Coordinate Geometry

Additional Notes

Coordinate Geometry Formulae

We need to be able to use the following formulae:

Distance between two points (x_1, y_1) and (x_2, y_2)

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Midpoint, M, between two points (x_1, y_1) and (x_2, y_2)

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Coordinate Geometry Formulae

Slope, m, of line containing two points (x_1, y_1) and (x_2, y_2)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Equation of a line with slope m and point (x_1, y_1)

$$y - y_1 = m(x - x_1)$$

General equation of line y=mx+c

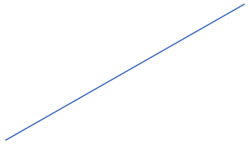
The equation of a line is represented in general form as y = mx + c where the m represents the slope of the line and the c is the y-intercept (where line cuts the y-axis.

The line y = 3x + 5 has a slope of 3 and cuts the y-axis at 5, ie. at point (0,5).

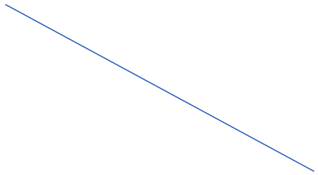
A positive slope means the line slopes upwards from left to right.

The line y = -2x + 6 has a slope of -2 and cuts the y-axis at 6 ie. at point (0,6).

A line with a negative slope slopes downwards from left to right.



Positive sloping line



Negative sloping line

Line with slope of 0

What is the slope of the line 2x + 3y = 12?

First put equation into form y = mx + c.

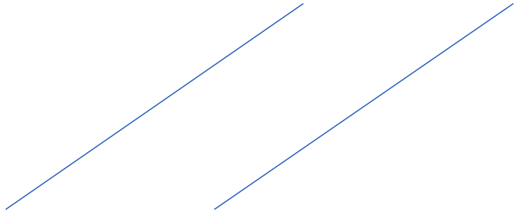
$$2x + 3y = 12$$

$$3y = -2x + 12$$

$$y = \frac{-2}{3}x + 4$$

So the slope of the line is $-\frac{2}{3}$ and y-intercept is (0,4)

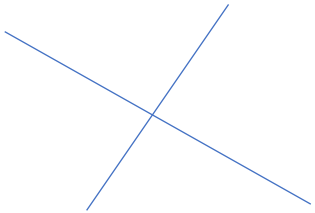
Parallel lines



Parallel lines have equal slopes. So L: $y = 3x + 7$ and M: $y = 3x - 5$ are both parallel.

We say $L \parallel M$.

Perpendicular lines



Perpendicular lines intersect at right angles. If L ⊥ M, then the product of the slopes is -1.

Perpendicular lines

The equation of line L is $y = 2x - 5$

The equation of line M is $y = -\frac{1}{2}x + 9$

The slope of L is 2, $m_1 = 2$.

The slope of M is $-\frac{1}{2}$, $m_2 = -\frac{1}{2}$

$m_1.m_2 = 2. -\frac{1}{2} = -1$

So L $\perp$ M

Perpendicular lines

Given a(4,3) b(2,8) and c(14,7) prove ab ⊥ ac

First find slope of ab.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Slope of ab = $\frac{8 - 3}{2 - 4} = \frac{5}{-2} = -\frac{5}{2}$

Slope of ac = $\frac{7 - 3}{14 - 4} = \frac{4}{10} = \frac{2}{5}$

$$-\frac{5}{2} \cdot \frac{2}{5} = -\frac{10}{10} = -1$$

So ab ⊥ ac

Intersection of two lines

Two lines intersect at a single point (x_1,y_1). To find the point of intersection we use simultaneous equations. See three examples on the following slides.

Simultaneous equations-example 1

Solve $y = 2x + 3$

$y = 7 - 2x$

In first equation $y = 2x + 3$ and in second equation $y = 7 - 2x$, so

$$2x + 3 = 7 - 2x$$

Solving for x

$$2x + 2x = 7 - 3$$

$$4x = 4$$

$$x = 1$$

Now substitute in x = 1 to either original equation to find y.

$$y = 2x + 3$$

$$y = 2(1) + 3 = 5$$

So solution x = 1 and y = 5

So point of intersection is (1,5)

Simultaneous equations-example 2

Solve $4x + 2y = 32$

$3x - y = 19$

To solve here we need to cancel the x or the y. To cancel a variable we need same number and opposite signs in front of the variable.

We will cancel the y terms. Why?

$4x + 2y = 32$

(x 2) $\underline{3x - y = 19}$

$$4x + 2y = 32$$
$$\underline{6x - 2y = 38}$$

$$4x + 2y = 32$$
$$\underline{6x - 2y = 38}$$
$$10x = 70$$
$$\mathbf{x = 7}$$

Substitute x = 7 into one of the original equations:

$$4x + 2y = 32$$
$$4(7) + 2y = 32$$
$$28 + 2y = 32$$
$$2y = 4 \text{ .....so } \mathbf{y = 2}$$

So point of intersection is (7,2)

Simultaneous equations-example 3

Solve $5x + 4y = 26$

$2x + 3y = 16$

To solve here we need to cancel the x or the y. To cancel a variable we need same number and opposite signs in front of the variable.

We will cancel the y terms. Why?

$$\begin{array}{rl} (3) & 5x + 4y = 26 \\ (-4) & \underline{2x + 3y = 16} \end{array}$$

$$15x + 12y = 78$$

$$-8x - 12y = -64$$

$$7x = 14$$

so $x = 2$ **$x = 2$**

Substitute x = 2 into one of the original equations.

$$5x + 4y = 26$$

$$5(2) + 4y = 26$$

$$10 + 4y = 26$$

$$4y = 26 - 10 = 16$$

$$y = 16/4$$ **$y = 4$**

So point of intersection is (2,4)

Drawing a line on the cartesian plane (the x – y axis)

To draw any line we need two points. Suppose we wish to draw the line $2x + 3y = 12$.

Find two points on the line…..where line cuts x-axis and y-axis.

<u>Line cuts x-axis at y=0…so let y=0 and solve for x.</u>

$2x + 3y = 12$

$2x + 3(0) = 12$

$2x = 12$……so $x = 6$

line cuts x-axis at (6,0)

Line cuts y-axis at x=0…so let x=0 and solve for y.

$2x + 3y = 12$

$2(0) + 3y = 12$

$3y = 12$……so $y = 4$

line cuts y-axis at (0,4)

Plot the two points and draw a line through the two points. See next slide.

$2x + 3y = 12$