

# CS 5173/4173 Computer Security

## Topic 2. Introduction to Cryptography

# Cryptography

- *Cryptography*: the art of secret writing
- Converts data into unintelligible (random-looking) form
  - Must be *reversible* (can recover original data without loss or modification)
- If cryptography is combined with compression
  - What is the right order?

# Cryptography vs. Steganography

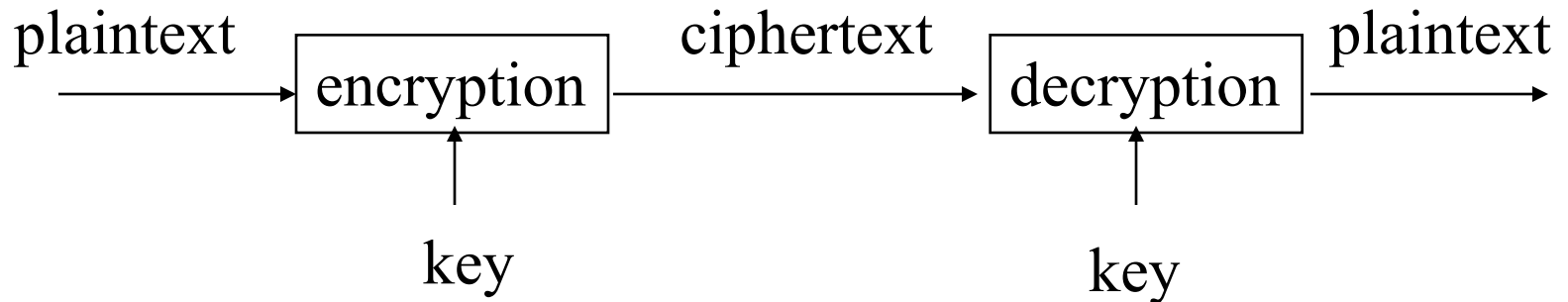
- *Steganography* concerns **existence**
  - Conceals the very existence of communication
    - Examples?

*A**p**parently **n**eutral's **p**rotest **i**s **t**horoughly **d**iscounted **a**nd **i**gnored.  
**I**sman **h**ard **h**it. **B**lockade **i**ssue **a**ffects **p**retext **f**or **e**mbargo **o**n **b**y-  
**p**roducts, **e**jecting **s**uets **a**nd **v**egetable oils.*

**Pershing sails from NY June 1**

- *Cryptography* concerns **what**
  - Conceals the contents of communication between two parties

# Encryption/Decryption



- Plaintext: a message in its original form
- Ciphertext: a message in the transformed, unrecognized form
- Encryption: the process that transforms a plaintext into a ciphertext
- Decryption: the process that transforms a ciphertext to the corresponding plaintext
- Key: the value used to control encryption/decryption.

# Cryptanalysis

- Cryptanalysis: the art of revealing the secret
  - Defeat cryptographic security systems
  - Gain access to the real contents of encrypted messages
  - Cryptographic keys can be unknown
- Difficulty depends on
  - Sophistication of the encryption/decryption
  - Amount of information available to the code breaker

# Ciphertext Only Attacks

- An attacker intercepts a set of ciphertexts
- Breaking the cipher: analyze patterns in the ciphertext
  - provides clues about the plaintext and key

# Known Plaintext Attacks

- An attacker has samples of both the plaintext and its encrypted version, the ciphertext
- Makes some ciphers (e.g., mono-alphabetic ciphers) very easy to break

# Chosen Plaintext Attacks

- An attacker has the capability to choose arbitrary plaintexts to be encrypted and obtain the corresponding ciphertexts
  - How could such attacks be possible?
  - Difference between known plaintext and chosen plaintext attacks



# Exercise

- Alice wants to send a message to Bob. The message content is “sell the business”.
- Alice encrypts the message by replacing each letter with the one 3 letters later in the alphabet (e.g., a -> d, b - > e)
  - What is the plaintext?
    - sell the business
  - What is the ciphertext?
    - vhoo wkh exvlqhv
  - How to encrypt?
  - How to decrypt?
  - What is the key in this cryptosystem?
    - 3
  - Cryptanalysis attacks?

# Perfectly Secure Ciphers

1. Ciphertext does not reveal any information about which plaintexts are more likely to have produced it
  - e.g., the cipher is robust against ciphertext only attacks
- and
2. Plaintext does not reveal any information about which ciphertexts are more likely to be produced
  - e.g, the cipher is robust against known/chosen plaintext attacks

# Computationally Secure Ciphers

1. The **cost** of breaking the cipher quickly exceeds the value of the encrypted information

**and/or**

2. The **time** required to break the cipher exceeds the useful lifetime of the information
- Under **the assumption** there is not a faster / cheaper way to break the cipher, waiting to be discovered

