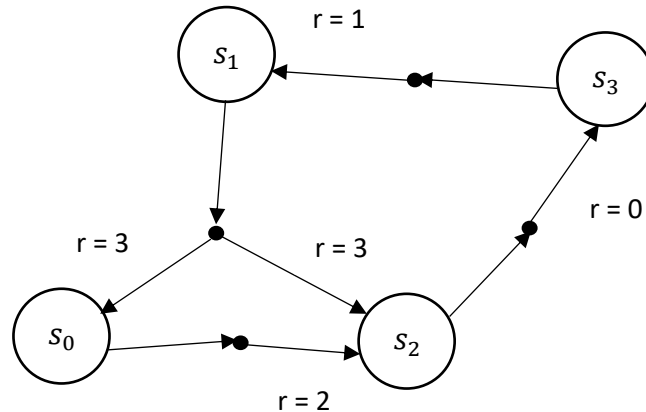


### Q1(a)

CID is 01328574. The trace generated is:

$$\tau = s_3 1 s_1 3 s_0 2 s_2 0 s_3 1 s_1 3 s_2 0$$

### Q1(b)



The above diagram shows the MDP process between states strictly according to the data generated. As can be concluded from the trace, for the 4 possible states there are a total of 5 unique state transitions:  $s_3 \rightarrow s_1, s_1 \rightarrow s_0, s_0 \rightarrow s_2, s_2 \rightarrow s_3, s_1 \rightarrow s_2$ . All state transitions from each state seem to be deterministic, except for state  $s_1$  where an action in that state can lead to either  $s_0$  or  $s_2$ . Note that the black dots in between the arrows represent the link between action chosen and the resulting next state, reflecting the transition dynamics. Rewards are labelled only after the black dot for each transition, representing the reward of entering the next state, conditioned on the previous state and action selected.

### Q1(c)(i)

From data and the trace above, the transition matrix  $T_{ij}$  can be expressed as:

$$T_{ij} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0.5 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

### Q1(c)(ii)

The immediate reward function of MDP takes the form of  $r_{(t+1)} = r(s_{t+1}, s_t, a_t)$ , and in this case the action for each step is fixed/constant, then we have:  $r_{(t+1)} = r(s_{t+1}, s_t)$ . Additionally, by inferring from the reward data generated, it is found that reward  $r_{(t+1)}$  is independent of the next state  $s_{t+1}$  and thus only dependent on current state  $s_t$ , i.e. at  $t = 2$ :  $r_{t=3} = r(s_0, s_1) = 3$ ; at  $t = 6$ :  $r_{t=7} = r(s_2, s_1) = 3$ , which indicates at step  $t = 2$  and  $t = 6$ , the immediate rewards are the same, as for both instances the current states  $s_t$  are  $s_1$ . This conclusion brings further simplification to the reward function:  $r_{(t+1)} = r(s_t)$ .

A reward matrix  $R_{ij}$  can be generated for this process, given the reward function's independence on action at each step.  $R_{ij}$  should be a 4x4 matrix, since there are only 4 possible states. Considering further that  $r_{(t+1)} = r(s_t)$ , across each row there should be constant value, since reward only depending on the current state. Finally each entry of the matrix shall only have non-negative value due to the way reward is generated. By only evaluating the matrix according to the data, we have:

$$R_{ij} = \begin{bmatrix} 2 & 2 & 2 & 2 \\ 3 & 3 & 3 & 3 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

### Q1(c)(iii)

The state value for the first state can be obtained via state value Bellman equation. The value function  $V_{\pi(s)}$  for each state  $s_t = s$  is defined as:

$$V_{\pi(s)} = E_{\pi}[R_t | S_t = s]$$

The expected value is obtained by summing all possible outcomes weighted by the probability that they occur, thus we can expand:

$$V_{\pi(s)} = \sum_a \pi(a|s) \sum_{s'} \sum_r p(s', r | s, a) [r + \gamma V_{\pi(s')}] \quad \text{where } s_{(t+1)} = s'$$

Since there is only one possible action for every state, the policy  $\pi$  is simply equal to 1 for every state, then the value function becomes:

$$V_{\pi(s)} = \sum_{s'} \sum_r p(s', r | s, a) [r + \gamma V_{\pi(s')}]$$

Now this is equivalent to the action value function which is defined as:

$$Q_{\pi(s,a)} = E_{\pi}[R_t | S_t = s, A_t = a] = \sum_{s'} \sum_r p(s', r | s, a) [r + \gamma V_{\pi(s')}]$$

Hence in this case, we have  $V_{\pi(s)} = Q_{\pi(s,a)}$ , which is of no surprise since the action  $a$  at each step can be omitted due to fixed singular action. Now to express the state value for the first state ( $s_3$ ), substituting in the discount factor, immediate reward  $r$ , the state value for next state  $s'$ , and the transition dynamics/probability  $p(s_1, r | s_3)$  which is assumed to be unclear as the environment is not fully known:

$$V_{(s_3)} = \sum_{s'} \sum_r p(s', r | s) [r + V_{\pi(s')}]$$

It can be seen that  $V_{(s_3)}$  depends on state values for all potential next states, meaning when after the terminal state is reached the state values can be back propagated and then the first state value can be evaluated.

## Q2(a)

Name: DING KE; CID: 01328574; Reward state:  $s_j = s_3$ ;  
 $p = 0.55$ ;  $\gamma = 0.55$

## Q2(b)

(1). A gridworld environment with transition matrix  $T$  and reward matrix  $R$  is specifically designed for the problem to be solved by DP. The optimal state values and policy are obtained by policy iteration algorithm(policy evaluation in policy improvement). Parameters to be set: transition probability  $p = 0.55$ , discount factor  $\gamma = 0.55$ , policy iteration threshold = 0.0001. Assumptions: assume principle of optimality.

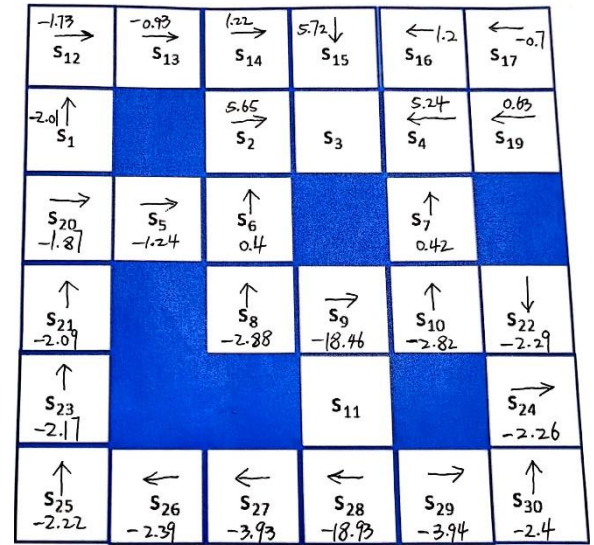
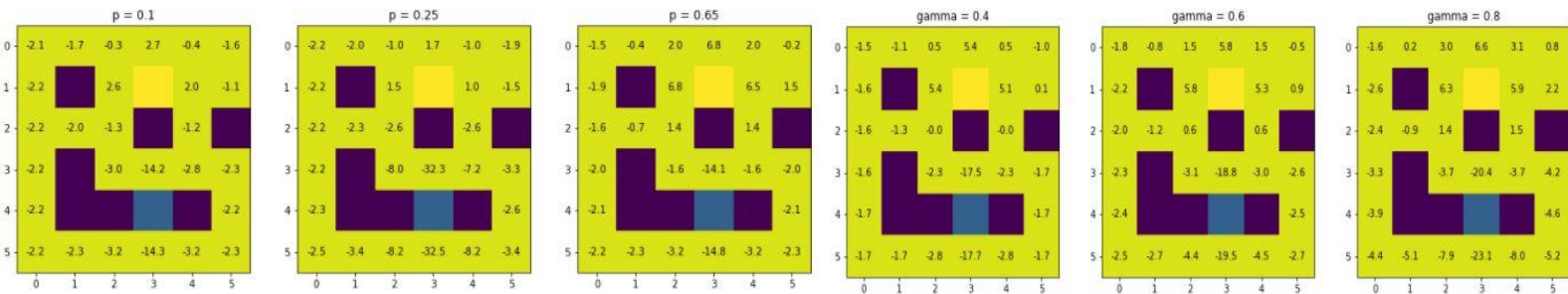


Figure 1. Optimal value function and policy for DP.

(2),(3). See figure 1.



Graphical representation of the policy for each p:

Graphical representation of the policy for each gamma:

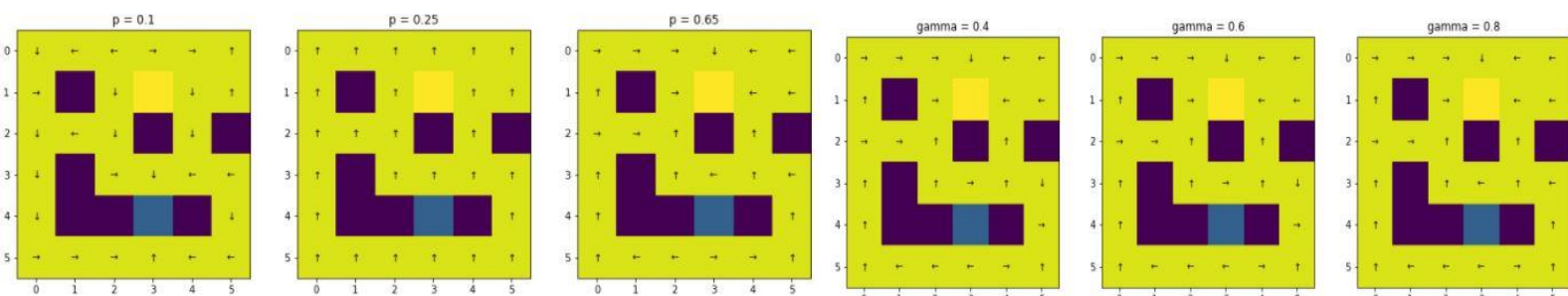


Figure 2. Change of optimal value function and policy with different  $p$ .

