COMP3340_A3

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First import the required libraries:

```
library("reticulate")# for incorporating Python code
library("png")
library("grid")
library("RWeka")
library("scatterplot3d")
```

Exercise 1

The regression dataset chosen is the Yacht Hydrodynamics dataset sourced from the UCI Machine Learning Repository on the 7th of November, with the .data file downloaded and saved as yacht_hydrodynamics.data in the working directory. The response variable will be removed. There are 6 remaining features, and 308 samples in total.

```
import pandas as pd
import numpy as np
import math
import networkx as nx
yacht_data = pd.read_csv("yacht_hydrodynamics.data", delim_whitespace=True)
# Remove the response variable
yacht_data = yacht_data.drop("7", axis = 1)
# # Define node names by row
i_names = [str(i) for i in yacht_data.index.values]
yacht_data.head()
```

```
##      1      2     3     4     5      6
## 0 -2.3  0.568  4.78  3.99  3.17  0.125
## 1 -2.3  0.568  4.78  3.99  3.17  0.150
## 2 -2.3  0.568  4.78  3.99  3.17  0.175
## 3 -2.3  0.568  4.78  3.99  3.17  0.200
## 4 -2.3  0.568  4.78  3.99  3.17  0.225
```

The variables are continuous and of different scales. Before defining the Euclidean Distance between points, the values will be standardised using a z-score transformation.

```
# normalise the data
norm_yacht_data = (yacht_data - yacht_data.mean())/yacht_data.std()
def euclid_distance(s1, s2):
```

```
    return math.sqrt(np.sum((s1-s2) ** 2))
# Distance matrix for samples:
samples_edist_m = [[euclid_distance(norm_yacht_data.iloc[y],
  norm_yacht_data.iloc[x]) for y in range(len(i_names))] for x in range(len(i_names))]
round(pd.DataFrame(samples_edist_m, columns = i_names, index = i_names), 3).head()
```

```
##        0      1      2      3      4  ...    303    304    305    306    307
## 0  0.000  0.248  0.495  0.743  0.991  ...  3.636  3.793  3.959  4.133  4.315
## 1  0.248  0.000  0.248  0.495  0.743  ...  3.489  3.636  3.793  3.959  4.133
## 2  0.495  0.248  0.000  0.248  0.495  ...  3.355  3.489  3.636  3.793  3.959
## 3  0.743  0.495  0.248  0.000  0.248  ...  3.234  3.355  3.489  3.636  3.793
## 4  0.991  0.743  0.495  0.248  0.000  ...  3.128  3.234  3.355  3.489  3.636
##
## [5 rows x 308 columns]
```

Defining an `Edge` class to be used throughout relevant exercises:

```
class Edge:
    def __init__(self, s, w, e):
        self.start = s
        self.weight = w
        self.end = e
    def get_edge(self):
        return (str(self.start) + '-' + str(self.weight) + '-' + str(self.end))
    def get_nx_edge(self):
        return ((self.start, self.end, {'label': str(self.weight)}))
```

Next, defining code to compute the RNG.

```
# Function to generate Relative Neighbourhood Graph
def relative_neighbourhood_graph(G):
    V = [i for i in range(len(G))]
    RNG = []
    for u in V: # start point
        for v in V: # end point
            dist = G[u][v]
            if (dist != 0):
                for r in V: # third point
                    if ((r != u) & (r != v)):
                        d_r_u = G[u][r]
                        d_r_v = G[v][r]
                        if ((d_r_u < dist) & (d_r_v < dist)):
                            break
                else:
                    RNG.append([u,v])
    return RNG
# Function to generate .gml for Relative Neighbourhood Graph
def get_RNG(matrix, names, output_file):
    E = relative_neighbourhood_graph(matrix)
    V = [i for i in range(len(matrix))]
    RNG = []
    node_labels = [names[n] for n in V]
```

```
    for e in range(1, len(E)):
        RNG.append(Edge(node_labels[E[e][0]], matrix[E[e][0]][E[e][1]], node_labels[E[e][1]]))
    G = nx.Graph()
    G.add_nodes_from(node_labels)
    G.add_edges_from([e.get_nx_edge() for e in RNG])
    nx.write_gml(G, output_file)
# Generate Relative Neighbourhood Graph for samples:
get_RNG(samples_edist_m, i_names, 'graph_1.gml')
```

The output file graph_1.gml was processed using yEd Graph Editor to produce the following visualisation in `Figure 1`:

```
graph_1 <- readPNG("graph_1.png")
grid.raster(graph_1)
```

Exercise 2

- "The classification problem could be of binary type" - does that mean it must be? I would like to use the Iris dataset, which has >2 classes in the response. The classification dataset to be used is the iris dataset from sklearn.datasets. It has four continuous features, a response variable with three classes, and 150 samples in total.

```
from sklearn import datasets
from sklearn.model_selection import train_test_split
iris = datasets.load_iris()
iris_data = pd.DataFrame(data= np.c_[iris['target'], iris['data']],
                     columns= ['CLASS'] + iris['feature_names'])
print(iris_data.head())
```

```
##    CLASS  sepal length (cm)  ...  petal length (cm)  petal width (cm)
## 0    0.0                5.1  ...                1.4               0.2
## 1    0.0                4.9  ...                1.4               0.2
## 2    0.0                4.7  ...                1.3               0.2
## 3    0.0                4.6  ...                1.5               0.2
## 4    0.0                5.0  ...                1.4               0.2
##
## [5 rows x 5 columns]
```

