Co-Ordinate Geometry

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**References**

1. Foundation Maths 4th edition, Anthony Croft and Robert Davison

# **3.1 Cartesian Plane**

**Cartesian Plane** - A 2 dimensional space made up of points which can be identified by their relation to the origin, the x-axis and the y-axis. It is also sometimes called the Co-ordinate plane.

**X-axis** – the horizontal number line in the coordinate system.

**Y-axis** – the vertical number line in the coordinate system.

**Origin** – a fixed point from which measurements are taken. The origin has coordinates (0,0) and is found at the point of intersection between the x-axis and the y-axis.

**Quadrant** – one of the four regions formed by the intersection of the x-axis and the y-axis.

Quadrant I – all points (+, +)

Quadrant II – all points (-, +)

Quadrant III – all points (-, -)

Quadrant IV – all points (+, -)

**Co-ordinates** – Numbers that define the position of a point or set of points. Every point has an x-coordinate and a y-coordinate.

**Ordered pair** – a pair of numbers (x-coordinate, y-coordinate) indicating the position of a point in the Cartesian Plane P (6,3) has an x-value of 6 and a y-value of 3.

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | QII |  |  |  | 5 |  |  | QI |  |  |
|  |  |  |  |  | 4 |  |  |  |  |  |
|  |  | (-3,4) |  |  | 3 |  |  |  |  |  |
|  |  |  |  |  | 2 |  |  | (3,2) |  |  |
|  |  |  |  |  | 1 |  |  |  |  |  |
| -5 | -4 | -3 | -2 | -1 | 1 | 2 | 3 | 4 | 5 |  |
|  |  |  |  |  | -1 |  |  |  |  |  |
|  |  |  |  |  | -2 |  |  |  |  | (5,-2) |
|  |  |  | (-2,-3) |  | -3 |  |  |  |  |  |
|  |  |  |  |  | -4 |  |  |  |  |  |
|  | QIII |  |  |  | -5 |  |  | QIV |  |  |

## *3.1.1 Distance between 2 points*

The distance between 2 points  and  is given by the formula



**Examples:**

1. Calculate the distance between the points (1,1) and (5,4)  
     
     
     
   
2. A circle is centered at the origin and passes through the point (0, -3). Which of the following points does the circle also pass through?

(3,3)

(-2, -1)

(2,6)

(1.5, 1.3)

(-3,4)

Since the circle is centered at the origin and passes through the point (0,-3), the radius of the circle is 3. If any other point is on the circle, the distance from that point to the center of the circle must also be 3. We will calculate the distance from the origin to each of the points above

***distance from (0,0) to (3,3)***









The distance from (0,0) to (3,3) = 4.24 the circle does not pass through the

point (3,3)

***distance from (0,0) to ***











The distance from (0,0) to (-2, -1) = 3  the circle passes through the point (-2 , -1)

***distance from (0,0) to (2,6)***









The distance from (0,0) to (2,6) = 6.32  the circle does not pass through the point (2,6)

***distance from (0,0) to (1.5,1.3)***









The distance from (0,0) to (1.5,1.3) = 1.98  the circle does not pass through the point (1.5,1.3)

***distance from (0,0) to (-3,4)***









The distance from (0,0) to (-3,4) = 5  the circle does not pass through the point (-3,4)

## *3.1.2 Midpoint between 2 points*

The midpoint M between 2 points (x1, y1) and (x2, y2) is given by the formula



We simply average the corresponding coordinates of the 2 points.

**Example:**

Find the coordinates of the midpoint of the line joining (1,3) and (3,5)



# **3.2 Linear Equations**

In general, a straight line is represented algebraically by a **linear equation** of the form



where  is the gradient or slope of the line and  is the -intercept. The gradient is also referred to as the slope and is a measure of the steepness of the line.

In a linear equation it is important to note that the power to which x is raised is 1. However in the equation , either *m* or *c* may have the value 0 so for example y = 2x is a straight line and y = 7 is also a straight line. The point at which the graph cuts the *y*-axis is called the *y*-intercept and can be found by letting *x = 0* which from the equation  we can see will give us *c,* so the point *(0,c)* is a point on the line and is also the *y*-intercept. The point at which the graph cuts the *x*-axis is called the *x*-intercept and can be found by letting *y = 0.*

**Example:**

Sketch the line described by the equation .

First, we need to rearrange the equation so that it is in the form . If we multiply both sides by 2,  becomes . From this equation, we can see that the gradient, , and the -intercept, . In order to sketch the curve, we need one other point. It is always useful to calculate the -intercept. This will be our other point. Let . Then  becomes . Solving this equation for  we find . Now we have two points: the -intercept (0,-4) and the -intercept (2,0).

