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HW 5

Counting Problems

1) Case 1: 1 "u" $5!$
Case 2: 2 "u" $\binom{4}{3} \cdot \frac{5!}{2!}$
Case 3: 3 "u" $\binom{4}{2} \cdot \frac{5!}{3!}$

a) There are $1 + \binom{4}{3} + \binom{4}{2}$ or 11 unique subsets of 5 letters.
b) You can make $5! + \binom{4}{3}\frac{5!}{2!} + \binom{4}{2}\frac{5!}{3!}$ or 480 possible strings.

2) $\binom{4}{2}\binom{4}{2}\binom{13}{2}\binom{44}{1} = \boxed{123{,}552 \text{ ways}}$

3) Stars and bars BUT some are fixed
stars = songs
Assuming the fighting couple is fixed,
$*|$

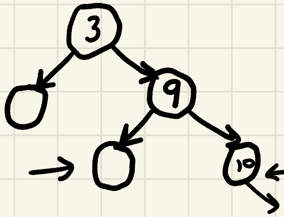
15 remaining stars
5 remaining bars $\binom{20}{5}$

OR

16 stars
5 bars $\binom{21}{5}$

$$\boxed{\binom{20}{5} + \binom{21}{5} = 35853 \text{ ways}}$$

4)



Left subtree: 2 options for 2 nodes

3, 2, 1 or 3, 1, 2

← 5 options for 3 nodes

10, 11, 12   10, 11, 12   12, 10, 11   12, 11, 10   11, 10, 12

a<b<c

87654

$\binom{5}{3}(2)(2)$

$\binom{5}{1}$

$a<b<c<d$

a: 5 options bcd   b: a, 2 options cd   c: ab 2 options, d   d: 5 options abc

$\underbrace{\qquad\qquad}_{\text{14 options for 4 node}}$

$a<b<c<d<e$

a: 14   b: a, 5   c: 2, 2   d: 5, e   e: 14

$\underbrace{\qquad\qquad}_{\text{42 options}}$

$$\begin{array}{r} 28 \\ 10 \\ 4 \\ \hline 42 \end{array}$$

$\underbrace{2}_{\text{Left subtree}} \cdot \underbrace{5}_{10, 11, 12} \cdot \underbrace{42}_{\text{everything else}} = \boxed{\text{420 possible trees}}$

5) Case 1: Lunch Break

↳ Reframe problem: how many ways to roll a 10 on 3 8-sided dice?

8, 1, 1
7, 1, 2
6, 1, 3   6, 2, 2
5, 1, 4   5, 2, 3

4, 4, 2
3, 3, 4

Case 2: no lunch break

↳ roll a 10 on 4 7-sided dice

7, 1, 1, 1
6, 1, 1, 2
5, 1, 1, 3   5, 1, 2, 2
4, 1, 1, 4   4, 1, 3, 1   4, 2, 2, 2

3, 3, 2, 2
3, 3, 3, 1
3, 4, 1, 2

$\boxed{\text{18 total combinations}}$