Of the 150 samples, 80% (120 rows) will be denoted the training set and 20% (30 rows) will be set aside for testing.

```
iris_train, iris_test = train_test_split(iris_data, test_size = 0.2)
print(iris_train.head())
```

```
##      CLASS  sepal length (cm)  ...  petal length (cm)  petal width (cm)
## 21     0.0                5.1  ...                1.5               0.4
## 45     0.0                4.8  ...                1.4               0.3
## 142    2.0                5.8  ...                5.1               1.9
## 16     0.0                5.4  ...                1.3               0.4
```

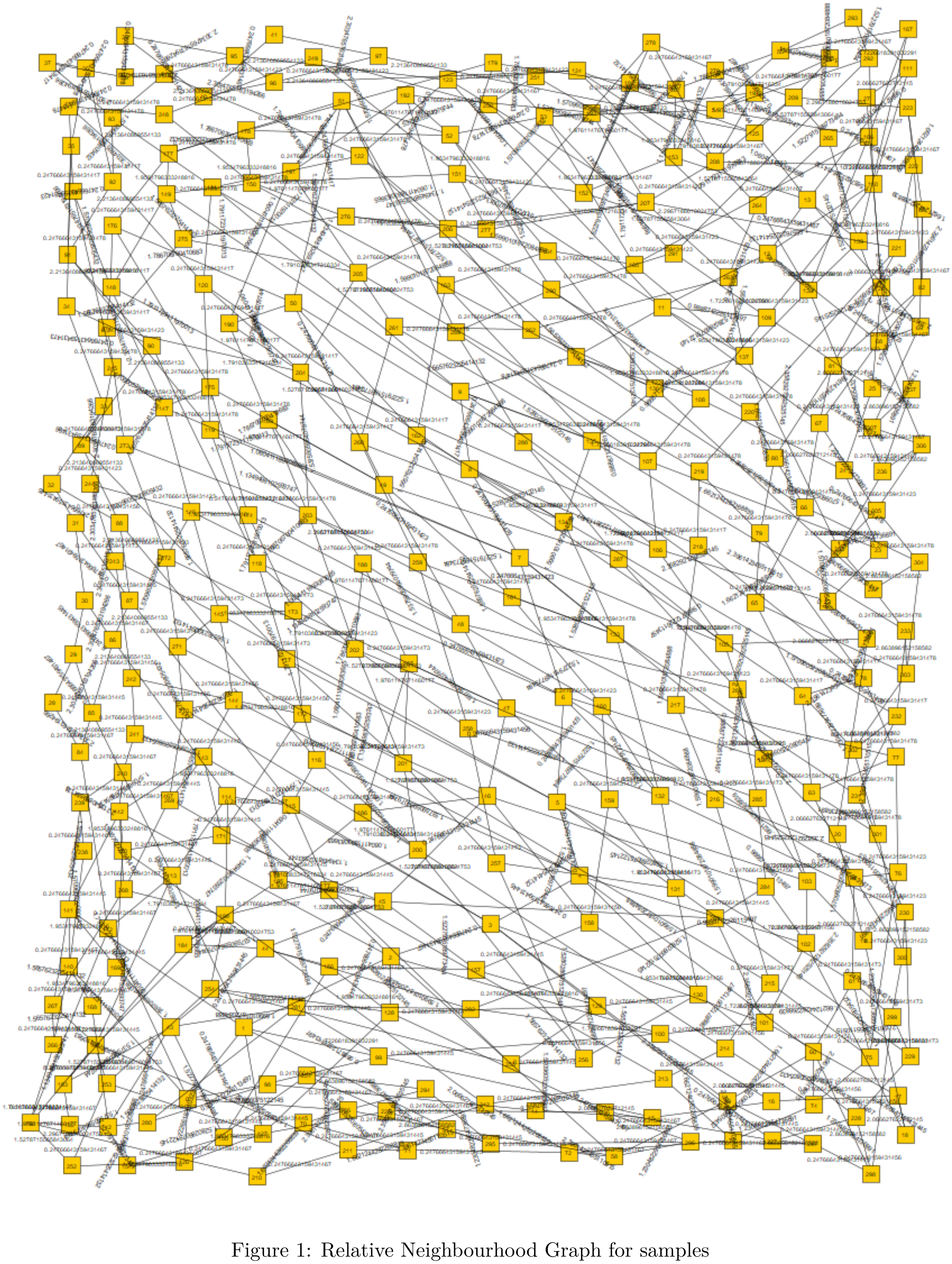


Figure 1: Relative Neighbourhood Graph for samples

```
## 39     0.0                5.1  ...                1.5                0.2
##
## [5 rows x 5 columns]
```

Next perform entropy-based Fayyad-Irani discretisation (https://www.ijcai.org/Proceedings/93-2/Papers/022.pdf) using `RWeka::Discretize`, removing the ambiguous features that result. Following the discretisation step, no features are removed the dataset.

```
py$iris_train$CLASS <- as.factor(py$iris_train$CLASS)
train_data <- Discretize(CLASS ~ ., data = py$iris_train)
# Remove any features that are identical after discretisation
train_data <- train_data[!duplicated(lapply(train_data, summary)) & !duplicated(lapply(train_data, summa
                                                                 fromLast = TRUE)]
head(train_data)
```

```
##   sepal length (cm) sepal width (cm) petal length (cm) petal width (cm) CLASS
## 1     '(-inf-5.45]'     '(3.35-inf)'     '(-inf-2.45]'     '(-inf-0.8]'     0
## 2     '(-inf-5.45]'    '(-inf-3.35]'     '(-inf-2.45]'     '(-inf-0.8]'     0
## 3     '(5.45-6.25]'    '(-inf-3.35]'      '(4.75-inf)'     '(1.65-inf)'     2
## 4     '(-inf-5.45]'     '(3.35-inf)'     '(-inf-2.45]'     '(-inf-0.8]'     0
## 5     '(-inf-5.45]'     '(3.35-inf)'     '(-inf-2.45]'     '(-inf-0.8]'     0
## 6     '(-inf-5.45]'    '(-inf-3.35]'     '(2.45-4.75]'     '(0.8-1.65]'     1
```

