

Modular Arithmetic

- Addition: O(n)
- Multiplication: O(n²) (naive)
- Multiplication: O(nlogn) (FFT)
- # of bits in x<sup>y</sup> = ylog<sub>2</sub>x ≤ 2<sup>n</sup> × n
- ∑<sub>i=0</sub><sup>∞</sup> r<sup>i</sup> =  $\frac{1}{1-r}$ , if r < 1

Formal Limit Proof

$\lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} :$   
 $\geq 0 \ (\infty) \Rightarrow f(n) \in \Omega(g(n))$   
 $< \infty \ (0) \Rightarrow f(n) \in O(g(n))$

$= c_{|0 < c < \infty} \Rightarrow f(n) \in \Theta(g(n))$

Logarithm Tricks

$\log_b x^p = p \log_b x$   
 $\frac{\ln(x)}{\ln(m)} = \log_m x$   
 $x^{\log_b y} = y^{\log_b x}$

Complexity Hierarchy

Exponential  
Polynomial  
Logarithmic

Constant

Master’s Theorem

$T(n) = aT(\frac{n}{b}) + O(n^d)$ , if  $a > 0, b > 1, d \geq 0$

$T(n) = \begin{cases} O(n^d) & \text{if } d > \log_b a \\ O(n^d \log_b n) & \text{if } d = \log_b a \\ O(n^{\log_b a}) & \text{if } d < \log_b a \end{cases}$

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