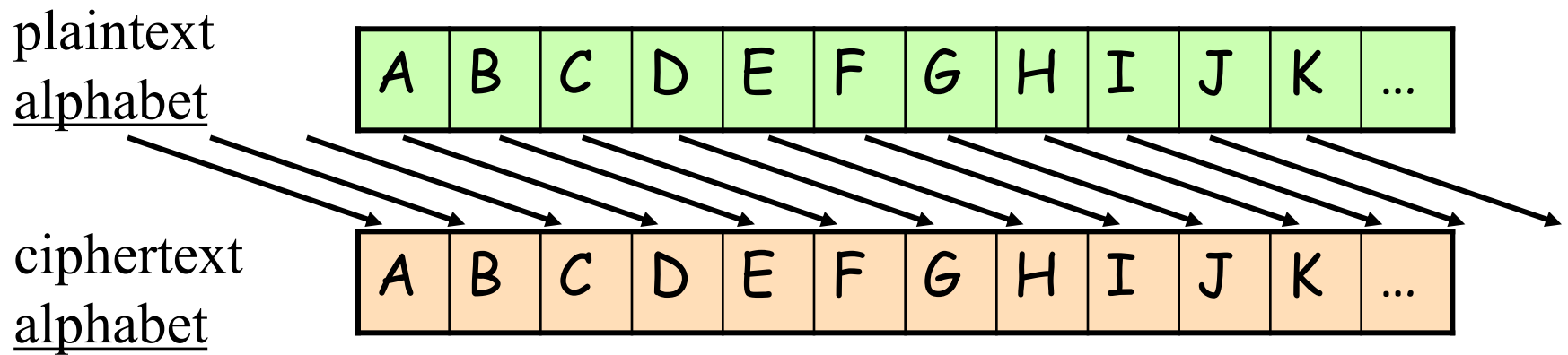
# Secret Keys v.s. Secret Algorithms

- Keep algorithms secret
  - We can achieve better security if we keep the algorithms secret
  - Hard to keep secret if used widely
- Publish the algorithms
  - Security depends on the secrecy of the keys
  - Less unknown vulnerability if all the smart (good) people in the world are examine the algorithms
- Military
  - Both secret key and secret algorithm

# Some Early Ciphers

# Caesar Cipher

- Replace each letter with the one **3** letters later in the alphabet



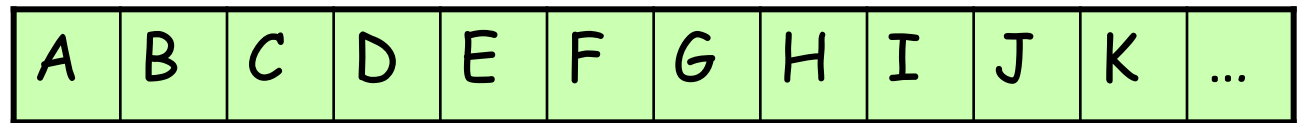
CAT → FDW

Trivial to break

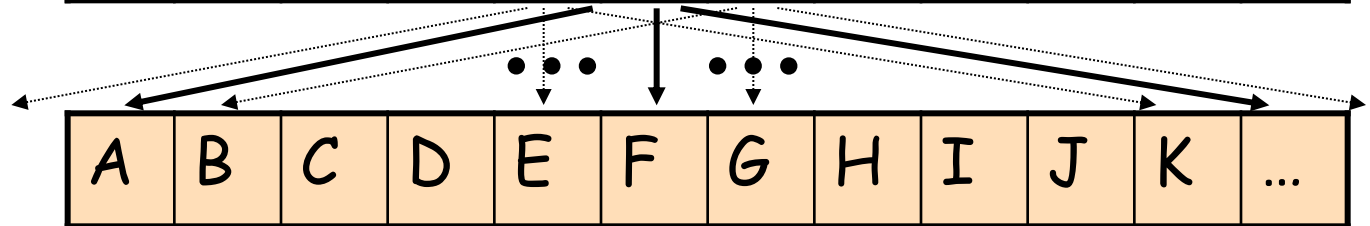
# A variant of Caesar Cipher

- Replace each letter by one that is  $\delta$  positions later, where  $\delta$  is selectable (i.e.,  $\delta$  is the key)
  - example: IBM  $\rightarrow$  HAL (for  $\delta=25$ )
- Also trivial to break with modern computers (how many possibilities?)

plaintext  
alphabet



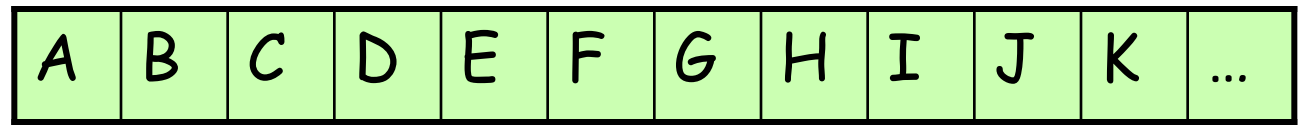
ciphertext  
alphabet



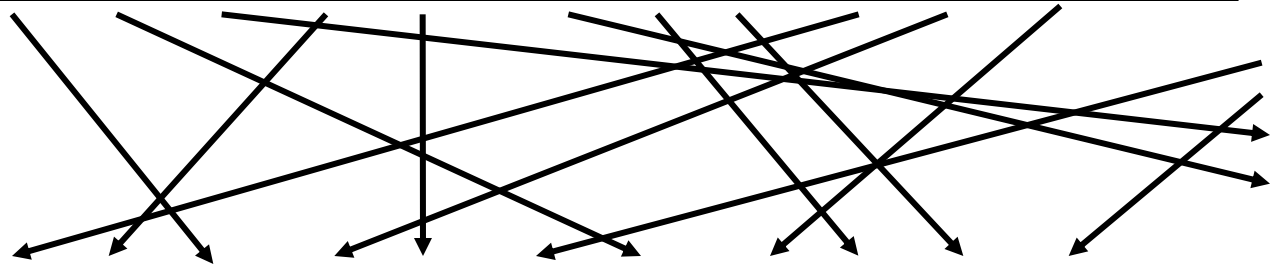
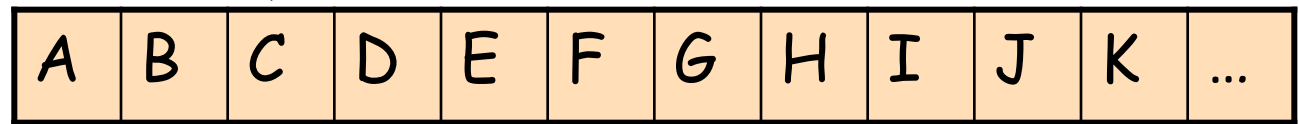
# Mono-Alphabetic Ciphers

