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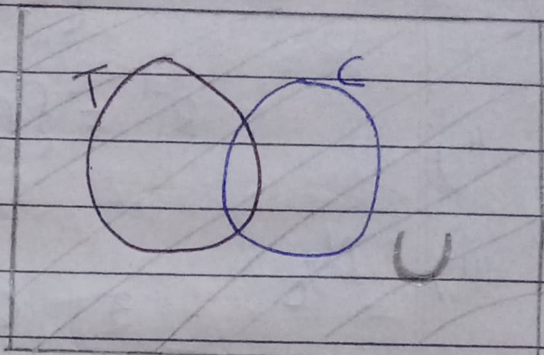
Q1) In a survey of 150 people, it was found that 75 liked tea, 85 liked coffee and 35 liked both. How many people didn't like any of the two?

Given,



$n(U) = 150$
$n(T) = 75$
$n(C) = 85$
$n(T \cap C) = 35$

number of people that did not like any of the two

$$\Rightarrow n(T' \cap C') = n(T \cup C)'$$

$$= n(U) - n(T \cup C)$$
$$= n(U) - [n(T) + n(C) - n(T \cap C)]$$
$$= n(U) - [75 + 85 - 35]$$
$$= 150 - [125]$$
$$= \underline{\underline{25}}$$

Q2) Let $A = \{3, 6, 7, 8\}$
$B = \{1, 2, 3, 4\}$

i) Find R defined $(a, b) \in R$ if $(a-b)$ divisible by 'a'

ii) R^{-1}

iii) M_R and M_R^{-1}

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i) $A \times B = \{(3,1),(3,2),(3,3),(3,4),(6,1),(6,2),(6,3),(6,4),$
$(7,1),(7,2),(7,3),(7,4),(8,1),(8,2),(8,3),(8,4)\}$

Now $R = \{(a,b),\ (a-b) \text{ divisible by } a\}$

$= \{(3,3)\}$

ii) $R^{-1} = \{(3,3)\}$

iii) $M_r =$

$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 3 & 0 & 0 & 1 & 0 \\ 6 & 0 & 0 & 0 & 0 \\ 7 & 0 & 0 & 0 & 0 \\ 8 & 0 & 0 & 0 & 0 \end{array}$$

$$M_r^{-1} = \begin{array}{c|cccc} & 3 & 6 & 7 & 8 \\ \hline 1 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 \\ 3 & 1 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 \end{array}$$

Q3, Let $A = \{1, 2, 3, 4\}$

$R = \{(1,2), (2,3), (2,4), (3,1), (4,1)\}$

$S = \{(1,3), (2,4), (3,1), (2,2)\}$

$Q = \{(1,4), (4,2), (2,1), (3,2)\}$

i) RoS

$RoS = \{(1,2), (2,3)(2,4), (3,1), (4,1)\} \circ \{(1,3), (2,4), (3,1), (2,2)\}$

$= \{(1,4), (1,2), (2,1), (3,3), (4,3)\}$

ii) SoR

$SoR = \{(1,3, (2,4), (3,1), (2,2)\} \circ \{(1,2), (2,3), (2,4), (3,1), (4,1)\}$

$= \{(1,1), (2,1), (3,2), (2,4), (2,3)\}$

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iii) $Q \circ S$

$\{(1,4),(4,2),(2,1),(3,2)\} \circ \{(1,3),(2,4),(3,1),(2,4)\}$

$= \{(4,2),(4,4),(2,3),(3,4),(3,2)\}$

(iv) $Q^2 = Q \circ Q$

$\{(1,4),(4,2),(2,1),(3,2)\} \circ \{(1,4),(4,2),(2,1),(3,2)\}$

$= \{(1,2),(4,1),(2,4),(3,1)\}$

(v) S^4

$S^2 = S \circ S$
$S^3 = S^2 \circ S$
$S^4 = S^3 \circ S$

$S^2 = \{(1,3),(2,4),(3,1),(2,2)\} \circ \{(1,3),(2,4),(3,1),(2,2)\}$

$= \{(1,1),(3,3),(2,4),(2,2)\}$

$S^3 = \{(1,1),(3,3),(2,4),(2,2)\} \circ \{(1,3),(2,4),(3,1),(2,2)\}$

$= \{(1,3),(3,1),(2,4),(2,2)\}$

$S^4 = \{(1,3),(3,1),(2,4),(2,2)\} \circ \{(1,3),(2,4),(3,1),(2,2)\}$

$= \{(1,1),(3,3),(2,4),(2,2)\}$

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(vi) S^2

$S^2 = \{(1,3),(2,4),(3,1),(2,2)\} \circ \{(1,3),(2,4),(3,1),(2,2)\}$

$= \{(1,1),(3,3),(2,2),(2,4)\}$

(vii) S^3

$S^3 = \{(1,3),(3,1),(2,4),(2,2)\}$ (solved in (v))

