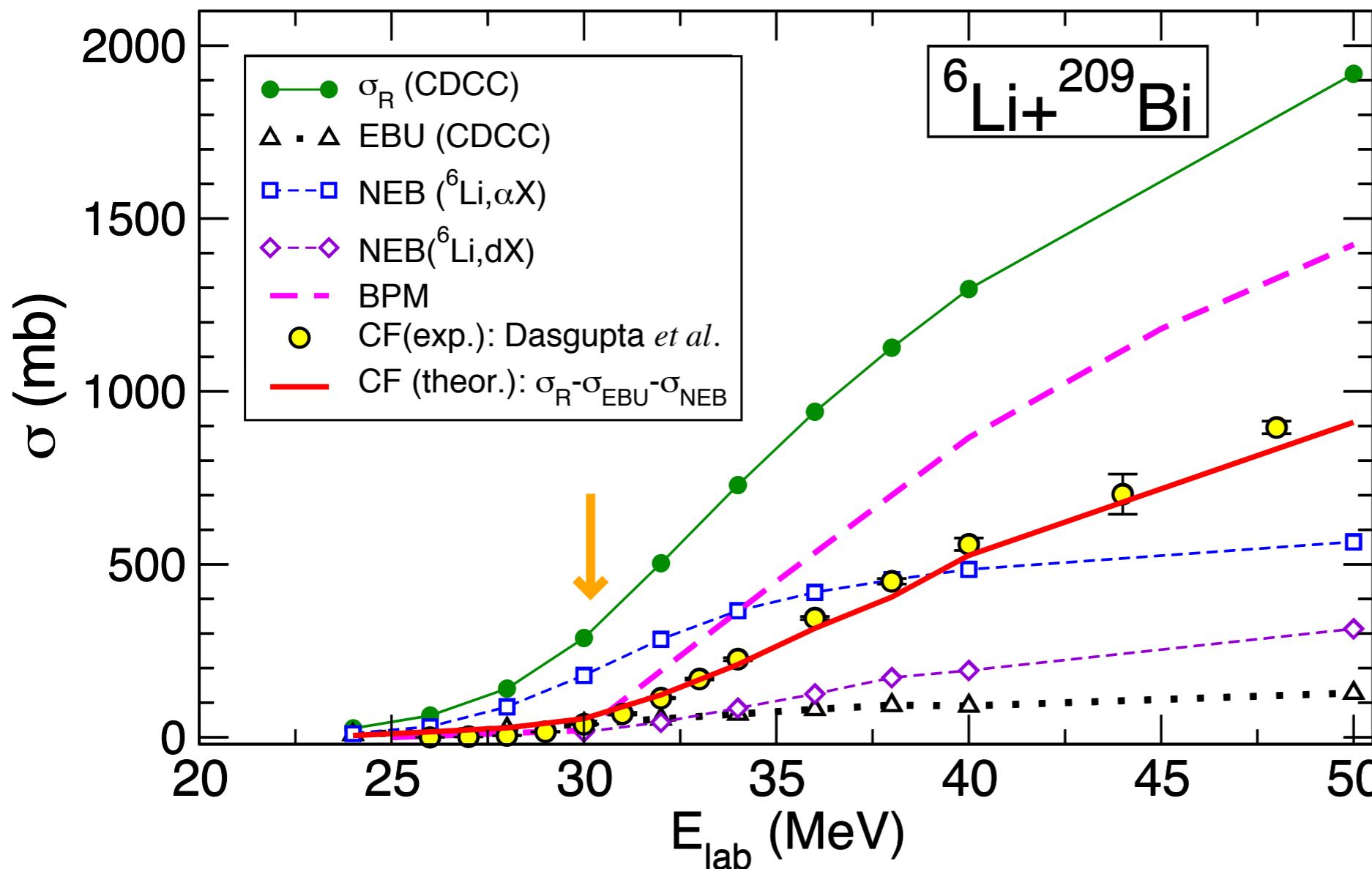
- Take ${}^6\text{Li} + A$ as an example



$$\sigma_R \approx \sigma_{CF} + \sigma_{EBU} + \sigma_{NEB}^{(b)} + \sigma_{NEB}^{(x)}$$

Study the fusion cross section through a three body model

$$\sigma_{\text{CF}} \approx \sigma_R - \sigma_{\text{EBU}} - \sigma_{\text{NEB}}^{(b)} - \sigma_{\text{NEB}}^{(x)}$$



