De Morgan's Law

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- ✓ This law can be used to change the function of logic gates, so that NAND gates (or NOR gates) can carry out any of the other standard logic functions of gates.

- ✓ The theorem comprises two laws that describe how inverting the inputs to a gate, changes the gate's function.

De Morgan's Law

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Law 1

$$\overline{A} + \overline{B} = \overline{A \bullet B}$$

Inverting the inputs to an OR gate changes its function to NAND.

Law 2

$$\overline{A} \bullet \overline{B} = \overline{A + B}$$

Inverting the inputs to an AND gate changes its function to NOR.

Boolean Laws

De Morgan's Law

AND	OR
$\overline{AB} = \bar{A} + \bar{B}$	$\overline{A + B} = \bar{A}\bar{B}$

De Morgan's Law Proof

$$\overline{AB} = \overline{A} + \overline{B}$$

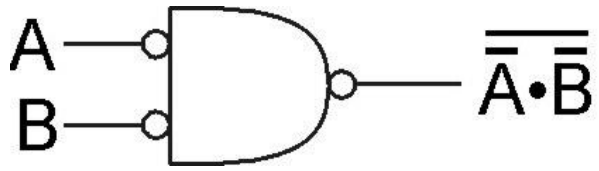
A	B	$\overline{AB}$	$\overline{A}$	$\overline{B}$	$\overline{A} + \overline{B}$
0	0	1	1	1	1
0	1	1	1	0	1
1	0	1	0	1	1
1	1	0	0	0	0

$$\overline{A + B} = \overline{A}\,\overline{B}$$

DeMorgan's Law

Converting AND to OR (with some help from NOT)

Consider the following gate:



A	B	$\overline{A}$	$\overline{B}$	$\overline{A} \cdot \overline{B}$	$\overline{\overline{A} \cdot \overline{B}}$
0	0	1	1	1	0
0	1	1	0	0	1
1	0	0	1	0	1
1	1	0	0	0	1

Same as A+B!

To convert AND to OR (or vice versa), invert inputs and output.

DeMorgan Shortcut

BREAK THE LINE, CHANGE THE SIGN

Break the *LINE* over the two variables,
and change the *SIGN* directly under the line.

$$\overline{A \cdot B} = \overline{A} + \overline{B}$$

Break the line, and change the AND function to an OR function. Be sure to keep the lines over the variables.

$$\overline{A + B} = \overline{A} \cdot \overline{B}$$

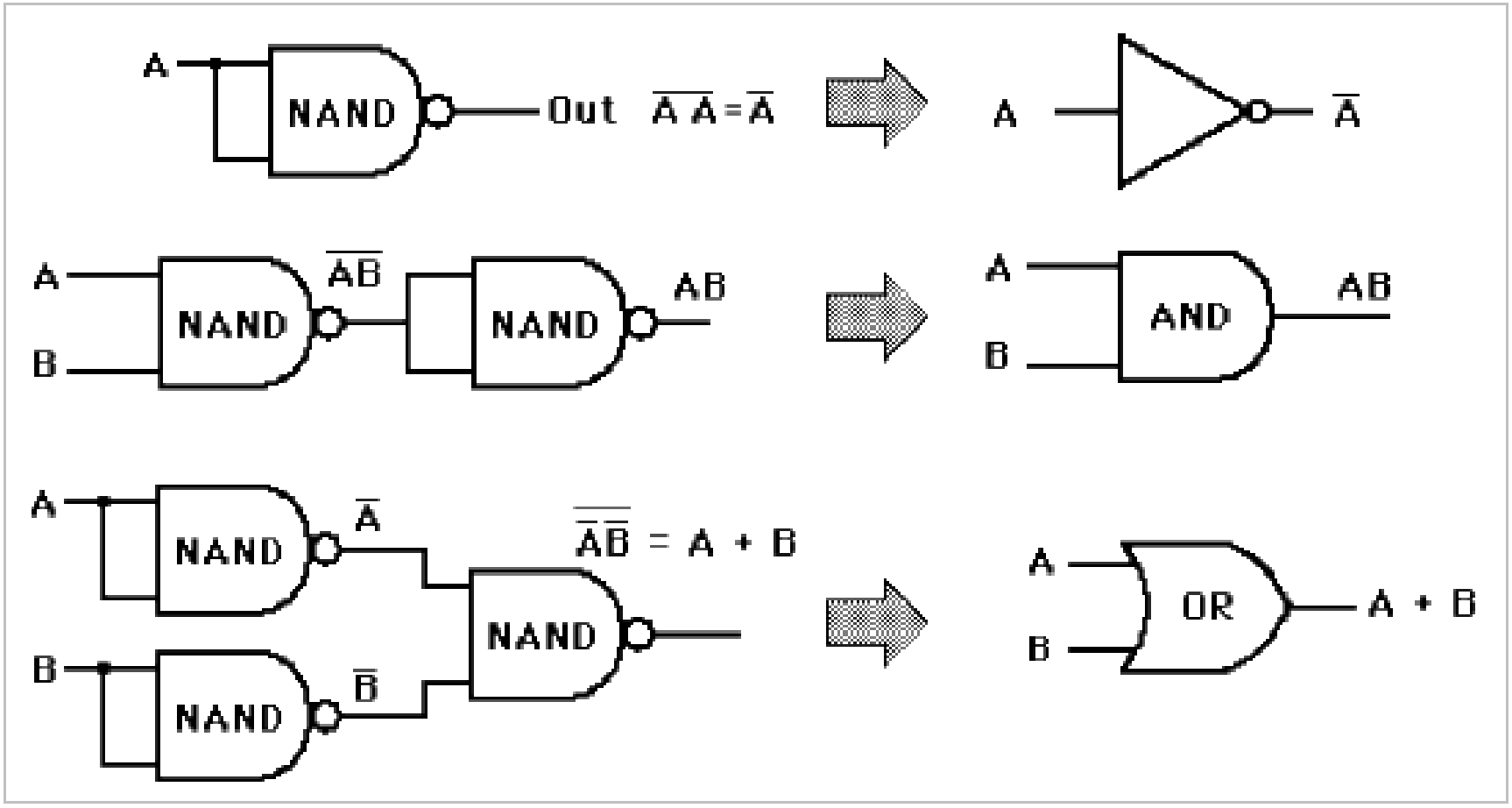
Break the line, and change the OR function to an AND function. Be sure to keep the lines over the variables.

NAND Gates

All computations can be done with a NAND Gate.

- How to build a NOT gate?
  - Connect all inputs to common input signal
- How to build an AND gate?
  - Use the invertor just built to invert the output of a NAND
- How to build an OR gate?
  - We can construct an OR gate from NAND gates by applying De Morgan's theorem

NAND Gates


A
NAND
Out
$\overline{A}\,\overline{A} = \overline{A}$
A
$\overline{A}$
A
B
NAND
$\overline{AB}$
NAND
AB
A
B
AND
AB
A
NAND
$\overline{A}$
$\overline{A}\,\overline{B} = A + B$
B
NAND
$\overline{B}$
NAND
A
B
OR
A + B