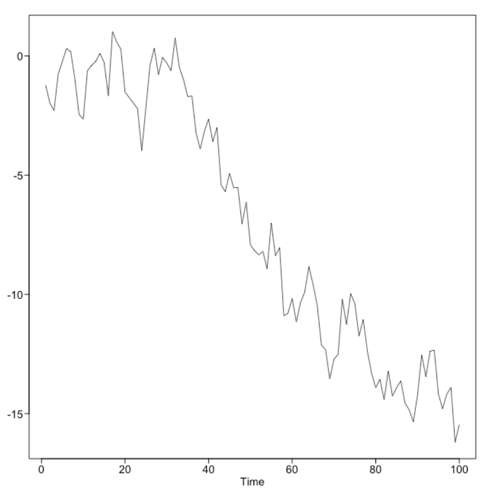
Final Exam Time Series Fall 2019 (75 pts): Solution

True or False (2 pts each):

1. If a spectral density has a peak at zero then it must come from a non-stationary process.
   False.
2. If a spectral density has a strong peak between 0 and .5 then the data must come from a non-stationary process.
   False.
3. Burg estimation of parameters of models of the airline data will always yield a stationary model.
   True. (Burg estimates are always stationary, only MLR can yield nonstationary models)
4. If the Ljung-Box test fails to reject the null hypothesis, then we know for sure that that set of data is white noise.
   False. Ljung-Box is a very low powered test.
5. A problem with the Dickey-Fuller test is that it will indicate that there is a (1-B) in the model when the data is actually from a stationary process more often than it should.
   True.
6. The Cochrane-Orcutt procedure is helpful in adjusting a linear model for serial correlation.
   True.
7. A model must yield white noise residuals to be useful.
   False. "All models are wrong, some are useful" – George Box
8. The model $X_t = .7a_{t-1} + a_t$ is already in GLP form.
   True.

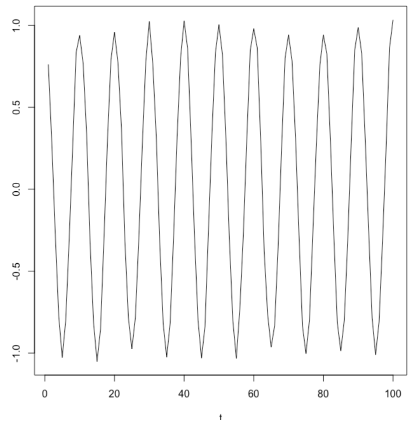
9. We know that an invertible model is desirable because, it ensures that the "present is related to the past in a sensible manner." Describe briefly what is meant by this statement. (3pts)
   In short, if a model is not invertible, the coefficients of the model will grow exponentially with increasing lags, which is not "sensible". Sensible models are decreasingly dependent upon the past, which requires thetas less than 1.

10. Consider the realization below and highlight the best answer. (3 pts)



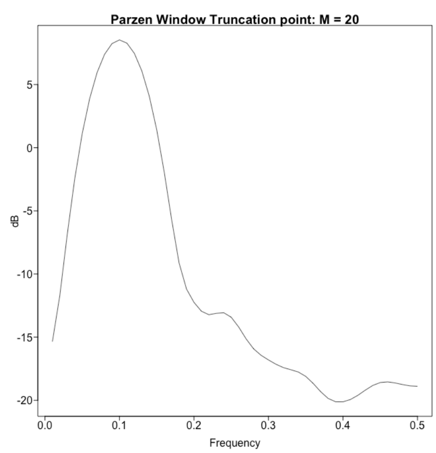
A. This data clearly comes from a stationary process.
B. This data clearly comes from a non-stationary process.
C. We cannot tell from a single realization
This could be from an AR(1) model!