Aside: Save the cutpoints defined here for use on the test data.

```
cutpoints <- list()
feature <- character()
for (i in seq(1, length(names(train_data))-1)){ # for each feature
  feature[i] <- names(train_data)[i]
  t <- table(train_data[,feature[i]], train_data$CLASS)
  for (j in seq(1, length(levels(train_data[,names(train_data)[i]])))){ # for each bin of a feature
    print(names(which.max(t[j,]))) # the label to assign to the bin
    print(levels(train_data[,names(train_data)[i]])) # the values of the bin itself
  }
}
```

```
## [1] "0"
## [1] "'(-inf-5.45]'" "'(5.45-6.25]'" "'(6.25-inf)'"
## [1] "1"
## [1] "'(-inf-5.45]'" "'(5.45-6.25]'" "'(6.25-inf)'"
## [1] "2"
## [1] "'(-inf-5.45]'" "'(5.45-6.25]'" "'(6.25-inf)'"
## [1] "1"
## [1] "'(-inf-3.35]'" "'(3.35-inf)'"
## [1] "0"
## [1] "'(-inf-3.35]'" "'(3.35-inf)'"
## [1] "0"
## [1] "'(-inf-2.45]'" "'(2.45-4.75]'" "'(4.75-inf)'"
## [1] "1"
## [1] "'(-inf-2.45]'" "'(2.45-4.75]'" "'(4.75-inf)'"
## [1] "2"
## [1] "'(-inf-2.45]'" "'(2.45-4.75]'" "'(4.75-inf)'"
```

```
## [1] "0"
## [1] "'(-inf-0.8]'" "'(0.8-1.65]'" "'(1.65-inf)'"
## [1] "1"
## [1] "'(-inf-0.8]'" "'(0.8-1.65]'" "'(1.65-inf)'"
## [1] "2"
## [1] "'(-inf-0.8]'" "'(0.8-1.65]'" "'(1.65-inf)'"
```

```
# feature <- character()
# cutpoint <- numeric()
# for (i in seq(1, length(names(train_data))-1)){
#   feature[i] <- names(train_data)[i]
#
#   cutpoint[i] <- as.numeric(regmatches(levels(train_data[, i])[2], gregexpr("-?\\d.\\d+", levels(trai
# }
# ( cutpoints <- data.frame(feature, cutpoint) )
```

Genuinely don't know how to go on with this question. The chi-squared test might not even remove any features. and if I have three classes at the end, what sort of distance measure could I define? Might need to go and find a binary dataset after all.

Exercise 3

Formal mathematical definition of the k-Feature Set Problem:
Input: A set $X = \{x^{(1)}, x^{(2)}, ..., x^{(m)}\}$ of m examples having $x^{(i)} = \{x_1^{(i)}, x_2^{(i)}, ..., x_n^{(i)}, t^{(i)}\} \in \{0,1\}^{n+1} \; \forall \; i$, and an integer $k > 0$.
Question: Does there exist a feature set S, where $S \subseteq \{1, ..., n\}$, $|S| = k$, and for all pairs of examples $i \neq j$: if $t^{(i)} \neq t^{(j)} \; \exists \; I \in S$ such that $x_I^{(i)} \neq x_I^{(j)}$?