CF: complete fusion

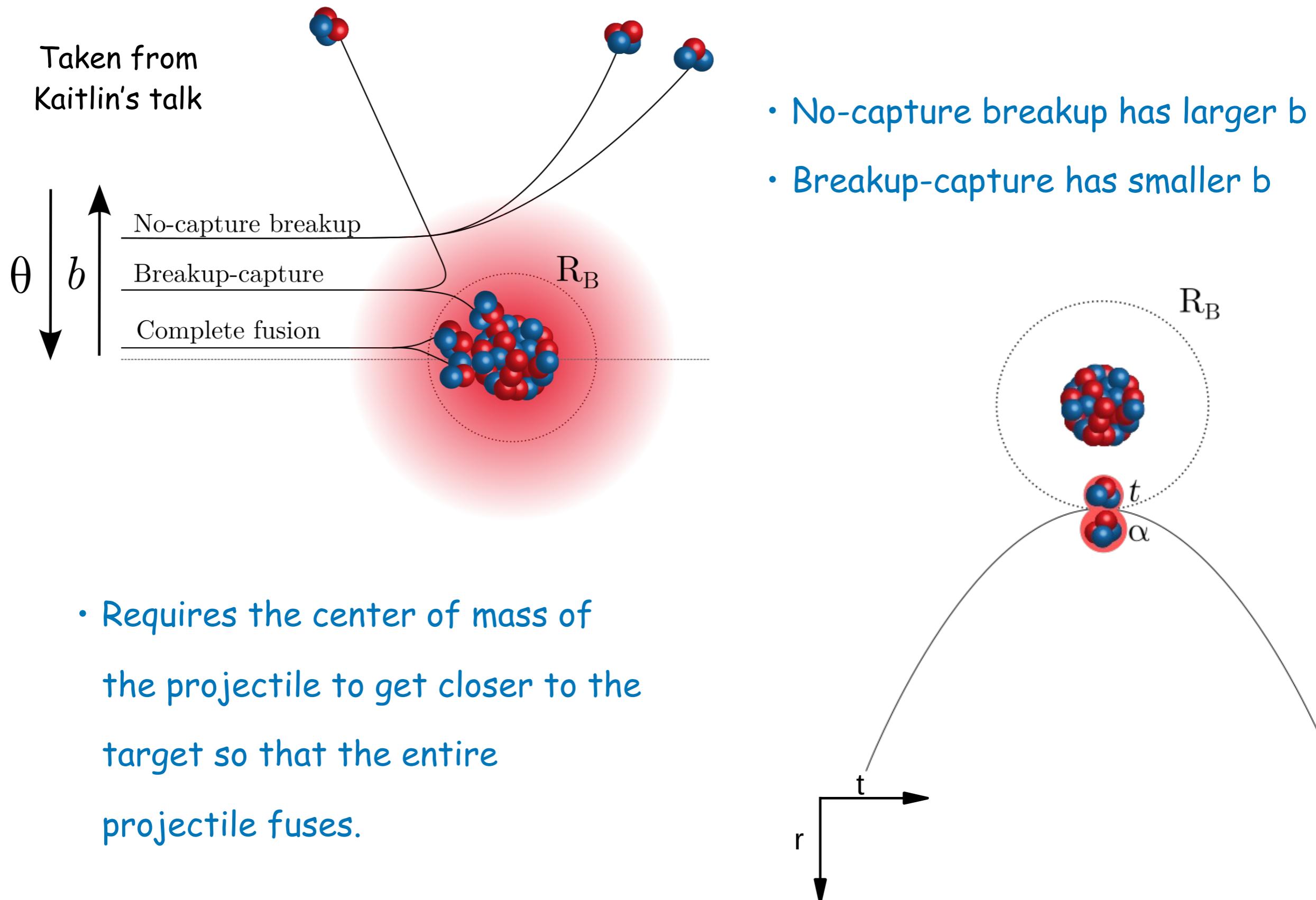
NEB: nonelastic breakup

EBU: elastic breakup

Data: M. Dasgupta *et al.*, Phys. Rev. C 70, 024606 (2004)

