Exercices

1. Solve the problem

$$maxf = 4x_1 + 3x_2$$

subject to

$$\begin{cases} 3x_1 + 4x_2 \leq 12 \\ 3x_1 + 3x_2 \leq 10 \\ 4x_1 + 2x_2 \leq 8 \\ x_i \geq 0 \end{cases}$$

2. Find the minimum of the function

$$f(x) = x_1 + 2x_2 + 3x_3 + 4x_4$$

subject to the constraints

$$\begin{cases} x_1 + x_2 + x_3 + x_4 = 1 \\ x_1 + x_3 - 3x_4 = 0.5 \\ x_i \geq 0 \end{cases}$$

by the simplex method.

Hint: add artificial variables x_5 and x_6, which are positive or zero, to the constraints and minimize the function $g(x) = x_5 + x_6$ by the simplex method. Justify the method and use it. Deduce the solution of the initial problem.

3. Solve the LP program

$$maxf = 2x_1 - x_2 + x_3$$

subject to

$$\begin{cases} -x_1 + x_2 - 4x_3 \leq 1 \\ -x_1 + x_2 + x_3 \leq -2 \\ x_1 + 3x_2 + 2x_3 = 3 \\ x_i \geq 0 \end{cases}$$

4.

$$min2x_1 - x_2 + x_3$$

subject to

$$\begin{cases} -x_1 + x_2 - 4x_3 \leq 1 \\ -x_1 + x_2 + x_3 \leq -2 \\ 2x_1 + 4x_2 + 5x_3 \leq 3 \\ -x_1 - x_2 - x_3 \leq -3 \\ x_1 + 3x_2 + 2x_3 = 3 \\ x_i \geq 0 \end{cases}$$

Solution $x_1 = 0.8, x_2 = 2.4, f = 10.4$.

5.

$$maxf = 5x_1 - 3x_2 + 4x_3$$

subject to

$$\begin{cases} x_1 - x_2 \geq 1 \\ 3x_1 + 2x_2 + 2x_3 \leq 1 \\ 4x_1 - x_3 = 1 \\ x_2 \geq 0, x_3 \geq 0 \end{cases}$$

The variable x_1 can take any value. Hint: pose $x_1 = x_{11} - x_{22}, x_{11} \geq 0, x_{22} \geq 0$. Solution: $x_1 = 0.6, x_2 = 0, x_3 = 1.4, f = 8.6$.

6. Find the minimum of the function

$$f(x) = 2x_1 + 3x_2$$

subject to the constraints

$$\begin{cases} 2x_1 + x_2 \geq 3 \\ x_1 + x_2 \geq 10 \\ x_1 + 3x_2 \geq 8 \\ x_i \geq 0, \forall i \end{cases}$$

7. Solve using the dual,

   - A company has two machines M_1 and M_2 and fabricates two products A,B. Producing A takes 1h of M_1 and 2h of M_2. Producing B takes 1h of M_1 and 1h of M_2. The first machine can operate for 70 hours and the second for 90 hours .The marginal cost per variable is 30 euros and 40 euros respectively for A and B. Find the number of products that can be produced to maximize the profit of the company. Show the dual problem and its solution.
   - Solve the following problem by passing from the dual.

$$\begin{cases} y_1 \geq 10, y_2 \geq 15, y_3 \geq 0, y_4 \geq 0 \\ y_1 + 2y_2 + y_3 \geq 60 \\ 2y_1 + y_2 + 2y_3 \geq 95 \\ min 2y_1 + 5y_2 + 3y_3 \end{cases}$$

   - A pharmacist wants to offer a portion containing a minimum quantity of the vitamins A,B,C using powder from two different laboratories.

100 gr of powder	lab 1	lab 2
vitamine A	20 units	5 units
vitamine B	30 units	20 units
vitamine C	5 units	10 units
cost	6	9

She needs at least 25 units of A, 60 units of B, and 15 units of vitamin C. Solve the LP problem.