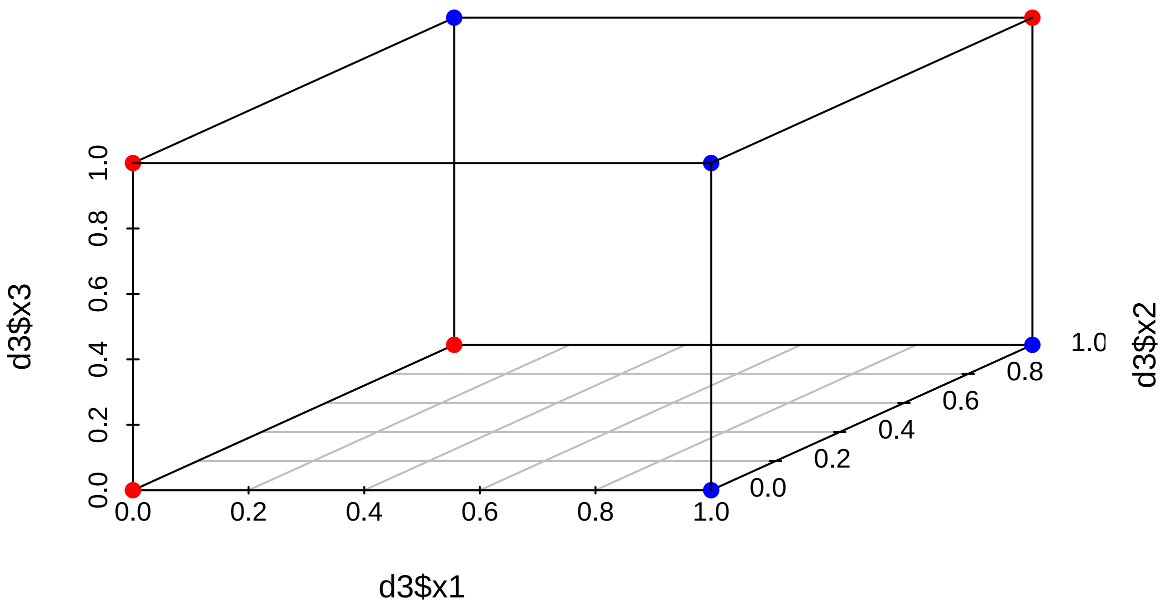
An example of the problem is as follows, where variables `x1`, `x2`, `x3`, `x4` represent characteristics of a student and `y` represents their grade on a test.

```
d3 <- data.frame(x1 = c(1,0,1,1,1,0,0,0),
                 x2 = c(0,1,1,0,1,1,0,0),
                 x3 = c(1,1,0,0,1,0,1,0),
                 x4 = c(0,1,0,1,0,1,0,1),
                 y  = as.factor(c(rep("pass", 4), rep("fail", 4))))
d3
```

```
##   x1 x2 x3 x4    y
## 1  1  0  1  0 pass
## 2  0  1  1  1 pass
## 3  1  1  0  0 pass
## 4  1  0  0  1 pass
## 5  1  1  1  0 fail
## 6  0  1  0  1 fail
## 7  0  0  1  0 fail
## 8  0  0  0  1 fail
```

It can be determined that this table has a 3-Feature set $F = \{x1, x2, x3\}$ and no 2-Feature sets. Additionally, the points are not linearly separable, as is demonstrated by the following 3-dimensional scatterplot (points of the "fail" class are plotted in red, while those in the "pass" class are plotted in blue):

```r
scatterplot3d(x = d3$x1, y=d3$x2, z=d3$x3, color = c("red", "blue")[d3$y], pch=19)
```



It is clear from inspecting `Figure X` that no single plane could be drawn that separates these points by colour. Therefore, the 3-Feature set $F = \{x1, x2, x3\}$ is not linearly separable.

Exercise 4

Formal mathematical definition of the l-Pattern Identification Problem:
Input: A finite alphabet Σ, two disjoint sets $Good, Bad \subseteq \Sigma^n$ of strings (where a string is a concatenation of symbols from that alphabet) and an integer $l > 0$.
Question: Is there a set P of patterns (where a pattern is a string s over an extended alphabet $\Sigma_* := \Sigma \cup \{*\}$) having $|P| \leq l$ and $P \to (Good, Bad)$?

A toy example of the problem is as follows, with factors contributing to a student achieving a passing or failing `grade` on a piano examination. The features consistute `level` the level of difficulty of the exam, `daily_practice` the amount of daily practice performed (>1hr, 1-2hrs, 2+hrs), `fitness` the regularity of the student's exercise, and `lesson_attendance` the student's regularity of lesson attendance. These are all measured on a scale with 3 levels - Low (L), Medium (M) and High (H).

```r
d4 <-  data.frame(level = c("L", "L", "L", "M", "L", "M", "H", "H"),
                  daily_practice = c("M", "H", "L", "H", "L", "L", "L", "M"),
                  fitness = c("M", "L", "M", "L", "M", "L", "H", "H"),
                  lesson_attendance = c("H", "M", "H", "L", "H", "L", "L", "L"),
                  grade = as.factor(c(rep("pass", 4), rep("fail", 4))))
d4
```

```
##   level daily_practice fitness lesson_attendance grade
## 1     L              M       M                 H  pass
## 2     L              H       L                 M  pass
## 3     L              L       M                 H  pass
## 4     M              H       L                 L  pass
## 5     L              L       M                 H  fail
## 6     M              L       L                 L  fail
## 7     H              L       H                 L  fail
## 8     H              M       H                 L  fail
```

The following 4-pattern solution is presented:
For `pass`,
$L * MH$
$* HL *$
For `fail`,
$* L * L$
$H * * L$
These two sets of patterns uniquely identify samples corresponding to their respective classes of `grade`, and each cover all existing samples in each group.
Examining the data shows no 3-pattern solutions.

Exercise 5

The Iris dataset from sklearn.datasets will be used for this task. The target cluster will be removed for the purpose of computing the MST.

```python
from operator import itemgetter
from sklearn import datasets
from sklearn.model_selection import train_test_split
iris = datasets.load_iris()
iris_data = pd.DataFrame(data= np.c_[iris['target'], iris['data']],
                     columns= ['CLASS'] + iris['feature_names'])
iris_features = iris_data.drop("CLASS", axis = 1) # remove the target from a copy of the data
i_names = [str(i) for i in iris_features.index.values] # Define node names by row
print(iris_features.head())
```

```
##    sepal length (cm)  sepal width (cm)  petal length (cm)  petal width (cm)
## 0                5.1               3.5                1.4               0.2
## 1                4.9               3.0                1.4               0.2
## 2                4.7               3.2                1.3               0.2
## 3                4.6               3.1                1.5               0.2
## 4                5.0               3.6                1.4               0.2
```

The remaining dataframe has 150 samples and four features.