Figure 3. Change of optimal value function and policy with different  $\gamma$ .

(4). A transition probability of 0.25 implies in effect exact random selection of actions regardless of the optimal policy, a low value of  $p$  is found to be able to introduce tremendous amount of noise. From figure 2, all other parameters are held constant while  $p$  varies, and the resulting state value function generally increase in value as  $p$  increases, and the optimal policy at higher  $p$  ( $p > 0.6$ ) is considerably less noisy. The discount factor  $\gamma$  controls how much weight long term reward can carry. As  $\gamma$  increases from below 0.5 to above 0.5, from figure 3, the optimal policy becomes less noisy as the agent becomes more far-sighted. The state values also become more negative as  $\gamma$  increases(more variance).

## Q2(c)

(1). A new interactive gridworld environment class(gridworldenv) with the core role of evaluating agent's action and generating reward is constructed for the two RL learners: MC and TD. The environment itself also encodes the transition uncertainty characterised by  $p$ . The MC iterative learning algorithm is used to obtain an estimate of the optimal action value function  $Q_{(s,a)}$ . The optimal value function  $V$  can then be obtained by maximising optimal action value function, according to Bellman optimality equation. The optimal policy is the greedy policy. Parameters:  $p = 0.55$ ,  $\gamma = 0.55$ ,  $\epsilon = 0.1$ , learning rate/stepsize  $\alpha = 0.2$ .

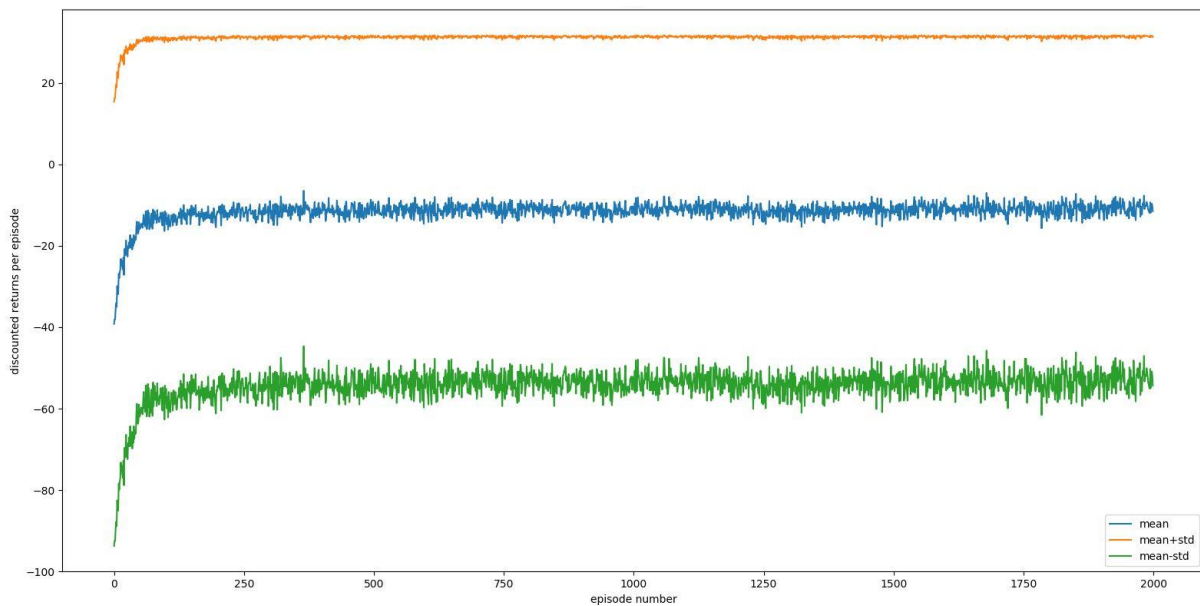
$\rightarrow$ $s_{12}$ -1.62	$\rightarrow$ $s_{13}$ -0.31	$\rightarrow$ $s_{14}$ 1.78	$\downarrow$ $s_{15}$ 8.36	$\downarrow$ $s_{16}$ 1.61	$\leftarrow$ $s_{17}$ -0.8
$\nwarrow$ $s_1$ -2.11		$\rightarrow$ $s_2$ 6.04	$s_3$	$\leftarrow$ $s_4$ 8.44	$\leftarrow$ $s_{19}$ 0.32
$\rightarrow$ $s_{20}$ -1.97	$\rightarrow$ $s_5$ -1.43	$\uparrow$ $s_6$ 1.73		$\nwarrow$ $s_7$ -1.9	
$\uparrow$ $s_{21}$ -2.16		$\uparrow$ $s_8$ 0.1	$\leftarrow$ $s_9$ -14.32	$\uparrow$ $s_{10}$ -8.1	$\downarrow$ $s_{22}$ -2.36
$\uparrow$ $s_{23}$ -2.21			$s_{11}$		$\downarrow$ $s_{24}$ -2.23
$\uparrow$ $s_{25}$ -2.22	$\leftarrow$ $s_{26}$ -2.63	$\leftarrow$ $s_{27}$ -2.71	$\rightarrow$ $s_{28}$ -30.46	$\downarrow$ $s_{29}$ -2.23	$\downarrow$ $s_{30}$ -2.23

Figure 4. The optimal value function and policy for MC.

(2). See figure 4.

(3). See figure 5. The sufficient number of 1000 iterations is determined when there is no further significant change in the standard deviation of the return and the maximum of the mean curve reaches steady level.

Figure 5. Backward discounted reward per episode for MC.



(4). From figure 6 and 7(with smoothing to help pick out trends), higher learning rate(above 0.2) leads to lower discounted rewards, and higher epsilon(0.25) also leads to reduced rewards. The learning rate in the incremental update controls how much of a “pull” is to be exerted to the value function estimation. If learning rate gets too large, the chance of overshooting increases and the learning curve will drop. The epsilon controls exploration frequency and it allows more states to be visited. A large value of epsilon may introduce too much unnecessary randomness and prevents the policy to converge to greedy.



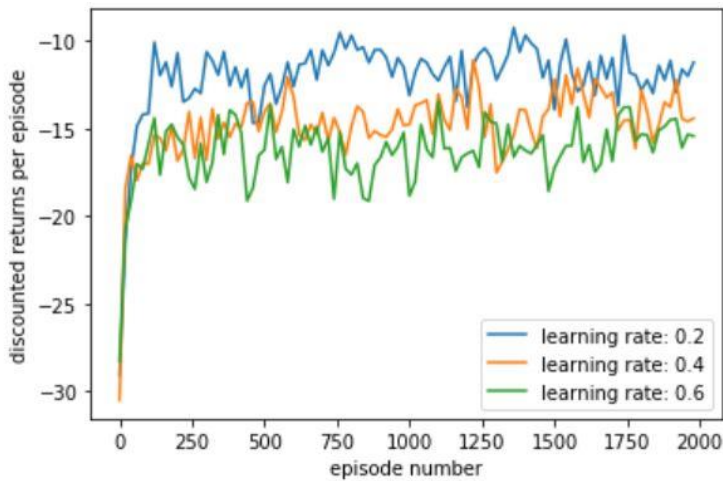


Figure 6. Effect of varying learning rate for learning curve, for MC.

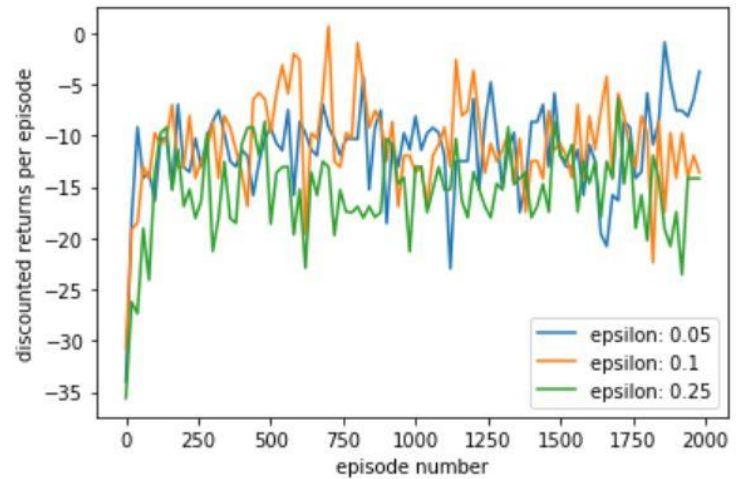


Figure 7. Effect of varying epsilon on learning curve, for MC.

### Q2(d)

(1). The same environment is used (gridworldenv) for TD. Off policy Q-learning method is implemented to estimate the optimal action value function. Use epsilon greedy as the behaviour policy and greedy target policy. Parameters:  $p = 0.55$ ,  $\gamma = 0.55$ ,  $\epsilon = 0.1$ , learning rate/stepsize  $\alpha = 0.2$ .

