March 9 | Note 5

$(x_i, y_i)_{i=1}^{N}$

$\Sigma (y_i - x_i^T b)^2$

$y = Xb + e$ ，估计 b

$$\min Q(b) = \|y - Xb\|^2 = \sum_{i=1}^{N} (y_i - x_i^T b)^2$$

对数据结构的假定：

1. 每个样本观测数据各占一项，并且不与其它样本有牵连，即假定了各次观测独立或至少不相关 $\mathrm{cov}(e_i, e_j) = 0$

2. 各样本观测数据在表达式 $Q(b)$ 中权重相等，即假定了各样本数据具有大致一样的方差 $\mathrm{Var}(e_i) = \sigma^2$

在 Gauss - Markov 条件下，OLS 显得更加合理.

(Gauss - Markov Thm)

Under G-M model, $\lambda^T b$ estimatable, then $\lambda^T \hat{b}_{OLS}$ is BLUE of $\lambda^T b$.

G-M 条件不满足时，$\hat{b}_{OLS}$ 还是一个“最好”的估计吗？

$\longrightarrow$ Aitken Model

$\mathrm{Var}(e) = \sigma^2 V$， $RVR^T = I$， $z = Ry = RXb + Re = Ub + f$，

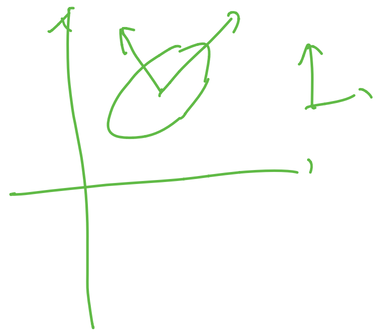
$\mathrm{Var}(f) = \sigma^2 I$

$\longrightarrow \hat{b}_{GLS}$

G-M

~~M-C~~ model $\hat{b}_{OLS} = \operatorname{argmin} d_1(y, Xb) = \underbrace{(y - Xb)^T (y - Xb)}_{\text{Euclidean distance}}$

Aitken model $\hat{b}_{GLS} = \operatorname{argmin} d_2(y - Xb) = \underbrace{(y - Xb)^T V^{-1} (y - Xb)}_{\text{Mahalanobis distance}}$



$$y = Xb + e \quad , \quad \underbrace{Var(e) = V}_{rank(V) = r < N} \text{ positive semi-definite}$$

$$\exists R, \text{ s.t. } \underline{RVR^T} = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} U_1 \\ U_2 \end{bmatrix} b + \begin{bmatrix} f \\ 0 \end{bmatrix} \quad , \quad Var(f) = I_r$$

$$y = Xb + e$$

$$\underline{Ry = RXb + \underline{Re}}$$

$$Var(Re) = R \, var(e) \, R^T = RVR^T = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}$$

$$z = Ub + f$$