Sketch:







O



2



-4

**Example:**

Sketch the line described by the equation .

From the equation , we can see that the gradient, , and the -intercept, . Again, we will calculate the -intercept to obtain a second point. When  we have . Solving this equation for  we find .

Now we have two points: the -intercept (0,4) and the -intercept (2,0).

Sketch:





-4



O

2



Note that the line in Figure 1 rises as we move from left to right and the line in figure 2 falls as we move from left to right. This is true for all lines. If *m* is positive the line will rise and if *m* is negative the line will fall as we move from left to right. If m = 0 the line will be horizontal as you can see from Figure 3 below.

*y*

-4



*x*

O

## *3.2.1 Gradient Formula*

The gradient (or slope) of a line measures the inclination of the line. By definition it is the ratio of the vertical change to the horizontal change. The vertical change is called the rise and the horizontal change is called the run. The slope is the rise over the run. The gradient of a line, , where  represents the change in  values or the difference between the *y*-coordinates and  represents the change in  values or the difference between the *x*-coordinates.

Given 2 points  and  the gradient is



**Examples:=**

1. Find the gradient of the line joining the 2 points (1,3) and (3,5)



1. Given the sketch in Figure 1 above, determine the gradient of the line (i.e. pretend that you do not know the equation of this line).

From the sketch we are given two points, that is (2,0) and (0,-4). So the gradient 

We can confirm that this answer is correct by again looking at the equation given in example 4, i.e. . We know from this equation that the gradient is 2.

## *3.2.2 The equation of a line using one point and the gradient*

The equation of a line which has gradient *m* and which passes through the point

 is



**Example:**

Find the equation of the line with gradient 2 passing through (1,4)

y – 4 = 2(x – 1)

y – 4 = 2x – 2

y = 2x + 2

# **3.3 Intersection of Lines**

Earlier we saw that the equation of a line is given by . In this section we will investigate the behavior of two lines. We will see there are only 3 possibilities for the intersection of two lines.

1. One point of intersection between two lines.
2. No intersection between two lines – parallel lines.
3. Infinitely many solutions – coincident lines.
4. One point of intersection between two lines.







O



From Figure 4 we can see that there is one point where the two lines meet.

The equations of the two lines are

 (1)

 (2)

It is possible to determine what the point of intersection is by solving the two *simultaneous equations*. To do this we must eliminate one of the unknowns i.e. we must liminate either *x* or *y*. This can be done by multiplying each equation by a suitable constant and then adding / subtracting, as follows:

 (3) [equation (1) \* 2]

 (4) [equation (2) \* 3]

 [equation (3) – equation (4)]

 [Solve for ]

Substituting this value for  back into either equation (1) or equation (2) and then solving for  will result in the solution. That is,

 [Substituting  into equation (1)]

 [Solve for ]

So the point of intersection between the two lines in (3,5).

Alternatively, we could have solved equations (1) and (2) by eliminating  first, as follows:

 (5) [equation (1) \* 3]

 (6) [equation (2) \* 2]

 [equation (5) + equation (6)]

 [solve for ]

Now substitute this value for  back into either equation (1) or equation (2). That is,

 [Substituting  into equation (1)]

 [Solve for ]

So the point of intersection is at (3,5) which agrees with our result above.

1. No intersection between two lines – parallel lines.







O



From Figure 5 we can see that there are no points where the two lines meet.

The equations of the two lines are

 (1)

 (2)

Let’s try to solve these two simultaneous equations.

 (3) [equation (1) \* 2]

 (2) [no change necessary to equation (2)]

 [equation (3) – equation (2)]



Notice what this results says. . Obviously this is not correct. That is, we have reached a *contradiction*. When this occurs we say that the original equations are *inconsistent,* and no solution exists (as illustrated in Figure 5).

1. Infinitely many solutions – coincident lines.







O



Figure 6 shows two lines that are superimposed on each other. In other words, the two lines are *coincident* (the same). Algebraically, this means that the two lines meet in infinitely many points along their lengths.

The equations of the two lines are

 (1)

 (2)

Again, try to solve these two simultaneous equations.

 (3) [equation (1) \* 2]

 (2) [no change necessary to equation (2)]

 [equation (3) – equation (2)]

Here it has not been possible to eliminate an unknown – we end up eliminating both unknowns. However, the final result of  is important. This tells us that it does not matter what values we substitute for  and ; we will always achieve this result of . That is, there are an *infinite* number of values that we could choose for  and . The equations will remain consistent. Whenever the result is  we say that there are an infinite number of solutions.