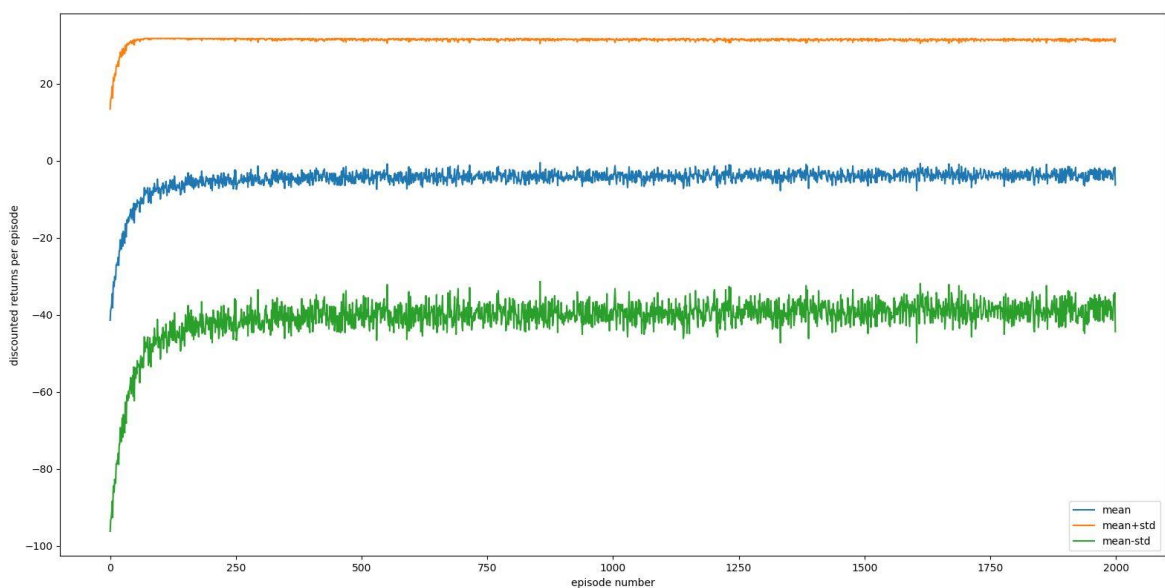
(2). See figure 8.

(3). See figure 9. The sufficient number of 1000 iterations is determined when there is no significant change in the standard deviation of the return and the maximum of the mean curve reaches steady level.

$\rightarrow$ $s_{12}$ -1.46	$\rightarrow$ $s_{13}$ -0.08	$\rightarrow$ $s_{14}$ -2.31	$\downarrow$ $s_{15}$ 6.82	$\leftarrow$ $s_{16}$ 2.9	$\leftarrow$ $s_{17}$ -0.25
$\uparrow$ $s_1$ -1.94		$\rightarrow$ $s_2$ 6.21	$s_3$	$\leftarrow$ $s_4$ 4.89	$\leftarrow$ $s_{19}$ 1.73
$\rightarrow$ $s_{20}$ -1.73	$\rightarrow$ $s_5$ -1.08	$\uparrow$ $s_6$ 0.44		$\uparrow$ $s_7$ 0.42	
$\uparrow$ $s_{21}$ -2.06		$\uparrow$ $s_8$ -2.87	$\rightarrow$ $s_9$ -11.14	$\uparrow$ $s_{10}$ -2.34	$\downarrow$ $s_{22}$ -2.28
$\uparrow$ $s_{23}$ -2.13			$s_{11}$		$\leftarrow$ $s_{24}$ -2.23
$\uparrow$ $s_{25}$ -2.2	$\leftarrow$ $s_{26}$ -2.5	$\leftarrow$ $s_{27}$ -3.18	$\leftarrow$ $s_{28}$ -12.92	$\rightarrow$ $s_{29}$ -3.41	$\uparrow$ $s_{30}$ -2.28

Figure 8. Optimal value function and policy for TD.

Figure 9. Backward discounted reward per episode for TD.



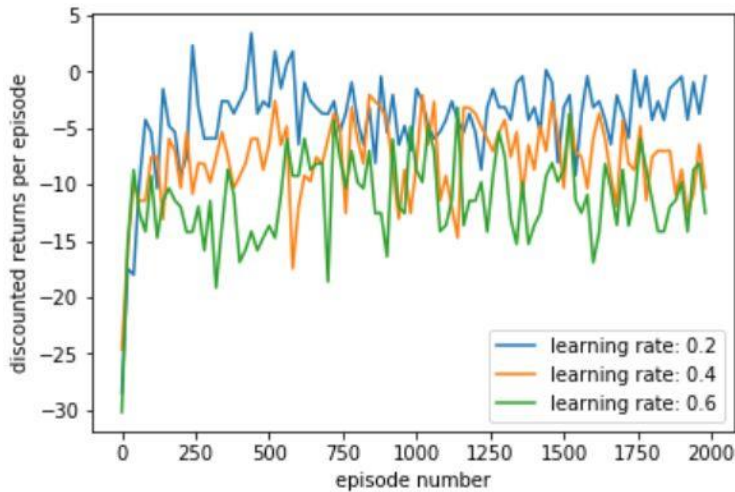


Figure 10. Effect of varying learning rate on learning curve, for TD.

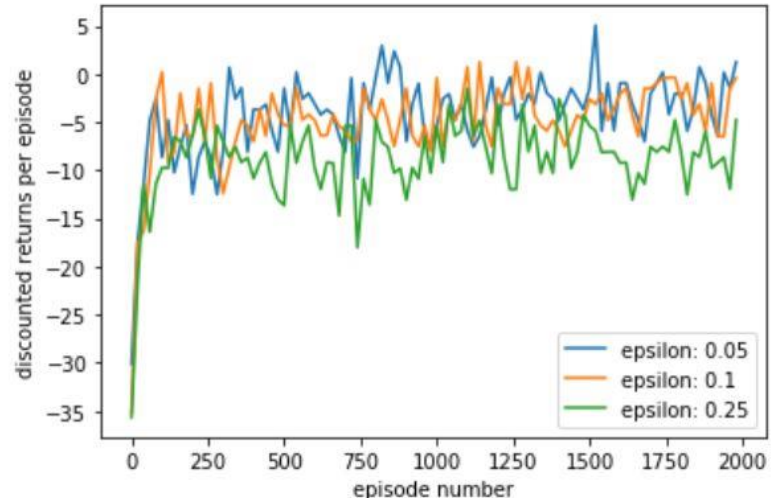


Figure 11. Effect of varying epsilon on learning curve, for TD.

(4). From figure 10 and 11 (with smoothing to help pick out trends), it can be seen that as learning rate increases, the return goes down; as epsilon increases, the reward decreases (although epsilon of 0.05 and 0.1 are arguably identical in performance, 0.25 becomes too large to impair performance). Again, the high learning rate could lead to overshooting when updating value function, and high epsilon leads to excessive exploration which may prevent convergence.

## Q2(e)

(1). From figure 12, RMSE on an episode basis for both MC and TD are plotted on the same graph, and for comparison purposes the plots are smoothed. Both MC and TD can quickly gain accuracy after just a few hundreds episodes, as can be seen by the steep drop at the start. It can also be observed that the orange curve (TD) consistently stays slightly below the blue curve (MC), with a very few exceptions when the orange curve shoots up, and they are likely due to random exploration and noise. This indicates that the TD method (Q-learning) has a slightly

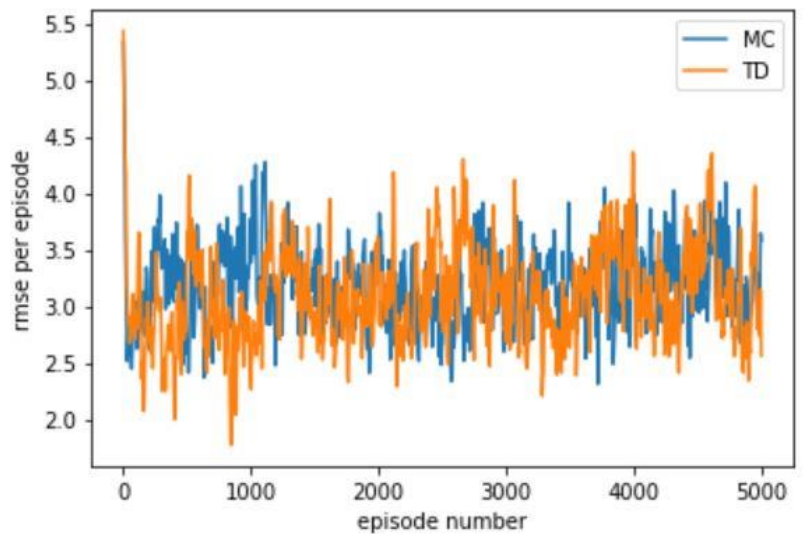


Figure 12. RMSE per episode for both MC and TD.

higher overall accuracy in estimating the value function than MC for the given episodes. However, the MC curve seems to be more stable along all the different episodes and the degree of fluctuation is less violent compared to that of TD. This observation could be explained by the fact the algorithms use "exploring start" which means randomly select a state to start an episode, while MC is not sensitive to initial value, TD can be heavily affected by initial value. Although MC should converge better to the true value (no bias) in the long run, in this experiment it seems that 5000 episodes are insufficient for MC to converge, thus TD shows better performance as it is faster and more efficient in limited episode numbers.