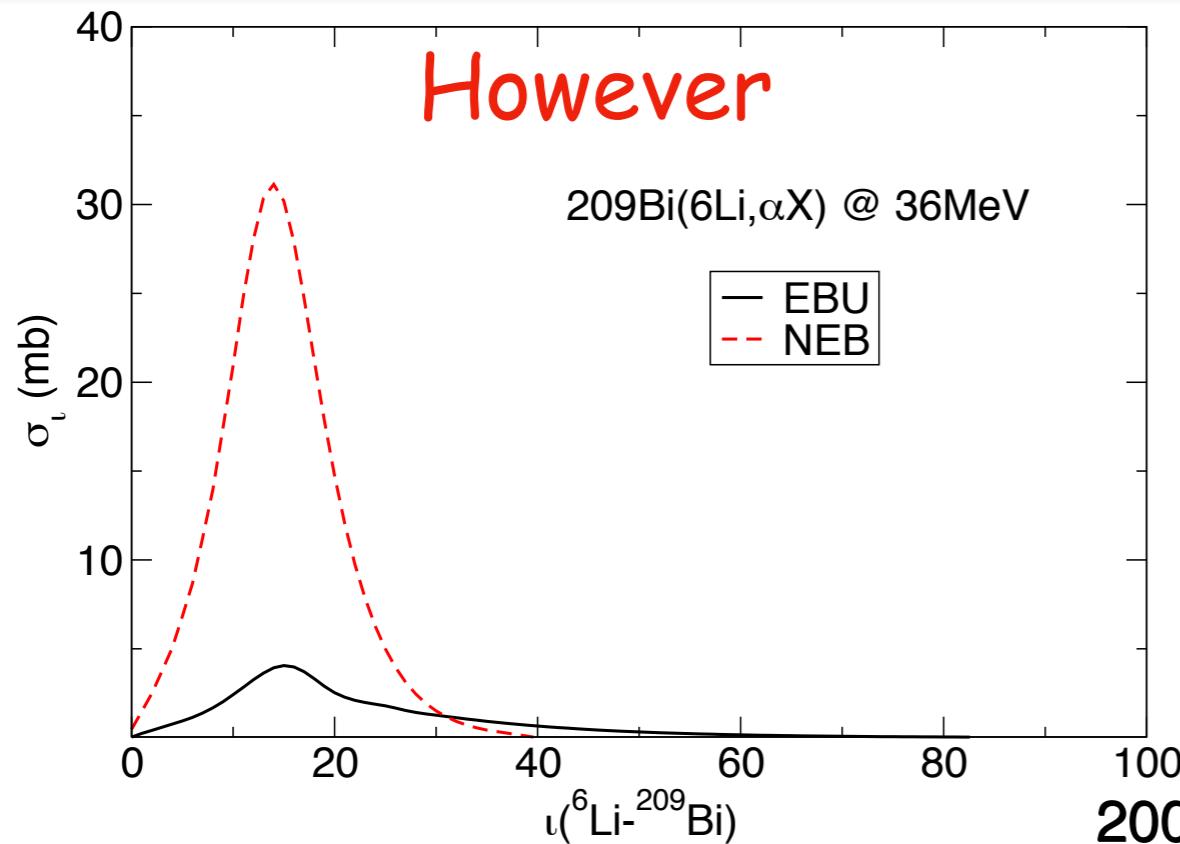
- Apply the above relation to ${}^6\text{Li}+{}^{209}\text{Bi}$ reaction around the Coulomb barrier
- Compare calculated fusion cross section with experiment
- EBU mechanism plays a minor role
- Dominant breakup mechanism in both reactions is alpha production due to $({}^6\text{Li}, \alpha X)$ NEB.

