

Domain method of estimating frequency data in time series

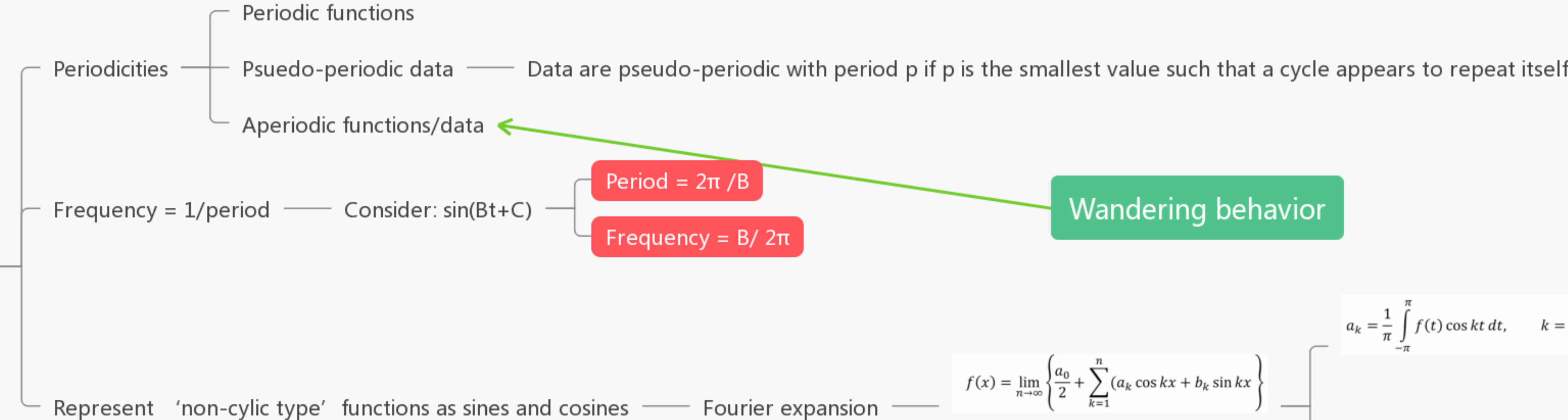
Frequency Domain/Spectral Density

Discover the frequency content of a set of data(or lack), better understand process generating data

1. Review:Trig Functions

$\begin{matrix} \text{Sin}(t) \\ \text{Cos}(t) \end{matrix} \left. \vphantom{\begin{matrix} \text{Sin}(t) \\ \text{Cos}(t) \end{matrix}} \right\} \text{Period} = 2\pi \quad \text{Horizontal Phase Shift: } (t+\Delta)$

2. The Frequency domain



Example:

Fourier Expansion for $f(x) = x$ — $x = -2 \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \sin(nx), \quad x \in (-\pi, \pi)$

Fourier Expansion for $f(x) = e^x$ — $e^x = \frac{\sinh \pi}{\pi} + \frac{2 \sinh \pi}{\pi} \sum_{k=1}^{\infty} \frac{(-1)^k}{1+k^2} [\cos kx - k \sin kx]$

3. Spectrum/Spectral density

Tool to identify the frequency content of a time series

Estimate spectral density — Smoothing spectrum

Choose $M(2\sqrt{n})$ and a window function(λ_k) — $\hat{S}(f) = \lambda_0 \cdot 1 + 2 \sum_{k=1}^M \lambda_k \hat{\rho}_k \cos 2\pi f k \quad |f| \leq .5$

Minimize impact of $\hat{\rho}_k$ as k increases

Properties

$\int_{-.5}^{.5} S(f) e^{2\pi i f k} df = \rho_k$

$\int_{-.5}^{.5} S(f) df = 1$

White noise

$S_x(f) = 1 + 2 \sum_{k=1}^{\infty} \rho_k \cos 2\pi f k$

$\hat{S}_x(f) = \lambda_0 \cdot 1 + 2 \sum_{k=1}^M \lambda_k \hat{\rho}_k \cos 2\pi f k$

$M = 2\sqrt{n}$

$|\lambda_k|$ decreases as k increases

$|f| \leq .5$

$S(f) = 1$

4. Nyquist frequency

$f > 0.5$ — Aliasing: Sampling at $t = 0, +1, +2, \dots$ the two curves are indistinguishable — Lower frequency is identified

5. Estimation of spectrum

Log spectral density — accentuate secondary peaks

Window:

1. Bartlett (triangular) window

$$\lambda_k = 1 - \left| \frac{k}{M} \right|, \quad k = 0, 1, \dots, M$$

$= 0, \quad k > M$

2. Tukey window

$$\lambda_k = \frac{1}{2} \left[1 + \cos \left(\frac{\pi k}{M} \right) \right], \quad k = 0, 1, \dots, M$$

$= 0, \quad k > M$

3. Parzen window

$$\lambda_k = 1 - 6 \left(\frac{k}{M} \right)^2 + 6 \left(\frac{k}{M} \right)^3, \quad 0 \leq k \leq \frac{M}{2}$$
$$= 2 \left[1 - \left(\frac{3k}{M} \right) \right]^3, \quad M/2 < k \leq M$$

$= 0, \quad k > M$