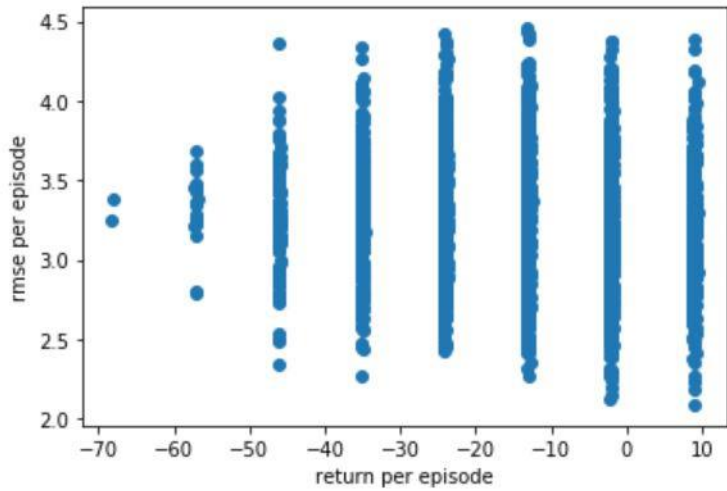


Figure 13. RMSE per episode against return per episode, for MC.

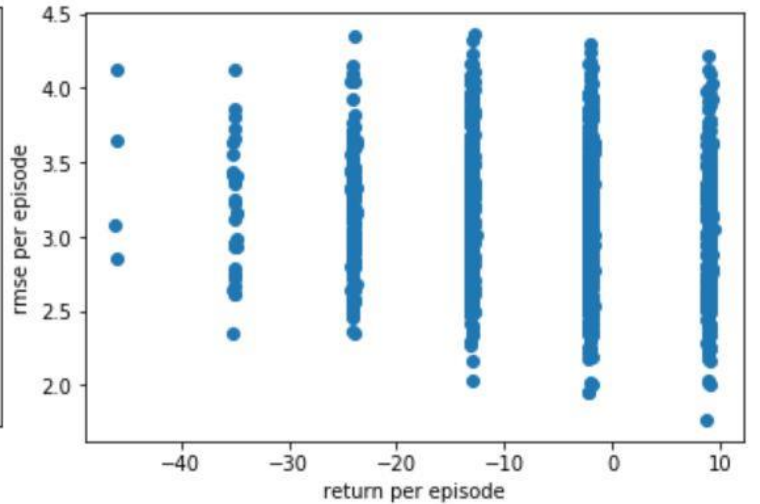


Figure 14. RMSE per episode against return per episode, for TD.

(2). After continuous training (more than 500,000 episodes) for each agent, experiments are carried out to store RMSE and discounted reward data pair in episode-by-episode basis, and the data is plotted as scatter plots, for both MC and TD. Both figure 13 and 14 show that at low error(RMSE) levels, more datapoints appear on the right half of the plot(higher rewards). This implies that an accurate(low RMSE) value function estimation leads to higher rewards. Since for model-free learners like MC/TD, the generalised policy iteration(GPI)'s policy evaluation and policy improvement cycle guarantees that the estimated value function( $V$  or  $Q$ ) is used to improve the current policy, and a good estimation will lead to more improved policy and thus higher rewards due to it.

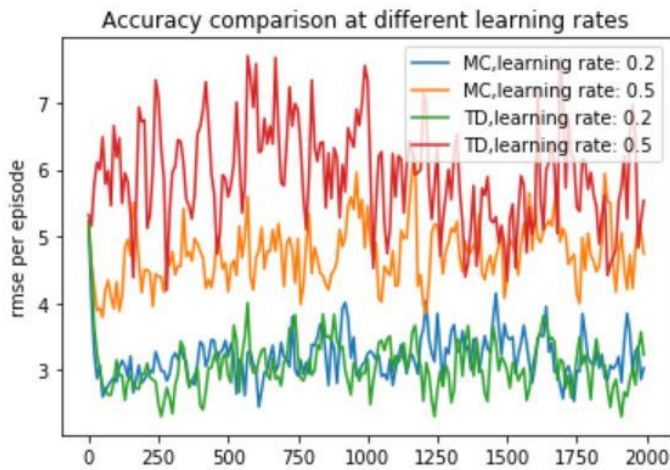


Figure 15. RMSE per episode with varying  $\alpha$ , for MC&TD comparison.

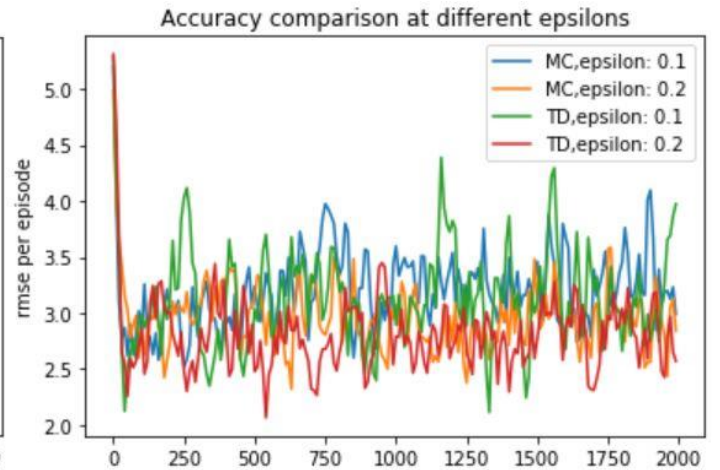


Figure 16. RMSE per episode with varying  $\epsilon$ , for MC&TD comparison.

(3). Figure 15 shows that, for low learning rate(0.2), TD performs better with slightly higher accuracy, but at high learning rate(0.5), MC performs significantly better with much higher accuracy. Both MC and TD learns equally quick, as the initial RMSE drops are almost identical. Figure 16 shows that the effect of epsilon change on both learners' performances is considerably less, as the curves are all closely tangled. Still one can pick out the trend: TD has slightly less RMSE(more accurate) at all epsilon values. The experiments show that TD is not systematically better than MC, especially not when facing high learning rates. However, given the relatively low episode number(2000), TD does appear to be overall more effective than MC, with the ability to learn equally quickly and has slightly better accuracy. Finally, TD is also more susceptible to noise.

## Appendix A: all source codes

### Codes for Q2(b):

**This part reuses many code segments from the given Lab\_SOLUTIONS.IPYNB file.**

```
import numpy as np

import matplotlib.pyplot as plt

%matplotlib inline

from tqdm.notebook import tqdm

class GridWorld(object):

    def __init__(self, p=0.55):

        # Shape of the gridworld

        self.shape = (6,6)


        # Locations of the obstacles

        self.obstacle_locs = [(1,1),(2,5),(2,3),(3,1),(4,1),(4,2),(4,4)]


        # Locations for the terminal/absorbing states

        self.absorbing_locs = [(4,3),(1,3)]


        # Rewards for each of the absorbing states

        self.special_rewards = [-100, 10] # Corresponds to each of the absorbing_locs


        # Reward for all the other states

        self.default_reward = -1


        # Action names

        self.action_names = ['N','E','S','W'] # Action 0 is 'N', 1 is 'E' and so on


        # Number of actions

        self.action_size = len(self.action_names)


        #action probabilty p

        self.p = p
```



```

# Get attributes defining the world
state_size, T, R, absorbing, locs = self.build_grid_world()

# randomly select a starting location
locs_available = locs.copy()
locs_available.remove(self.absorbing_locs[0])
locs_available.remove(self.absorbing_locs[1])
self.starting_loc = locs_available[np.random.choice(range(len(locs_available)))]

# Number of valid states in the gridworld (there are 22 of them - 5x5 grid minus obstacles)
self.state_size = state_size

# Transition operator (3D tensor)
self.T = T # T[st+1, st, a] gives the probability that action a will
            # transition state st to state st+1

# Reward function (3D tensor)
self.R = R # R[st+1, st, a] gives the reward for transitioning to state
            # st+1 from state st with action a

# Absorbing states
self.absorbing = absorbing

# The locations of the valid states
self.locs = locs # State 0 is at the location self.locs[0] and so on

# Number of the starting state
self.starting_state = self.loc_to_state(self.starting_loc, locs);

# Locating the initial state
self.initial = np.zeros((1, len(locs)));
self.initial[0, self.starting_state] = 1

```

```

# Placing the walls on a bitmap
self.walls = np.zeros(self.shape);
for ob in self.obstacle_locs:
    self.walls[ob] = -150

# Placing the absorbers on a grid for illustration
self.absorbers = np.zeros(self.shape)
for ab in self.absorbing_locs:
    self.absorbers[ab] = -1

# Placing the rewarders on a grid for illustration
self.rewarders = np.zeros(self.shape)
for i, rew in enumerate(self.absorbing_locs):
    self.rewarders[rew] = self.special_rewards[i]

def build_grid_world(self):
    # Get the locations of all the valid states, the neighbours of each state (by state number),
    # and the absorbing states (array of 0's with ones in the absorbing states)
    locations, neighbours, absorbing = self.get_topology()

    # Get the number of states
    S = len(locations)

    # Initialise the transition matrix
    T = np.zeros((S,S,4))

    # for action in range(4):
    #     for effect in range(4):
    #         # Randomize the outcome of taking an action
    #         outcome = (action+effect+1) % 4
    #         if outcome == 0:
    #             outcome = 3

```

```

#         else:
#             outcome -= 1
#             prob = [0.8,0.1,0,0.1][effect]
#             for prior_state in range(S):
#                 post_state = neighbours[prior_state, outcome]
#                 post_state = int(post_state)
#                 T[post_state,prior_state,action] = T[post_state,prior_state,action]+prob

```

#define the transition dynamics

```

for prior_state in range(S):
    for action in range(4):
        for effect in range(4):
            if action == effect:
                post_state = int(neighbours[prior_state, action])
                T[post_state,prior_state,action] += self.p
            else:
                post_state = int(neighbours[prior_state, effect])
                T[post_state,prior_state,action] += (1-self.p)/3