Unraveling the mechanisms leading to fusion suppression



Unraveling the mechanisms leading to fusion suppression

16



However

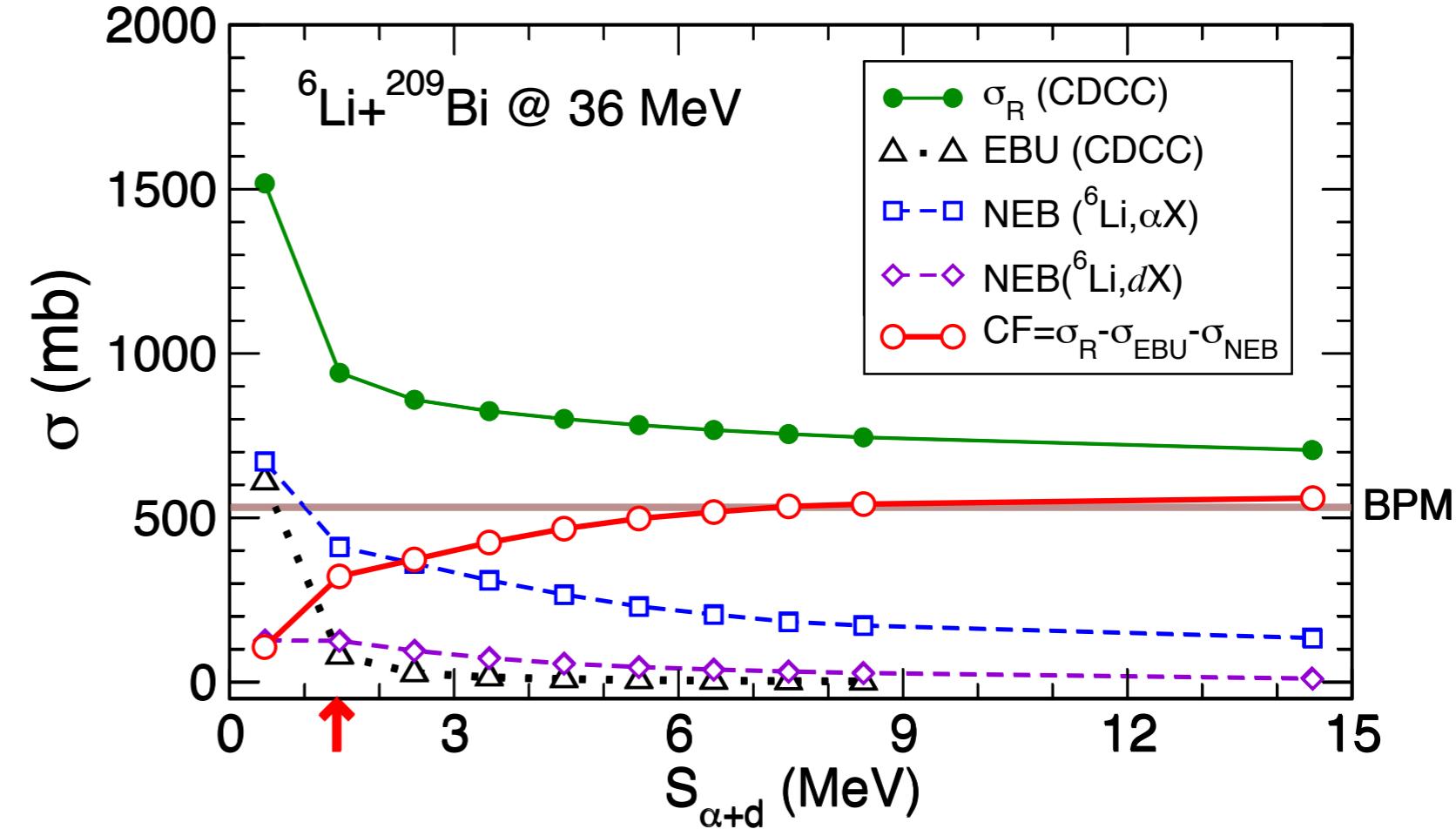
209Bi(6Li, α X) @ 36MeV

— EBU
— NEB

Use a toy model to study
effects of separation energy

- vary the binding energy of
 ${}^6\text{Li}(\alpha+d)$ in the projectile.

When the binding energy becomes larger, the calculated cross section approaches the barrier penetration model (BPM)