11. Consider the realization below and highlight the best answer. (3 pts)



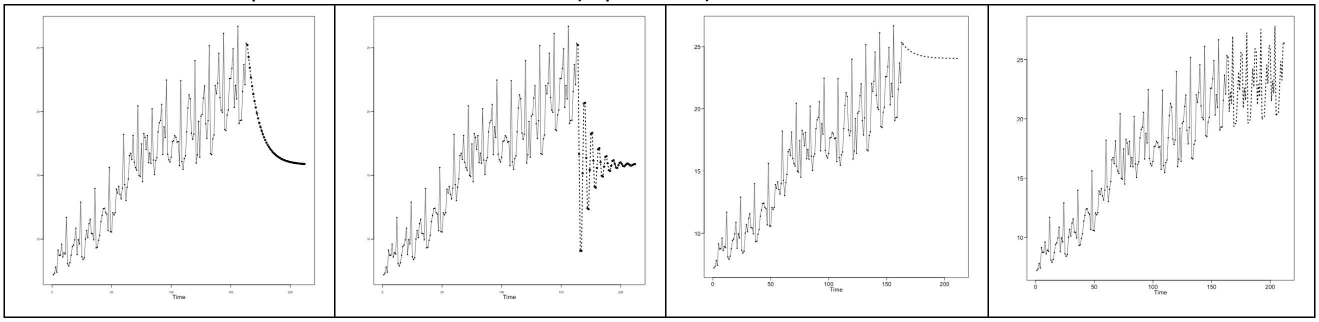
A. This data clearly comes from a stationary process.
B. This data clearly comes from a non-stationary process.
C. We cannot tell from a single realization.

12. You have been hired by a sleep scientist to study the cycles of REM sleep. REM (Rapid Eye Movement) is the part of sleep in which people are thought to dream. The scientist's hypothesis is that on a certain drug, these REM cycles should happen fairly regularly. In order to test this hypothesis, she measures the muscle activity in 5-minute intervals (observations are taken 5 minutes apart) and records the number of eye movements in those 5 minutes. Below is the spectral density of the resulting data set. The scientist would like to know if there is any evidence of consistent periodic behavior of the REM cycle and if so, what the period is thought to be. Write a 1 to 3 sentence response addressing if there is evidence of periodic behavior of the REM cycle and, if so, provide an estimate of the time amount of time (in minutes) between each REM cycle. (4 pts)



There is strong evidence of a periodic behavior of the REM cycle. The frequency is approximately 0.1, which results in a period of 10. The data is recorded in five-minute intervals, which suggest an approximately 50-minute cycle.

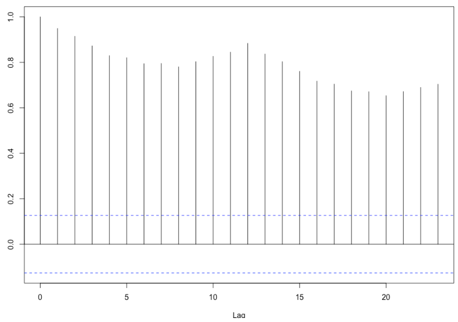
13. Below are the monthly retail debit card usage in Iceland from January 2000 - August 2013 and four different forecasts of the next four years. Match the forecast with the corresponding model that produced those forecasts. (2 pts each)



__D_ __C_ _B__ __A_

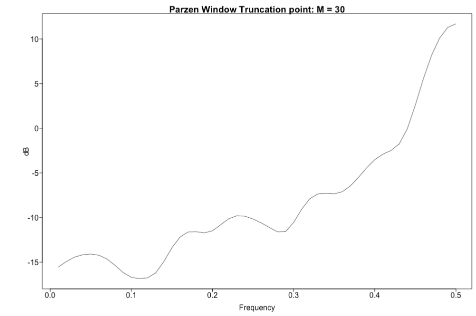
A. Airline Model
B. $\phi(B)(1-B)X_t = \theta(B)a_t$
C. AR(2) Complex Roots
D. AR(2) Real Roots
E. $\phi(B)(1-B^s)X_t = \theta(B)a_t$
F. $(1-B)X_t = a_t$

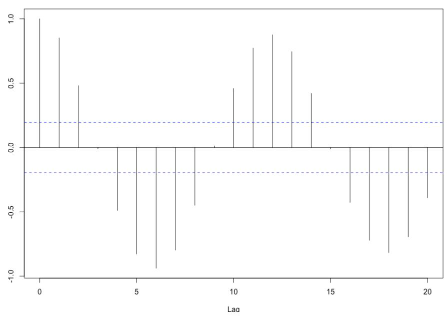
14. Match the ACF on the left with the corresponding spectral density or realization on the right. Simply place the letter next to the corresponding number below the plots. (2 pts each)



1.