```

# Build the reward matrix

```

R = self.default_reward*np.ones((S,S,4))
for i, sr in enumerate(self.special_rewards):
    post_state = self.loc_to_state(self.absorbing_locs[i],locations)
    R[post_state,,:,]= sr

```

return S, T,R,absorbing,locations

def get\_topology(self):

```

    height = self.shape[0]
    width = self.shape[1]

```

index = 1

```

locs = []
neighbour_locs = []

for i in range(height):
    for j in range(width):
        # Get the locaiton of each state
        loc = (i,j)

        #And append it to the valid state locations if it is a valid state (ie not absorbing)
        if(self.is_location(loc)):
            locs.append(loc)

            # Get an array with the neighbours of each state, in terms of locations
            local_neighbours = [self.get_neighbour(loc,direction) for direction in ['nr','ea','so', 'we']]
            neighbour_locs.append(local_neighbours)

# translate neighbour lists from locations to states
num_states = len(locs)
state_neighbours = np.zeros((num_states,4))

for state in range(num_states):
    for direction in range(4):
        # Find neighbour location
        nloc = neighbour_locs[state][direction]

        # Turn location into a state number
        nstate = self.loc_to_state(nloc,locs)

        # Insert into neighbour matrix
        state_neighbours[state,direction] = nstate;

# Translate absorbing locations into absorbing state indices

```



```

absorbing = np.zeros((1,num_states))

for a in self.absorbing_locs:
    absorbing_state = self.loc_to_state(a,locs)
    absorbing[0,absorbing_state] =1

```

```

return locs, state_neighbours, absorbing

```

```

def loc_to_state(self,loc,locs):
    #takes list of locations and gives index corresponding to input loc
    return locs.index(tuple(loc))

```

```

def is_location(self, loc):
    # It is a valid location if it is in grid and not obstacle

    if(loc[0]<0 or loc[1]<0 or loc[0]>self.shape[0]-1 or loc[1]>self.shape[1]-1):
        return False

    elif(loc in self.obstacle_locs):
        return False

    else:
        return True

```

```

def get_neighbour(self,loc,direction):
    #Find the valid neighbours (ie that are in the grif and not obstacle)

    i = loc[0]
    j = loc[1]

    nr = (i-1,j)
    ea = (i,j+1)
    so = (i+1,j)
    we = (i,j-1)

    # If the neighbour is a valid location, accept it, otherwise, stay put

```

```

if(direction == 'nr' and self.is_location(nr)):
    return nr
elif(direction == 'ea' and self.is_location(ea)):
    return ea
elif(direction == 'so' and self.is_location(so)):
    return so
elif(direction == 'we' and self.is_location(we)):
    return we
else:
    #default is to return to the same location
    return loc

def get_transition_matrix(self):
    return self.T

def get_reward_matrix(self):
    return self.R

#policy evaluation and iteration
def policy_iteration(self, discount=0.55, threshold = 0.0001):
    ## Slide 139 of the lecture notes for pseudocode ##

    # Transition and reward matrices, both are 3d tensors, c.f. internal state
    T = self.get_transition_matrix()
    R = self.get_reward_matrix()

    # Initialisation
    policy = np.zeros((self.state_size, self.action_size)) # Vector of 0
    policy[:,0] = 1 # Initialise policy to choose action 1 systematically
    epochs = 0
    policy_stable = False # Condition to stop the main loop

    while not(policy_stable):

```

```

# Policy evaluation
V, epochs_eval = self.policy_evaluation(policy, threshold, discount)

epochs += epochs_eval # Increment epoch


# Set the boolean to True, it will be set to False later if the policy prove unstable
policy_stable = True


# Policy iteration
for state_idx in range(policy.shape[0]):

    # If not an absorbing state
    if not(self.absorbing[0,state_idx]):

        # Store the old action
        old_action = np.argmax(policy[state_idx,:])

        # Compute Q value
        Q = np.zeros(4) # Initialise with value 0
        for state_idx_prime in range(policy.shape[0]):
            Q += T[state_idx_prime,state_idx,:] * (R[state_idx_prime,state_idx, :] + discount *
V[state_idx_prime])

        # Compute corresponding policy
        new_policy = np.zeros(4)
        new_policy[np.argmax(Q)] = 1 # The action that maximises the Q value gets probability 1
        policy[state_idx] = new_policy


    # Check if the policy has converged
    if old_action != np.argmax(policy[state_idx]):
        policy_stable = False


return V, policy, epochs

```

```

def policy_evaluation(self, policy, threshold, discount):

    # Make sure delta is bigger than the threshold to start with
    delta= 2*threshold

    #Get the reward and transition matrices
    R = self.get_reward_matrix()
    T = self.get_transition_matrix()

    # The value is initialised at 0
    V = np.zeros(policy.shape[0])

    # Make a deep copy of the value array to hold the update during the evaluation
    Vnew = np.copy(V)

    epoch = 0

    # While the Value has not yet converged do:
    while delta>threshold:
        epoch += 1
        for state_idx in range(policy.shape[0]):
            # If it is one of the absorbing states, ignore
            if(self.absorbing[0,state_idx]):
                continue

            # Accumulator variable for the Value of a state
            tmpV = 0

            for action_idx in range(policy.shape[1]):
                # Accumulator variable for the State-Action Value
                tmpQ = 0

                for state_idx_prime in range(policy.shape[0]):

```



```
tmpQ = tmpQ + T[state_idx_prime,state_idx,action_idx] * (R[state_idx_prime,state_idx,
action_idx] + discount * V[state_idx_prime])
```

```
tmpV += policy[state_idx,action_idx] * tmpQ
```

```
# Update the value of the state
```

```
Vnew[state_idx] = tmpV
```

```
# After updating the values of all states, update the delta
```

```
# Note: The below is our example way of computing delta.
```

```
# Other stopping criteria may be used (for instance mean squared error).
```

```
# We encourage you to explore different ways of computing delta to see
```

```
# how it can influence outcomes.
```

```
delta = max(abs(Vnew-V))
```

```
# and save the new value into the old
```

```
V=np.copy(Vnew)
```

```
return V, epoch
```

```
#drawing functions
```

```
def draw_deterministic_policy(self, Policy):
```

```
# Draw a deterministic policy
```

```
# The policy needs to be a np array of 22 values between 0 and 3 with
```

```
# 0 -> N, 1->E, 2->S, 3->W
```

```
plt.figure()
```

```
plt.imshow(self.walls+self.rewarders +self.absorbers) # Create the graph of the grid
```

```
#plt.hold('on')
```

```
for state, action in enumerate(Policy):
```

```
if(self.absorbing[0,state]): # If it is an absorbing state, don't plot any action
```

```
continue
```

```
arrows = [r"$\uparrow$",r"$\rightarrow$", r"$\downarrow$", r"$\leftarrow$"] # List of arrows
corresponding to each possible action
```

```
action_arrow = arrows[action] # Take the corresponding action
```

```

location = self.locs[state] # Compute its location on graph

plt.text(location[1], location[0], action_arrow, ha='center', va='center') # Place it on graph


plt.show()


def draw_value(self, Value):
    # Draw a policy value function
    # The value need to be a np array of 22 values
    plt.figure()

    plt.imshow(self.walls+self.rewarders +self.absorbers) # Create the graph of the grid
    for state, value in enumerate(Value):
        if(self.absorbing[0,state]): # If it is an absorbing state, don't plot any value
            continue

        location = self.locs[state] # Compute the value location on graph
        plt.text(location[1], location[0], round(value,2), ha='center', va='center') # Place it on graph

    plt.show()


def draw_deterministic_policy_grid(self, Policy, title, n_columns, n_lines):
    # Draw a grid of deterministic policy
    # The policy needs to be an arraya of np array of 22 values between 0 and 3 with
    # 0 -> N, 1->E, 2->S, 3->W
    plt.figure(figsize=(20,8))
    for subplot in range (len(Policy)): # Go through all policies
        ax = plt.subplot(n_columns, n_lines, subplot+1) # Create a subplot for each policy
        ax.imshow(self.walls+self.rewarders +self.absorbers) # Create the graph of the grid
        for state, action in enumerate(Policy[subplot]):
            if(self.absorbing[0,state]): # If it is an absorbing state, don't plot any action
                continue

            arrows = [r"$\uparrow$", r"$\rightarrow$", r"$\downarrow$", r"$\leftarrow$"] # List of arrows
corresponding to each possible action

            action_arrow = arrows[action] # Take the corresponding action

```

```

        location = self.locs[state] # Compute its location on graph

        plt.text(location[1], location[0], action_arrow, ha='center', va='center') # Place it on graph

    ax.title.set_text(title[subplot]) # Set the title for the graoh given as argument

plt.show()

```

```

def draw_value_grid(self, Value, title, n_columns, n_lines):

```

```

    # Draw a grid of value function

```

```

    # The value need to be an array of np array of 22 values

```

```

    plt.figure(figsize=(20,8))

```

```

    for subplot in range (len(Value)): # Go through all values

```

```

        ax = plt.subplot(n_columns, n_lines, subplot+1) # Create a subplot for each value

```

```

        ax.imshow(self.walls+self.rewarders +self.absorbers) # Create the graph of the grid

```

```

        for state, value in enumerate(Value[subplot]):

```

```

            if(self.absorbing[0,state]): # If it is an absorbing state, don't plot any value

```

```

                continue

```

```

            location = self.locs[state] # Compute the value location on graph

```

```

            plt.text(location[1], location[0], round(value,1), ha='center', va='center') # Place it on graph