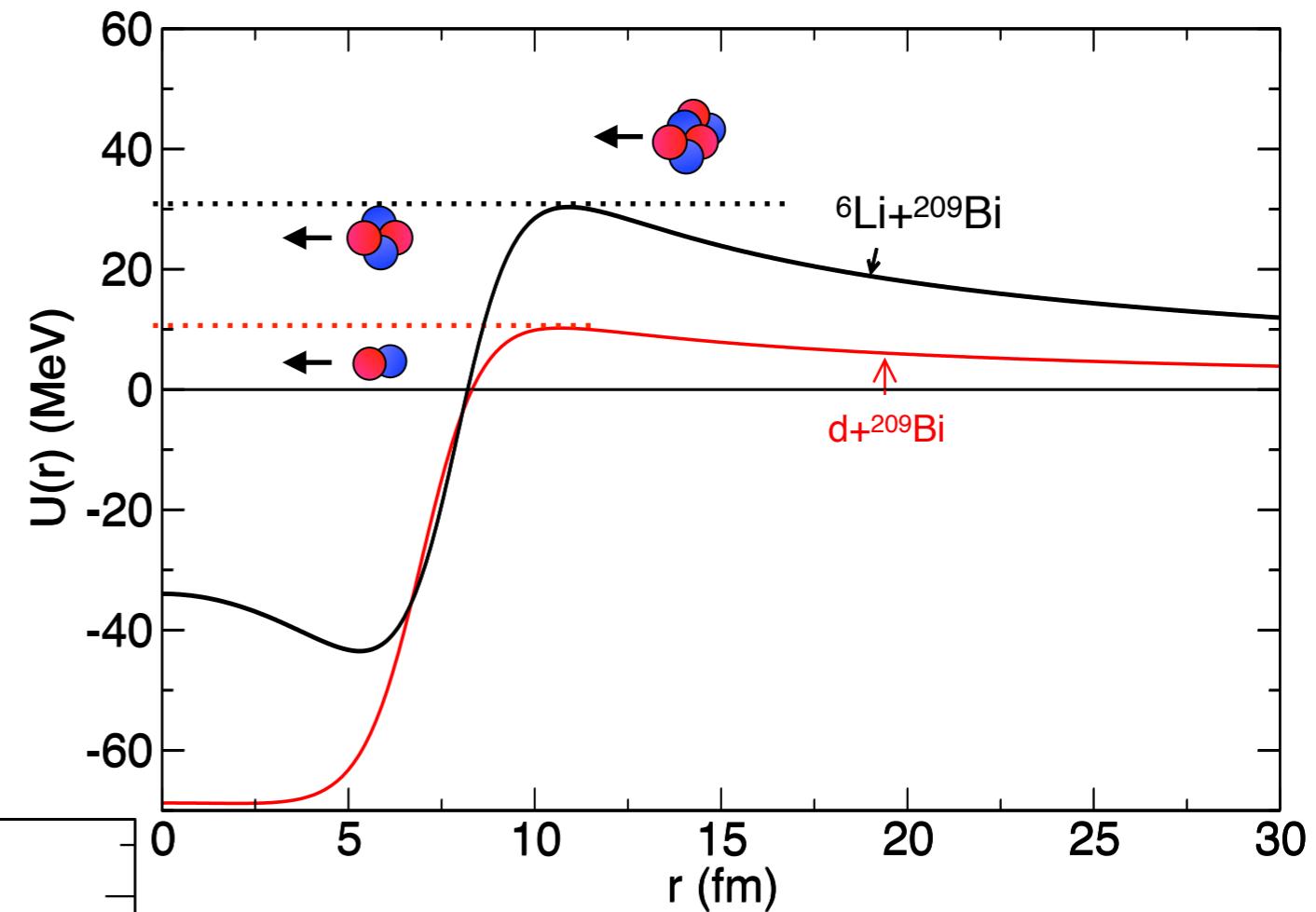
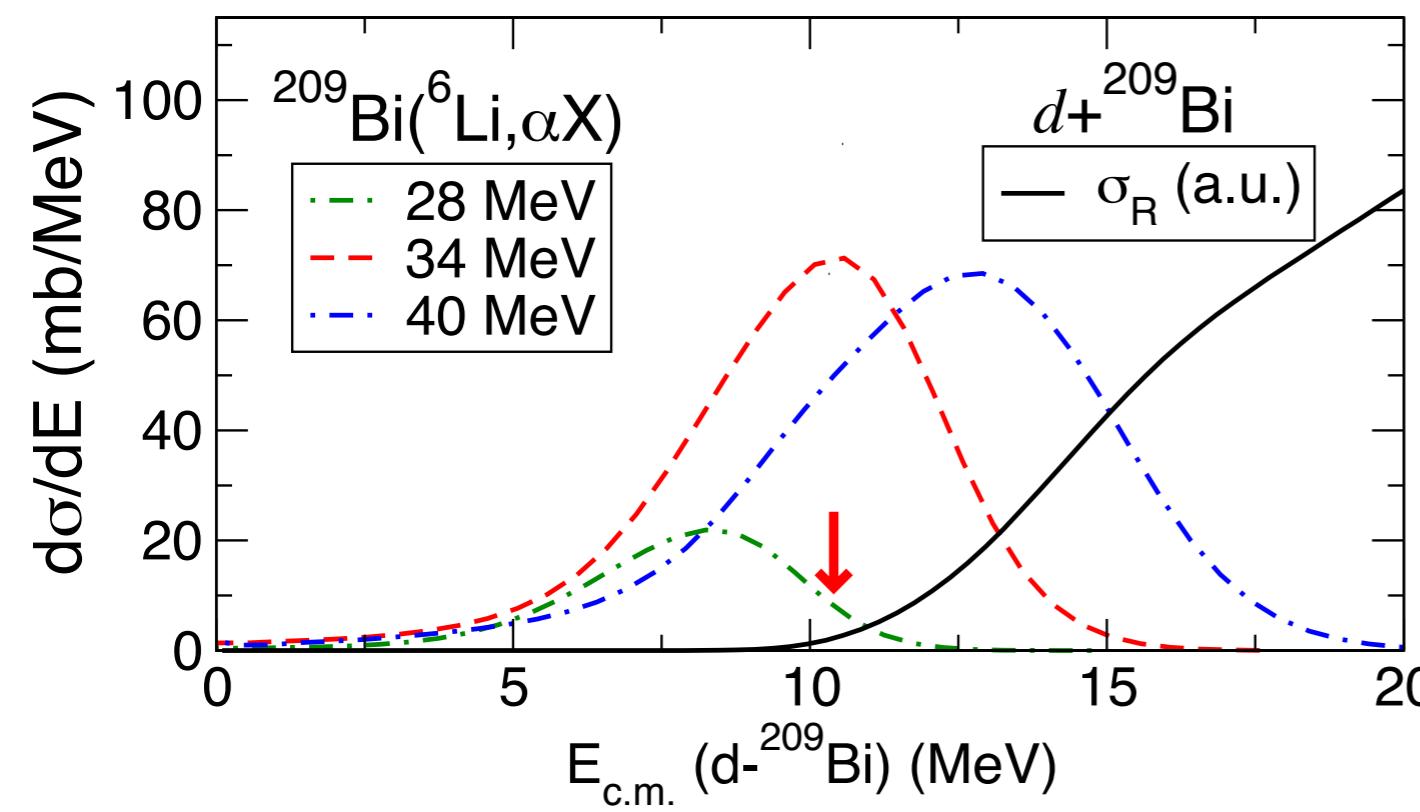
JL and Antonio M. Moro,
Phys. Rev. Lett. 122, 042503 (2019)