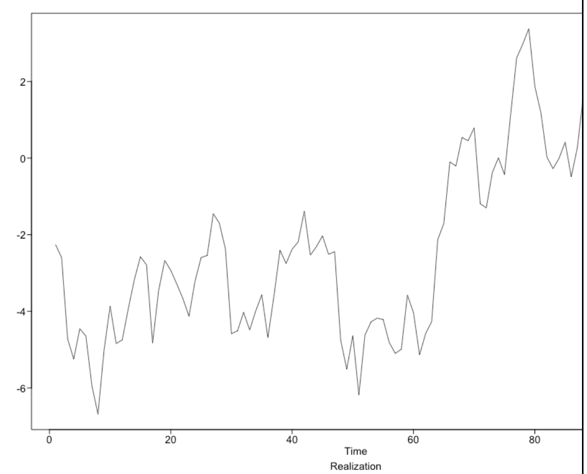
a.



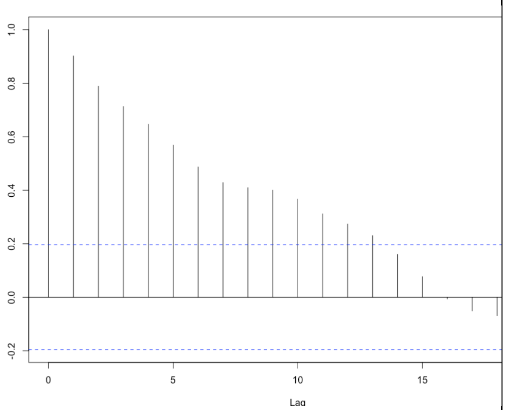


2.

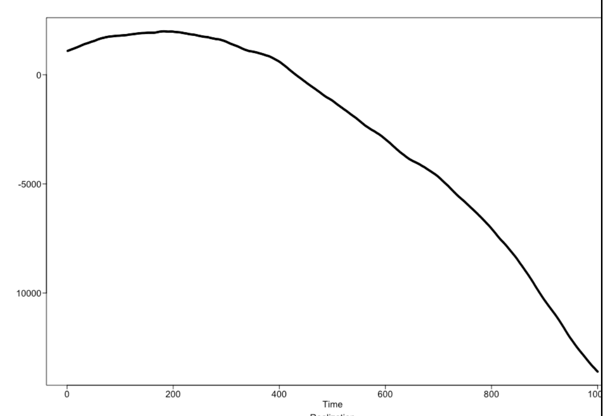
b.

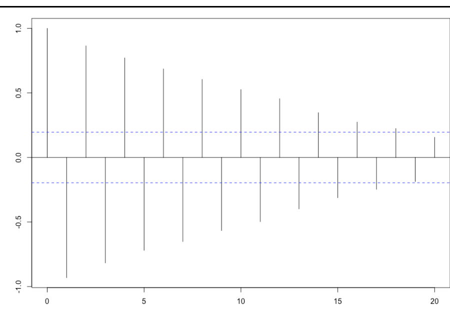


3.



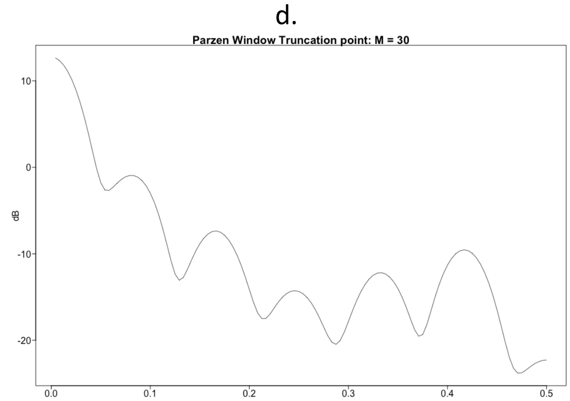
c.