```

```

        ax.title.set_text(title[subplot]) # Set the title for the graoh given as argument

```

```

    plt.show()

```

```

#get the optimal value function and optimal policy

```

```

grid = GridWorlddp()

```

```

V_opt, pol_opt, epochs = grid.policy_iteration()

```

```

print("\n\nIts graphical representation is:\n")

```

```

grid.draw_value(V_opt)

```

```

print("\n\nIts graphical representation is:\n")

```

```

grid.draw_deterministic_policy(np.array([np.argmax(pol_opt[row,:]) for row in range(grid.state_size)]))

```

```

#test different gamma values

```

```

gamma_range = [0.4, 0.6, 0.8]

```

```

# Use policy iteration for each gamma value

```

```

#initialize

```

```

epochs_needed = []

```

```

pols_needed = []

```

```

vals_needed = []

```

```

titles = []

for gamma in gamma_range:
    grid = GridWorldddp()
    V_opt, pol_opt, epochs = grid.policy_iteration(discount = gamma)
    epochs_needed.append(epochs)
    pols_needed.append(np.array([np.argmax(pol_opt[row,:]) for row in range(grid.state_size)]))
    vals_needed.append(V_opt)
    titles.append(f"gamma = {gamma}")

print("\nGraphical representation of the value function for each gamma:\n")
grid.draw_value_grid(vals_needed, titles, 1, 4)

print("\nGraphical representation of the policy for each gamma:\n")
grid.draw_deterministic_policy_grid(pols_needed, titles, 1, 4)

#test different p values
p_range = [0.1, 0.25, 0.65]
#policy iteration for each p value
#initialize
epochs_needed = []
pols_needed = []
vals_needed = []
titles = []

for p_val in p_range:
    grid = GridWorldddp(p=p_val)
    V_opt, pol_opt, epochs = grid.policy_iteration()
    epochs_needed.append(epochs)
    pols_needed.append(np.array([np.argmax(pol_opt[row,:]) for row in range(grid.state_size)]))
    vals_needed.append(V_opt)
    titles.append(f"p = {p_val}")

print("\nGraphical representation of the value function for each p:\n")
grid.draw_value_grid(vals_needed, titles, 1, 4)

```



```
print("\nGraphical representation of the policy for each p:\n")
grid.draw_deterministic_policy_grid(pols_needed, titles, 1, 4)
```

```
#DP optimal
```

```
grid5 = GridWorlddp()
```

```
V_opt_dp, P_opt_dp, _ = grid5.policy_iteration()
```

```
V_OPT = V_opt_dp #the true optimal state values (ground truth) set as global variable
```

[Codes for Q2\(c\), Q2\(d\), Q2\(e\):](#)

```
class GridWorldenv(object):
```

```
    def __init__(self, p=0.55, curr_state=28):
```

```
        #shape of the grid
```

```
        self.shape = (6,6)
```

```
        #Locations of the obstacles
```

```
        self.obstacle_locs = [(1,1),(2,5),(2,3),(3,1),(4,1),(4,2),(4,4)]
```

```
        #all possible actions
```

```
        self.action_names = ['N','E','S','W'] # Action 0 is 'N', 1 is 'E' and so on
```

```
        #Number of actions
```

```
        self.action_size = len(self.action_names)
```

```
        #action probability p
```

```
        self.p = p
```

```
        #all locations
```

```
        self.locs = self.get_all_locs()
```

```
        #number of states
```

```
        self.num_state = len(self.locs)
```

```
        #Locations for the terminal/absorbing states
```

```
        self.absorbing_locs = [(4,3),(1,3)]
```

```
        self.absorbing_states = []
```

```
        for loc in self.absorbing_locs:
```

```
            self.absorbing_states.append(self.loc_to_state(loc, self.locs))
```

```
        #current location/state
```

```
        self.curr_state = curr_state
```

```
        self.curr_loc = self.locs[self.curr_state]
```

```

#for visualisation of the gridworld:

# Placing the walls on a bitmap
self.walls = np.zeros(self.shape);
for ob in self.obstacle_locs:
    self.walls[ob] = -150

# Placing the absorbers on a grid for illustration
self.absorbers = np.zeros(self.shape)
for ab in self.absorbing_locs:
    self.absorbers[ab] = -1

# Placing the terminal states on a grid for illustration
self.rewarders = np.zeros(self.shape)
for i, rew in enumerate(self.absorbing_locs):
    self.rewarders[rew] = [-100,10][i]

def get_all_locs(self):
    #get all valid locs
    locs = []
    for i in range(self.shape[0]):
        for j in range(self.shape[1]):
            if self.is_location((i, j)):
                locs.append((i, j))
    return locs

def loc_to_state(self,loc,locs):
    #takes list of locations and gives index corresponding to input loc
    return locs.index(tuple(loc))

def is_location(self, loc):

```

```

# It is a valid location if it is in grid and not obstacle
if(loc[0]<0 or loc[1]<0 or loc[0]>self.shape[0]-1 or loc[1]>self.shape[1]-1):
    return False

elif(loc in self.obstacle_locs):
    return False

else:
    return True

def env_start(self):
    #start the environment and randomly pick a starting loc(1/27)
    locs_available = self.locs.copy()
    locs_available.remove(self.absorbing_locs[0])
    locs_available.remove(self.absorbing_locs[1])
    self.curr_loc = locs_available[np.random.choice(range(len(locs_available)))]
    self.curr_state = self.loc_to_state(self.curr_loc, self.locs)

def env_step(self, action):
    #env decides the action [n,e,s,w], with non-deterministic transition
    self.curr_loc = self.locs[self.curr_state]
    possible_next_locs = [(self.curr_loc[0]-1,self.curr_loc[1]),
                          (self.curr_loc[0],self.curr_loc[1]+1),
                          (self.curr_loc[0]+1,self.curr_loc[1]),
                          (self.curr_loc[0],self.curr_loc[1]-1)]

    if action == 0:
        if np.random.uniform(0,1)<=self.p:
            next_loc = possible_next_locs[0]
            actual_action = action
        else:
            next_loc = possible_next_locs[np.random.choice([1,2,3])]
            actual_action = possible_next_locs.index(next_loc)

    elif action == 1:
        if np.random.uniform(0,1)<=self.p:

```

```

        next_loc = possible_next_locs[1]

        actual_action = action
    else:
        next_loc = possible_next_locs[np.random.choice([0,2,3])]
        actual_action = possible_next_locs.index(next_loc)
    elif action == 2:
        if np.random.uniform(0,1)<=self.p:
            next_loc = possible_next_locs[2]
            actual_action = action
        else:
            next_loc = possible_next_locs[np.random.choice([0,1,3])]
            actual_action = possible_next_locs.index(next_loc)
    elif action == 3:
        if np.random.uniform(0,1)<=self.p:
            next_loc = possible_next_locs[3]
            actual_action = action
        else:
            next_loc = possible_next_locs[np.random.choice([0,1,2])]
            actual_action = possible_next_locs.index(next_loc)
    else:
        raise Exception(self.action_names[action]+'is not in available actions!')

#check validity of the move
if self.is_location(next_loc):
    next_position = next_loc
else:
    next_position = self.curr_loc

#reward function
if next_position == self.absorbing_locs[0]:
    reward = -100
    done = True
elif next_position == self.absorbing_locs[1]:

```

```

        reward = 10

        done = True

    else:

        reward = -1

        done = False

    next_state = self.loc_to_state(next_position, self.locs)

    return next_state, reward, done

#drawing functions
def draw_deterministic_policy(self, Policy):
    # Draw a deterministic policy

    # The policy needs to be a np array of 22 values between 0 and 3 with
    # 0 -> N, 1->E, 2->S, 3->W

    plt.figure()

    plt.imshow(self.walls+self.rewarders +self.absorbers) # Create the graph of the grid
    #plt.hold('on')

    for state, action in enumerate(Policy):

        if state in self.absorbing_states: # If it is an absorbing state, don't plot any action
            continue

        arrows = [r"$\uparrow$", r"$\rightarrow$", r"$\downarrow$", r"$\leftarrow$"] # List of arrows
corresponding to each possible action

        action_arrow = arrows[action] # Take the corresponding action

        location = self.locs[state] # Compute its location on graph

        plt.text(location[1], location[0], action_arrow, ha='center', va='center') # Place it on graph

    plt.show()

def draw_value(self, Value):
    # Draw a policy value function

    # The value need to be a np array of 22 values