The data will be normalised using a z-score transformation, and a distance matrix defined using the Euclidean distance measure.

```python
# Normalise the data
norm_iris_data = (iris_features - iris_features.mean())/iris_features.std()
def euclid_distance(s1, s2):
    return math.sqrt(np.sum((s1-s2) ** 2))
```

```python
# Distance matrix for samples:
iris_samples_edist_m = [[euclid_distance(norm_iris_data.iloc[y],
  norm_iris_data.iloc[x]) for y in range(len(i_names))] for x in range(len(i_names))]
round(pd.DataFrame(iris_samples_edist_m, columns = i_names, index = i_names), 3)
```

```
##          0      1      2      3      4  ...    145    146    147    148    149
## 0    0.000  1.172  0.843  1.100  0.259  ...  4.156  4.062  3.793  3.813  3.324
## 1    1.172  0.000  0.522  0.433  1.382  ...  4.117  3.648  3.734  4.004  3.203
## 2    0.843  0.522  0.000  0.283  0.988  ...  4.303  3.960  3.923  4.059  3.369
## 3    1.100  0.433  0.283  0.000  1.246  ...  4.297  3.875  3.910  4.084  3.329
## 4    0.259  1.382  0.988  1.246  0.000  ...  4.282  4.239  3.923  3.878  3.446
## ..     ...    ...    ...    ...    ...  ...    ...    ...    ...    ...    ...
## 145  4.156  4.117  4.303  4.297  4.282  ...  0.000  1.356  0.462  1.104  1.169
## 146  4.062  3.648  3.960  3.875  4.239  ...  1.356  0.000  1.185  2.146  1.253
## 147  3.793  3.734  3.923  3.910  3.923  ...  0.462  1.185  0.000  1.068  0.773
## 148  3.813  4.004  4.059  4.084  3.878  ...  1.104  2.146  1.068  0.000  1.197
## 149  3.324  3.203  3.369  3.329  3.446  ...  1.169  1.253  0.773  1.197  0.000
##
## [150 rows x 150 columns]
```

5. a)

```python
class Edge:
    def __init__(self, s, w, e):
        self.start = s
        self.weight = w
        self.end = e
    def get_edge(self):
        return (str(self.start) + '-' + str(self.weight) + '-' + str(self.end))
    def get_nx_edge(self):
        return ((self.start, self.end, {'label': str(self.weight)}))

def minimum_spanning_tree_Prims(G):
    V = [i for i in range(len(G))] # nodes in the graph
    W = [] # nodes in the MST
    adj_weights = [0] + [float('inf')] * (len(G) - 1) # the [0] initalises the first node into the MST
    u = [-1] * len(G)
    def closestNode(weights, added_nodes): # find the nearest node in the 'adjacent weights' list
        m = float('inf') # minimum edge distance
        v = -1 # node to return
        for i in range(len(weights)):
            if ((i not in added_nodes) & (weights[i] < m)):
                m = weights[i]
                v = i
        return v

    while (set(W) != set(V)):
        v = closestNode(adj_weights, W)
        if (v != -1):
            W.append(v)
        for i in range(len(G)): # for each node:
```

```
            if ((i not in W) & (G[v][i] < adj_weights[i])):
                u[i] = v
                adj_weights[i] = G[v][i]
    return W, u

def get_mst(matrix, names, output_file):
    W, u = minimum_spanning_tree_Prims(matrix)
    node_labels = [names[n] for n in W]
    F = []
    for f in range(1, len(matrix)): # unordered
        F.append(Edge(names[u[f]], matrix[f][u[f]], names[f]))
    G = nx.Graph()
    G.add_nodes_from(node_labels)
    G.add_edges_from([f.get_nx_edge() for f in F])
    nx.write_gml(G, output_file)
    return G
five_a_result = get_mst(iris_samples_edist_m, i_names, 'graph_5a.gml')
```

The output file graph_5a.gml was processed using yEd Graph Editor to produce the following visualisation in `Figure XXXX`:

```
graph_5a <- readPNG("graph_5a.png")
grid.raster(graph_5a)
```

5. b)

```
def k_NNG(G, K):
    P = [i for i in range(len(G))]
    kNNG = []
    for p in P:
        dist = []
        for q in P:
            if (p != q):
                dist.append([p, G[p][q], q])
        sorted_dist = sorted(dist, key = itemgetter(1))
        max_dist = sorted_dist[K-1][1]
        for d in sorted_dist:
            if d[1] <= max_dist:
                kNNG.append(d)
    return kNNG

def get_k_NNG(k, matrix, names, output_file):
    E = k_NNG(matrix, k)
    V = [i for i in range(len(matrix))]
    kNNG = []
    node_labels = [names[n] for n in V]
    for e in range(len(E)):
        kNNG.append(Edge(node_labels[E[e][0]], matrix[E[e][0]][E[e][2]], node_labels[E[e][2]]))
    G = nx.Graph()
    G.add_nodes_from(node_labels)
```