Q4) If $A = \{(1,2,3,4)\}$ and

$R = \{(1,1),(1,2),(2,1),(3,2)(4,1),(2,2)\}$

Find reflexive, symmetric, transitive closure of R

i) Reflexive closure of R

$R^r = R \cup I_A$

$= \{(1,1),(1,2),(2,1),(3,2),(4,1),(2,2)\} \cup \{(1,1),(2,2),(3,3),(4,4)\}$

$= \{(1,1),(1,2),(2,1),(3,2),(4,1),(2,2),(3,3),(4,4)\}$

ii) Symmetric closure of R

$R^s = R \cup R^{-1}$

$$R^S = \{(1,1),(1,2),(2,1),(3,2),(4,1),(2,2)\} \cup \{(1,1),(2,1),(1,2),(2,3),(1,4),(2,2)\}$$

$$= \{(1,1),(1,2),(2,1),(3,2),(4,1),(2,2),(2,3),(1,4)\}$$

iii) Transitive closure of R

$$R^T = R \cup R^2 \cup R^3 \cup R^4$$

$$= \{(1,1),(1,2),(2,1),(3,2),(4,1),(2,2)\}$$

$$\cup \{($$

iii) Transitive closure of R

$$R^T = R \cup R^2 \cup R^3 \cup R^4$$

$$R^2 = R \circ R$$

$$R^2 = \{(1,1),(1,2),(2,1),(3,2),(4,1),(2,2)\} \circ \{(1,1),(1,2),(2,1),(3,2),(4,1),(2,2)\}$$

$$= \{(1,1),(1,2),(2,1),(2,2),(3,1),(3,2),(4,1),(4,2)\}$$

$$R^3 = R^2 \circ R$$

$$= \{(1,1),(1,2),(2,1),(2,2),(3,1),(3,2),(4,1),(4,2)\} \circ \{(1,1),(1,2),(2,1),(3,2),(4,1),(2,2)\}$$

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$$= \{(1,1),(1,2),(2,1),\overset{(3,1)}{\wedge}(3,2),(4,1),\overset{(4,2)}{\wedge}(2,2)\}$$

$R^4 = R^3 \circ R$

$= \cancel{\{(1,1),(1,2),(2,1),(3,2),(4,1),(2,2)\} \circ \{(1,1),(1,2),(2,1),(2,2),(3,1),(3,2),(4}$

$= \{(1,1),(1,2),(2,1),(3,2),(4,1),(2,2)\} \circ \{(1,1),(1,2),(2,1),(3,2),(4,1),(2,2)\}$

$= \{(1,1),(1,2),(2,1),(2,2),(3,1),(3,2),(4,1),(4,2)\}$

Since, $R^3 = R^4$, $R^T = R^4$

Q5) Given the relational matrix of a relation on the set of $\{a,b,c\}$. Find R^{-1}, R^2, R^3, $R \circ R^{-1}$ where

(a)
$$M_R = \begin{array}{c} \\ a \\ b \\ c \end{array} \begin{array}{c} \begin{array}{ccc} a & b & c \end{array} \\ \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \end{array}$$

(b)
$$M_R = \begin{array}{c} \\ a \\ b \\ c \end{array} \begin{array}{c} \begin{array}{ccc} a & b & c \end{array} \\ \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \end{array}$$

(a)(i) $R = \{(a,a),(a,c),(b,a),(b,b),(c,a),(c,b),(c,c)\}$

$R^{-1} = \{(a,a),(c,a),(a,b),(b,b),(a,c),(b,c),(c,c)\}$

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(ii) $R^2 = R \circ R$

$= \{(a,a),(a,c),(b,a),(b,b),(c,a),(c,b),(c,c)\} \circ$

$\{(a,a),(a,c),(b,a),(b,b),(c,a),(c,b),(c,c)\}$

$= \{(a,a),(a,b),(a,c),(b,b),(b,a),(c,c),(c,b),$

$(b,c),(c,a)\}$

(iii) $R^3 = R^2 \circ R$

$= \{(a,a)(a,b),(a,c),(b,b),(b,a),(c,c),(c,b),(b,c),(c,a)\}$

$\circ \{(a,a),(a,c),(b,a),(b,b),(c,a),(c,b),(c,c)\}$

$= \{(a,a),(a,b),(a,c),(b,a),(b,b),(b,c),(c,a),(c,b),(c,c)\}$

(iv) $R \circ R^{-1}$

$= \{(a,a),(a,c),(b,a),(b,b),(c,a),(c,b),(c,c)\}$

$\circ \{(a,a),(c,a),(a,b),(b,b),(a,c),(b,c),(c,c)\}$

$= \{(a,a),(a,b),(a,c),(b,a),(b,b),(b,c),(c,a),$

$(c,b),(c,c)\}$

(b) (i) $R = \{(a,c), (b,c), (c,a)\}$

$R^{-1} = \{(c,a), (c,b), (a,c)\}$

(ii) $R^2 = R \circ R = \{(a,c), (b,c), (c,a)\} \circ \{(a,c), (b,c), (c,a)\}$

$= \{(a,a), (b,a), (c,c)\}$

(iii) $R^3 = R^2 \circ R$

$= \{(a,a), (b,a), (c,c)\} \circ \{(a,c), (b,c), (c,a)\}$

$= \{(a,c), (b,c), (c,a)\}$

(iv) $R \circ R^{-1}$

$= \{(a,c), (b,c), (c,a)\} \circ \{(c,a), (c,b), (a,c)\}$

$= \{(a,a), (a,b), (b,a), (b,b), (c,c)\}$