- Generalized substitution cipher: randomly map one letter to another (How many possibilities?)
  - $26!$  ( $\approx 4.0 \cdot 10^{26} \approx 2^{88}$ )
- The key must specify which permutation; how many bits does that take?
  - $\log_2(26!) \approx 90$  bits

plaintext  
alphabet



ciphertext  
alphabet





# Attacking Mono-Alphabetic Ciphers

- Known plaintext attacks
- Frequency of single letters in English language, taken from a large corpus of text:

|                   |                  |                  |                  |
|-------------------|------------------|------------------|------------------|
| A $\approx$ 8.2%  | H $\approx$ 6.1% | O $\approx$ 7.5% | V $\approx$ 1.0% |
| B $\approx$ 1.5%  | I $\approx$ 7.0% | P $\approx$ 1.9% | W $\approx$ 2.4% |
| C $\approx$ 2.8%  | J $\approx$ 0.2% | Q $\approx$ 0.1% | X $\approx$ 0.2% |
| D $\approx$ 4.3%  | K $\approx$ 0.8% | R $\approx$ 6.0% | Y $\approx$ 2.0% |
| E $\approx$ 12.7% | L $\approx$ 4.0% | S $\approx$ 6.3% | Z $\approx$ 0.1% |
| F $\approx$ 2.2%  | M $\approx$ 2.4% | T $\approx$ 9.1% |                  |
| G $\approx$ 2.0%  | N $\approx$ 6.7% | U $\approx$ 2.8% |                  |

# Attacking... (Cont'd)

- Suppose the attacker intercepts the following message

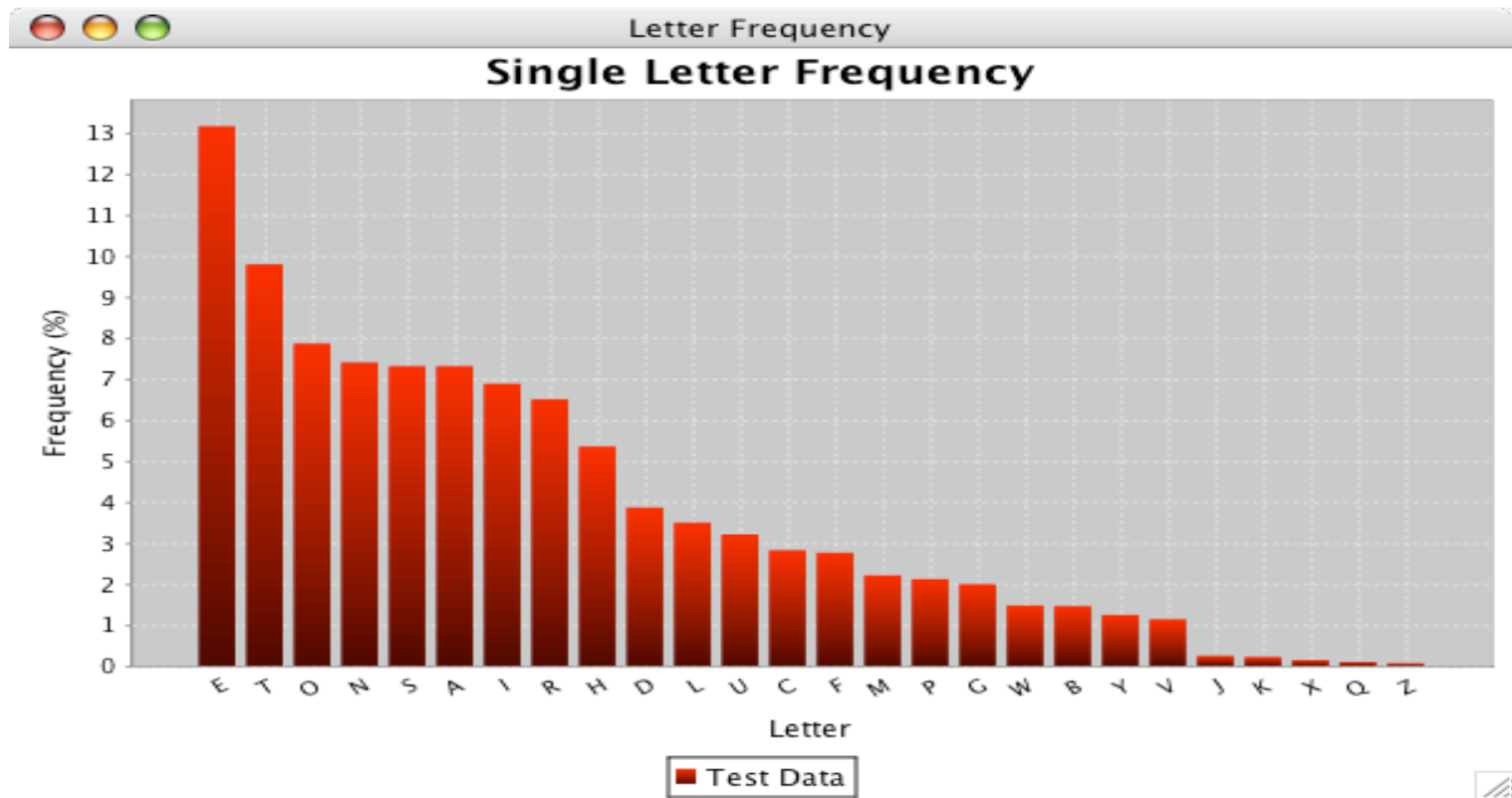
UXGPOGZCFJZJTFADADAJEJNDZMZHBBGZGGKQGVVGXCDIWGX

| A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 3 | 2 | 2 | 4 | 1 | 2 | 8 | 1 | 1 | 4 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 2 | 1 | 3 | 0 | 5 |

|                   |                  |                  |                  |
|-------------------|------------------|------------------|------------------|
| A $\approx$ 8.2%  | H $\approx$ 6.1% | O $\approx$ 7.5% | V $\approx$ 1.0% |
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FREQUENCY  
ANALYSIS IS  
AMAZING NOW  
WE NEED  
BETTER CIPHER

# Letter Frequencies



# Vigenere Cipher

- A **set** of mono-alphabetic substitution rules (shift amounts) is used
  - the key determines what the sequence of rules is
  - also called a *poly-alphabetic* cipher
- Ex.: key = (**3 1 5**)
  - i.e., substitute first letter in plaintext by letter+**3**, second letter by letter+**1**, third letter by letter+**5**
  - then repeat this cycle for each 3 letters

# Vignere... (Cont'd)

- Ex.: plaintext = "BANDBAD"

plaintext message

|   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|
| B | A | N | D | B | A | D |
|---|---|---|---|---|---|---|

shift amount

|   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|
| 3 | 1 | 5 | 3 | 1 | 5 | 3 |
|---|---|---|---|---|---|---|

ciphertext message

|   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|
| E | B | S | G | C | F | G |
|---|---|---|---|---|---|---|

What are the possible attacks?