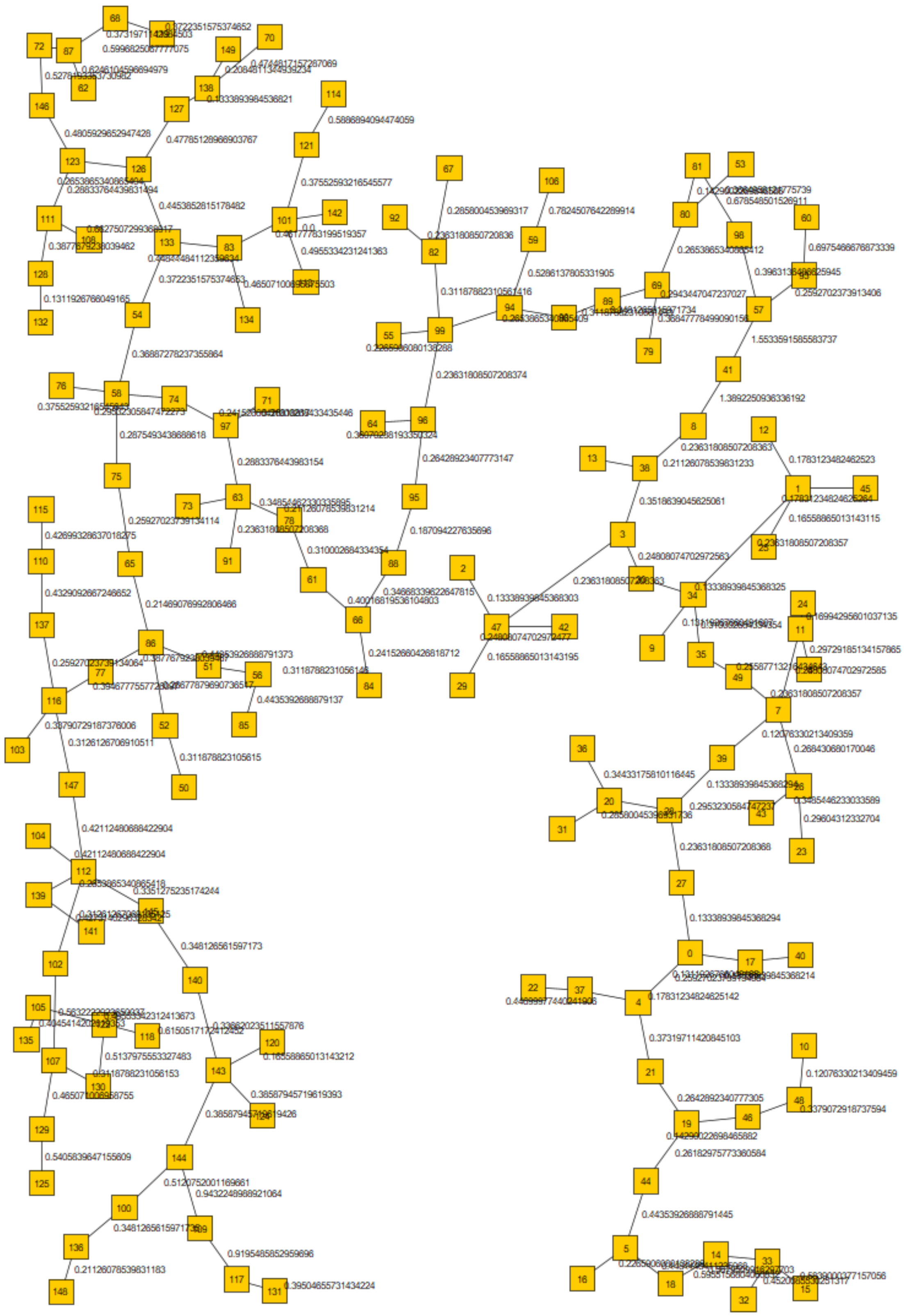


Figure 2: Minimum Spanning Tree for Iris dataset samples

```
    G.add_edges_from([e.get_nx_edge() for e in kNNG])
    nx.write_gml(G, output_file)
    return G
five_b_result = get_k_NNG(3, iris_samples_edist_m, i_names, 'graph_5b.gml')
```

The output file graph_5b.gml was processed using yEd Graph Editor to produce the following visualisation in `Figure XXXX`:

```
graph_5b <- readPNG("graph_5b.png")
grid.raster(graph_5b)
```