Why NEB is so important

The NEB of α production



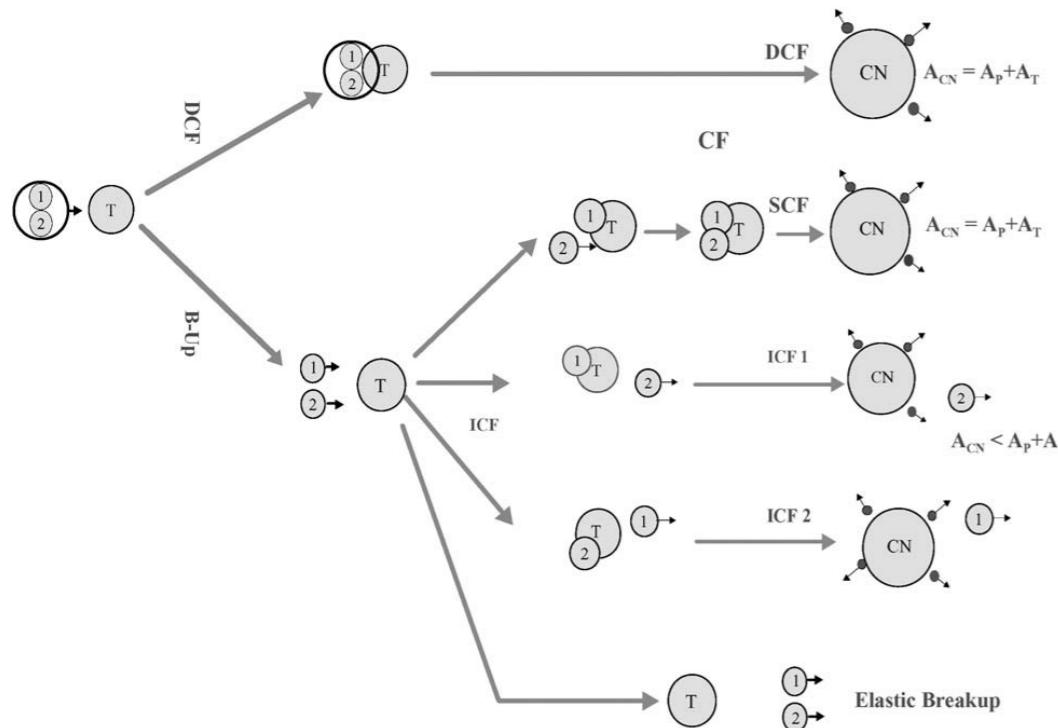
Compare with



Trojan Horse Effect

JL and Antonio M. Moro,
Phys. Rev. Lett. 122, 042503 (2019)

Incomplete fusion



L.F. Canto et al. Physics Reports 424 (2006) 1 – 111

Incomplete fusion is part of the projectile absorbed by the target

By definition, ICF is part of NEB, for example

$$\begin{aligned} \sigma(6\text{Li}+209\text{Bi} \rightarrow \alpha + 211\text{Po}^*) &= \sigma(\text{NEB}) - \\ \sigma(6\text{Li}+209\text{Bi} \rightarrow d+\alpha + 209\text{Bi}^*) - \\ \sigma(6\text{Li}+209\text{Bi} \rightarrow n+p+\alpha + 209\text{Bi}) - \\ \sigma(6\text{Li}+209\text{Bi} \rightarrow n+p+\alpha + 209\text{Bi}^*) - \text{others} \end{aligned}$$

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PHYSICAL REVIEW LETTERS

20 OCTOBER 1980

pheric neutrino experiments conducted in a South African gold mine. The ratio of the observed to the expected horizontal flux of product muons was determined

to be $0.62^{+0.21}_{-0.12}$. F. Reines, in Proceedings of the Sixteenth International Cosmic Ray Conference, Kyoto, August 1979 (unpublished).

Breakup-Fusion Description of Massive Transfer Reactions with Emission of Fast Light Particles

T. Udagawa and T. Tamura

Department of Physics, University of Texas, Austin, Texas 78712
(Received 30 June 1980)

that we describe the massive transfer reaction as a two-step process. Take again the above example. The first step is then the breakup of ^{14}N into $\alpha + ^{10}\text{B}$. This is then followed by the second step, in which ^{10}B is fused into ^{159}Tb .

suggested two step process

PHYSICAL REVIEW LETTERS 122, 102501 (2019)

Origins of Incomplete Fusion Products and the Suppression of Complete Fusion in Reactions of ^7Li

