Algorithm Analysis

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Recursion Trees

- Missed some of this because I was running late but I took screenshots of what he wrote when I joined on my phone so I'll update my notes later

Heapsort

- What is a heap?
    - Basically an array viewed as an almost complete binary tree

Max Heap

- Max heap is a heap where the parent is greater than or equal to the child, e.g. $A[\text{parent}] \geq A[\text{child}]$
    - LEFT$(i) = 2i$
    - RIGHT$(i) = 2i + 1$
    - PARENT$(i) = \lfloor \frac{i}{2} \rfloor$
- Is the heap $\langle 4, 16, 10, 14, 7, 9, 3, 2, 8, 1 \rangle$ a max heap? No, $16 > 4$
    - However, if you take two subtrees where the roots are 16 and 10, the subtrees are max heaps
- There is an algorithm that'll turn a heap into a max heap

Max-Heapify

- The algorithm is top-down
- Assume that two subtrees are max heaps
- What is the worst-case time complexity for this algorithm? $\Theta(\lg(n))$
- What about the space complexity? Just $\Theta(1)$, it doesn't require any extra space
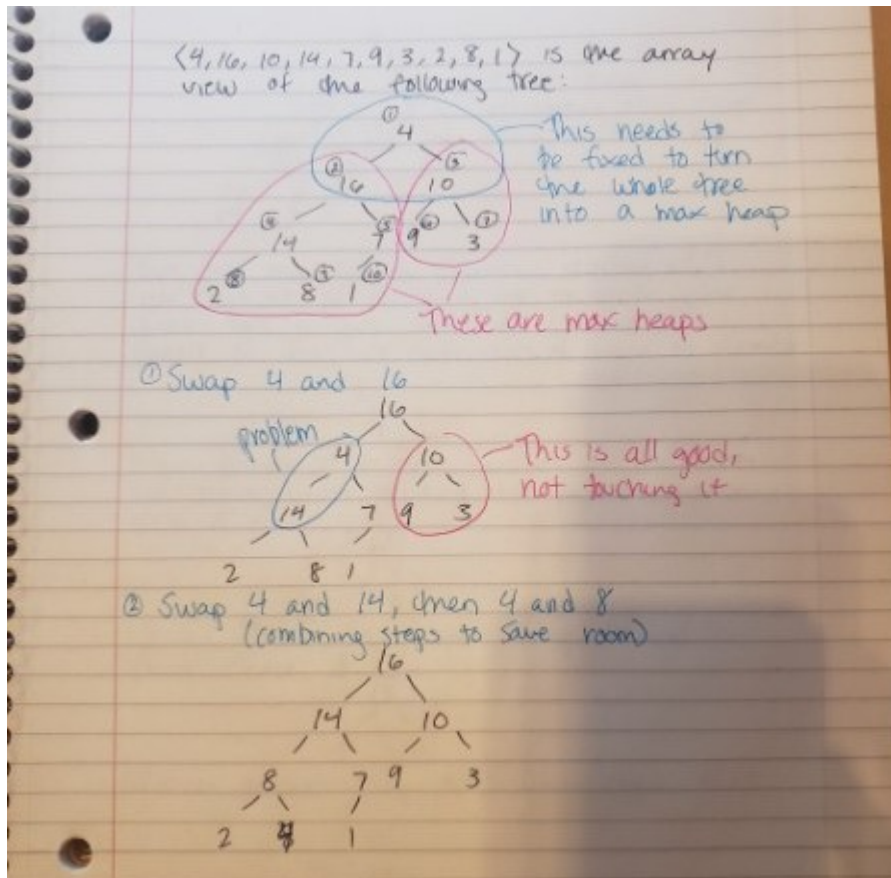
Build-Max-Heap

- Algorithm for our computer problem on the quiz
- Take the heap $\langle 4, 1, 3, 2, 16, 9, 10, 14, 8, 7 \rangle$
- Space complexity is $\Theta(1)$
- Time complexity is $O(n \lg(n))$ but it isn't precise, the precise one is $\Theta(n)$
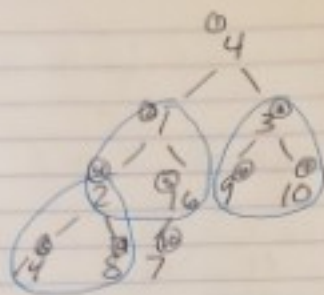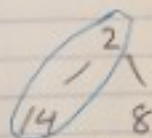
All of these are
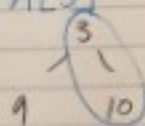preventing this from
being a max-heap

Run the max-heapify algorithm, starting w/ 16.
We start at 16 b/c it is the maximum index
that has a child. Start at ⌊A.length/2⌋.
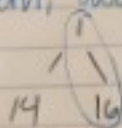Parent of the last node. Lots of ways to remember
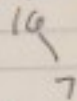
When looking at a node, you look at its subtree.
No issues with 16 and its children, move
onto A[4]
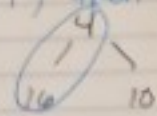

Uh oh! There's a problem. Swap 14 and 7.
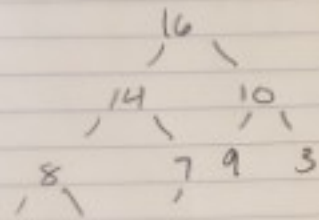Then check A[3]

Another problem, Swap 3 + 10 then check A[2]

Swap 1 + 16, do we move onto A[1] or continue
dropping 1? Continue dropping 1.

```
        1
       /
      7
```

Swap 1 + 7, then go back to A[1]

```
      4
     / \
   16   10
```

Swap 4 + 16. continue to drop 4. Final tree:

```
        16
       /  \
      14   10
     / \   / \
    8   7 9   3
   / \  /
  2   4 1
```

New array: ⟨16, 14, 10, 8, 7, 9, 3, 2, 4, 1⟩

Quiz

- We'll need to know the build-max-heap algorithm, it'll be the computer problem
- Given an input array, give the output at a specific index
- The human problem will be asking about parents/children/grand parents/leaves/etc