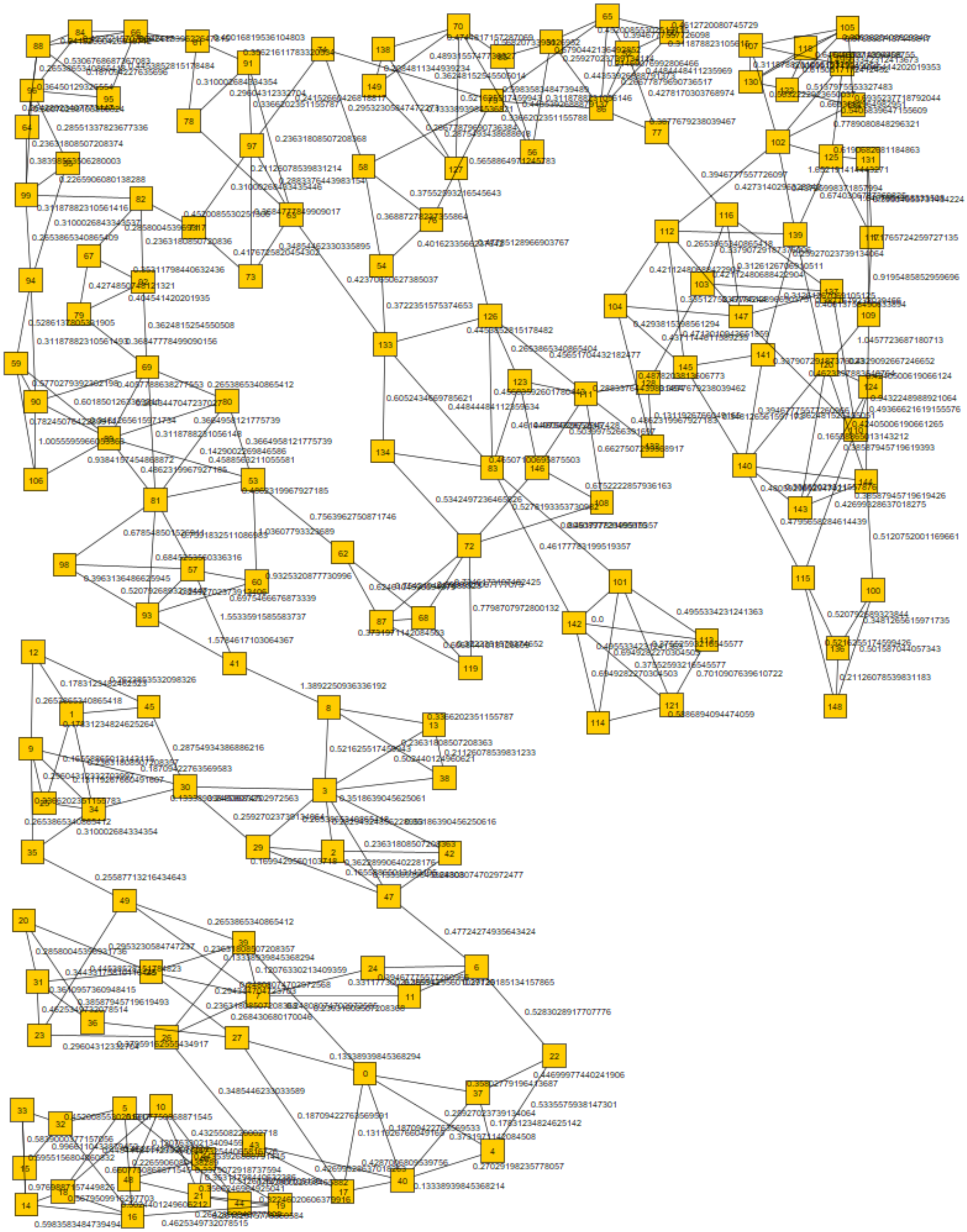


Figure 3: k-nearest-neighbours for Iris dataset samples

5. c)

```
MST_edges = list(five_a_result.edges)
kNNG_edges = list(five_b_result.edges)
common_edges = list(set(MST_edges).intersection(kNNG_edges))

# yEd output
G = nx.Graph()
G.add_nodes_from(i_names)
G.add_edges_from(common_edges)
nx.write_gml(G, 'graph_5c.gml')

# tabulate the clusters
cluster = []
for i in i_names:
  cluster.append(sorted(list(nx.node_connected_component(G, i))))
u = [list(x) for x in set(tuple(x) for x in cluster)]
count = len(u)
clusters = list(zip(range(count), u))
clusters = pd.DataFrame(clusters, columns = ['cluster_label', 'samples'])
```

5. d)

The output file graph_5c.gml was processed using yEd Graph Editor to produce the following visualisation in `Figure XXXX`:

```
graph_5d <- readPNG("graph_5d.png")
grid.raster(graph_5d)
```

The results were tabulated in part c, and the results printed below:

```
print(clusters)
```

```
##     cluster_label                    samples
## 0               0                       [10]
## 1               1            [1, 12, 25, 45]
## 2               2  [11, 24, 26, 43, 49, 7]
## 3               3                       [74]
## 4               4                       [32]
## ..            ...                        ...
## 68             68                      [118]
## 69             69                      [124]
## 70             70                   [65, 86]
## 71             71                       [60]
## 72             72                      [108]
##
## [73 rows x 2 columns]
```