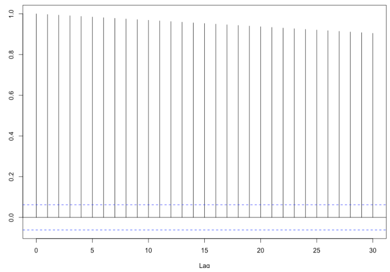


4.

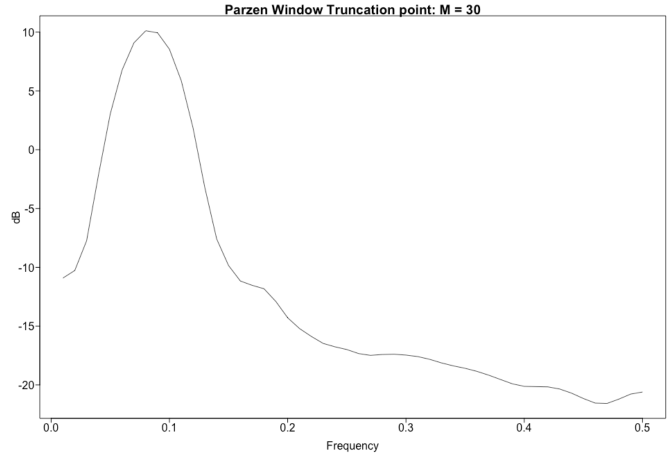
d.



5.



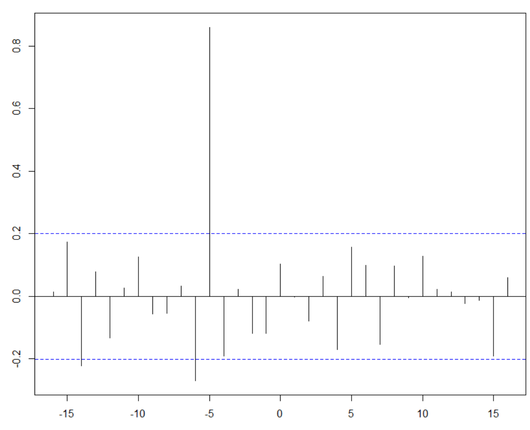
e.



1. _d__ 2._e__ 3. __b_ 4. _a__ 5. __c__

15. Consider the data in the file: FinalExamData.csv. This file contains a column X_t and a column Z_t. Your goal is to simply model this data the using a vector autoregressive model (VAR) and a multi-layer perceptron model (MLP). You ultimately want to forecast X_t with a horizon of 10 using Z_t if it is useful. Provide the following information in your response:
    a. Identify the relationship/association between X_t and Z_t. Specifically, is there evidence of a relationship/association between X_t and a lagged Z_t? … if so, what is the lag?) What evidence do you have to support this relationship/association? (4 pts)

    There seems to be a strong correlation between Xt and Zt at a lag of 5. See the following cross-correlation plot from: ccf(tsZt, tsXt)

    The correlation between these two now is also large when lagged (ccf no longer works with the na's introduced)

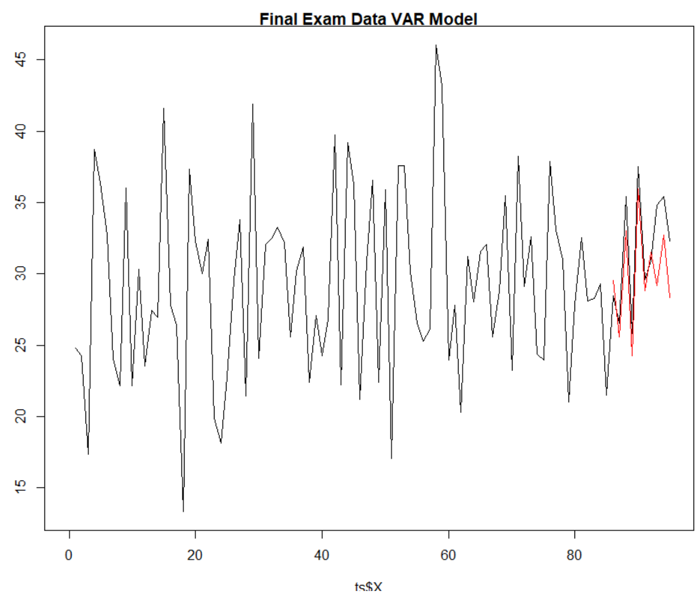
```
> cor(ts$Xt, Zt_lag, use = "complete.obs")
[1] 0.9358085
```

