1. Let $G = (V, E, p)$ be a Markov chain and M its stochastic matrix. If G is strongly connected then M^T and M are irreducible.

2. Compute the eigenvalue centrality of an n directed cycle and a k regular graph (non directed).

3. The agdacency matrix A of a digraph is nilpotent ($\exists k$ such that $A^k = 0$ the zero matrix). Give an example of such a graph with 5 vertices and compute the eigenvalue centrality.

4. Find the eigevector centrality of a tree with two leaves and two edges.

5. There are n fish in a lake some of which are green and the rest blue. Marc catches one fish with equal probability of catching a blue or a green one. He throws back the fish but he paints every green fish blue before throwing it back. Let G_i be the event that there are i green fish left in the lake. Describe the situation with a Markov chain whose distribution function must be precised.

6. You are given a digraph with two vertices C and V standing for consonants and vowels respectively. The graph has a self loop at C happening with probability 0.2 and a self loop at V with probability 0.5. We have programmed a computer to print letters based on the transition probabilities of this graph. Given that the first letter is a consonant what is the probability that the fourth letter is a consonant? Determine the invariant distribution and interpret the result.

7. We are given the digraph $G = (V, E, p)$ with

   $$\{E = (A, B), (A, C), (A, D), (B, C), (C, A), (C, E), (D, B), (D, C), (D, E), (E, A)\}.$$

   Draw the graph and rank the vertices using PageRank with damping factor $d = 0.5$ and without normalization. Do at least 10 iterations to determine ther ranking.

8. You are given a digraph $G = (V, E, p)$ with edge set

   $$\{(1, 2), (1, 3), (2, 1), (2, 3), (3, 2), (4, 3), (4, 5), (4, 6), (6, 4), (6, 5)\}.$$

   Calculate the PageRank without dumping factor.