

CS4510 Automata and Complexity

Spring 2020 Section A

Test 4*Instructor: Richard Peng*

11:20am - 2:10pm, Monday, Apr 27, 2020

- This test is **online**, posted via Canvas and the course homepage, and handed in via GradeScope.
- The GradeScope submission page will close at 2:30pm (Eastern Time), but we suggest that you start wrapping up by around 2:10pm.
- You have 170 minutes to earn up to $1 + 6 + 4 + 5 + 5 + 5 = 26$ points, the test is graded out of 25.
- This booklet contains **5 questions on 6 pages**, including this one.
- Write your solutions in the space provided. If you run out of space, continue your answer on the back of the last page, and make a notation on the front of the sheet.
- You may use any written or locally stored resources.
- However, the only internet resources that you could use during the test are: specifically:
 1. The BlueJeans Office Hours link at <https://gatech.bluejeans.com/242024735>, where clarifications will be posted in the chat.
 2. Piazza Test 4 clarification page: <https://piazza.com/class/k4xbfrttfnc687?cid=661>.
- You may use any of the theorems/facts/lemmas from the lecture notes, homeworks, or textbook without re-proving them unless explicitly stated otherwise. However, we may check with you about the citation / reference afterwards, possibly via regrades.
- If necessary make reasonable assumptions but please be sure to state them clearly
- Do not spend too much time on any one problem. Generally, a problem's point value is a good indication of how many minutes to spend on it.
- Good luck!

0. (/1 point) Submit your test through GradeScope before 2:30pm, under the right name/id, and with the pages corresponding to each problem clearly indicated.

1. (/6 points, 3 each) Provide solutions to the following two instances of the post correspondence problem. **Make sure to specify your answer as the sequence of indices.**

Recall that the post correspondence problem is given strings $a_1 \dots a_n$ and $b_1 \dots b_n$, find a sequence of (possibly repeating) indices $i_1 \dots i_m$ such that

$$a_{i_1} a_{i_2} \dots a_{i_m} = b_{i_1} b_{i_2} \dots b_{i_m}.$$

Here the strings are provided as $\frac{a_i}{b_i}$.

(a)

$$1 : \frac{baaa}{bb}, \quad 2 : \frac{ab}{a}, \quad 3 : \frac{a}{aa}.$$

(b)

$$1 : \frac{000000}{0}, \quad 2 : \frac{0}{00000}.$$

Name:

User ID (NOT the 9 digit number):

2. (/5 points) Show that the following language is undecidable:

$$A = \{M \mid M \text{ is a Turing machine and } M \text{ halts on all inputs}\}.$$

You may reduce from any undecidable problem that we discussed, or in the text book.

3. (/5 points) Consider the language over the binary alphabet $\Sigma = \{0, 1\}$:

$$L = \{w \in \{0, 1\}^* \mid \text{every prefix of } w \text{ has more 0s than 1s}\}$$

Use pumping lemma to show that L is not regular.

Note that ‘more’ here means strictly more, and does not include the equality case.

4. (/5 points)

Consider the PDA below with alphabet $\Sigma = \{0, 1\}$ and stack symbols $\Gamma = \{x_0, x_1, \$\}$.

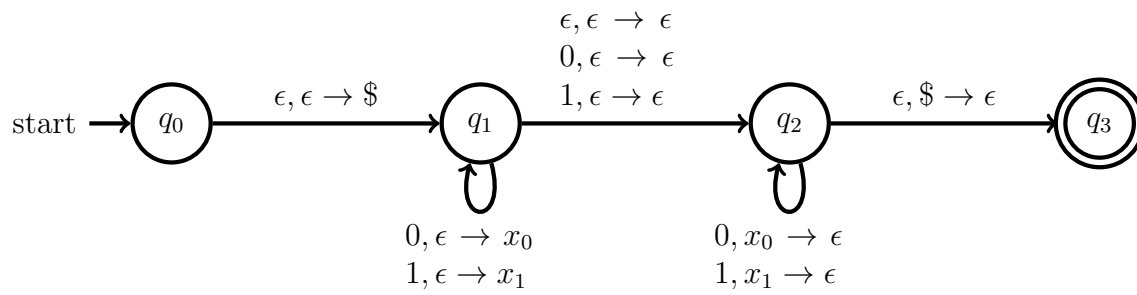


Exhibit a sequence of states / stack states by which this PDA accepts the input string

0110.

Make sure to indicate the direction of the stack in the stack states.

5. (/5 points)

Describe a deterministic Turing machine that takes as input (encoded as binary):

- (a) a context free grammar G (given as variables, terminals/alphabet, rules, starting variable),
- (b) a string s
- (c) a positive integer k

and returns whether one can modify at most k characters from s to create a string that's in $L(G)$ (the set of strings that can be generated from the grammar G).

You may use any modifications to Turing machines discussed in the text book, or in class. However, you need to be specific about how you manipulate the given CFG grammar. Furthermore, note that the problem is asking for a **deterministic** Turing machine.