    b. For each model (VAR / MLP) (10 pts each model):
        i. Provide the code you used to fit the model and make the predictions.
        ii. Provide a plot of the 10 forecasts (time points 96 – 105). You do not need prediction intervals for this question.

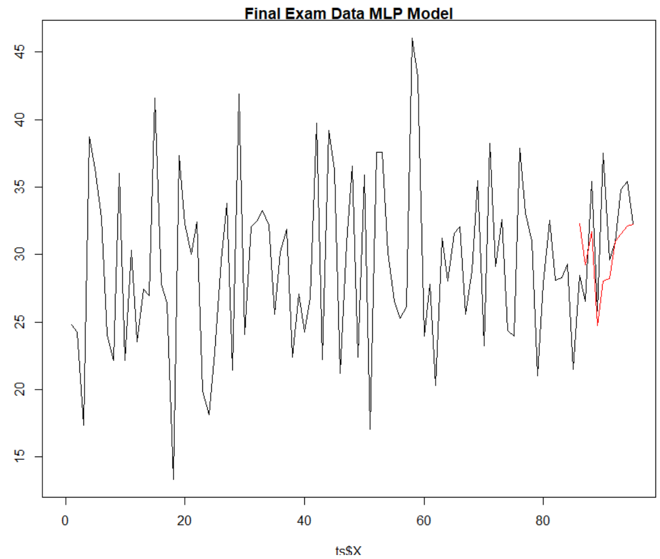
I'm going to lose points here, but I can't do this with extra regressors and don't know how. To predict 96-105 we need more observations of Zt. I'll do 86-95.

iii. Find the ASE for each model using the last 10 observations of the dataset. Include your code for this as well.

```
#####VAR MODEL#####

ccf(ts$Zt, ts$Xt)

VARselect(ts_nox, lag.max = 10, type = "both")

Zt_lag = dplyr::lag(ts$Zt, 5)

#VARSelect suggests p = 7, but we know 5 is best!

VAR_mod <- VAR(ts_nox, type = "both", p = 5)
var_preds <- predict(VAR_mod, n = 10)

plot(ts$X, ts$Xt, type = "l",xlim = c(0,105), ylab = "Xt/Yt", main = "Final Exam Data VAR Model")
lines(seq(96,105,1), var_preds$fcst$Xt[,1], type = "l", col = "red")
```




```
#####MLP MODEL#####
library(nnfor)
ts_Xt <- ts(ts$Xt)
ts_Zt_df <- data.frame(ts(ts$Zt))

mlp_model <- mlp(ts_Xt, xreg = ts_Zt_df, hd.auto.type = "cv")
mlp_preds <- forecast(mlp_model, h = 10)
plot(ts$X, ts$Xt, type = "l",xlim = c(0,95), ylab = "Xt/Yt", main = "Final Exam Data MLP Model")
lines(seq(86,95,1), mlp_preds$mean, type = "l", col = "red")
```



ASE:

```
> ASE = mean((ts$Xt[86:95] - var_preds$fcst$Xt[,1])^2)
> ASE
[1] 7.673009
> ASE = mean((ts$Xt[86:95] - mlp_preds$mean)^2)
> ASE
[1] 15.08779
```

c. Make a quick statement about which model you feel is most useful and why. (4 pts)

The VAR model seems most useful. Better performance may have been achieved in the neural network model with more time and more layers, but VAR outperformed it with the extra regressors. As we knew they are highly correlated, and the VAR model seems to have captured this well.