K. J. Cook,^{*} E. C. Simpson, L. T. Bezzina, M. Dasgupta, D. J. Hinde, K. Banerjee,[†]

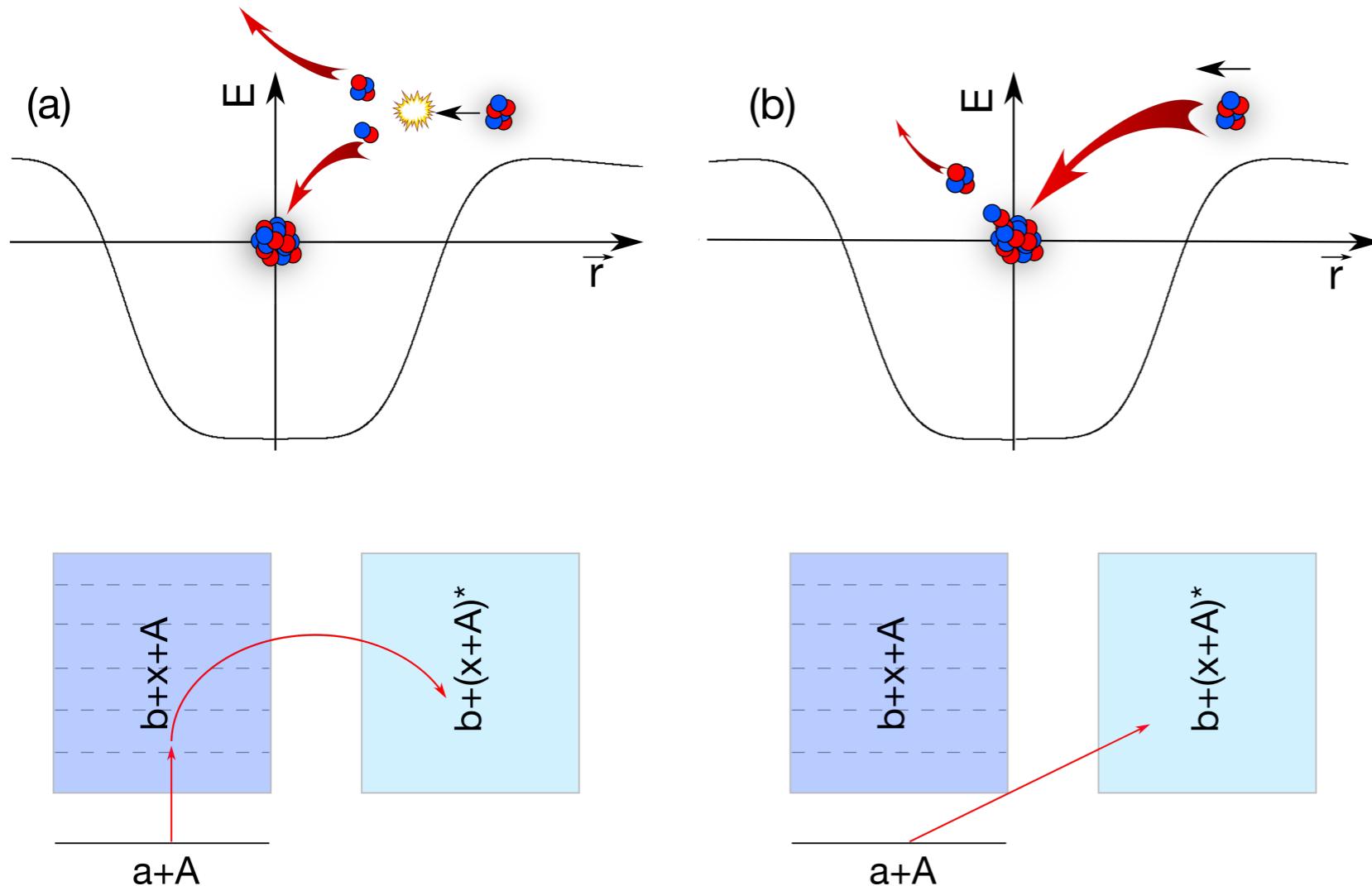
A. C. Beriman, and C. Sengupta

Department of Nuclear Physics, Research School of Physics and Engineering, The Australian National University, Canberra ACT 2601, Australia

breakup, inconsistent with expectations for breakup capture. We have unambiguously identified that ^{212}Po is produced by direct triton cluster transfer, and demonstrated that the measured distributions of all unaccompanied α particles are broadly consistent with triton transfer. This is

suggested one step process

Exploring the reaction path for incomplete fusion



Incomplete fusion: part of the projectile absorbed by the target

Two-step: projectile is inelastically excited into its continuum and then fuses with the target

One-step: fragment fuses with the target directly from its ground state

Resolve this puzzle by studying nonelastic breakup (incomplete fusion is a part)

