ARIMA Model and Seasonality

Non-stationary model

1. Signal-plus-noise model

$X_t = s_t + Z_t$

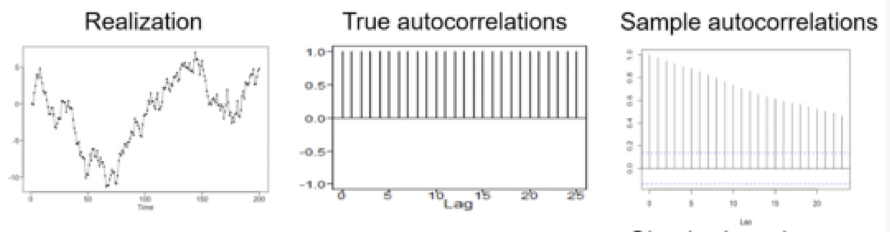
- s_t is a deterministic signal
- Z_t is a zero-mean, stationary process

Example signals:

- $s_t = a + bt$ — **Example signals:** $X_t = .1 + .7t + Z_t$ Z_t are AR(1)
- $s_t = a + bt + ct^2$ — $X_t = .1 + .7t + .07t^2 + Z_t$ $Z_t \sim$ N(0,500)
- $s_t = A\cos(2\pi ft + C)$ C constant — $X_t = 5\cos(2\pi(.1)t + 2.5) + Z_t$ $Z_t \sim$ ARMA(1,1)

2. ARIMA models

- Properties and Characteristics — ARIMA(p,d,q) — The *autoregressive integrated moving average process* of orders p, d, and q (denoted ARIMA(p,d,q)) is a process, X_t, whose differences $(1-B)^d X_t$ satisfy a (stationary) ARMA(p,q) model, where d is a non-negative integer. — $\varphi(B)(1-B)^d X_t = \theta(B)a_t$
- ARIMA(0,1,0) — $(1-B)X_t = a_t$ — $X_t - X_{t-1} = a_t$ wandering behavior



Strictly wandering since there is one factor with a root of 1

Identically equal to $\rho_k = 1$

Slowly damping, even though true values don't damp

- **Differency** — Account for a (1-B) factor in the ARIMA — Make ARIMA stationary — EX. ARIMA(0,1,0) — $(\tilde{X}_t = X_t - X_{t-1})$ first difference, equals to white noise(stationary) — $\tilde{X}_t = a_t$ Remove (1-B)
- **Summary**
  - (1-B)^d dominates the stationary parts
    - Realizations
    - Autocorrelations
    - spectral density all have peaks at f=0
  - True acf is 1, sample is damping — indication of ARIMA model

3. Seasonal models

$\varphi(B)(1-B^s)X_t = \theta(B)a_t$

- Contain factor $(1-B^s)$
- Monthly data $(1-B^{12})$, quarterly data $(1-B^4)$,…

Quartely — S=4

Factor	Roots	Abs Recip	f
$1-B$	1	1	0
$1+B^2$	$\pm i$	1	.25
$1+B$	1	1	.5

Montly — S=12

Factor	Root(s)	Abs Recip	f
$1-B$	1	1	0
$1-\sqrt{3}B+B^2$	$.866 \pm .5i$	1	.083
$1-B+B^2$	$.5 \pm .866i$	1	.167
$1+B^2$	$+i$	1	.25
$1+B+B^2$	$-.5 \pm .866i$	1	.333
$1+\sqrt{3}B+B^2$	$-.866 \pm .5i$	1	.417
$1+B$	-1	1	.5

$\varphi(B)(1-B)^d(1-B^s)X_t = \theta(B)a_t$

Note that this model has:

- Stationary factors: $\varphi(B)$ and $\theta(B)$
- An ARIMA-type factor: $(1-B)^d$
- A seasonal factor: $1-B^s$

More general seasonal model

Q： Why bi-annual s=6 rather than s=2?