HAVE YOU SEEN?

Below are some explanations or examples of the four advanced concepts mentioned in the questionnaire. Hopefully these clarify whether you have seen these or not.

- **Integration by parts:** Given an integral, the "integration by parts formula" is $\int u\,dv = uv - \int v\,du$. Thus, to integrate $\int xe^x\,dx$, one might allow $u = x$, $dv = e^x dx$, resulting in $du = dx$ and $v = e^x$. One can then complete the integration process by using the above formula.

- **Improper integrals:** An improper integral is one with either an infinite value in the lower or upper limit of the integral, or the integral is over an interval $[a, b]$ and the integrand takes on infinite values (positive or negative or both) somewhere in this interval. Examples: $\int_0^\infty x^2\,dx$ or $\int_0^4 \frac{3}{x-2}\,dx$.

- **Sequences and Series:** A sequence is a list of numbers, often represented by a pattern. For example, $1, 1/2, 1/4, 1/8, 1/16, \ldots$ is a sequnce. Questions to ask about sequences include finding a description of the n^{th} term (in this case $\frac{1}{2^{n-1}}$) and determining the limit (in this case 0). A series can be loosely thought of as the sum of a sequence. For example, the series corresponding to the sequence above is $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots$. We ask questions about convergence for series as well. In other words, given an infinite series, does it converge or not. We might also ask to what it converges, if it converges. Here, our series converges to 2. Another similar series, $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \ldots$, called the harmonic series does not converge.

- **Taylor polynomials and series:** We just saw how series of numbers are defined but sometimes we also consider series involving variables. For example, we may also be interested in series that can represent functions for some values of x. We then ask about the radius of convergence. For example, the function e^x can be represented by $1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots$ for all values of x. This is the Maclaurin series for e^x, a special case of a Taylor series. A Taylor polynomial is "finite version" of a Taylor series.