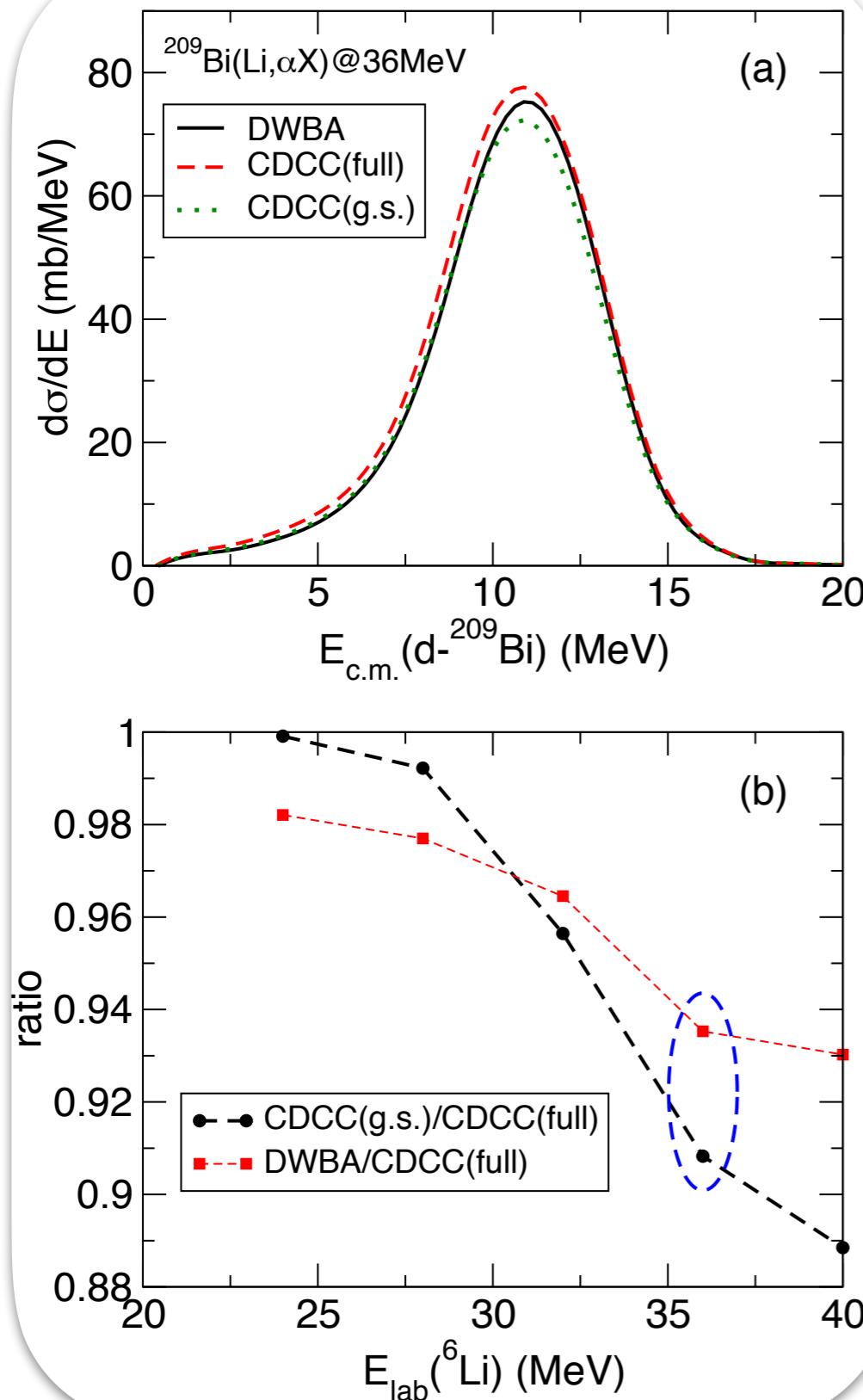
Use CDCC wave-function in the IAV model:

$$\varphi_x(\mathbf{k}_b, \mathbf{r}_x) = \int G_x(\mathbf{r}_x, \mathbf{r}'_x) \langle \mathbf{r}'_x \chi_b^{(-)} | V_{post} | \Psi^{\text{CDCC}(+)} \rangle d\mathbf{r}'_x$$

$$\Psi^{\text{CDCC}(+)}(\mathbf{r}_a, \mathbf{r}_{bx}) = \sum_b \phi_a^b(\mathbf{r}_{bx}) \chi_a^{b(+)}(\mathbf{r}_a) + \int d\mathbf{k} \phi_a^{\mathbf{k}}(\mathbf{r}_{bx}) \chi_a^{\mathbf{k}(+)}(\mathbf{r}_a)$$

- Continuum and ground states are separated
- Allows to study continuum effects on the NEB
- Test validity of DWBA

Apply to ${}^6\text{Li}$ induced reaction



$$\Psi^{\text{CDCC}(+)}(\mathbf{r}_a, \mathbf{r}_{bx}) = \sum_b \phi_a^b(\mathbf{r}_{bx}) \chi_a^{b(+)}(\mathbf{r}_a) + \int d\mathbf{k} \phi_a^{\mathbf{k}}(\mathbf{r}_{bx}) \chi_a^{\mathbf{k}(+)}(\mathbf{r}_a)$$

$$\Psi^{\text{CDCC}(+)\text{g.s.}}(\mathbf{r}_a, \mathbf{r}_{bx}) = \sum_b \phi_a^b(\mathbf{r}_{bx}) \chi_a^{b(+)}(\mathbf{r}_a)$$

$$\Psi^{\text{DWBA}(+)}(\mathbf{r}_a, \mathbf{r}_{bx}) = \phi_a(\mathbf{r}_{bx}) \chi_a^{(+)}(\mathbf{r}_a)$$

- DWBA is a good approximation compared to CDCC
- Nonelastic breakup (incomplete fusion) is mixture of one-step (>90%) and two-step (<10%) processes

Summary and outlook

- Summary
 - Studied ${}^6\text{Li}$ induced reactions
 - Found the reaction mechanism for complete fusion suppression
 - Investigated the reaction path for incomplete fusion (nonelastic breakup)
- Outlook
 - Find a suitable theory to extract incomplete fusion cross sections for deuteron (surrogate reaction) and ${}^6\text{Li}$
 - Apply the IAV model for knockout reaction to verify semi-classical model
 - Study uncertainties caused by effective interactions