- Known plaintext? Frequency analysis?

# Hill Ciphers

- Encrypts  $m$  letters of plaintext at each step
  - i.e., plaintext is processed in blocks of size  $m$
- Encryption of plaintext  $p$  to produce ciphertext  $c$  is accomplished by:  $c = Kp$ 
  - the  $m \times m$  matrix  $K$  is the key
  - decryption is multiplication by inverse:  $p = K^{-1}c$
  - *remember: all arithmetic mod 26*

# Hill Cipher Example

- For  $m = 2$ , let  $K = \begin{pmatrix} 1 & 2 \\ 3 & 5 \end{pmatrix}$ ,  $K^{-1} = \begin{pmatrix} 21 & 2 \\ 3 & 25 \end{pmatrix}$

Plaintext  $p =$

| A | B | X  | Y  | D | G |
|---|---|----|----|---|---|
| 0 | 1 | 23 | 24 | 3 | 6 |

$$(1*0+2*1) \bmod 26$$

$$(21*15+2*13) \bmod 26$$

$$(3*23+5*24) \bmod 26$$

Ciphertext  $c =$

|   |   |    |   |    |    |
|---|---|----|---|----|----|
| 2 | 5 | 19 | 7 | 15 | 13 |
| C | F | T  | H | P  | N  |

What about encrypting "CS"?

# Hill... (Cont'd)

- Fairly strong for large  $m$
- Possible attacks
  - Ciphertext only?
  - Known/Chosen plaintext attack?
    - Choose  $m$  plaintexts, generate corresponding ciphertexts
    - Form a  $m \times m$  matrix  $\mathbf{X}$  from the plaintexts, and  $m \times m$  matrix  $\mathbf{Y}$  from the ciphertexts
    - Can solve directly for  $\mathbf{K}$  (i.e.,  $\mathbf{K} = \mathbf{Y} \mathbf{X}^{-1}$  )
    - How many (plaintext, ciphertext) pairs are required?



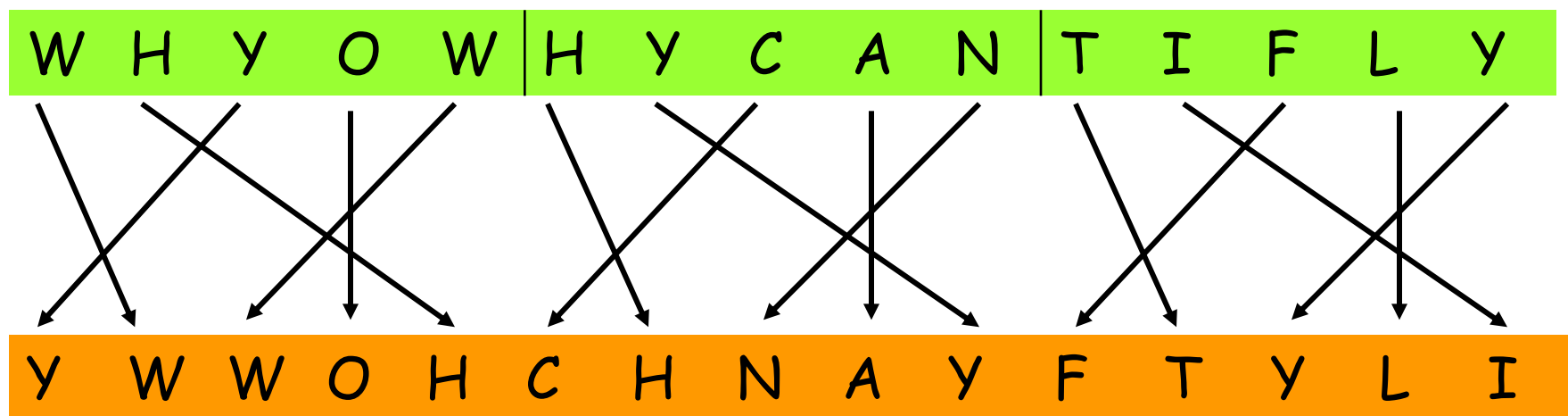
# Permutation Ciphers

- The previous codes are all based on substituting one symbol in the **alphabet** for another symbol in the alphabet
- **Permutation cipher**: permute (rearrange, transpose) the letters in the **message**
  - the permutation can be fixed, or can change over the length of the message

# Permutation... (Cont'd)

- Permutation cipher ex. #1:
  - Permute each successive block of 5 letters in the message according to position offset  $\langle +1, +3, -2, 0, -2 \rangle$

