1. 验证 $\langle Pa, b\rangle = \langle a, Pb\rangle$

P正交投影

$$a = a_p + a_N \quad , \quad a_p = Pa \perp a_N = I - Pa$$

$$b = b_p + b_N$$

$$\langle Pa, b\rangle = \langle a_p, b_p + b_N\rangle$$

$$= \langle a_p, b_p\rangle \qquad \langle a, Pb\rangle = \langle a_p, b_p\rangle$$

$$\underline{\langle a, Pb\rangle = \langle Pa, b\rangle}$$

$$\Rightarrow \quad (Pa)^T b = a^T P b$$

$$\Rightarrow \quad a^T P^T b = a^T P b$$

$$\Rightarrow \quad P^T = P \qquad\qquad P^H = P$$

2.

$$\hat{b} = X^* y + (I - X^* X) z$$

$$= (X^T X)^- X^T y + \cdots -$$

minimal norm

$$(X^T X)^+ X^T y \overset{?}{=} X^+ y$$

验证 $X^+ = (X^T X)^+ X^T$ X'

1、 $X X' X = X$ 显然

2、 $X' X X' = X'$

$$\underline{(X^T X)^+ X^T X (X^T X)^+} X^T = \underline{(X^T X)^+} X^T$$

3、 $X' X$ symmetric

$$(X^T X)^+ X^T X$$

4. $X X'$ symmetric

$P_X = X \quad X^T X)^+ X^T$ Thm 3.9