Exercices

1. Snakes eat frogs and birds eat spiders; birds and spiders both eat insects; frogs eat snails, spiders and insects. Draw a digraph representing this predatory behaviour.

2. Draw

   - A simple graph,
   - a non-simple graph with no loops,
   - a non-simple graph with no multiple edges

   each with five vertices and eight edges.

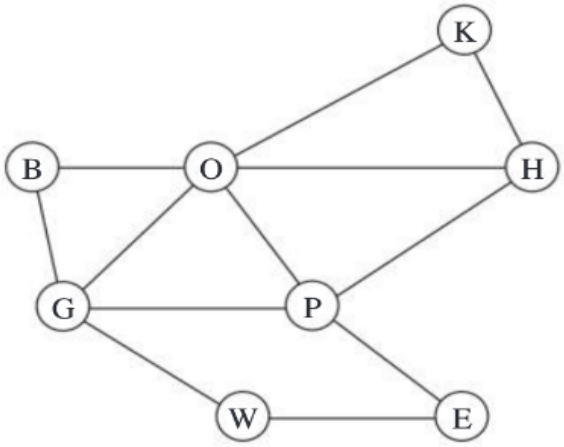
3. Let $G = (V, E)$ be a non-empty, non-trivial graph. Then G has at least one pair of vertices with equal degree.

4. Let $G = (V, E)$ be a non-empty, non-trivial graph. There is an even number of vertices in V with odd degree.

5. Let $G = (V, E)$ be a graph with $|V| = n$. Then

$$0 \leq |E| \leq \binom{n}{2}.$$

6. The following graph is the plan of the metro stations in London.



   The vertex names stand for : B Bond street, E embankment, G green park, H Holborn, K King's cross st Pancras, O oxford circus, P picadilly circus, W westminster.

   - Determine if the graph is connected, complete?
   - Is the graph Eulerian? If yes find a tour. Otherwise search for a eulerian trail.

- Find the adjacency matrix putting the vertices in alphabetical order.
- You are given

$$M^3 = \begin{pmatrix} 2 & 3 & 6 & 4 & 2 & 7 & 3 & 1 \\ 3 & 0 & 1 & 1 & 2 & 3 & 6 & 4 \\ 6 & 1 & 4 & 4 & 4 & 9 & 10 & 6 \\ 4 & 1 & 4 & 4 & 5 & 8 & 8 & 3 \\ 2 & 2 & 4 & 5 & 2 & 7 & 3 & 1 \\ 7 & 3 & 9 & 8 & 7 & 8 & 10 & 3 \\ 3 & 6 & 10 & 8 & 3 & 10 & 4 & 1 \\ 1 & 4 & 6 & 3 & 1 & 3 & 1 & 0 \end{pmatrix}$$

  A tourist is in Holborn and wants to go the green park using exactly three lines. Without using the graph give the numbe of possible paths. List these paths.

7. You are given the following graph

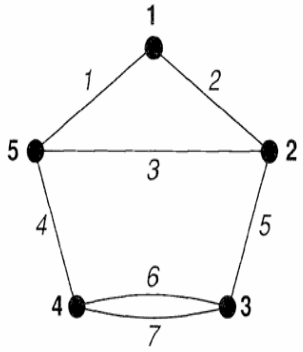


Fig. 2.24

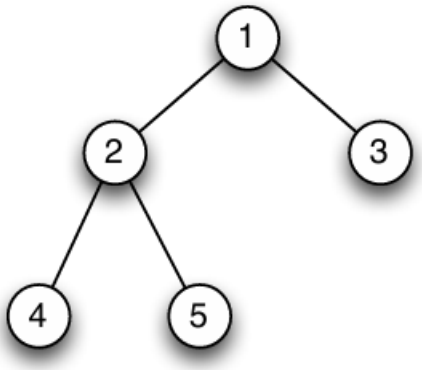
and the matrix

$$\begin{bmatrix} 2 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

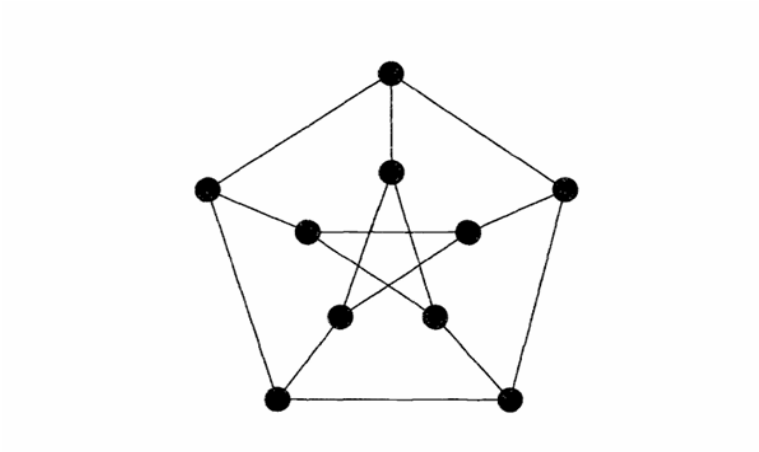
- Write down the adjacency matrix of the graph.

- Draw the graph whose adjacency matrix is given above.

8. Compute the centrality of all vertices and their geodesic centrality.



9. Prove that the Petersen graph is not Hamiltonian but it has a hamiltonian path.



10. Let G be a regular graph. Assume all vertices have degree 3. What can we say about the number of vertices in this graph? Show that for all $p \geq 2$, there is a regular graph with $2p$ vertices each having degree 3.

11. Let $G = (V, E)$ be a graph with $|E| = 2p$. Assume that the degree of every vertex is at least p. Prove that the graph is connected.

12. True or False

    a) There exists a tree where at least half the vertices have degree 1.

    b) There exists a tree where at least half the vertices have degree 2.

c) There exists a tree where at least half the vertex have degree 3.

d) Suppose that a tree T has no vertex with degree 2. Then, at least half of its vertices have degree 1.