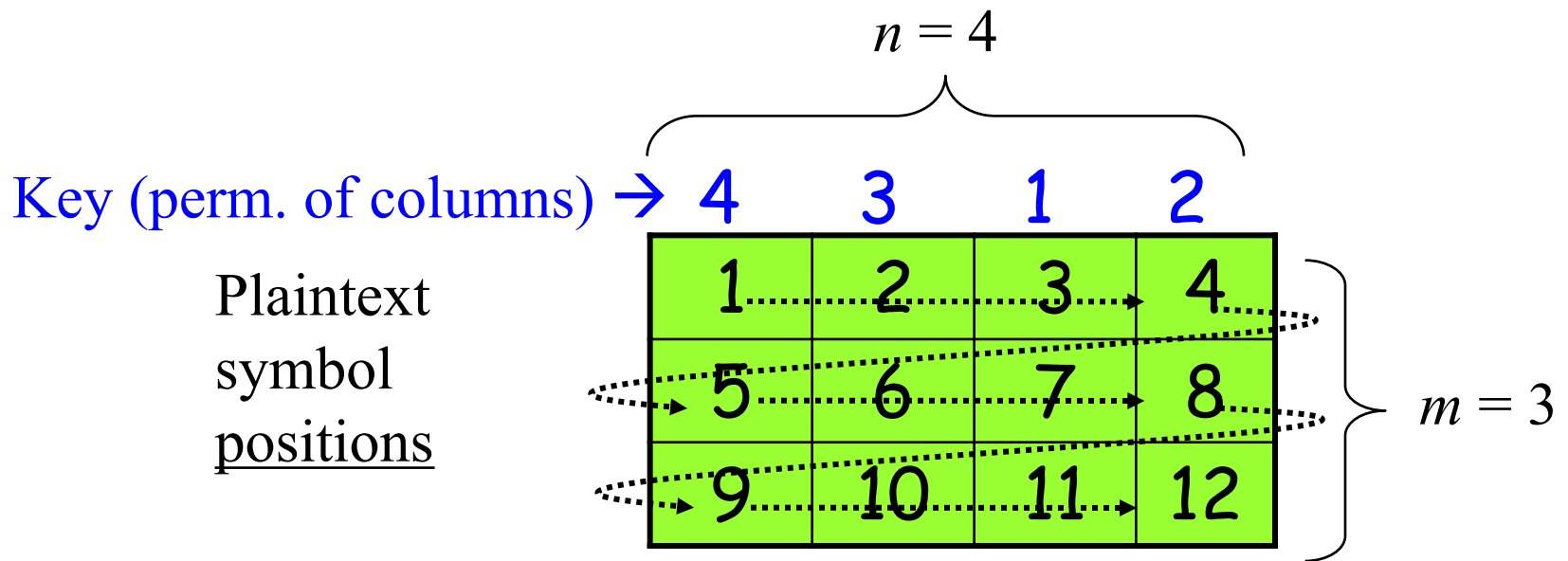
plaintext message



ciphertext message

# Permutation... (Cont'd)

- Permutation cipher [ex. #2](#):
- arrange plaintext in blocks of  $n$  columns and  $m$  rows
- then permute columns in a block according to a key  $K$



ciphertext sequence (by plaintext position) for one block

3    7    11    4    8    12    2    6    10    1    5    9

# Permutation... (Cont'd)

- A longer example: plaintext =  
“ATTACK POSTPONED UNTIL TWO AM”

Key:

4 3 1 2 5 7 6

plaintext

|   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|
| A | T | T | A | C | K | P |
| O | S | T | P | O | N | E |
| D | U | N | T | I | L | T |
| W | O | A | M | X | Y | Z |

ciphertext

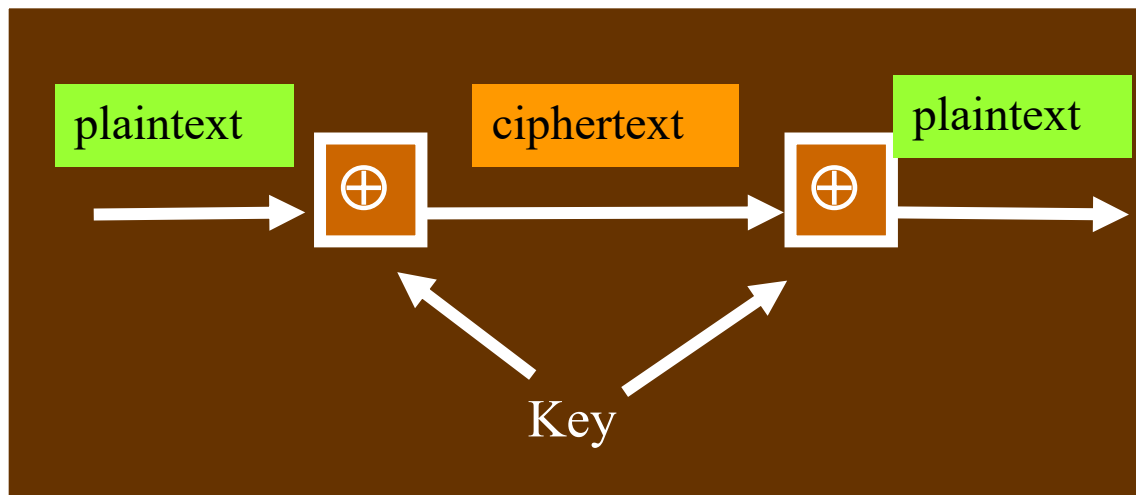
TTNA APTM TSUO AODW COIX PETZ KNLY

# A Perfectly Secure Cipher: One-Time Pads

- According to a theorem by Shannon, a perfectly secure cipher **requires**:
  - a key length **at least as long as the message** to be encrypted
  - the key **can only be used once** (i.e., for each message we need a new key)
- Very limited use due to need to negotiate and distribute long, random keys for every message