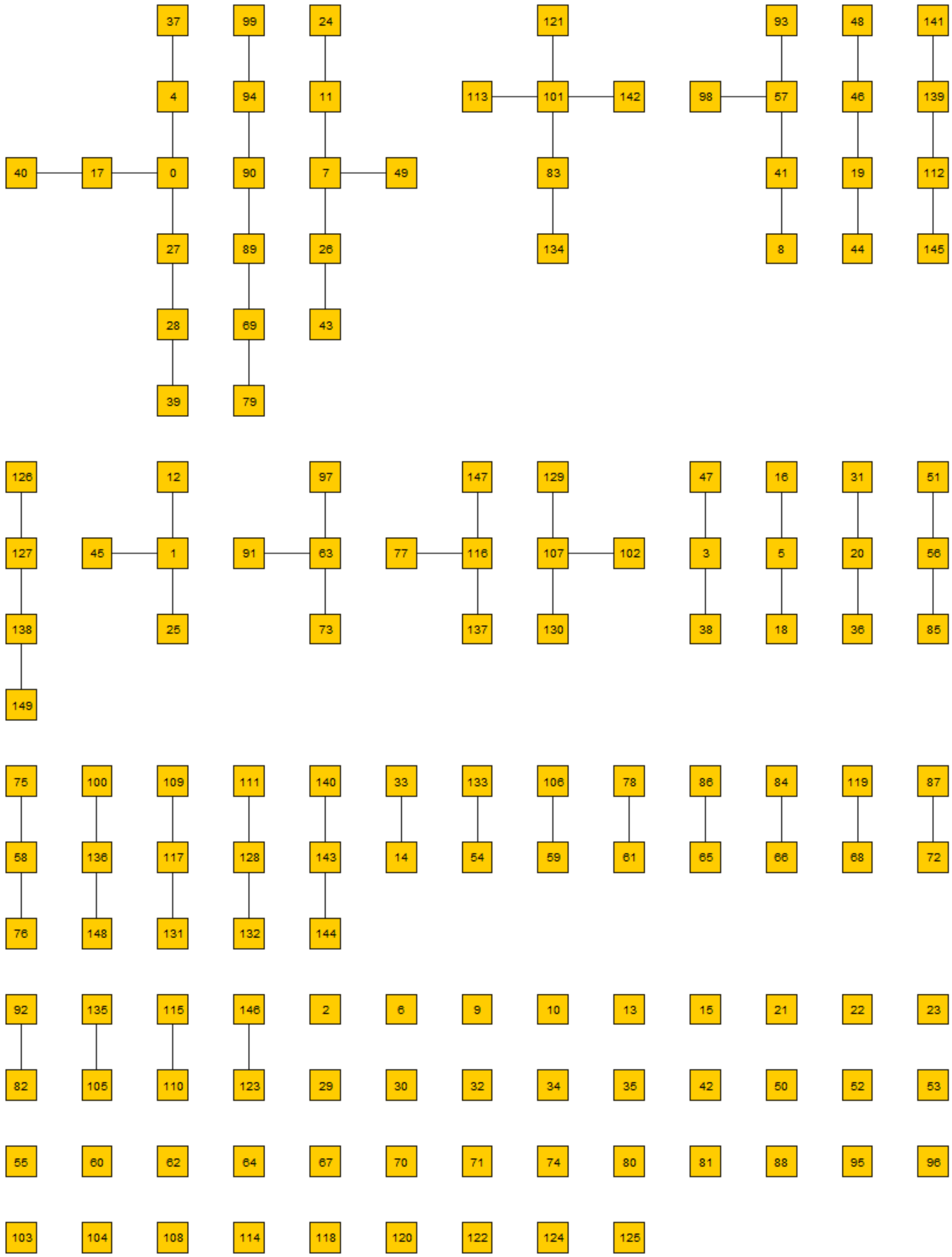


Figure 4: MST-kNN for Iris dataset samples

Exercise 6

6. a)

6. b)

6. c)

Exercise 7

7. a)

7. b)

Exercise 8

- How is this different from Exercise 6? Should we incorporate those 2 things and a last one?

Exercise 9

9. a)

```r
pres_data <- read.csv("USPresidency.csv")
pres_data[pres_data$Target == 0,]
```

```
##    Year Q1 Q2 Q3 Q4 Q5 Q6 Q7 Q8 Q9 Q10 Q11 Q12 Target
## 19 1860  1  0  1  1  0  0  1  0  1   0   0   0      0
## 20 1876  1  1  0  1  0  1  0  0  0   1   0   0      0
## 21 1884  1  0  0  1  0  0  1  0  1   0   1   0      0
## 22 1892  0  0  1  0  1  0  0  1  1   0   0   1      0
## 23 1896  0  0  0  1  0  1  0  1  1   0   1   0      0
## 24 1912  1  1  1  1  1  0  1  0  0   0   0   0      0
## 25 1920  1  0  0  1  0  1  0  1  1   0   0   0      0
## 26 1932  1  1  0  0  1  1  0  0  1   0   0   1      0
## 27 1952  1  0  0  1  0  0  0  0  0   1   0   1      0
## 28 1960  1  1  0  0  0  1  0  0  0   0   0   1      0
## 29 1968  1  1  1  1  0  0  1  1  1   0   0   0      0
## 30 1976  1  1  0  1  1  0  0  0  0   1   0   0      0
## 31 1980  0  0  1  1  1  1  1  0  0   1   0   1      0
```

```r
pres_data[pres_data$Target == 1,]
```

```
##    Year Q1 Q2 Q3 Q4 Q5 Q6 Q7 Q8 Q9 Q10 Q11 Q12 Target
## 1  1864  0  0  0  0  1  0  0  1  1   0   0   0      1
## 2  1868  1  1  0  0  0  0  1  1  1   0   1   0      1
## 3  1872  1  1  0  0  1  0  1  0  0   0   1   0      1
## 4  1880  1  0  0  1  0  0  1  1  0   0   0   0      1
```

```
## 5  1888  0  0  0  0  1  0  0  0  0   0   0   0      1
## 6  1900  0  1  0  0  1  0  1  0  0   0   0   1      1
## 7  1904  1  1  0  0  1  0  1  0  0   0   1   0      1
## 8  1908  1  1  0  0  0  1  0  1  0   0   0   0      1
## 9  1916  0  0  0  0  1  0  0  1  0   0   0   0      1
## 10 1924  0  1  1  0  1  0  1  1  0   1   0   0      1
## 11 1928  1  1  0  0  0  0  1  0  0   0   0   0      1
## 12 1936  0  1  0  0  1  1  1  1  0   0   1   0      1
## 13 1940  1  1  0  0  1  1  1  1  0   0   1   0      1
## 14 1944  1  1  0  0  1  0  1  1  0   0   1   0      1
## 15 1948  1  1  1  0  1  0  0  1  0   0   0   0      1
## 16 1956  0  1  0  0  1  0  1  0  0   0   1   0      1
## 17 1964  0  0  0  0  1  0  1  0  0   0   0   0      1
## 18 1972  0  0  0  0  1  0  0  1  1   0   0   0      1
```

9. b)

Exercise 10

Exercise 11

Exercise 12

12. a)

12. b)

12. c)