```

```
plt.figure()
```

```
plt.imshow(self.walls+self.rewarders +self.absorbers) # Create the graph of the grid
```

```
for state, value in enumerate(Value):
```

```
    if state in self.absorbing_states: # If it is an absorbing state, don't plot any value
```

```
        continue
```

```
    location = self.locs[state] # Compute the value location on graph
```

```
    plt.text(location[1], location[0], round(value,2), ha='center', va='center') # Place it on graph
```

```
plt.show()
```

```
class MC_agent(object):
```

```
    def __init__(self, epsilon=0.1, step_size=0.2):
```

```
        self.agent_env = GridWorldenv()
```

```
        self.actions = list(range(self.agent_env.action_size)) #[0,1,2,3]
```

```
        self.step_size = step_size
```

```
        self.epsilon = epsilon
```

```
        self.discount = 0.55
```

```
        self.samples = []
```

```
        self.qvalues = np.zeros((self.agent_env.num_state,len(self.actions)))
```

```
    def argmax(self, q_values):
```

```
        #argmax with random tie-breaking
```

```
        top=float("-inf")
```

```
        ties=[]
```

```
        for i in self.actions:
```

```
            if q_values[i]>top:
```

```
                top=q_values[i]
```

```
                ties=[]
```

```
            if q_values[i]==top:
```

```
                ties.append(i)
```

```
return np.random.choice(ties)
```

```
def choose_action(self, state):
```

```
    #agent choose an action given the current state, according to epsilon-greedy
```

```
    if np.random.uniform(0,1)<self.epsilon:
```

```
        #take random action
```

```
        action = np.random.choice(self.actions)
```

```
    else:
```

```
        #take action according to the q function
```

```
        action = self.argmax(self.qvalues[state])
```

```
    return int(action)
```

```
def update(self):
```

```
    #update the q function at the end of episode
```

```
    G = 0
```

```
    for tup in self.samples[::-1]:
```

```
        state, action = (tup[0],tup[1])
```

```
        G = self.discount*G + tup[2]
```

```
        self.qvalues[state][action] = (self.qvalues[state][action] + self.step_size*(G - self.qvalues[state][action]))
```

```
def reset(self):
```

```
    self.agent_env = GridWorldenv()
```

```
def play(self, num_episode=2000):
```

```
    self.total_reward = np.zeros(num_episode)
```

```
    self.total_rms = np.zeros(num_episode)
```

```
    #start to generate a trace
```

```
    for i in range(num_episode):
```

```
        self.agent_env.env_start()
```

```
        state = self.agent_env.curr_state
```

```
        action = self.choose_action(state)
```

```

#cumulative_reward = 0

while True:
    next_state, reward, done = self.agent_env.env_step(action)
    self.samples.append((self.agent_env.curr_state,action,reward))
    self.agent_env.curr_state = next_state
    #cumulative_reward += reward

    action = self.choose_action(next_state)

#now update the q function at the end of each episode
if done:
    #print(f"episode : {i}")
    self.update()
    self.total_rms[i] = self.get_rms()
    self.total_reward[i] = self.back_discounted_reward(sample=self.samples)
    if i < (num_episode-1):
        self.samples = []
    self.reset()
    break

def back_discounted_reward(self, sample):
    #find the backward discounted reward for one episode
    G_return = 0
    for tup in sample:
        reward = tup[2]
        G_return = self.discount*G_return + reward
    return G_return

def get_rms(self):
    V_estimate = np.max(self.qvalues,axis=1)
    rms = np.sqrt(np.mean((V_estimate-V_OPT)**2))
    return rms

```



```

def train_q_values(self, q_values):

    #takes in q_values for training

    self.qvalues = q_values


#train the MC agent
agent0 = MC_agent()
agent0.play(num_episode=500000)
TRAINED_Q = agent0.qvalues

grid3 = GridWorldenv()

V_opt_mc = np.max(agent0.qvalues,axis=1) #the optimal value function
P_opt_mc = np.argmax(agent0.qvalues,axis=1)
grid3.draw_value(V_opt_mc)
grid3.draw_deterministic_policy(P_opt_mc)


#the smoothing function for helping visualising trends in plots
def smooth_rewards(total_rewards, smoothing_interval):

    data = []

    for i in range(0,len(total_rewards),smoothing_interval):

        data.append(np.mean(total_rewards[i:i+smoothing_interval]))

    smooth_index = list(range(0,len(total_rewards),smoothing_interval))

    return smooth_index, data


#plot learning curve for Q2c3
iterations = 1000
episode_num = 2000
repeat_reward = np.zeros((iterations,episode_num))
for i in (range(iterations)):

    agent = MC_agent()

    agent.play(episode_num)

    repeat_reward[i] = agent.total_reward

mean_reward = np.mean(repeat_reward, axis=0)
std_reward = np.std(repeat_reward, axis=0)

```

```

plus = mean_reward + std_reward
minus = mean_reward - std_reward
#no smoothing
idx, val = smooth_rewards(mean_reward,1)
_, val1 = smooth_rewards(plus,1)
_, val2 = smooth_rewards(minus,1)
fig0 = plt.figure()
plt.plot(idx, val, label="mean")
plt.plot(idx, val1, label="mean+std")
plt.plot(idx, val2, label="mean-std")
plt.xlabel('episode number')
plt.ylabel('discounted returns per episode')
plt.title('MC')
plt.legend()
plt.show()

#parameter adjusting for learning rate a for Q2c4
iterations = 100
episode_num = 2000
stepsizes = [0.2, 0.4, 0.6]
repeat_reward = np.zeros((len(stepsizes),iterations,episode_num))
for i in (range(iterations)):
    for idx,a in enumerate(stepsizes):
        agent = MC_agent(step_size=a)
        agent.play(episode_num)
        repeat_reward[idx][i] = agent.total_reward
mean_reward = np.mean(repeat_reward[0], axis=0)
mean_reward1 = np.mean(repeat_reward[1], axis=0)
mean_reward2 = np.mean(repeat_reward[2], axis=0)
idx, val = smooth_rewards(mean_reward,20)
_, val1 = smooth_rewards(mean_reward1,20)
_, val2 = smooth_rewards(mean_reward2,20)
plt.plot(idx, val, label="learning rate: 0.2")

```

```

plt.plot(idx, val1, label="learning rate: 0.4")
plt.plot(idx, val2, label="learning rate: 0.6")
plt.xlabel('episode number')
plt.ylabel('discounted returns per episode')
plt.legend()
plt.show()

```

```

#parameter adjusting for epsilon e for Q2c4
iterations = 10

episode_num = 2000
epsilons = [0.05, 0.1, 0.25]
repeat_reward = np.zeros((len(epsilons),iterations,episode_num))
for idx,e in enumerate(epsilons):
    for i in (range(iterations)):
        agent = MC_agent(epsilon=e)
        agent.play(episode_num)
        repeat_reward[idx][i] = agent.total_reward
mean_reward = np.mean(repeat_reward[0], axis=0)
mean_reward1 = np.mean(repeat_reward[1], axis=0)
mean_reward2 = np.mean(repeat_reward[2], axis=0)
idx, val = smooth_rewards(mean_reward,20)
_, val1 = smooth_rewards(mean_reward1,20)
_, val2 = smooth_rewards(mean_reward2,20)
plt.plot(idx, val, label="epsilon: 0.05")
plt.plot(idx, val1, label="epsilon: 0.1")
plt.plot(idx, val2, label="epsilon: 0.25")
plt.xlabel('episode number')
plt.ylabel('discounted returns per episode')
plt.legend()
plt.show()

```

```

#plot rmse against return per episode for MC after training for Q2e2
iterations = 10

```

```

episode_num = 2000

repeat_data = np.zeros((2,iterations,episode_num))

for i in (range(iterations)):

    agent = MC_agent()

    agent.train_q_values(TRAINED_Q)

    agent.play(episode_num)

    repeat_data[0][i] = agent.total_rms

    repeat_data[1][i] = agent.total_reward

mean_rms = np.mean(repeat_data[0], axis=0)

mean_reward = np.mean(repeat_data[1], axis=0)

# _, val = smooth_rewards(mean_rms,5)

# _, val1 = smooth_rewards(mean_reward,5)

plt.scatter(mean_reward, mean_rms)

plt.xlabel('return per episode')

plt.ylabel('rmse per episode')

plt.show()

```

```

class TD_agent(object):

    def __init__(self, epsilon=0.1, step_size=0.2):

        self.agent_env = GridWorldenv()

        self.actions = list(range(self.agent_env.action_size)) #[0,1,2,3]

        self.step_size = step_size

        self.epsilon = epsilon

        self.discount = 0.55

        self.trace = []

        self.q_values = np.zeros((self.agent_env.num_state,len(self.actions)))

    def argmax(self, q_values):

        #argmax with random tie-breaking

        top=float("-inf")

        ties=[]

```

```

for i in self.actions:
    if q_values[i]>top:
        top=q_values[i]
        ties=[]

    if q_values[i]==top:
        ties.append(i)

return np.random.choice(ties)

```

```

def agent_start(self, state):
    #agent choose an action given the current state, according to epsilon-greedy
    if np.random.uniform(0,1)<self.epsilon:
        #take random action
        action = np.random.choice(self.actions)
    else:
        #take action according to the q function
        action = self.argmax(self.q_values[state])
    self.prev_state = state
    self.prev_action = action
    return int(action)

```

```

def agent_step(self, reward, state):
    # Choose action using epsilon greedy.
    if np.random.uniform(0,1)<self.epsilon:
        action = np.random.choice(self.actions)
    else:
        action = self.argmax(self.q_values[state])

    #perform q update
    self.q_values[self.prev_state][self.prev_action] +=
self.step_size*(reward+self.discount*self.q_values[state][action]-
self.q_values[self.prev_state][self.prev_action])

```

```

self.prev_state = state

self.prev_action = action

return int(action)

def reset(self):

    self.trace = []

    self.agent_env = GridWorldenv()

    self.prev_state = None

    self.prev_action = None

def play(self, num_episode=2000):

    self.total_reward = np.zeros(num_episode)

    self.total_rms = np.zeros(num_episode)

    for i in range(num_episode):

        self.agent_env.env_start()

        #cumulative_reward = 0

        action = self.agent_start(self.agent_env.curr_state)

        state, reward, done = self.agent_env.env_step(action)

        self.agent_env.curr_state = state

        self.trace.append((self.agent_env.locs[state], action, reward))

        #counter = 0

        while not done:

            #cumulative_reward += reward

            action = self.agent_step(reward=reward,state=self.agent_env.curr_state)

            state, reward, done = self.agent_env.env_step(action)

            self.agent_env.curr_state = state

            self.trace.append((self.agent_env.locs[state], action, reward))

            #counter += 1

        #episode has terminated, update q table

        self.q_values[self.prev_state][self.prev_action] += self.step_size*(reward+0-
self.q_values[self.prev_state][self.prev_action])

        #cumulative_reward += reward