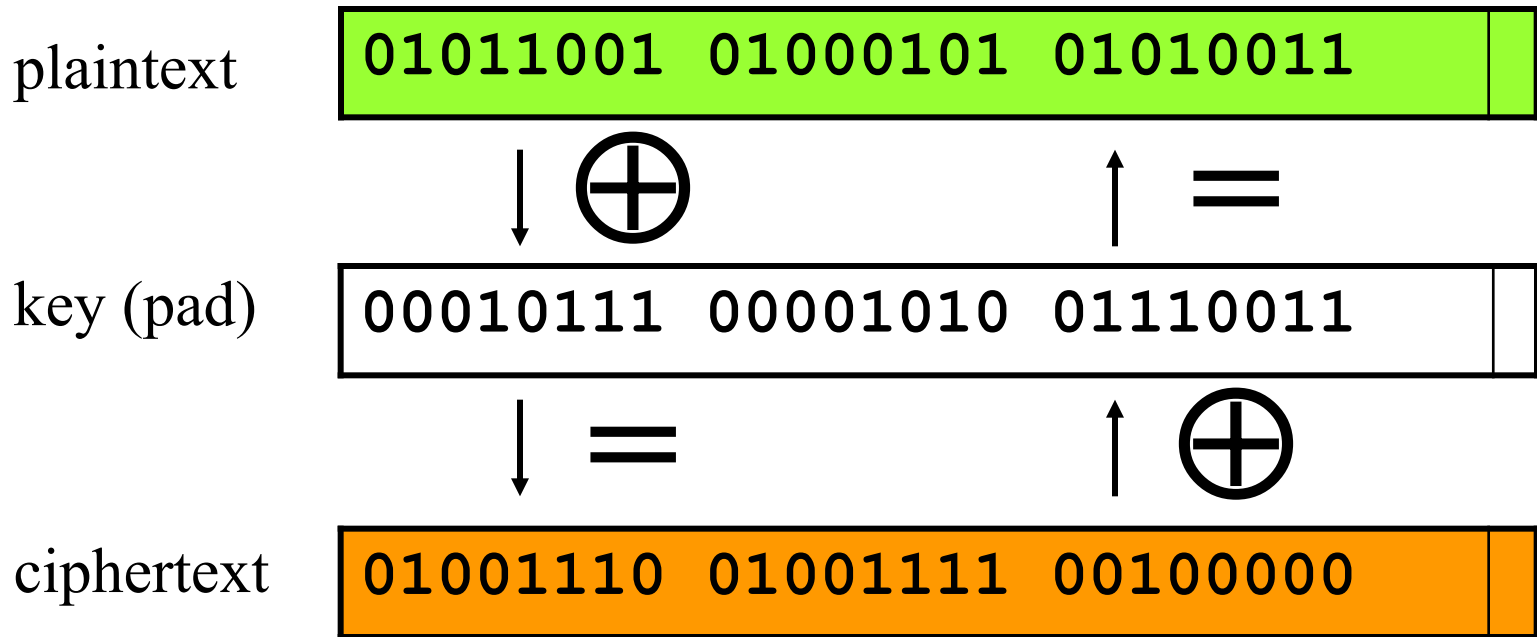
# OTP... (Cont'd)

- Idea
  - generate a **random** bit string (the key) as long as the plaintext, and share with the other communicating party
  - **encryption**: XOR this key with plaintext to get ciphertext
  - **decrypt**: XOR same key with ciphertext to get plaintext



$$\begin{array}{lcl} 0 & \oplus & 0 = 0 \\ 0 & \oplus & 1 = 1 \\ 1 & \oplus & 0 = 1 \\ 1 & \oplus & 1 = 0 \end{array}$$

# OTP... (Cont'd)



- Why can't the key be reused?
  - if  $C = P \oplus K$ , then  $K = C \oplus P$

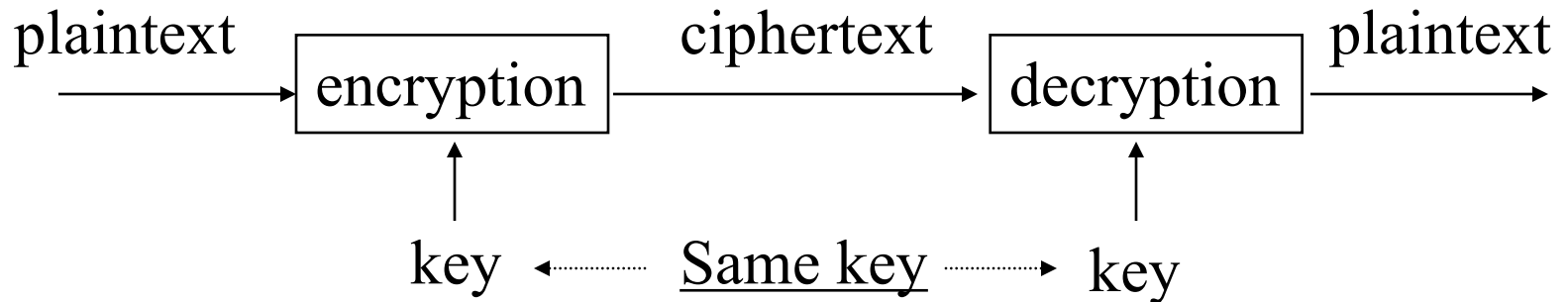
# Some “Key” Issues



# Types of Cryptography

- Number of keys
  - Hash functions: no key
  - Secret key cryptography: one key
  - Public key cryptography: two keys - public, private
- The way in which the plaintext is processed
  - Stream cipher: encrypt input message **one symbol** at a time
  - Block cipher: divide input message into **blocks** of symbols, and processes the blocks in sequence
    - May require **padding**

