BIMCT Individual Round High School Division

BIMCT Team

October 2022

Individual Round

1. Alice and Bob share the same birthday. Six years ago, Alice was 5 times as old as Bob. If Alice is only 3 times as old as Bob now, how old will Bob be when Alice is 57 years old?

2. Workers Charlie and Doyle take 40 minutes and 60 minutes to paint a wall by themselves, respectively. When they work together, they each work 50% slower because they constantly talk. If N is the time in minutes it takes for Charlie and Doyle to paint a wall together, find N.

3. Bob rolls two standard 6-sided dice and adds the sum of the faces. If the probability that the sum of the faces is a perfect square can be expressed as $\frac{a}{b}$, where a and b are relatively prime, find the value of $a + b$.

4. Amanda has a pile of n candies that she wants to distribute to 4 friends. The $i-$th friend will receive $\frac{1}{i+1}$ of the remaining candies that Amanda has and will be the $i-$th person to receive their share of the candy. What is the minimum value of n such that each friend receives an integer amount of candies?

5. 7 students are voting for one of two activities. Each student independently votes for one of the activities, with a $1/2$ chance of picking either activity. The teacher then selects the activity with more votes. If the expected number of people who voted for the activity selected is $\frac{a}{b}$, what is the value of $a + b$?

6. Zack is walking along a number line from 0 to 10. He starts at position 4. Every minute, he flips a coin. If the coin shows heads, he moves forward by 1, and if the coin shows tails, he moves backward by 1. If the probability that he arrives at 10 before he arrives at 0 can be expressed as $\frac{a}{b}$, where a and b are relatively prime, find $a + b$.

7. In triangle ABC, altitudes AD, BE, and CF intersect at H. Given that $AB = 24$, $AH = 20$, and $BH = 8$, find the value of $AE \cdot EC$.

8. The newest world cup soccer ball is a polyhedron consisting only of faces which are regular octagons, regular heptagons, and squares. At each vertex, there is one octagon, one heptagon, and one square. If there are a octagons, b heptagons, and c squares, what is $10000a + 100b + c$?

9. How many ordered pairs of (not necessarily distinct) permutations $(\sigma_1,\ \sigma_2)$ of the set $S = \{1, 2, 3, 4\}$ satisfy
$$\sigma_1(\sigma_2(i)) = \sigma_2(\sigma_1(i))$$
for all $i \in S$? (A permutation σ is a function that associates each $i \in S$ to some $j \in S$, such that no two $i \in S$ are associated with the same j. An example permutation σ of the set $S = \{1, 2, 3, 4\}$ is $\{1, 2, 3, 4\} \rightarrow \{2, 1, 3, 4\}$. In this case $\sigma(1) = 2, \sigma(2) = 1, \sigma(3) = 3, \sigma(4) = 4$.

10. Let w, x, y, and z be positive real numbers that satisfy the following equations

$$\begin{aligned} 4x^2 + y^2 + xy &= 496 \\ x^2 + z^2 + xz &= 121 \\ 4z^2 + w^2 - zw &= 496 \\ w^2 + y^2 - yw &= 484 \end{aligned}$$

Then, $xw + yz$ can be expressed as $a\sqrt{b}$, where b is squarefree. Find $a + b$.