```

```

self.total_rms[i] = self.get_rms()

self.total_reward[i] = self.back_discounted_reward(self.trace)


#reset the env at the end of an episode
if i<(num_episode-1):
    self.reset()


def back_discounted_reward(self, sample):
    #find the backward discounted reward for one episode

    G_return = 0
    for tup in sample:
        reward = tup[2]

        G_return = self.discount*G_return + reward

    return G_return


def get_rms(self):
    V_estimate = np.max(self.q_values,axis=1)
    rms = np.sqrt(np.mean((V_estimate-V_OPT)**2))

    return rms


def train_q_values(self, q_values):
    #takes in q_values for training

    self.q_values = q_values


#train the TD agent
agent1 = TD_agent()
agent1.play(num_episode=500000)
TRAINED_Q_TD = agent1.q_values #make the trained data as global variable
grid4 = GridWorldenv()
V_opt_td = np.max(agent1.q_values,axis=1) #the optimal value function
P_opt_td = np.argmax(agent1.q_values,axis=1)
grid4.draw_value(V_opt_td)
grid4.draw_deterministic_policy(P_opt_td)

```

```

#plot learning curve for Q2d3

iterations = 100
episode_num = 2000
repeat_reward = np.zeros((iterations,episode_num))
for i in (range(iterations)):
    agent = TD_agent()
    agent.play(episode_num)
    repeat_reward[i] = agent.total_reward
mean_reward = np.mean(repeat_reward, axis=0)
std_reward = np.std(repeat_reward, axis=0)
plus = mean_reward + std_reward
minus = mean_reward - std_reward
#no smoothing
idx, val = smooth_rewards(mean_reward,1)
_, val1 = smooth_rewards(plus,1)
_, val2 = smooth_rewards(minus,1)
fig1 = plt.figure()
plt.plot(idx, val, label="mean")
plt.plot(idx, val1, label="mean+std")
plt.plot(idx, val2, label="mean-std")
plt.xlabel('episode number')
plt.ylabel('discounted returns per episode')
plt.title('TD')
plt.legend()
plt.show()

#parameter adjusting for learning rate a for Q2d4
iterations = 10
episode_num = 2000
stepsizes = [0.2, 0.4, 0.6]
repeat_reward = np.zeros((len(stepsizes),iterations,episode_num))
for i in (range(iterations)):
    for idx,a in enumerate(stepsizes):

```



```

agent = TD_agent(step_size=a)

agent.play(episode_num)

repeat_reward[idx][i] = agent.total_reward

mean_reward = np.mean(repeat_reward[0], axis=0)
mean_reward1 = np.mean(repeat_reward[1], axis=0)
mean_reward2 = np.mean(repeat_reward[2], axis=0)

idx, val = smooth_rewards(mean_reward,20)
_, val1 = smooth_rewards(mean_reward1,20)
_, val2 = smooth_rewards(mean_reward2,20)

plt.plot(idx, val, label="learning rate: 0.2")
plt.plot(idx, val1, label="learning rate: 0.4")
plt.plot(idx, val2, label="learning rate: 0.6")

plt.xlabel('episode number')
plt.ylabel('discounted returns per episode')
plt.legend()
plt.show()

#parameter adjusting for epsilon e for Q2d4

iterations = 10

episode_num = 2000

epsilons = [0.05, 0.1, 0.25]

repeat_reward = np.zeros((len(epsilons),iterations,episode_num))

for idx,e in enumerate(epsilons):
    for i in (range(iterations)):
        agent = TD_agent(epsilon=e)
        agent.play(episode_num)
        repeat_reward[idx][i] = agent.total_reward

mean_reward = np.mean(repeat_reward[0], axis=0)
mean_reward1 = np.mean(repeat_reward[1], axis=0)
mean_reward2 = np.mean(repeat_reward[2], axis=0)

idx, val = smooth_rewards(mean_reward,20)
_, val1 = smooth_rewards(mean_reward1,20)
_, val2 = smooth_rewards(mean_reward2,20)

```

```

plt.plot(idx, val, label="epsilon: 0.05")
plt.plot(idx, val1, label="epsilon: 0.1")
plt.plot(idx, val2, label="epsilon: 0.25")
plt.xlabel('episode number')
plt.ylabel('discounted returns per episode')
plt.legend()
plt.show()

```

```

#plot rms per episode for MC,TD, for Q2e1
iterations = 10
episode_num = 5000
repeat_data1 = np.zeros((iterations,episode_num))
repeat_data2 = np.zeros((iterations,episode_num))
for i in (range(iterations)):
    agent1 = MC_agent()
    agent1.play(episode_num)
    repeat_data1[i] = agent1.total_rms
    agent2 = TD_agent()
    agent2.play(episode_num)
    repeat_data2[i] = agent2.total_rms
mean_data1 = np.mean(repeat_data1, axis=0)
idx, val1 = smooth_rewards(mean_data1,5)
mean_data2 = np.mean(repeat_data2, axis=0)
idx, val2 = smooth_rewards(mean_data2,5)
plt.plot(idx, val1, label="MC")
plt.plot(idx, val2, label="TD")
plt.xlabel('episode number')
plt.ylabel('rmse per episode')
plt.legend()
plt.show()

```

```

#plot rmse against return per episode for TD after training for Q2e2

iterations = 10

episode_num = 2000

repeat_data = np.zeros((2,iterations,episode_num))

for i in (range(iterations)):

    agent = TD_agent()

    agent.train_q_values(TRAINED_Q_TD)

    agent.play(episode_num)

    repeat_data[0][i] = agent.total_rms

    repeat_data[1][i] = agent.total_reward

mean_rms = np.mean(repeat_data[0], axis=0)

mean_reward = np.mean(repeat_data[1], axis=0)

#_, val = smooth_rewards(mean_rms,5)

#_, val1 = smooth_rewards(mean_reward,5)

plt.scatter(mean_reward, mean_rms)

plt.xlabel('return per episode')

plt.ylabel('rmse per episode')

plt.show()

```

```

#comparison of MC and TD: learning accuracy(rmse) for Q2e3

#for different learning parameter epsilon e

iterations = 10

episode_num = 2000

epsilons = [0.1, 0.2]

repeat_reward1 = np.zeros((len(epsilons),iterations,episode_num))

repeat_reward2 = np.zeros((len(epsilons),iterations,episode_num))

for i in (range(iterations)):

    for idx,s in enumerate(epsilons):

        agent1 = MC_agent(epsilon=s)

        agent1.play(episode_num)

        repeat_reward1[idx][i] = agent1.total_rms

        agent2 = TD_agent(epsilon=s)

        agent2.play(episode_num)

```

```

        repeat_reward2[idx][i] = agent2.total_rms
mean_mc_1 = np.mean(repeat_reward1[0], axis=0)
mean_mc_2 = np.mean(repeat_reward1[1], axis=0)
mean_td_1 = np.mean(repeat_reward2[0], axis=0)
mean_td_2 = np.mean(repeat_reward2[1], axis=0)
idx, val1 = smooth_rewards(mean_mc_1,10)
_, val2 = smooth_rewards(mean_mc_2,10)
_, val3 = smooth_rewards(mean_td_1,10)
_, val4 = smooth_rewards(mean_td_2,10)
plt.plot(idx, val1, label="MC,epsilon: 0.1")
plt.plot(idx, val2, label="MC,epsilon: 0.2")
plt.plot(idx, val3, label="TD,epsilon: 0.1")
plt.plot(idx, val4, label="TD,epsilon: 0.2")
plt.xlabel('episode number')
plt.ylabel('rmse per episode')
plt.title('Accuracy comparison at different epsilons')
plt.legend()
plt.show()

```

```

#comparison of MC and TD: learning accuracy(rmse) for Q2e3
#for different learning parameter learning rate a
iterations = 10
episode_num = 2000
stepsizes = [0.2, 0.5]
repeat_reward1 = np.zeros((len(stepsizes),iterations,episode_num))
repeat_reward2 = np.zeros((len(stepsizes),iterations,episode_num))
for i in (range(iterations)):
    for idx,a in enumerate(stepsizes):
        agent1 = MC_agent(step_size=a)
        agent1.play(episode_num)
        repeat_reward1[idx][i] = agent1.total_rms
        agent2 = TD_agent(step_size=a)
        agent2.play(episode_num)

```

```
repeat_reward2[idx][i] = agent2.total_rms
mean_mc_1 = np.mean(repeat_reward1[0], axis=0)
mean_mc_2 = np.mean(repeat_reward1[1], axis=0)
mean_td_1 = np.mean(repeat_reward2[0], axis=0)
mean_td_2 = np.mean(repeat_reward2[1], axis=0)
idx, val1 = smooth_rewards(mean_mc_1,10)
_, val2 = smooth_rewards(mean_mc_2,10)
_, val3 = smooth_rewards(mean_td_1,10)
_, val4 = smooth_rewards(mean_td_2,10)
plt.plot(idx, val1, label="MC,learning rate: 0.2")
plt.plot(idx, val2, label="MC,learning rate: 0.5")
plt.plot(idx, val3, label="TD,learning rate: 0.2")
plt.plot(idx, val4, label="TD,learning rate: 0.5")
plt.xlabel('episode number')
plt.ylabel('rmse per episode')
plt.title('Accuracy comparison at different learning rates')
plt.legend()
plt.show()
```