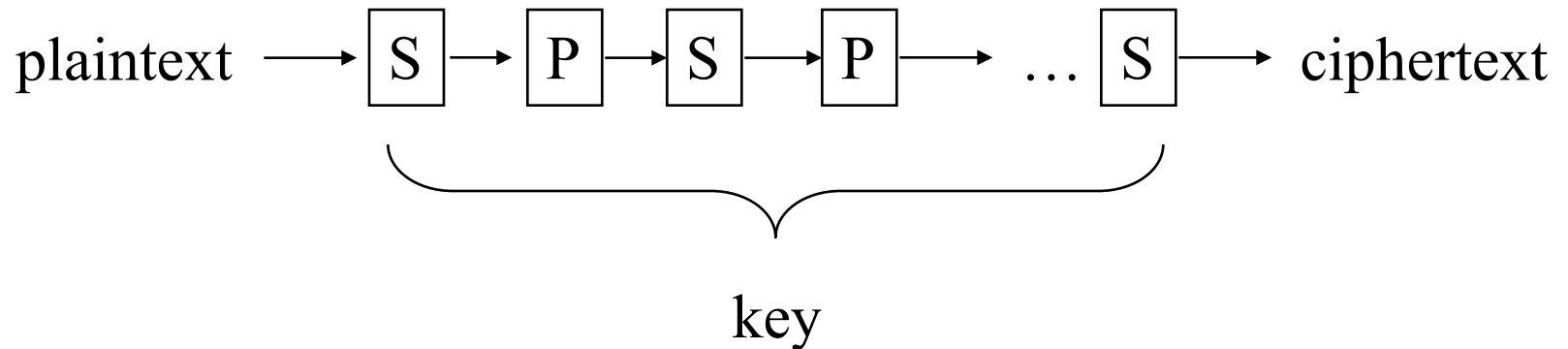
# Secret Key Cryptography



- Same key is used for encryption and decryption
- Also known as
  - Symmetric cryptography
  - Conventional cryptography

# Secret Key Cryptography (Cont'd)

- Basic technique
  - Product cipher:
    - Multiple applications of interleaved substitutions and permutations



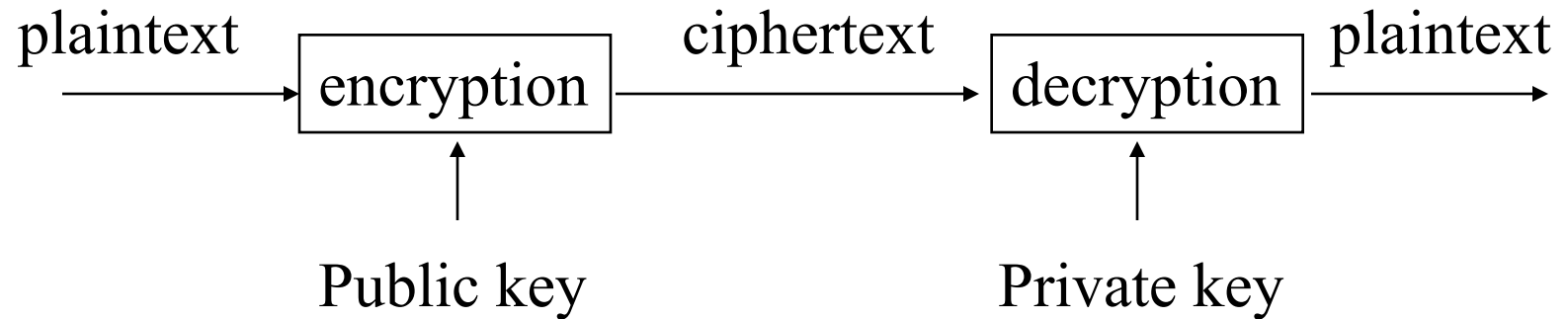
# Secret Key Cryptography (Cont'd)

- Ciphertext approximately the same length as plaintext
- Examples
  - Stream Cipher: RC4
  - Block Cipher: DES, IDEA, AES

# Applications of Secret Key Cryptography

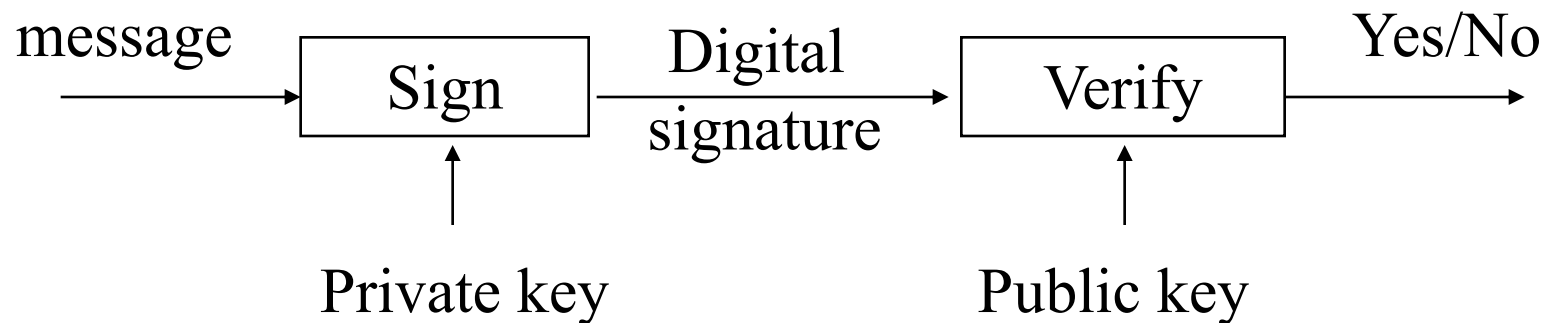
- Transmitting over an insecure channel
  - Challenge: How to share the key?
- Authentication
  - Challenge-response
  - To prove the other party knows the secret key
  - Must be secure against chosen plaintext attack
- Integrity check
  - Message Integrity Code (MIC)
    - a.k.a. Message Authentication Code (MAC)

# Public Key Cryptography



- Invented/published in 1975
- A public/private key pair is used
  - Public key can be publicly known
  - Private key is kept secret by the owner of the key
- Much slower than secret key cryptography
- Also known as
  - Asymmetric cryptography

# Public Key Cryptography (Cont'd)



- Another mode: digital signature
  - Only the party with the private key can create a digital signature.
  - The digital signature is verifiable by anyone who knows the public key.
  - The signer cannot deny that he/she has done so.

