Lemma 5.5. *The estimator* $\mathbf{t}^\intercal\mathbf{y}$ *is the BLUE for its expectation* $\mathsf{E}(\mathbf{t}^\intercal\mathbf{y})$ *if and only if* $\mathsf{Cov}(\mathbf{t}^\intercal\mathbf{y}, \mathbf{c}^\intercal\mathbf{y}) = 0$ *for all* $\mathbf{c}$ *such that* $\mathbf{c}^\intercal\mathbf{y}$ *is an unbiased estimator of zero.*

Corollary 5.6. *Under the Aitken model, the estimator* $\mathbf{t}^\intercal\mathbf{y}$ *is the BLUE for its expectation* $\mathsf{E}(\mathbf{t}^\intercal\mathbf{y})$ *if and only if* $\mathbf{Vt} \in \mathcal{C}(\mathbf{X})$.

Theorem 5.7. *Under the Aitken model, all ordinary least squares estimator* $\boldsymbol{\lambda}^\intercal\hat{\mathbf{b}}_{OLS}$ *are the BLUE for the corresponding (linearly) estimable* $\boldsymbol{\lambda}^\intercal\mathbf{b}$ *if and only if there exists a matrix* $\mathbf{Q}$ *such that* $\mathbf{VX} = \mathbf{XQ}$.

Result 4.1 *The BLUE* $\lambda^T\hat{\mathbf{b}}$ *of estimable* $\lambda^T\mathbf{b}$ *is uncorrelated with all unbiased estimators of zero.*

$E(h^Ty) = h^TXb = 0$, for all b.

$h^TX = 0$

$h \in N(X^T)$.

$Cov(t^Ty, h^Ty) = \sigma^2 t^TVh = 0$

$Vt \in C(X)$

$Vt = Vt_1 + Vt_2$, $\quad Vt_1 \in N(X^T)$, $Vt_2 \in C(X)$.

$Cov(t^Ty, h^Ty) = \sigma^2 t^TVh = \sigma^2 t_1^TVh + \underbrace{\sigma^2 t_2^TVh}_{\to 0}$

$= \sigma^2 t_1^TVh$

$E(t^TVy) = t^TVXb = \underbrace{t_1^TVXb}_{0} + t_2^TVXb = t_2^TVXb$