# Applications of Public Key Cryptography

- Data transmission:
  - Alice encrypts  $m_a$  using Bob's public key  $e_B$ , Bob decrypts  $m_a$  using his private key  $d_B$ .
- Storage:
  - Can create a safety copy: using public key of trusted person.
- Authentication:
  - No need for verifiers to store secrets, only need public keys.
  - Secret key cryptography: need to share secret key for every person to communicate with.



# Applications of Public Key Cryptography (Cont' d)

- Digital signatures
  - Sign hash  $H(m)$  with the private key
    - Authorship
    - Integrity
    - Non-repudiation: can't do with secret key cryptography
- Key exchange
  - Establish a common session key between two parties
  - Particularly for encrypting long messages

# Hash Algorithms



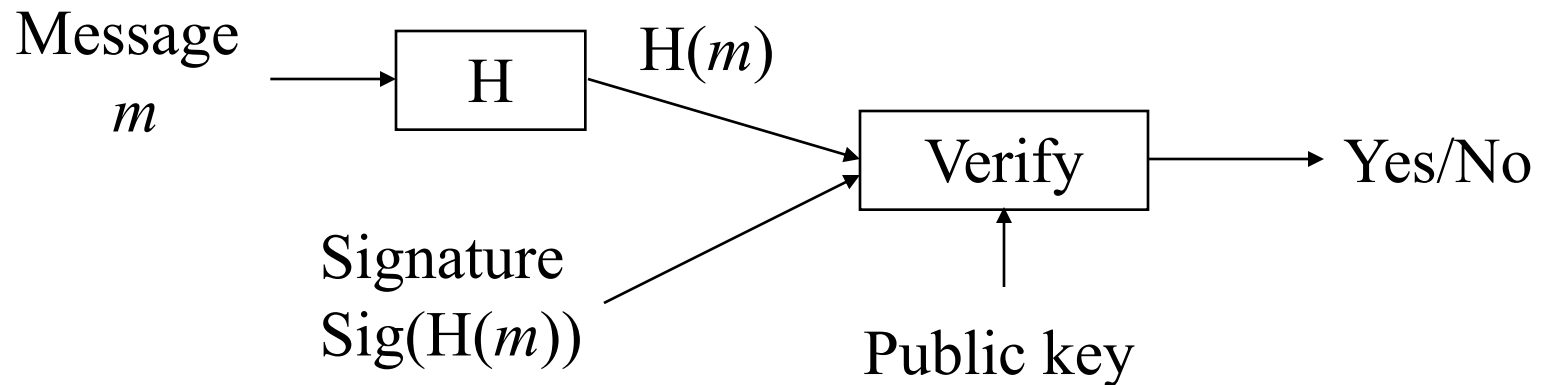
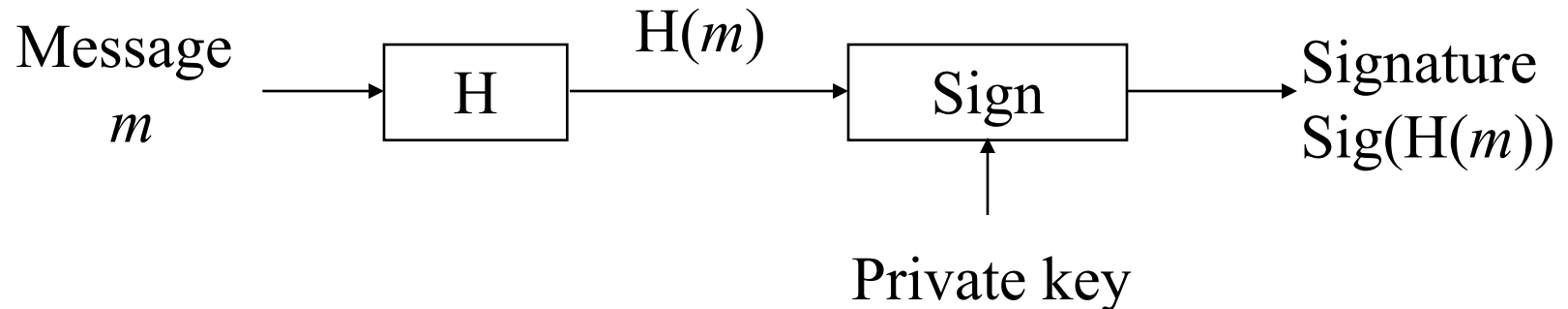
- Also known as
  - Message digests
  - One-way transformations
  - One-way functions
  - Hash functions
- Length of  $H(m)$  much shorter than length of  $m$
- Usually fixed lengths: 128 or 160 bits

# Hash Algorithms (Cont'd)

- Desirable properties of hash functions
  - Performance: Easy to compute  $H(m)$
  - One-way property: Given  $H(m)$  but not  $m$ , it's difficult to find  $m$
  - Weak collision free: Given  $H(m)$ , it's difficult to find  $m'$  such that  $H(m') = H(m)$ .
  - Strong collision free: Computationally infeasible to find  $m_1, m_2$  such that  $H(m_1) = H(m_2)$

# Applications of Hash Functions

- Primary application
  - Generate/verify digital signatures



# Applications of Hash Functions

## (Cont' d)

- Password hashing
  - Doesn't need to know password to verify it
  - Store  $H(\text{password} + \text{salt})$  and salt, and compare it with the user-entered password
  - Salt makes dictionary attack more difficult
- Message integrity
  - Agree on a secret key  $k$
  - Compute  $H(m | k)$  and send with  $m$
  - Doesn't require encryption algorithm, so the technology is exportable

# Applications of Hash Functions

## (Cont' d)

- Message fingerprinting
  - Verify whether some large data structures (e.g., a program) has been modified
  - Keep a copy of the hash
  - At verification time, recompute the hash and compare
  - Hashing program and the hash values must be protected separately from the large data structures

# Summary

- Cryptography is a fundamental, and most carefully studied, component of security
  - not usually the “weak link”
- “Perfectly secure” ciphers are possible, but too expensive in practice
- Early ciphers aren’t nearly strong enough
- Key distribution and management is a challenge for any cipher